PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XVI. 1888-89. No. 129.

THE 106TH SESSION.

GENERAL STATUTORY MEETING.

Monday, 26th November 1888.

The following Council were elected :—

President.

SIR WILLIAM THOMSON, F.R.S.

Vice-Presidents.

JOHN MURRAY, Esq., LL.D.
Prof. Sir DOUGLAS MACLAGAN, M.D.
Hon. Lord MACLAREN, LL.D., F.R.A.S.
Rev. Professor FLINT, D.D.
Professor CHRYSTAL, LL.D.
THOMAS MUIR, Esq., LL.D.

General Secretary—Professor TAIT.

Secretaries to Ordinary Meetings.

Professor Sir W. TURNER, F.R.S.
Professor CRUM BROWN, F.R.S.

Treasurer—ADAM GILLIES SMITH, Esq., C.A.

Curator of Library and Museum—ALEXANDER BUCHAN, Esq., M.A., LL.D.

Ordinary Members of Council.

Sir ARTHUR MITCHELL, K.C.B., LL.D.
STAIR AGNEW, Esq., C.B.
R. M. FERGUSON, Esq., Ph.D.
A. FORBES IRVINE, Esq. of Drum, LL.D.
Dr J. BATTY-TUKE, F.R.C.P.E.
Professor BOWER, M.A., F.L.S.
Dr G. SIMS WOODHEAD, F.R.C.P.E.
ROBERT COX, Esq. of Gorgie, M.A.
Professor ISAAC B. BALFOUR, F.R.S.
Professor EWING, F.R.S.
Professor JACK, LL.D.
Professor JAMES GEIKIE, LL.D., F.R.S.

By a Resolution of the Society (19th January 1880), the following Hon. Vice-Presidents, having filled the office of President, are also Members of the Council :—

HIS GRACE THE DUKE OF ARGYLL, K.G., K.T., D.C.L., LL.D.
THE RIGHT HON. LORD MONCREIFF of Tulliebole, LL.D.

PROFESSOR SIR DOUGLAS MACLAGAN, Vice-President, in the Chair.

Chairman's Opening Address.

(Read December 3, 1888.)

It is a time-honoured custom in this as in other learned societies, that the annual Session shall be inaugurated and its last meeting closed by an address from the Chair, and it is my duty now to say a few words to you on the occasion of the opening of our 106th Session.

I would have shrunk from undertaking this duty, but for the reason that our indefatigable Secretary, Professor Tait, has expiscated the fact that I am the only one of the Society's Vice-Presidents who has never given any address from the Chair, and thus the Council of the Society was led to insert in their minutes a request—almost equivalent to a command—that I should say something to you on the present occasion. I comply with this, not from any feeling that I am qualified to do so, but on the principle that those who accept the honours of the Society have no right to do so without attempting, however imperfectly, to discharge the relative duties.

I can at all events congratulate you on the prosperous condition of the Society as regards the number of Fellows. The roll of the Society at present contains 494 Ordinary Fellows, 32 Foreign Honorary Fellows, and 19 British Honorary Fellows. In reference to our Ordinary Fellowship, I cannot refrain from adverting to an opinion which I have heard expressed in reference to their number, that there must be too great facilities for admission to their ranks. I demur to this opinion. It is not necessary that we should maintain that all our Fellows are special cultivators of science; but, setting aside for a moment the department of literature, I am persuaded that all who are admitted here have shown before they have passed the somewhat prolonged ordeal of the Council, and have been put before the Society for ballot, have taken a real interest, by study or otherwise, in the progress of science, and are desirous, so far as lies in their power, to promote its advancement.

This is not the place or time for discussing what is the best basis upon which the election of Fellows to such a Society as ours should rest, whether by our system of petition—recommendation by four existing Fellows, and subsequent scrutiny by the Council of evidence in the candidate's favour—or by the selection of a limited number of names of those who have, so to speak, won their spurs by original work. We must bear in mind that science is not nowadays the *arcanum* of the philosopher, and that every day shows more and more how indissolubly science is mixed up with those great practical works which so powerfully influence the material prosperity of the country—(we need not go for this beyond the realms of Physics and Chemistry)—and that there are many among us whose occupations or inclinations do not permit of their working at scientific research, but who yet know the value of it for practical purposes, and who are deeply interested in learning, as directly as possible, either by the hearing of the ear from the scientist himself, or by the perusal of the papers in our *Transactions* and *Proceedings*, what he has to tell them. I am always glad to welcome such men into our ranks.

As regards financial matters, those who have looked into the abstract of the accounts of our excellent Treasurer will see that the income for the past sessional year was exceeded by the expenditure to the extent of about £235. We need not be alarmed at this. Most of it is accounted for by the expenditure on one item, the important volume of the "Botany of Socotra;" and the finances otherwise are in a satisfactory state.

Of course the heaviest item in our annual expenditure is the printing of the *Transactions* and *Proceedings*, which amounted last year to about £780. This cannot be avoided. These publications are an essential object of our existence, but assuredly every endeavour is made to have this done as economically as is consistent with the prime requisite of accuracy.

This allusion to our publications leads me to notice certain new arrangements which the Council have made with a view to expediting the issue of the *Transactions* and *Proceedings*. With this view they have resolved that each memoir in the *Transactions* shall be printed separately, and be obtainable as soon as ready, without waiting for the completion of a part of a volume. Those Fellows who

desire it can, as at present, have the *Transactions* in the volume form. The *Proceedings* will be issued in parts as soon as sixty-four pages are in type and passed for press, so that they may thus sooner be in the hands of Fellows. Occasional delays in bringing out *Transactions* and *Proceedings* are unavoidable, especially in the case of the former, which may have to be illustrated by plates requiring great nicety and skill on the part of the artist; but that need hardly occur in regard to the *Proceedings*, if authors will only second the endeavours of the Council by promptitude in furnishing abstracts and reports of papers. The issuing of parts will not be stopped when sixty-four pages are ready, because some author may not send in his MS.; and if it occurs that such papers are not made public till after the issue of others which should not have preceded them, the author must blame himself. If due attention is paid to promptitude, the issue of *Proceedings* may be made as regular as that of a serial publication.

A further new arrangement is that writers of papers who will furnish, when the paper is sent in, an abstract of not more than twelve lines, may have this inserted in the billet of the meeting at which the paper is to be read, and thus inform the Society of the nature and scope of the subject to be discussed.

There is one point regarding which the Council is not in a condition to congratulate the Society, viz., on its local habitation. The want of accommodation for our books, now forming a large and most important scientific library, has long been a subject of much lamentation, and those who may chance to find their way into the penetralia to the south of the Hall, and find a staircase filled with books to the roof, must have occasionally experienced the dread of their possibly being overwhelmed by a literary avalanche, and have wondered how in such circumstances our worthy Librarian can, without personal danger, find anything there that may be desired of him. We must, however, in the meantime live in hope that the building which the munificence of an unknown donor is erecting in Queen Street, and to which some of the contents of the Royal Institution are to be removed, may enable the Trustees of the Board of Manufactures to find some extension of accommodation for the Royal Society.

It is impossible, when mentioning our library, to avoid noticing

a bibliographic event in which we all have a deep interest. I allude to the near completion of the Challenger Reports.

This monumental work, so interesting from the importance and novelty of the results it has given to the world, is the most extensive and perhaps the most valuable record of a scientific voyage ever published by any nation, and it is due to the skilful supervision and indomitable energy of the editor, my brother Vice-President, Dr John Murray, that the work, notwithstanding its unparalleled magnitude, has been brought so soon and successfully to a close.

To another matter of great public interest, though not directly connected with the work of the Royal Society, a short reference may be permitted. The Universities (Scotland) Bill has long been before the nation, and has been the subject of much discussion and some opposition. It would, of course, be entirely out of place for me to say anything either in the way of approval or criticism of a measure now before Parliament, and I would not have alluded to it but for a single noteworthy circumstance.

Among the original provisions was one proposing the transference of the Royal Observatory and the Royal Botanic Garden to the custody of the University of Edinburgh, which threatened to produce opposition from many quarters, including among others our municipal authorities. These bones of contention have happily been taken out of the way, the one directly and the other indirectly, by the munificent action of the Earl of Crawford.

Our learned Vice-President, Lord M'Laren, whose absence from among us, owing to bad health, we all so much regret, in his opening address to the Society this time last year, expressed the hope that the Edinburgh Observatory, in accordance with the agreement between its founders and the Government of the day, would be maintained as a national institution properly endowed and adequately equipped. This wish of the learned Lord is in prospect of being realised in a way that could not have been thought of twelve months ago. By his gift to the nation of his celebrated Observatory at Dunecht, with the powerful instruments and valuable apparatus, which the noble Earl has accumulated at an immense cost, along with his astronomical library, the most complete collection in the kingdom of works referring to his favourite science, Lord Crawford has secured for Scotland a national observatory, not by the retention of

the old one on the Calton Hill, but by the erection of a new one in the vicinity of Edinburgh. Government has accepted the gift, has undertaken to provide a suitable site and buildings, and to increase the endowment. It is to be hoped that as little delay as possible will occur in carrying this out, and that before long Scotland may be put in possession of this princely gift—the bestowal of which by her premier Earl is worthy to stand beside the noblest of the achievements that have been recorded in the Lives of the Lindsays.

I have no occasion to take any retrospect of the past Session, because that was done on 16th July in his closing address by Professor Flint, who then summarised what had been done in the Society, and also noticed those Fellows who had during the Session been removed by death. Only one addition has been made to our death-list of Fellows since the close of last Session, in the person of Dr William Wallace, F.C.S., public analyst to the City of Glasgow, which took place at his residence at Hillhead on 5th November. For two years Dr Wallace had been in somewhat feeble health, and for a month he was entirely confined to the house. While his death thus did not come unexpectedly to his more intimate friends, the sad intelligence was received with surprise by many people not only in the City of Glasgow, but throughout all the country, by whom, both on professional and personal grounds, he was highly esteemed. Dr Wallace, who had reached his 56th year, began his professional career in Glasgow as assistant to the late Dr Penny at the Andersonian College. He held this appointment for a number of years. He was next appointed Lecturer to the Mechanics' Institute (now the College of Science and Arts), and this position he held for several years. In 1870 he entered into partnership with Mr Tatlock and Dr Clarke, and for many years a large and important business was carried on by the firm. The office of Public Analyst for the City of Glasgow was conferred on Dr Wallace in 1874, and, besides discharging the duties of this appointment to the satisfaction of the Town Council, he held a number of similar offices, in all of which, by his great skill as a chemist, and his courteous and agreeable manner to all with whom he came in contact, he extended his reputation and secured the esteem of a wide circle of friends. In conjunction with Dr Russell, the medical officer of health for the city, he was the author

of a series of papers published in 1879 dealing with the subjects of "The Prevention and Control of Infectious Diseases," and "Air, Water Supply, Sewage Disposal, and Food," the portion under the latter title being contributed by Dr Wallace. He was also the author of a very large number of pamphlets, and of papers contributed to Societies on a great variety of subjects. Dr Wallace was for many years an active and valuable member of the Glasgow Philosophical Society, of which some years ago he was elected President.

Having no retrospect to take, I look to the future with a special regard to those places where work has been, or is being done, and from which we may expect contributions to the work of the Society during the present Session. I say nothing of what is being done in the quiet of the laboratories by the physicist or chemist, which as yet is known only to the workers, and may furnish abundant materials for papers at our meetings. I would speak of those operations which are more or less in the public eye, and which may equally contribute to furnish us with papers of interest and value. The general survey entitles me to say that there is no lack of scientific activity in Scotland at present.

I shall perhaps be pardoned if, in a spirit of *esprit de corps*, I first mention the Laboratory of Research of the Royal College of Physicians of Edinburgh, under the able superintendenceship of Dr Woodhead. It is a personal gratification to me to think that the movement for establishing this Laboratory was inaugurated and carried out when I had the honour of filling the Chair of the Royal College; but the merit is in no respect due to me, but to the energy and perseverance of a member of our Council, Dr Batty-Tuke. It is a great satisfaction to know that not only have we the prospect of the original investigations made there being communicated to the world through the Royal Society, but that we have had an earnest for the future in the shape of papers already read before us, and now in the course of publication; and I may be permitted to add, as an office-bearer of the Royal College, that this development is an evidence, among other instances, that medical corporations do not gather funds for selfish purposes, but are ready and willing to use them for the advancement of scientific knowledge.

Next, I would point to another field of scientific work as one

which strongly appeals not only to those who take special interest in the rapidly progressing science of Meteorology, but to thousands beyond the limits of Scotland, and especially to those who go down to the sea in ships. My look here is directed up to the highest inhabited spot in Great Britain—to our Observatory on Ben Nevis. Every daily newspaper keeps under our eyes the work that is done there, and I am sure that all of you will join with me in paying a tribute of admiration to the unremitting self-denying labours of the superintendent, Mr Omond, and those who are associated with him. It will interest the Society to learn from authentic sources what has recently been done there.

The work of observing at the Ben Nevis Observatory continues to be prosecuted by night and by day with energy and success, and five years' observations have now been made. These observations, combined with those made at Fort William, contribute the best data yet available for the investigation of those fundamental questions in atmospheric physics, viz., the rate of diminution of the temperature with height, and of pressure with height, for air temperatures and sea-level pressures. The valuable results already arrived at have been utilised in the construction of the new isothermal and isobaric charts of the globe now being prepared for the Challenger Expedition Reports.

Data of invaluable character have been contributed for the solution or elucidation of such important questions as the diurnal changes in the velocity and direction of the wind; the relation between temperature and wind; the hygrometry of the atmosphere; earth currents; glories, halos, coronæ, and other optical phenomena in their relations to cloud crystals. Photography has recently been added to the observers' instrumental means on Ben Nevis, and it is interesting to note that a few days ago a case of St Elmo's Fire at the Observatory was successfully photographed.

I venture to quote a sentence from the Report of a Committee of the Royal Society appointed to co-operate with the Scottish Meteorological Society in making meteorological observations on Ben Nevis.

The Directors of the Observatory are maturing a plan for a thorough discussion of the Ben Nevis and Fort William observations in their scientific and practical bearings. This plan will

require for the carrying of it out a small additional staff, working in conjunction with the office in Edinburgh and the staff of the Ben Nevis Observatory, for a period of at least three years.

In connection with the practical side of the inquiry, the Directors refer with the greatest satisfaction to the publication, by General Greely, Chief Signal-Officer of the United States Army, of daily Weather Charts of the Atlantic, beginning with October 1886. These charts have been partially examined in connection with the Ben Nevis Observations, and it is not possible to over-estimate their importance in the large inquiry now in contemplation by the Directors as to the relations of these observations to the weather of North-Western Europe, which is truly an international undertaking. With the United States charts will be conjoined the observations of storms and other phenomena made at the Scottish lighthouses, as described in previous reports, and the bi-daily charts of the Meteorological Office. One of the points to which attention will be specially given will be to ascertain the earliest time at which storms, seen to be advancing over the Atlantic towards Europe, could be signalled from the Ben Nevis observations in combination with observations at lower levels; and further, to endeavour from an investigation of the bearings of Ben Nevis observations on the movements and courses of anticyclones, to ascertain the path the advancing cyclone will take, whether to the north of, across, or to the south of the British Islands in its easterly course.

The Directors have from the commencement insisted on the necessity of an observatory at Fort William, near sea-level, at which hourly observations can be recorded, in order that the observations made at the top of Ben Nevis shall be properly utilised in their scientific and practical bearings.

It is especially satisfactory to note that the Meteorological Council have granted £250 towards the maintenance of the low-level observatory, and that the Committee of the Edinburgh International Exhibition of 1886 have dedicated £1000 of the surplus funds to this important object. Is it too much to expect that some of the surplus of the magnificent Glasgow Exhibition, on the success of which we can so cordially congratulate the City of the West, may, by favourable meteorological influence, be wafted to Fort William?

The whole of the hourly and other observations made at the Observatory and at Fort William are now through the press, but the Report accompanying the observations is not yet completed. It is anticipated that the Report will be finished and through the press by the end of January next.

In the department of Biology there is also a large amount of scientific activity, which is of hopeful augury as to the furnishing of papers for our meetings.

There are a great many young scientists now occupied with the study of Microbiology—a study most important not merely as regards its scientific interest, but from its practical bearing on pathology and public health. Its dealings with pathological questions will most properly be brought under the notice of the strictly professional societies; but it is needless to say that, in reference to hygiene, it is of high practical interest to everybody, and any results which may be obtained may appropriately find their way to the public through our *Proceedings.*

The Society does not need to be informed as to the work which for four years has been carried on at the Scottish Marine Station, both at Granton on the east and at Millport on the west coast, an interesting account of which has been given by Mr Hoyle in No. II. of the *Journal of the Marine Biological Association.*

At the Marine Station, Granton, the chemico-biological work has been steadily carried on on the same lines as during last year. The investigations as to the secretion of carbonate of lime by animals, and the solubility of lime (in its different molecular states) in sea-water, have yielded results of considerable interest and importance as bearing upon the formation of coral and shell deposits, and on their rescue from destruction in the ocean. Experiments have been made as to the actual composition of sea-water; such results as indicate any advance in this direction will, it is hoped, be brought before the Society during the Session.

Not less interesting and important are the investigations which have been made, and are still going on, at the instance of the Fishery Board, as bearing upon the enormously important industry of the sea.

The scientific work in connection with the fisheries of Scotland may be said to have taken rise with the establishment of the

present Fishery Board, which succeeded the old Board of Commissioners for the British White Herring Fisheries in 1882; and the scientific work has been undergoing gradual expansion year by year since the new Board was formed, largely owing to the energy of Professor Ewart and Sir James Maitland.

The work which has been accomplished may be conveniently referred to under three heads—(1) General Fishery Questions, (2) Biological Observations, and (3) Physical Observations.

Among general questions, which do not so much concern us here, may be mentioned the investigations of the "Garland" into the influence of various modes of fishing, especially in territorial waters, the preservation of fish, specially studied by Professor Ewart, and various other matters which are set forth in the last two Reports of the Fishery Board. At the present time experiments are being carried on regarding the artificial cultivation of mussels on the French or *bouchot* system, and investigations are in progress on the relative efficiency of different baits, and on the possibility of devising artificial substitutes.

The biological investigations have naturally ranged over a wide variety of subjects, and these can only be referred to summarily. One of the earliest questions to engage the attention of the new Board was the natural history of the herring, and several papers have been published in the various Reports on the structure, spawning, development, food, and migrations of this familiar fish, which still forms the greatest source of wealth in connection with the fisheries of Scotland. These inquiries were extended later to other groups of edible fishes; and the most complete account yet given of the food of most of them, such as the cod, haddock, whiting, &c., will be found in the Reports of the Scottish Board; and an inquiry into the structure of the digestive system and the cognate processes of digestion in fishes, was also made the subject of a special investigation by Professor Stirling of Owens College.

The vertebrate and invertebrate fauna of several districts, especially in the Clyde and in the Firth of Forth, have been very completely worked up, a considerable number of species, either new to science or new to Britain, have been recorded; and the occurrence of rare fishes has been described from time to time. Inquiries have also been carried on into pathological questions connected with fish

life, and papers have been published dealing specially with the fungi and micro-organisms which occur in the waters of our salmon rivers, and which are more or less inimical to their inhabitants.

Investigations were also made into the development of the common mussel, which is so important in connection with the supply of bait to fishermen.

The Fishery Board has also carried on a series of inquiries into the temperature and other changing physical conditions of the sea, which are known to exert an important influence on the movements and habits of fish. These investigations, the results of which are published in the various Reports, have been partly conducted at certain stations on the coast, partly during the routine work of the cruisers engaged in fishery service, and partly during special expeditions of these vessels.

In connection with the physical work done under the auspices of the Fishery Board, a short but interesting expedition started from Granton in the beginning of September, under the guidance of Dr John Gibson, the Admiralty granting the use of H.M.S. "Jackal," which was duly fitted up with a laboratory for the occasion. One of the objects of the expedition, which lasted for about six weeks, was to establish communications with Norwegian, Danish, and German observers, and for this purpose the "Jackal" visited Bergen, Copenhagen, and Kiel.

Another object kept in view was to arrange for new observation stations on the east coast of Scotland, the Orkneys and Shetland, and at the same time to make some further investigations regarding sea-water. These observations embraced the temperature, alkalinity, specific gravity, and amount of carbonic acid, both free and combined, at different points in the North Sea, Baltic, and Sound; further, the amount of carbonic acid in the atmosphere was as often as possible also determined, the micro-organisms in the air were also looked for, and the organisms in the surface water got by the tow-net.

The observations made are in course of being completed, and it may be hoped that the Fishery Board will not object to some of the results being laid before the Royal Society.

A large amount of important biological work has also been done at the St Andrews Marine Laboratory, under Professor M'Intosh, of

which also a report will be found in No. II. of the *Journal of the Marine Biological Association.*

Reference may be made here to a further work now being carried on in England in connection with marine biology. The Laboratory of the Marine Biological Association of the United Kingdom, which owes its existence mainly to the labours of Professor Lankester, was opened a few months ago at Plymouth, and in point of size and equipment takes rank among the first of such institutions throughout the world, considering the short time the laboratory has been established. A considerable amount of work has been already done, and the lines of future investigations have been laid down. A list of the fauna and flora of Plymouth Sound has been compiled, and researches have been begun on the breeding of the sole, pilchard, herring, conger, &c., and the development of the lobster, various shrimps, and other crustaceæ. It is proposed to make an exhaustive study of the fauna of the Sound, and of the connection between varying physical conditions and the distribution and migratory movements of the organisms. There can be little doubt that, now that it has been fairly started, this laboratory will prove an important means of extending our knowledge of the biology and physics of the sea. What I have said regarding scientific work has been confined to what is being done in our own country, perhaps chiefly from the selfish view that we are thence most likely to get contributions to our meetings and *Proceedings.* I should, however, be doing violence to my own, and I am sure also to your feelings, if I were to pass over without note the brilliant achievement of Dr Nansen of Bergen, with Lieutenant Dietrichsen and Mr Sverdrup, in crossing the inland ice of Greenland from east to west. We recognise in this not merely scientific zeal, but that indomitable Scandinavian "pluck" which filled the nation with terror at the prowess of the Norsemen of old, but now deserves the admiration of the civilised world for their hardy stalwart descendants. It is, of course, to be expected that whatever Dr Nansen may have to communicate will be told to the scientific societies of his own country; but I think that I may venture in your name to assure him that if he chooses to send anything to us in Danish, we shall easily find means of putting it in form to appear in our own *Proceedings.*

The *Proceedings* of the past and previous years show that there

is no lack of papers now offered to the Society, and no reason to apprehend that there shall be any scarcity in the present Session.

In looking back at the history of the Royal Society of Edinburgh, we find that it, like other similar bodies, has had its periods of scientific famine and mournful inactivity, but that has not been the case for a length of time. It is curious to look back at such a record as the following, given by Sir David Brewster in his Presidential Address in December 1864.

In the closing years of the last and in the first decade of the present century the Society was in a very languid condition. In each of the years 1799, 1802, 1803, 1808, and 1809, only one of the papers read at its meetings was published in the *Transactions*, and in 1801 and 1806 not a single paper read in these years was published. While our *Transactions* were thus scantily supplied with papers, those actually read were few in number, and often too abstruse to excite a general interest. The regular meetings of the Society had frequently no other business than to read the minutes, elect members, and receive donations, and this was sometimes done in the presence of only the Secretary and one or two Members of Council. In such circumstances the Secretary summoned the members by a billet, printed in red ink when a paper was to be read, and one in black when he had nothing to communicate.

Of course, from the immensity of its domain, and the number of questions which it offers for investigation, Physical Science must always furnish by far the largest proportion of communications read before us. It is, however, I think, to be regretted that so few literary papers are presented, and there is a marked contrast between the state of matters in the earliest and present position of the Society in this respect. It is to be borne in mind that this never was a Society devoted exclusively to physical and natural science, but that originally it consisted of two distinct sections—the Physical and the Literary. In its earlier years the literary class was actually more numerous than the physical, and was at least as active, seeing that the number of papers printed in the *Transactions* was about equally divided between the two classes. But this was not to continue. It is remarkable that many of the most illustrious men in the literary

section, though it includes Robertson, Reid, David Hume, Fergusson, and Adam Smith, scarcely contributed to the pages of the *Transactions* at all. Nay, even the elevation of Sir Walter Scott to the presidency in 1820 did not bring about any literary revival, and it does not appear that the author of *Waverley* ever contributed a paper. The literary section had practically ceased to exist in 1808, and was finally abolished as a distinct class in 1827. I find that between 1882 and 1887, while there were a few interesting papers on social questions, only three were read which can truly be said to be literary. It is no disparagement to any one to say that in the sphere of physical and natural science he may hear papers read which are of too technical or abstruse a nature to be interesting to him, but literary papers seem to be generally attractive, perhaps from their very rarity, and from the pleasant variety which they give to our meetings. I would fain hope, considering the original constitution of the Society, that though it has no longer a distinct literary class, literature may not altogether disappear from the meetings. Be this wish fulfilled or not, there seem to be abundant sources from which we might get valuable contributions from the domain of science, and I trust that this Session will be in all respects one of prosperity.

On *Pseudalius alatus*, Leuckart, collected by Mr Robert Gray in the Arctic Seas, and other species of the Genus. By Dr O. v. Linstow. *Communicated by* Dr John Murray. (With a Plate.)

(Read December 3, 1888.)

Dr John Murray was good enough to send me numerous specimens of *Pseudalius alatus*, obtained in the summer of 1888 by Mr Robert Gray, on the coast of Greenland, from *Monodon monoceros*.

This Nematode has been only once described—by Leuckart,* but that at a time when the methods of investigation were still imperfect. My results are thus essentially different from his. Leuckart called the species *Strongylus alatus*, and noted as its habitat the cranial cavity of *Monodon monoceros*.

* *Arch. f. Naturgeschichte*, Bd. xiv., 1848, pp. 26–28, pl. ii. fig. 3.

The body is elongated, more markedly attenuated posteriorly than anteriorly. The colour is brown. At the head end there is a shallow mouth cavity, and besides this on the lateral line at each side a very prominent papilla; somewhat further back there is another weakly developed. No transverse wrinkling of the cuticle is to be observed. The musculature is very strongly developed, and is referable to the Holomyarian type.

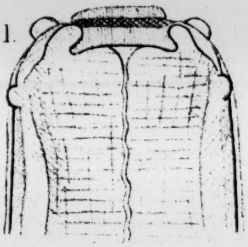
The male measures on an average 16 mm. in length by 4 mm. in breadth, and is much attenuated posteriorly. The œsophagus is short, and occupies only $\frac{1}{25 \cdot 5}$ of the total length. The tail has a spoon-shaped termination, which is supported by three finger-shaped processes (fig. 2). The two laterals each bear three papillæ, the median two; and in front of the cloaca on each side there is another papilla with a finger-shaped stalk. The two cirri are 0·84 mm. in length. A bursa, with rolled margin (fig. 4), occurs on lateral edges of the tail in the male forms. Among the longitudinal muscles there are strong oblique bursal muscles, which extend very far forward, for 3·2 mm. There is a remarkable longitudinal strand, which consists of shining spherules, and lies in the ventral line in the plane of the longitudinal muscles, whence it gives off lateral branches at right angles to right and left (fig. 4).

The female measures 16·5 mm. in length by 4·8 mm. in breadth. It is viviparous. The unpaired short vagina and the long paired uteri opening into it are filled with very numerous embryos. These measure 0·36 mm. in length by 0·013 mm. in breadth. The œsophagus occupies $\frac{1}{27 \cdot 6}$ of the total length, and is therefore very short. Close in front of the apex of the tail, which divides into two roundish processes, lies the anus (figs. 5 and 6, *b*). In front of this one notices an oval vesicular elevation of the cuticle, in the middle of which the vulva appears in the form of a transverse slit (figs. 5 and 6, *a*). It is bordered anteriorly by a slight protrusion of the parenchyma of the body, posteriorly by a similar thicker structure (fig. 6).

The six known species of *Pseudalius* appear to flourish where there is an abundant supply of oxygen, since they live in air-containing organs or in the blood-vascular system.

Ps. alatus, in the pharyngeal cavities, mouth, and Eustachian tube of *Monodon monoceros*.

Fig 1.



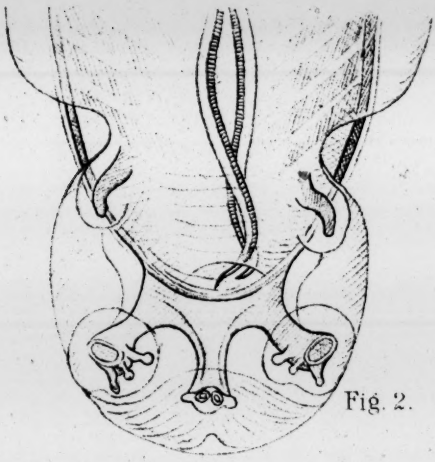


Fig. 2.

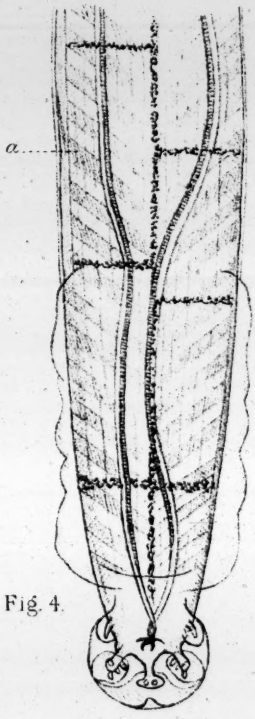


Fig. 4.

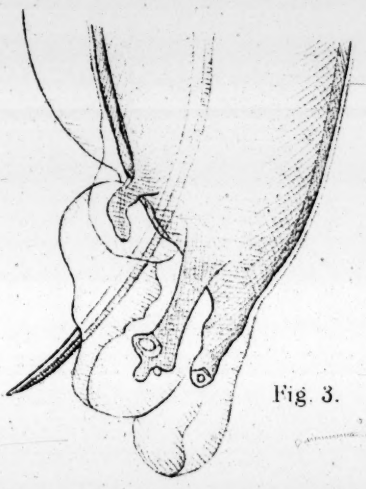


Fig. 3.

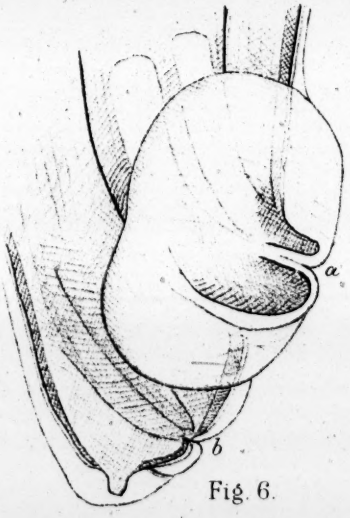


Fig. 6.

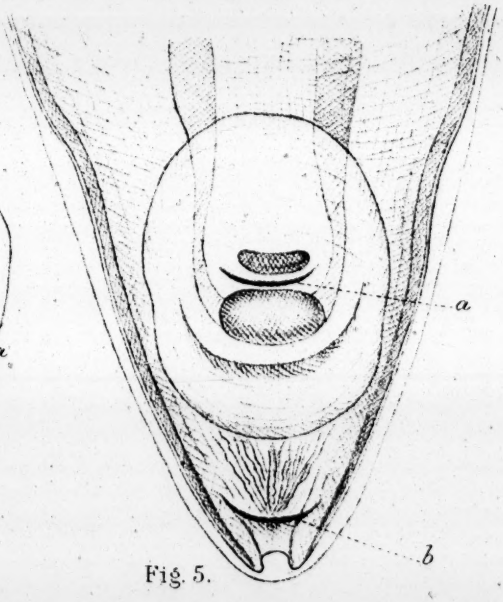


Fig. 5.

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M.cFarlane & Erskine Lith.rs Edin.r

PSEUDALIUS ALATUS, Leuckart.

Ps. inflexus, Duj.,* in the bronchi, gullet, heart, veins, and arteries of *Phocæna communis.*

Ps. minor, Kuhn,* in the cavum tympani, a cavity beneath the eyes, the bronchi, heart, and veins of *Phocæna communis.*

Ps. tumidus, Schneider,* in the alveoli of the lung of *Phocæna communis.*

Ps. convolutus, Kuhn,* in the bronchi and pulmonary vessels of *Phocæna communis* and *Globiocephalus svineval.*

Ps. ovis pulmonalis, Koch,† in the bronchi of the sheep.

In *Ps. minor*, *tumidus*, and *inflexus* the spicula are short and leaf-like, occupying only $\frac{1}{125}$ of the entire length of the animal. In *Ps. alatus*, *convolutus*, and *ovis pulmonalis* they measure about $\frac{1}{20}$ of the length of the body. The habitat of *Ps. ovis pulmonalis* is very different from that of *Ps. alatus* and *convolutus*, while the two last-named species are distinguished among other things by the fact that on the tail end of the body in the male of *Ps. convolutus* the bursa touches the round terminal lappets,‡ and exhibits striking curved lines; but in *Ps. alatus* the bursa is separate from the terminal lappets, and forms no such lines. Furthermore, the cuticle of *Ps. convolutus* is covered with longitudinal markings, which are absent in *Ps. alatus;* and again, *Ps. convolutus* measures 40–50 mm., and *Ps. alatus* 16–17 mm. in length.

Explanation of Plate.

Fig. 1. Head end.

Fig. 2. Terminal lobes of the tail end of the male, seen from the ventral surface.

Fig. 3. The same, seen from the left side.

Fig. 4. Posterior end of the male, seen from the ventral surface, less magnified than figs. 2 and 3. *a*, muscles of the bursa.

Fig. 5. Posterior end of the female, seen from the ventral surface. *a*, vulva; *b*, anus.

Fig. 6. The same, seen from the right side. *a*, vulva; *b*, anus.

* v. Linstow, *Compendium der Helminthologie*, Hannover, 1878, p. 59.

† A. Koch, "Die Nematoden der Schaflunge," *Oester. Monatschr. f. Thierheilkunde*, Wien, 1883, pp. 9–19.

‡ Schneider, *Monographie der Nematoden*, p. 174, pl. xii. fig. 3.

Abstract of the Results of an Inquiry into the Causation of Asiatic Cholera.

By **Neil Macleod**, M.D. Edin., and **Walter J. Milles**, F.R.C.S. Eng. *Communicated by* Dr WOODHEAD.

(Read December 17, 1888.)

This inquiry was commenced in Shanghai at the end of 1884, soon after Koch had published some of his results, and as he was then almost the only worker in the bacteriological field of whom it could be said that he had not published anything, subsequently upset, attention was directed at once to the "comma bacillus."

Slightly alkaline peptonised meat jelly on plates was used for purposes of cultivation, this being kept at a temperature of 60° to 70° Fahr. by means of an ice chest in hot weather.

The investigators worked independently, but the majority of the cases of cholera under observation were investigated by both.

For determining identity, it was assumed that the same organism grown under the same conditions as to medium, temperature, and time should have the same macroscopic, and being subjected to the same conditions as to stain, treatment in mounting and examining, the same microscopic characters.

The characters of identification laid down by Koch for this organism are now accepted by all observers as sufficient, the name alone being altered. The "comma bacillus" is now regarded as a "spirillum," but is best known by the former name. The characters are—

1st. It occurs in the characteristic cholera stools, and after death in the similar contents of the ileum.

2nd. It is from one-half to two-thirds the size of the tubercle bacillus, somewhat thicker than the latter, and slightly curved. Sometimes it occurs in groups so as to form a half circle, an S-shaped curve, or a wavy line of varying length.

3rd. It grows in and liquefies slightly alkaline gelatine, more slowly in neutral, scarcely at all in slightly, and not at all in markedly acid gelatine. On a gelatine plate cultivation the individual colonies are round, lie in a funnel-shaped cavity; viewed by transmitted light and magnified, they look like ground glass, and

the edge of the colony is finely notched. In a gelatine tube there forms a funnel-shaped cavity at the top of the puncture made by the inoculating wire, and lying in this cavity what looks like an inverted air-bubble with its top on a level with the surface of the jelly, and open to the air; along the puncture the gelatine liquefies, and in this may be seen the whitish mass of colonies, more particularly at the lowest part; in from three to four weeks liquefaction spreads to the whole mass, the bacilli falling to the bottom as a greyish-white sediment, having a faint orange tint in certain lights, and if undisturbed, a perfectly transparent liquid separates a whitish scum on the top from the sediment below.

4th. A freezing temperature does not destroy it; it grows scarcely at all below 50°, slowly at 60°, rapidly from 80° to 100° Fahr. in gelatine cultures.

(For ordinary work these characters suffice for identification.)

5th. It is actively mobile in fluid media.

6th. Grown on agar-agar, it forms a brownish, glistening scum; but this appearance is common to other organisms.

7th. On potato slices, at a temperature of from 90° to 100° Fahr., it forms glistening greyish-brown colonies, but it does not grow at all at ordinary room temperature.

8th. It liquefies blood serum.

9th. Growth ceases in the absence of air in gelatine cultivation.

10th. It is reproduced by fission. Spores have not been observed.

Nearly all observers of this organism have described what are called involution forms. These are characterised by irregularities of shape, so that it requires re-inoculation and growth under ordinary favourable conditions to determine that the specimens having these appearances are indeed pure cultivations of comma bacilli.

Ceci and others have described certain spore-like appearances in involution forms, but they are now regarded as dying or dead parts of the rods, such parts staining very slightly or not at all.

Hueppe has described what he regards as "lasting forms" (Dauerformen), and calls "arthrospores" as distinguished from "endospores." These he found in cultivations which were becoming exhausted of the material for nourishment, and he examined only unstained specimens. If the drying test for spores is to be regarded as an absolute one, they are not spores.

One of the authors of this inquiry found an appearance like that of endospore formation in actively growing cultivations in a neutral medium, kept for three days at 70° Fahr.

These are clear, round or oval spots in stained bacilli, one spot in each individual organism of typical shape. They are to be seen also in spirilla, and in slowly growing cultivations, in involution or irregularly-shaped forms. The appearance is that seen in tubercle and leprosy bacilli, and usually associated with the presence of spores, hundreds being seen in a single field. After forty-eight hours' drying, growth took place, but after no longer interval.

No conclusion has as yet been arrived at as to the nature of these spots, further than that they are not spore appearances.

For rapid examination of a fluid containing the organism, a thin film spread and dried on a cover glass can be stained by a watery solution of fuchsin in a few seconds, washed with water, and while yet wet, explored with an oil immersion twelfth or objective of similar power.

PART II.—In forty-four cases of cholera examined microscopically, and by plate and tube cultivations, the comma bacillus was found in forty. Of these thirty ended fatally, and in six of these examined after death the same organism was found. In one case examined by both observers there were numerous comma bacilli on the cover glass preparations of the stool, but not a single colony could be found on the plate cultivations, the stools having been mixed with carbolic acid when collected. The other three failures occurred with non-characteristic stools during the early part of the investigation.

One drop taken from a typical stool during the stage of collapse, or of the contents of the ileum of a patient who had succumbed during the same stage, was, with few exceptions, sufficient to furnish demonstration of the organism by cultivation, and these exceptions disappeared on the examination of a second drop. From such a stool, recently passed, colonies were almost always present on the third plate, which contained only one-three or four-thousandth part of the original drop. Later than the collapse stage, when the stool or bowel contents were no longer typical, but slimy, fæcal, or even only opaque, without being either of the former, the difficulty of finding the organism was very great, and often it could not be found. The authors point out that possibly for this reason the English

Cholera Commission in Spain frequently failed to demonstrate the presence of the comma bacillus, and also that Drs Roy, Graham Brown, and Sherrington made their examinations after death, when, it is urged, such examination, alone, is not sufficient to determine the absence of the organism in a given case, unless this be supplemented by failure to find it in the typical stools passed during life. They further observe that, unless the cases were under the observation of the members of the Commission during life, they could not be certain that they were dealing with the true collapse stage.

The so-called "rice-water" stool is almost transparent or only slightly opalescent when seen in a glass vessel, having no odour, or but a faint meaty smell, and containing white flakes containing masses of epithelium, which, however, are not present in sufficient quantity to render the fluid opaque. This fluid yielded an almost pure cultivation of the comma bacillus by the plate method, whilst in the non-typical stools several different organisms were found.

In typical cases it was found that in the intestinal follicles and in the spaces formed by the separation of the follicular epithelium from the underlying membrane numerous comma bacilli were to be demonstrated, and occasionally a few in the tissues near the follicles. Klein has arrived at the conclusion, that if the bacilli enter the tissues at all during life, it is only during prolonged agony before death, and not as a result of the proper activity of the bacillus,—but he does not, however, advance sufficient proof of this statement.

The authors found, on examination of a case in which death took place ten hours from the apparent onset of the disease (the *post-mortem* being made half an hour after death), that the small intestine was distended and externally was rose-red, the wall having a lax appearance as if paralysed, or as if it had been over-distended, and had not recovered its contractile power.

The contents were the clear fluid before referred to as the typical stool, the ileum being more particularly distended therewith. The mucous membrane was swollen, its epithelium stripped to a considerable extent, and red spots marked the mouths of the follicles. On microscopic examination these follicles were seen to be, many of them, filled with broken-down epithelium, others with the epithelium detached from the underlying membrane, and the epithelium of the

lumen of the intestine was absent in places. Comma bacilli were present in the follicles and sub-epithelial spaces.

In a case of thirty hours' duration, examined six and a half hours after death, the small bowel presented an appearance like the case just described, but was more deeply congested, there being hæmorrhages in the mucous membrane. Comma bacilli were present, as in the last case.

In a third case, thirty-two hours ill, examined two and a half hours after death, the ileum contained a thin, slightly blood-stained fluid, not in the least slimy or fæcal, there being intense congestion and swelling of the mucous lining, with much stripping of the epithelium.

In a fourth case, where death took place on the fourth day after very feeble reaction, and the stools had become slimy, dark brown, scanty, very offensive, and with a slight fæcal odour, the bowel contents were of the same nature as the stools, but more manifestly blood-stained. Here, stripping of the epithelium, ulceration, hæmorrhages, and everywhere great congestion, were the features in the small gut. The comma bacillus, which had been plentiful in the characteristic stools, was not found in the follicles or bowel contents; other organisms being abundant in the latter.

So far as the presence of the organism in the tissue of the intestinal wall is concerned, its relationship to the disease is not thereby materially influenced, even if it does not penetrate the tissue of the wall in every case.

Other organisms could be demonstrated in the intestinal wall occasionally, and in one instance a fungus network had penetrated deep into the tissues,—probably a *post-mortem* intruder.

The sections were left in Koch's solution of methylene blue for twenty-four hours at the ordinary temperature of the room.

PART III.—The authors then proceed to the third step of the investigation. The search for the comma bacillus in other than choleraic conditions of the human body was carried out in six cases of diarrhœa, one of dysentery, specimens of saliva from four healthy subjects, scrapings from three tonsils, one specimen of a healthy stool, material removed from the ileum of two cases of phthisis, two cases of malignant disease of the stomach, one each of tubercular peritonitis and pneumonia, cerebellar disease, strangulated hernia, cardiac disease, tubercular meningitis, acute bronchitis, and empyema.

In several of these, comma-shaped organisms were seen on microscopic examination, but never in any number, and in no single instance did cultivations on plates furnish such an organism. It has to be remembered that this material was subjected to the same process of examination as the cholera material, and also that there are few organisms more easily detected by plate cultivation than the comma bacillus. In a fluid containing comma bacilli alive, examination by plate cultivations will detect them when microscopic examination fails to do so.

Other bacilli have been advanced from time to time as identical with the comma bacillus, but on closer acquaintance their claims have been set aside. Finkler's and Prior's "cholera nostras" bacillus and Deneke's "cheese" bacillus are the only ones that have attracted much attention, but their characters of identity have also been found to be different from those of the comma bacillus, and it is not now necessary to particularise them.

The latest claims of this kind have been put forth by Klein (*Practitioner*, March 1887, p. 182), who thinks that in two instances out of many he has succeeded in cultivating bacilli similar to those observed by T. R. Lewis, and which appear "to be in their manner of growth, in nutritive gelatine, strikingly similar to the choleraic comma bacilli." He also describes a bacillus found in Noma as having similar properties. The characters given by Klein are, however, not sufficient to identify either of these bacilli with that described by Koch.

As regards the relation of the bacillus to the causation of the disease, it must be admitted that, if it is not in the body before the disease arises, it must either enter the body each time that the cholera poison does so, or originate spontaneously in the intestine of each case of the disease. Practically, therefore, this objection can only imply that the comma bacillus is already an inhabitant of the human bowel, but that it requires the choleraic process to demonstrate its presence there, since no one has yet succeeded in doing so apart from the disease. This is an objection with not an atom of proof, unless the following experiment adduced by Klein can be accepted as such. It is stated to be an analogous condition experimentally produced in an animal. His own words are—"That a pathological state of the intestine has a good deal to do with the

multiplication of comma bacilli, I have proved by direct experiment. In a monkey which had received the previous day a dose of castor oil, and had diarrhœa therefrom, the abdomen was opened under the spray; a loop of the lower ileum, just above the ileo-cæcal valve, and about 4 to 6 inches long, was ligatured above and below, care being taken not to include in the ligatures the large vessels. With a Pravaz syringe a droplet of mucus was withdrawn from the interior of the loop, and on examination no comma bacilli could with certainty be discovered. With another syringe, about 2 c.cm. of a saturated solution of magnesium sulphate was injected, the loop replaced, and the wound stitched up and dressed antiseptically, the whole operation being done under the spray. Immediately afterwards the animal received subcutaneously 1 gramme of chloral hydrate dissolved in 1 or 2 c.cm. of distilled water. . . . . Our animal was killed after forty-eight hours. On *post-mortem* examination the ligatured loop was found much injected, its cavity filled with, and distended by, mucus containing streaks of blood and numerous flakes. On microscopic examination these flakes contained, besides amorphous mucus and detached epithelial cells, longer or shorter straight thickish bacilli. . . . . There were present numerous comma bacilli, some single, others in dumb-bells. . . . . On microscopic comparison, it was found that they were of the same character as the choleraic comma bacilli, except that perhaps they looked a trifle smaller than those in the choleraic mucus flakes. Cultivations were made of them on six gelatine plates, and in one of these, after three days, there were no doubt a few colonies which corresponded with those of the choleraic comma bacilli; this was proved to be the case after two more days. Cultivations in gelatine tubes and in agar-agar tubes yielded growths indistinguishable from the cholera comma bacillus" (*Practitioner*, 1887, pp. 324–326).

In the first place, notwithstanding the failure to find comma bacilli before the pathological state was set up, they are assumed to be present; secondly, the material retained in the bowel for two days between a couple of ligatures cannot be regarded as representative of the material flowing so unobstructedly from the bowel as it does in cholera; thirdly, nothing is said about sterilisation of the needle and syringes, so that what they might introduce into the bowel is an open question; fourthly, the microscopic examination of one

"droplet" with a negative result can scarcely be compared with both microscopic and cultivation examinations of several drops, with positive results.

Klein's experiment by itself, if it proves anything, proves that he was dealing with an example of spontaneous generation! Further, it is not quite clear whether Klein claims that he was dealing with Koch's organism, or only with one identical as to the characters given.

PART IV.—*The Reproduction of the Disease by Neil Macleod, M.D. Edin.*—If Koch's comma bacillus be the cause of Asiatic cholera, the terms of reproduction of the disease may be stated thus:—There being no doubt that the organism finds conditions favourable to growth and multiplication in the human small intestine; there being as little doubt that it must have found its way there alive; and, since Asiatic cholera in man is capable of recognition by certain signs, then on pure cultivations of the living comma bacillus being introduced into the small intestine of a lower animal, the growth and multiplication of the organism in the small gut must be associated with the signs requisite for the recognition of the disease in man, but it must be proved that these signs themselves are not the result of the means by which the organism was introduced into the bowel.

While the early part of Koch's work was deemed of sufficient importance to be examined into by several British workers, and all of these, with the exception of Drs Roy, Graham Brown, and Sherrington, have practically confirmed Koch's facts up to the point of reproduction of the disease in this country, Klein and Watson Cheyne alone have experimentally examined Koch's claim of having reproduced the disease, as set forth in the Second Berlin Congress of 1885, although several have been content to criticise. Watson Cheyne confirms Koch's results after Nicati and Rietsch's method, and the conclusion drawn therefrom, but does not appear to have made experiments after Koch's own method. Klein admits that Koch's bacillus is pathogenic to guinea pigs, but opposes the conclusion that it is the cause of Asiatic cholera.

At the present time the general medical opinion in this country as to the relationship of the comma bacillus to Asiatic cholera may be stated thus: though easily identifiable, found as a constant con-

comitant of the collapse stage of Asiatic cholera in Europe, Egypt, India, &c.; though it has not been found apart from the disease; though it occurs in that part of the body specially affected,—the intestinal canal,—and disappears from it with the disease; while it may be regarded as pathognomic, it has yet to be proved to be the cause of the disease.

Many workers have failed to reproduce the disease. Nicati and Rietsch, believing that Koch's organism did not reach the intestine alive when given by the mouth, opened the abdominal cavity of dogs, and injected cultivations of the bacillus into the duodenum, both with and without ligation of the bile duct. Under these conditions, they announced that they had reproduced the disease. Van Ermengen, Koch, Watson Cheyne, and others repeated the experiments with the same results, but the operation was regarded by many as open to strong objection, and more probably the cause of death than cholera. The experiment is not free from objection.

At the second Berlin Congress in 1885, Koch announced that he had succeeded in reproducing the disease in guinea pigs without opening the abdomen. Finding the reaction of the contents of the guinea pig's stomach always strongly acid, he neutralised this reaction by injecting into the stomach a solution of soda. He then found that though the organism was thus enabled to pass through the stomach alive, no disturbance followed its presence in the small intestine. Many experiments of this kind failed; only one animal took sick, and died with signs like those in cholera, and this animal had aborted shortly before the experiment. It was observed, too, that there was some degree of peritonitis present as one of the results of the abortion. Koch then made some experiments, observing the condition of the animal's digestion, more especially in the small intestine, when he found that its contents passed through that part of the gut in a very short time. Coloured foods were seen to pass from the stomach to the cæcum in a few minutes. It occurred to him that, as in Nicati and Rietsch's experiments, the more the bowel was handled the more successful they were, and *vice versa*, their success might be due to the lessening of the peristaltic action of the gut, in consequence of the handling. The success in the guinea pig that had aborted, and in which peritonitis was present —a condition, too, accompanied by interference with peristalsis—

suggested the same explanation, viz., that the normal peristalsis in this animal did not afford sufficient time for the multiplication of the organisms, so that they and their products might be hurried through the small gut before they had multiplied to the extent necessary to set up the changes constituting the disease, and from the acid reaction of the contents of the cæcum they were unable to multiply and do mischief in that part of the bowel. Recognising in opium a drug that would lessen the peristalsis of the small gut, he made experiments with it, and finally decided that the injection of opium tincture into the peritoneal cavity was the surest means of bringing about this loss of peristalsis, without otherwise injuring the animal. In this way Koch claims that he has reproduced the disease, but his experiments are not as yet regarded in this country as sufficient, and the objections urged are :—

1. The symptoms produced in the guinea pig are not those of cholera in man, there being no purging, vomiting, or cramps.

There seems to be little doubt that Pasteur has produced hydrophobia both in dogs and rabbits by material taken from the disease in man; but there seems to be as little doubt that the symptoms of that disease in man, the dog, and the rabbit, do not correspond in any two out of the three. Is it to be expected that cholera in the guinea pig will present the same symptoms as in man? Will any one maintain that a given case in man without vomiting, purging, or cramps is not one of Asiatic cholera? Such cases are certainly met with in man, and what is here the exception may be the rule in the guinea pig.

2. It is objected that death and the *post-mortem* appearances in the animal experimented with may be the result of some other cause than cholera.

There is some truth in this objection, as animals may die, for instance, from mechanical injuries with the catheter used to inject the stomach; from the injection of fluid into the windpipe; from needle injuries in the abdomen; from an overdose of opium; from peritonitis or septicæmia. Animals dying from these causes, and even those presenting any injury which might not have been of itself sufficient to cause death, have been rejected from the experiments recorded below. Such accidents might have been passed over without comment; but in the interest of any one having to

repeat any of the experiments, it seems wiser to give the details by which they and the needless sacrifice of animals thereby entailed may be avoided.

With a careful dissection of the pharynx, the use of an English No. 3 catheter without the stilette, a wooden gag with a hole in it for the catheter, and notched to receive the upper and lower incisors, the avoidance of force and the exercise of a little patience, the passage of the catheter into the stomach of a guinea pig is attended with as little injury as its introduction into the human bladder in careful hands. The entrance of the catheter into the trachea may be recognised by the character of the breathing, and the shorter distance to which it passes without resistance as compared with the œsophagus. If fluid be injected into the windpipe, the animal dies in a few minutes.

Death occurred several times at first when giving the dose of opium tincture recommended by Koch, viz., 1 c.cm. to 200 grammes of the animal's weight, in which cases the animals never awoke from the opium sleep. No deaths were caused by giving in repeated doses 1 c.cm. or less of the tincture, till the animal was stupefied sufficiently to lie on the side or back for ten minutes when placed in that position. Several times the dose recommended by Koch had to be given, but usually a smaller one sufficed. Of the first seven animals dosed with opium alone, by means of a Pravaz syringe disinfected with corrosive sublimate solution, and that only before the first injection, two animals died on the afternoon of the second day, having been lively that morning. These deaths were attributed to septicæmia, as no special injury could be detected, and none of fourteen animals died that were subsequently injected by means of a Koch's syringe, the needle of which was heated red in the Bunsen flame and then allowed to cool before each injection, and the glass sterilised in the hot box. Koch's is more easily sterilised than the Pravaz syringe, there being no piston.

The animal is held on its back in the left palm, the front abdominal walls being projected forwards and made tense on both sides by the thumb and fingers, when it is easy to just enter the peritoneal cavity with the needle. An assistant should hold the animal's fore and hind quarters. All vessels with which the needle and syringe came in contact, as well as all those that contained laudanum and

other fluids to be injected, were sterilised in the hot box before being used.

In all 142 experiments were made. Some of them were carried out at the Royal College of Physicians' New Laboratory for Research in Edinburgh, but owing to restriction of the number of the experiments in terms of the vivisection licence, and delays in extending or renewing the latter, the rest of the experiments were carried out at the Hygienic Institute of the Berlin University.

The organism used was taken from the stools of a case of Asiatic cholera in Shanghai, October 1887; pure cultivations being made from plates and fresh gelatine tubes being inoculated at intervals of from three to six weeks. The organism retained its virulence, killing guinea pigs in the following October.

Thirteen healthy guinea pigs were killed, and the contents of the ileum were examined, both microscopically and by means of gelatine cultivations on plates. Normally the small intestine of these animals was almost empty and contracted. Occasionally a little food could be seen in it, but it was never distended, as is seen after an animal has been treated with opium as before described, in which case its contents are very similar to those of the stomach. In no instance was there any organism obtained by cultivation of the materials taken from the ileum that could be called comma-shaped, though the microscopic examination sometimes furnished such forms in small numbers, as well as spirilla, and large, curved, worm-like bodies, which latter were most numerous in the cæcal contents.

In all the animals injected with cholera material, as well as those for control experiments, 5 c.cm. of a 5 per cent. solution of soda (Na_2CO_3) were injected into the stomach by means of a catheter, ten or twenty minutes later a quantity of cholera material or sterilised broth, and then the opium tincture was introduced into the peritoneal cavity, in the quantity, and with the precautions already mentioned. The strongly acid reaction of the normal contents of the guinea pig's stomach is by this means neutralised for six hours, the opium prevents too rapid passage of the organism through the small intestine, and thus affords opportunity for its multiplication. Koch states that alcohol has a similar effect on peristaltic movement. Klein denies that opium tincture lessens intestinal peristalsis.

Four guinea pigs were injected, two with tincture of opium and two with rectified spirit, into the peritoneal cavity, in repeated doses, till the animals were sufficiently stupefied to lie on the side when placed there. Two c.cm. of spirit were used in one case, and 2·75 c.cm. in the other; their weights being respectively 730 and 750 grammes. They were killed at intervals of from six to nine hours afterwards, when relaxation of the intestine and arrest of peristaltic movement were amply demonstrated in all four by distension of the small intestine with food to a degree never seen in health. In the animals that received the spirit the stupefaction lasted longer, and there was increased vascularity of both peritoneal and pleural serous membranes, resembling that present in the conjunctiva, this vascularity being absent in the opium-treated animals. The effect of alcohol on vascularity is a well-known one. Klein's experiments, in which he gave the opium in other ways than by the peritoneal cavity, and then injected the cholera bacillus with negative results, are inconclusive, as he never made a control experiment with his material to see whether he was able to produce Koch's results under Koch's conditions. The well-known attenuation of virus through cultivation is left out of consideration, and no details of these thirty experiments as to doses, age of the cultivations, or conditions of administration are given.

In seven cholera experiments, varying quantities of a gelatine tube cultivation of comma bacilli were added to 5 c.cm. of sterilised broth and injected into the animal's stomach. In the next twenty-seven experiments a tube, containing about 5 c.cm. of slightly alkaline sterilised broth, was inoculated from a gelatine cultivation of the bacilli, and incubated at 98° Fahr. for twenty-four or more hours, and varying quantities of this were injected. Of these animals twenty-one died, thirteen recovered. The twenty-one animals recovered from the effects of the opium, sat up, ran about, and those that had small doses of the cholera material, fed. On the following day they became sick, refused food, the coat *stared*, the edges of the eyelids became dry and gummy, and the hind legs were dragged, later becoming as if paralysed. Some animals died on the second day, others on the third or fourth.

Of eleven control animals treated exactly in the same way, but receiving sterilised instead of cholera broth, all recovered.

Three animals, after injection into the stomach of soda solution alone, seemed in no way affected.

From one of the animals that died after a dose of cholera material, the small gut contents were collected in a sterilised vessel, and injected in doses of 2 c.cm. into the stomachs of two other animals. These two animals died, and the contents of their small intestine were used in the same way, and so on through ten generations. Of twenty-one animals thus treated two recovered, nineteen died. The dose varied from 0·5 to 2·5 c.cm.

As control experiments to these, two animals received 5 c.cm. of the soda solution, 5 c.cm. of sterilised broth, and thereafter the usual dose of opium tincture. After six hours these animals were killed, and the contents of their small intestine collected and injected with the same precautions in doses of 2 c.cm. Four animals thus treated all recovered.

These results with bowel contents dispose of Klein's objection that the action of "a chemical poison present in certain cultivations of the cholera comma bacillus," is necessary to kill guinea pigs, since no cultivations were used.

The cæcum of the cholera-infected animals was usually distended with watery, and if the animal died early, greenish contents, and contained comma bacilli in great numbers, among a multitude of other forms. From an animal where this was the case, 2 c.cm. of the contents of the ileum were injected into the stomach of one animal, and a similar quantity of the contents of the cæcum into that of each of the three others. The former died; the three latter recovered without a sign of sickness. Unfortunately no plate cultivations were made of those cæcal contents to ascertain whether the comma bacilli seen there were capable of growth like those of the ileum. Koch's contention that these organisms are killed in the cæcum of these animals is so far borne out by these three experiments.

Of the fifteen animals that recovered after injection of the cholera material, seven were manifestly sick, two were slightly so, and six were not apparently affected.

At the present time even Koch's most energetic opponent (Klein) will allow that the proof of Asiatic cholera in man is the presence of the comma bacillus either in the stools of a person taken suddenly

ill, with or without vomiting or cramps, or if there has been no purging, in the contents of the more or less distended, congested, rose-red, and paralysed-looking small intestine; these contents being watery, and containing white flakes.

If these appearances were present in man after having been dosed with a pure cultivation of comma bacilli, in a country where no cholera was about, would they not be accepted as proof of the presence of Asiatic cholera, even if that man had swallowed a dose of soda solution, and had received a dose of opium tincture into his peritoneal cavity?

If this test is to be accepted in man, why is it not to be accepted in the guinea pig?

On *post-mortem* examination of those animals, so dosed with comma bacilli, and death following, as described, the blood was fluid, thicker and darker than natural; the tissues of the thoracic and abdominal walls were markedly dry; the small intestine was throughout distended, congested, and paralysed-looking, and occupied a much larger proportion of the abdominal cavity than usual. The cæcum was distended with fluid or semi-fluid contents. If the animal had died early, the fluid was not quite clear in the small gut, there being present traces of food; still the watery character was very manifest, and mucous flakes were abundant. If the animal had died on the second or third day, no food remains were to be seen, and the fluid in the small gut the counterpart of the typical cholera stool of man. In either case the comma bacilli were demonstrated microscopically, and by cultivation, as in man. While the organism in the broth injected could be frequently counted in a microscopic field, in a drop of the small bowel contents from an animal having received such broth the bacilli might be so numerous that counting them without dilution of the fluid examined was an impossibility. On floating the bowel in water, the stripping of the epithelium could be well demonstrated. Two animals, not included amongst those already mentioned, died on the fifth and eighth days respectively after injection of cholera material. In them no trace of comma bacilli could be found. From the appearance of the bowel it might be inferred that they had died in the reaction stage. In one case (that of death on the eighth day) the intestine was lax, congested, the epithelial lining extensively stripped, and Peyer's patches could

be seen standing out prominently and ulcerated. This animal was the only one that had had loose stools.

While 2 c.cm. of the cholera small gut contents was a deadly dose, the same quantity of cholera broth from the original cultivations was by no means so deadly. This might be due either to increased virulence of the organism after its passage through the animal, or to the greater number of bacilli present in the one than the other. The organisms were certainly more numerous in the bowel contents than in the broths injected. As plate cultivations were made from the former, and broth cultivations grown from these as nearly as possible under the same conditions, as to quantity inoculated, time and temperature in the incubator, the following experiments are worthy of record, though they were not made with the intention of comparing the relative virulence of the cultivations, and the quantities injected are not very suitable for comparison. Broth cultivations derived from the original cultivations brought from China, and therefore cultivated in gelatine for a year after being taken from the human stool in doses of

5 c.cm. killed 5 out of 7 animals.
2 „ 0 „ 4 „
1 „ 1 „ 3 „

Similar cultivations of bacilli which had passed through three generations of the guinea pig, reckoning the first generation the first animal inoculated with bowel contents, in doses of

3 c.cm. killed 4 out of 4 animals.
2·5 „ 1 „ 2 „
2 „ 0 „ 2 „

Similar cultivations of organisms passed through six generations in doses of

2 c.cm. killed 3 out of 5 animals.

Koch states that the organisms obtained during the last European outbreak of cholera, and kept under cultivation in his laboratory, have lost their virulence so far as guinea pigs are concerned, so that his former results are no longer attainable.

In the guinea pig the larger the dose of cholera material given

the earlier the onset of sickness and death, so that the latter may take place within twenty-four hours.

It is objected by Klein and Gibbes "that the strongest reason for not admitting this kind of bacillar relation to the disease is this, that no bacilli exist in the blood or any other tissue of patients suffering from cholera" (page 32 of Government Report of Cholera Investigation in India), ignoring the fact that the first manifestations of the disease are in the alimentary canal, and are only followed by the general constitutional disturbances, while a marked departure from the normal condition is met with in the small intestine and its contents after death. Later, Klein admits that the organism is pathogenic in guinea pigs, though it acts from the intestine, and is not found elsewhere (*Practitioner*, March 1887, p. 200). So that what he applies to animals he will not allow to be applied to man.

Another of Klein's objections runs thus—"If, as many believe, the cholera dejecta *per se* were possessed of infective power, then it would be quite impossible to understand how it happens that the attendants, nurses, and physicians of cholera patients, those that handle the cholera dejecta, the friends and relatives living in the same room, remain so often exempt" (*Practitioner*, April 1887, p. 275).

In the above passage replace "syphilitic" and "discharges" by "cholera" and "dejecta," and the objection supplies its own answer.

Of a similar character and weight are his objections based on the failure of cholera to spread in certain districts into which it has been introduced, the cessation of cholera in certain seasons, &c.

Summary of Conclusions.

1. The comma bacillus of Koch is invariably present, and associated with certain changes, in the small intestine in cases of Asiatic cholera.

2. There is no evidence to show that it is a normal inhabitant of the human alimentary canal, and therefore no proof for the assertion that it is a result of the disease.

3. The means used to introduce the comma bacillus into, and those used to lessen the peristalsis of, the small intestine of the guinea pig, cannot be regarded as causing appearances like those of

Asiatic cholera, or as causing the death of the animal, far less a mortality of over 60 per cent.

4. Pure cultivations of the comma bacillus, introduced into the stomach under the precautions described, are pathogenic to the guinea pig.

5. Injected with similar precautions, the contents of the ileum from those animals killed by injections of pure cultivations of the bacilli, act in the same manner as pure cultivations of that organism.

6. The organism multiplies in the small intestine of the animal, and there is associated therewith changes similar to those in man in Asiatic cholera.

7. As there are conditions which favour the passage alive of the bacillus through the stomach of the guinea pig, and also conditions which favour its multiplication in the small intestine of that animal; so in man, as there cannot be a doubt that the organism finds conditions favourable to its multiplication in his small intestine, it must have found conditions favourable to its entrance alive, through, in all probability, the mouth and the stomach.

8. There is strong evidence, therefore, for regarding the comma bacillus of Koch as the cause of Asiatic cholera.

Preliminary Remarks on the Homologies of the Mesenteries in Antipatharia and other Anthozoa. By George Brook, *Lecturer on Comparative Embryology in the University of Edinburgh.*

(Read December 17, 1888.)

The mesenteries of many Anthozoa are usually regarded as being arranged in pairs, the members of a pair being adjacent mesenteries. The common shore Anemones, belonging to the Hexactiniæ, show this arrangement very well. In a typical case, the mouth and stomodæum are elongated, and the pairs of mesenteries are arranged in multiples of six. There are six primary pairs passing from the body-wall to the stomodæum, which are *complete;* six secondary pairs, which may also reach the stomodæum; twelve pairs of a third series, which are not complete; forty-eight incomplete pairs of a fourth series, and so on. Of the primary pairs, two have the

longitudinal muscles arranged on different aspects to those of all the other mesenteries, whatever may be their number. These are known as "directives," and are situate one pair at each end of the long axis of the mouth; on these the retractor muscles occupy the outer (interseptal) surfaces; on all the other pairs the retractor muscles are placed on the inner (intraseptal) surfaces.

In a very large number of cases, however, it is impossible to explain the arrangement in harmony with this plan. In the whole of the *Alcyonaria*, for instance, there are eight complete mesenteries. At one end of the stomodæum a pair of directives occur having the same arrangement of muscles as in Hexactiniæ; the other pair of directives, however, have the retractor muscles on the inner (intraseptal) surfaces; whilst the intermediate mesenteries do not form pairs in the usual sense, their muscles not being on adjoining or opposite sides, but on the same side. This arrangement may be explained simply by stating that all the retractor muscles are situated on the abaxial (ventral) surfaces of the mesenteries. In *Edwardsia* and its allies there is an arrangement which does not correspond either with the Alcyonarian or Hexactinian type. Of the eight mesenteries, two pairs of "directives" show the Hexactinian arrangement; the other four mesenteries have the same arrangement of retractor muscles as in the corresponding mesenteries of *Alcyonaria*. In Cerianthidæ the mesenteries are numerous—not in multiples of six—and, according to the current view, not arranged in pairs. The musculature of the mesenteries is rudimentary.

These four instances will serve to show the variation in the arrangement of the mesenteries in some of the leading groups of the Anthozoa. Many Madreporaria show the Hexactinian arrangement, but others again do not. The Antipatharia are generally supposed to be degenerate, and to have lost a considerable number of the mesenteries present in their ancestors, but very little is known concerning the morphology of this group. Dr John Murray has very kindly placed the "Challenger" collection of Antipatharia in my hands for identification, and I have now been enabled to make sections of twenty-three species of Antipathidæ, including the only two which had previously been examined by Lacaze Duthiers and G. von Koch. A careful comparison of the number and relative

development of the mesenteries in all these forms has led me to believe that their arrangement cannot be explained in accordance with the current views on the subject. With the exception of two pairs of directives, the mesenteries do not appear to show the paired arrangement usually looked for. The mouth is usually elongated, and is supported at each end by a pair of directives. The musculature is, however, very imperfectly developed, and I have not yet made out the position of the retractor muscles. In *Cladopathes*, n. gen., there are only six mesenteries present. Two pairs of directives occupy the extremities of the stomodæum, as in other types, and two other mesenteries are situated in the transverse axis—that is to say, at right angles to the long axis of the stomodæum. These bear the reproductive organs. These six mesenteries I have termed "primary;" all are of considerable length; those in the transverse axis are, however, somewhat longer than the others. In *Antipathes*, &c., there are ten mesenteries—six primary ones as in *Cladopathes*, and four short "secondary" mesenteries, one on each side of the transverse mesenteries. In *Leiopathes* there are twelve mesenteries, six primary and six secondary. In horizontal sections passing just beneath the mouth, the transverse axis is seen to be occupied by an interseptal space, and not by a mesentery. On each side of the stomodæum there are six mesenteries, three on each side of the interseptal space occupying the transverse axis. A little lower down, however, it is seen that two mesenteries bordering the transverse axis, one on each side of the stomodæum, become lost, and as they do so the other two become more important and approach the middle of the stomodæum so as to occupy the transverse axis. The arrangement is then as in *Antipathes*.

It appears to me that this arrangement receives its explanation by a comparison with the order in which the first twelve mesenteries are developed in Hexactiniæ according to Lacaze Duthiers. In *Actinia* and *Heliactis* the first twelve mesenteries are developed in pairs which are not adjoining mesenteries, but are situated one on each side of the stomodæum. The order in which they are developed in *Heliactis bellis* (and in *Actinia equina?*) precisely corresponds *with their relative length* in *Leiopathes*. The first pair to be developed are those corresponding to the transverse mesenteries in Antipathidæ; next follow the two pairs of directives, and afterwards

the three pairs which I have termed secondary in Antipathidæ. The shortest mesenteries in *Leiopathes* are the last of the six pairs to be developed in *Heliactis*. Evidently, then, the mesenteries forming a pair are originally *opposite* mesenteries, and not adjacent ones. We thus have in forms with an elongated stomodæum a true bilateral symmetry. The two pairs of directives limit an anterior and a posterior unpaired chamber. Between these two the cœlenteron may be imperfectly divided into any number of paired lateral chambers. On this interpretation the arrangement in Alcyonaria, Edwardsiæ, Cerianthidæ, Madracis, &c., is also easily understood; all are modifications of one plan.

In the Hexactiniæ the simple bilateralism is masked, but a careful study of the order in which the mesenteries are developed shows clearly how this is brought about. In all types the mesenteries of a pair are originally on opposite sides of the stomodæum. The two pairs of "directives" come to be *adjacent* mesenteries, for the reason that no new mesenteries are ever formed between them, and with a further development of mesenteries they come to be pushed closer together. As is clearly seen from Hertwig's figures of the embryonic condition in *Peachia*, the other so-called "pairs" of primary mesenteries are not pairs developmentally, as they consist of mesenteries of different ages. They are called pairs because they are arranged in couples, having the retractor muscles on their inner surfaces. In Hexactiniæ the further increase in the number of mesenteries takes place in a modified way. Buds appear which are on opposite sides of the stomodæum, between existing "pairs," but instead of giving rise to a single mesentery as before, each gives rise to two with the retractor muscles on their inner surfaces.

The general plan of development I consider to be as follows:—The mesenteries have a radiate arrangement in forms with a round stomodæum; this arrangement becomes bilateral by an elongation of the stomodæum in one axis, the sagittal. In this case the anterior and posterior pairs (directives) come to consist of adjoining mesenteries, whilst the intermediate pairs consist of opposite mesenteries. So long as the folds of the body-wall give rise to only one mesentery each, the simple bilateral arrangement is retained, as in Cerianthidæ (I refer to a bilateral arrangement of parts, irrespective of the outline of the polyp). In case the mesenterial rudiments give rise

(after the formation of the first twelve mesenteries) to two mesenteries instead of one, the Hexactinian type is reached. In certain Madreporaria (e.g., *Lophohelia*, *Mussa*, and *Euphyllia*) the radiate arrangement appears never to be lost. At any rate, according to Fowler and Bourne, there are no mesenteries distinguishable from the others as "directives," and there is a perfectly radiate symmetry.

Such a general plan of development in the Anthozoa is found in another form, for instance, in *Peripatus* and *Vertebrata*. In *Peripatus* the blastopore becomes elongate, and closes in the centre, but its two extremities remain open, as the mouth and anus. The mesoblastic somites are formed in the region in which the blastopore has closed, and these become more numerous as the two extremities become more and more separated. At present I am only able to indicate the bearing of these views in outline. I hope, however, shortly to make a more detailed communication on the subject.

On Certain Bodies, apparently of Organic Origin, from a Quartzite Bed near Inveraray. By His Grace The Duke of Argyll, K.G., K.T.

(Read January 14, 1889.)

The magnificent sections of the Archæan and Palæozoic rocks which are presented on the north-west coast of Scotland, have long been known to all British geologists. They occur principally in the counties of Sutherland and Wester Ross, from Cape Wrath on the north to Loch Kishorn on the south. They are not less striking in an artistic, than they are instructive in a scientific point of view. The height of the mountains, their abrupt and precipitous forms, and the thinness, or almost total absence of, superficial covering, are characteristics which combine with great variety and richness of colouring, to produce scenery which stands altogether by itself. The great rock masses to which its peculiarities are due are principally three, differing widely in geological age, in composition, and in texture, and yet often so piled upon the top of each other that very often one single landscape, and

sometimes even one single mountain, may present the whole of them to the eye in a splendid succession of precipices and of slopes. The general or normal order in that succession is exhibited in many of the finest hills. At the base there is that peculiar gneiss, which no geologist who has ever seen it can again mistake—sometimes called Hornblendic, from one of its most characteristic minerals—sometimes Laurentian, from its immense local development on the northern shores of the St Laurence—and more recently termed the Archæan, from its being the oldest sedimentary deposit as yet known in the world. On the top of this very ancient rock—with all the marks and aspect of a most hoar antiquity on its face—there rise immense, but isolated and broken masses of a deep chocolate-red sandstone supposed to be of Cambrian age—sometimes in low hills or sloping promontories, but more generally in fine mountains of very peculiar and very steep and sudden outlines—each separated from its nearest neighbour by deep glens or arms of the sea, or lakes of fresh water—and each looking as if it had come there by some inexplicable accident, seeing that it bears no obvious relation to the other rocks upon which it sits, or to the general aspect of the country out of which it rises. Then, lastly, in some places, but not everywhere, on the top of these steep and violent mountain forms—cut out, as it would seem, from some greater mass which has been swept away—we have a cap or summit composed of a pure white quartzite beautifully stratified—sometimes grey with lichens and mosses, but very often also unstained and glittering, like mountains whitened by an early fall of snow. The stratification of this quartzite is always unconformable with that of the red sandstone on which it lies, and as those mountains are all nearly naked of vegetation at the top, the difference in the lines of bedding is almost as conspicuous to the eye as the difference in colouring between the two rocks.

The lowest of these three great rock masses, which is the floor or basement rock along the whole of the west coast of Sutherland,—although in Canada it has furnished certain bodies which have been supposed to be, and probably are, of organic origin,—the so-called Eozoon,—has not yet in Scotland supplied even a doubtful indication of animal or of vegetable life. It is, so far as yet observed,

absolutely azoic; and from the intense mineralisation of its substance it is not very probable that it will ever afford us any glimpse into the great secrets connected with the origin of life upon our planet.

The enormous mass of red and gritty Cambrian sandstones and conglomerates which have been deposited upon the upturned edges of the Archæan gneiss, is in a very different mineral condition. I found in it myself, very lately, some thinly bedded deposits, on which the ripple-mark of the waters in which it was accumulated were as well marked and as fresh, apparently, as the ripple-marks left upon any of our shores by the tides of yesterday. There seems to have been no important alteration in its mineral constituents since the time when its sands and gravelly conglomerates were thrown down in seas or lakes having a bed of the primeval Archæan rocks. Yet through thousands of feet of thickness, until very lately, no trace of organic life had been detected; and even now, as the result of the most careful examination by the Geological Survey officers in all the beds of this vast deposit, the indications of life are not only few, but exceedingly obscure.

The case, however, is very different when we ascend to the highest of this triple group of rocks—when we go to the top of the red sandstone mountains, and examine, or even carelessly walk upon the white and flaky quartzites which lie upon them. It is impossible not to notice the curiously spotted, or pitted surfaces of the white slabs, which in many places look like nothing so much as the soles of a labourer's shoe, covered with projecting hob-nails. On examination, one finds that these little knobs, or nail-heads, are the projecting ends or tops of cylindrical rods which penetrate through the whole substance of the rock, so that if we lift one layer, and look at the next floor of rock below it in the bedding, we can trace the repetition of similar rods down,—down,—down—from the very top to the very bottom of the deposit. The extreme roughness which they sometimes present upon the surface is thus found to be due to the fact that the quartz graining or structure of the rods is harder than the graining of the intervening spaces,—so that this surface weathers away the fastest, and the tops of the rods stand out like the nail-heads in the shoe. But these rods can often be detected when there is no such indication on the surface. I

shall never forget my own surprise when I first went to examine a fresh section of this rock which was not parallel with the bedding, but across it, or perpendicular to it, and which being a well-protected surface in a very sheltered spot, showed at a little distance a perfectly smooth and homogeneous face of fracture. It was nearly as pure white as the finest loaf-sugar. Yet when I looked closely into it, I could distinctly see the parallel and perpendicular lines which indicated the rods, or little cylindrical columns which I had seen projecting on the weathered surfaces on the tops of the hills. Again, there are some other conditions in which these rods are especially conspicuous, and this is where the quartzite itself is not white, but more or less stained by the oxide of iron. Not far from the spot last referred to, I was living some years ago in a shooting lodge built near the foot of a precipitous line of crag which stretched straight up the hill behind the house. It was of a dull brown or chocolate colour, and, for a long time, it did not occur to me that this crag was an escarpment of the same quartzite, only differently coloured. On finding from its texture that it was so, I set to work to find the rods, and I very soon found a good many, which, to my surprise, were pure white—thus presenting a most conspicuous contrast with the surrounding rock. They were, however, not nearly so numerous, although of precisely the same form and character. For many years there was much doubt and speculation as to the nature and origin of these rods—whether they were some form of mineral concretion, or whether they were the marks and indications of some organism. Macculloch, in his well-known and admirable work on the Geology and Mineralogy of the Hebrides and adjacent Coasts, published in 1819, devotes some pages to a careful and very accurate account of this quartzite rock, in connection with his account of the Island of Jura, and mentions almost all the principal places in Scotland where it appears, and where it maintains a substantial identity of mineral composition and of structure. The passage in which he alludes to the appearances now under discussion, is curious as showing how little he regarded them as at all characteristic, and how little he thought them capable of any definite explanation. "Occasionally," he says, "also it (the quartz rock) contains those prolonged concretions and other well-known forms, which are common in the secondary sandstones, and which have been supposed

in these to originate in organic remains or impressions."* At the same time it is to be remembered, and it seems to have been much forgotten, that Macculloch and not Mr Peach was the first discoverer of the now famous fossils of Durness and of Eriboll in Sutherland,—some of which are described by him as occurring in quartz rock, intimately associated with beds of limestone. It is indeed most singular that an interval of thirty-five years should have been allowed to elapse between the publication of Macculloch's work and the rediscovery of those fossils in the Durness limestones by Mr Peach—because not only had Macculloch detected in the associated beds of Loch Eriboll certain definite organisms which he conjectured to be fragments of an Orthoceratite,—not only had he recognised those beds to be the same as those at Durness,—but he had also observed that the calcareous beds of Durness presented on their weathered surfaces "a variety of singular forms" which he suspected to be of organic origin, and he had expressly indicated the probability that a further search in the same beds, which it was impossible for him to conduct in a cursory visit, would in all probability reveal more abundant organic remains.† It is in connection with this early discovery of such remains by Macculloch at Durness that he mentions "vermicular forms" as observable in the base of the limestones, and says they are very similar to similar forms found in the Devonian Marbles at Babbicombe, near Torquay. The "conical bodies" which Macculloch discovered in the quartz-rock of Loch Eriboll have been since assigned to that class of Annelids which live in calcareous tubes, and Macculloch's name has been most properly associated with the peculiar species which he discovered. The bed of quartzite is now known as the "Serpulite Grit." It has been identified in other places in Sutherland, and is of great value in the detailed work of the Geological Survey in that most difficult section of country.

It was, however, perfectly natural that Macculloch should have laid little stress, or should have altogether failed to notice, the extraordinary abundance of the rod-like bodies in the quartzite of Sutherland and Ross, because the special area of his investigation was the Islands, and not the Mainland, of the north-western coast

* Macculloch's *Western Islands*, vol. ii. p. 222.

† *Ibid.*, pp. 512–13.

of Scotland. It was only in examining a very small island, called Garvh, lying about one mile off the nearest shores of the Bay of Durness,—an island hardly belonging to the Hebridean group at all, and really a mere fragment of the Sutherland Mainland, that he was led to observe the fossiliferous strata at Durness and Eriboll.* And here we come upon the curious fact, that although quartzite rocks of great thickness occur far to the south of the remarkable developments of it in Sutherland and Ross-shire, and although Macculloch did not fail to identify it elsewhere as the same rock, both mineralogically and stratigraphically, yet out of the classical region where it is so conspicuous it ceases altogether to exhibit the same rod-like bodies and "vermicular forms" which Macculloch did observe at one spot, and of which he spoke so doubtfully. It has long now been the accepted interpretation by all geologists that these bodies represent the tubes or burrows made by marine worms of many species in the intertidal zone of shores of the sea. This interpretation is not without its difficulties of detail. We are all familiar with the appearance of worm-castings on our sandy and muddy shores; and more attentive observers must have sometimes been surprised by the immense numbers of these burrowing annelids in particular areas. But the natural supposition would seem to be that although each one of these worms must make a tube in the sand or in the mud in which it feeds and burrows, yet these tubes must be as evanescent as the passage of the annelid, because the pressure of wet sand on its own particles, or the transferring and transfusing effects of water permeating its substance, would seem to be causes sure to effect a rapid obliteration of any temporary vacuity made by the passage of the worm. Still it is certain that internal tubes and passages must be made by such worms in the course of their ceaseless operations, and that they must be made in enormous numbers when the sand or the mud, or a combination of both, is sufficiently charged with organic matter, to support the annelids in such abundance as we constantly see upon the shore. It is perfectly natural, therefore, that upon primeval shores on which fine sand was constantly being formed and deposited during periods of time which were more or less prolonged, worms should have burrowed as they burrow now, if this form of organic life had been at that time established. The

* Macculloch's *Western Islands*, vol. ii. p. 508.

only difficulty is to understand the conditions under which these old burrows were so stable as to be capable of preservation when the sandy mud was in course of solidification, and when in many cases it must have been placed under the great pressure of later deposits. This is one of these difficulties which are felt instinctively even by those who do not consciously reason upon it, and it is not surprising that it should have long delayed the full acceptance of the annelid interpretation of rod-like bodies in a very hard and very ancient rock. I do not think that the difficulty of following the *modus operandi* in the preservation of these assumed annelid burrows has been altogether removed. But undoubtedly some facts connected with the peculiar structure and functions of the skins of annelids tend to diminish the difficulty in a great degree. We now know that, by virtue of a power to secrete and exude a very glutinous slime, some marine worms which do not burrow at all, do nevertheless make for themselves tubes of sand grains agglutinated together into a texture so firm that they are capable of very great resisting power. The congeners of these worms are in many cases creatures which are themselves naked, but are protected by always burrowing under ground. These are said to have, though in a less degree, the same power of secreting an agglutinating slime, so that in every tube which they dig out, or along which they pass, or in which they may lie, they make and leave behind them an internal wall, or caked surface, which has a certain coherence—just enough to constitute a definite mechanical difference between these tubes and the surrounding matrix. This is perfectly intelligible. But it is less easy to see how these tubes could come to be filled up by the surrounding sand, and yet without obliterating the impression of the walls. It seems as if the interior of the tube could only be filled by the destruction of its walls—unless indeed, we are to suppose that sand of some other texture, not agglutinated by slime, and not discoloured by contact with organic matter, had somehow come to be introduced into the tube from the top or surface or external aperture, and so forced down all its length as to take a cast of it along all its course. This really would seem to be the only process by which we can explain those cases in which the matrix is coloured red, and the tube converted into a rod of pure white quartz rock. It is not very easy to follow in detail the natural process by which

this result has been accomplished, although perhaps it is not more inconceivable that a good many other cases in which organic structure itself of the most delicate kind has been wholly converted into, and reproduced in, the mineral substances of lime and silex. There is one process which has sometimes suggested itself to me, although this also is beset with many difficulties. It is conceivable that a sandy sea beach, well bored by annelid tubes, may have been lifted above the sea-level by some earth movement. That all marine deposits now converted into rock have been so lifted at some time or other is certain. Such movements, involving only a very few feet of elevation, may have been sudden, and would be quite sufficient. It is conceivable that the open ends of the tubes might be thus exposed to desiccation, and to the penetration of blowing sand in a dry condition, or possibly of sand carried into them by floods of fresh water. In either way, it is conceivable that the tubes may thus have been filled with grains of sand capable of retaining their special colouring and texture under all the subsequent changes to which they, together with the surrounding matrix, were exposed in being hardened into rock.

But here we come upon another fact which seems to justify all our difficulties—in proving that only under conditions very exceptional indeed was it possible for such tubes to be preserved. In all parts of Scotland where this quartzite rock appears, except in Sutherland and Wester Ross, the annelid tubes have either been wholly obliterated, or else they have never existed. This last alternative is highly improbable. There is good stratigraphical evidence that the quartzite rock which appears elsewhere to the south belongs to the same horizon in geological time, and must have been accumulated under similar conditions. In the island of Jura, mountains of 3000 feet of elevation are entirely composed of it from top to bottom. In Islay also, the same rock reappears, and with more of its characteristic whiteness than in Jura. The same strata are prolonged towards the north-west in the line of the Great Glen, or of the Caledonian Canal. A string of small islets on the eastern side of Lismore are composed of it, in close proximity to other islets of limestone against which they are faulted in a position of extreme unconformability. The bed which they represent passes straight on under the hills of Ballichulish, near which it has been

quarried and exported for uses connected with potteries in England. To the north of Ballichulish, again, it rises to the summit of a high mountain in the ranges which terminate in Ben Nevis. This quartzite ridge not being the one nearest the shores of the Linneh Loch, but a ridge behind, is concealed from the eye in passing from Oban to Fort William. But it is visible from the passage between Lismore and the Morven shore. I never observed it at all until the present year, when the illumination of a setting sun, and the foil of a dark cloud behind, revealed some high and jagged peaks of a pure white, recalling the special scenery of Loch Torridon, of Loch Assynt, and Loch Maree. Now in all these southern masses of quartzite, so far as I have examined them myself, and so far as I know from others, no vestige has been seen of the annelid rods or presumed tubes of the western quartzite. In three places I have had special opportunities of observing it. First, on the western side of the island of Jura, where an extensive series of raised beaches, too little known to geologists, have collected together an immense assemblage of the rolled and weathered fragments of the quartzite, and where, from its reddish colour, any white worm tubes such as I have referred to in similar rocks in Sutherland, would be specially conspicuous; secondly, on two of the islets of white quartzite which belong to my own property in Lismore, and which I have examined with some care; and lastly, on the prolongation of the same beds south of Ballichulish, at a spot where the rock has been worked and quarried for commercial uses. In all these places, so far as I could see, no annelid tubes are to be seen. It looks as if they had been squeezed out under enormous pressure, or obliterated by some other of the unknown causes which seem to have had the same effect in many other cases of strata once full of the remains of organic life.

I now come to the special subject of my communication to the Society to-day. It was the result of a fresh visit made this last autumn to the wonderful escarpment sections of Western Ross, in Loch Torridon and Loch Kishorn, that I was led to think over and over again whether a closer search might not reveal a survival elsewhere of the organic remains of the quartzite. It was the well-known generalisation of Murchison, from the section of country across the county of Sutherland from east to west, that the great mass of mica-schists, and

more recent gneisses, which constitute the bulk of the Highlands from Loch Eriboll to the Firth of Clyde, represent the same strata, which lie between the chocolate-red Cambrian sandstones of the west coast, and the Old Red Sandstones on the eastern side, of that county. The rediscovery by Peach of the fossil remains so long before detected and indicated by Macculloch, enabled Murchison to present this generalisation as firmly resting both on stratigraphical and on palæontological evidence. I have never seen the least reason to suppose that the late marvellous discoveries of the Geological Survey in Sutherland cast the least doubt on Murchison's general conclusion. Those discoveries, indeed, are of immense interest, and teach us many lessons on the dynamics of geology—lessons, I think, which were greatly needed. But it does seem to me that the bearing of these discoveries on the conclusions of Murchison is almost *nil*. They prove indeed that, in a middle region between the extreme west and the extreme east, there has been local disturbance of an extraordinary kind, and that in that region the present order of superposition cannot be taken as determining the geological age of the rocks, owing to reversing and overlapping faults—"thrust-planes," and other phenomena of earth movements so peculiar that they seem to have already called forth a completely new nomenclature as requisite to describe them. It is a curious fact that Macculloch, with a penetrating and almost prophetic eye, seems to have suspected this area of country to be one of unusual disturbance. He speaks of "two distinct deposits of gneiss, of which one is placed in a reverse position to the other." He speaks of "essential disturbances among the primary rocks," due to "one or more revolutions," separated by "long intervals of time." He speaks of "the science of geology being not yet sufficiently advanced" to afford any complete explanation of the peculiar phenomena of that region.* All this sounds like a dim prevision of the wonderful inversions of the natural order of the rocks in that part of Sutherland, due to most "revolutionary" movements of subterranean force, which have only just been discovered and laboriously traced by the Geological Survey. Nothing but such a mapping and tracing of every bed in the greatest detail could have unravelled such inversions, conversions, distortions, and horizontal displacements. Murchison did not know of these, and he

* Macculloch's *Western Islands*, vol. ii. pp. 202–3.

consequently mistook the real meaning of certain confused sections in the middle zone of that disturbance. But, after all, the horizon of the Archæan gneiss on the west coast, with that of the Cambrian sandstones overlying it, and the other horizon of the Old Red on the east coast, with the Oolites and Lias on the top of it, remain fixed points in the interpretation of the whole section; whilst the discovery of a copious fossil fauna exactly where Macculloch suspected it would be found, leave no doubt of the Lower Silurian age of the great mass of strata overlying the Cambrian sandstones and the white quartzite on the west, and presumably underlying the Old Red upon the east.

The result of this conviction as to the substantial truth of Murchison's generalisation, has led me to anticipate with some hopeful expectation the discovery of organic remains, some day, among the metamorphic rocks of Argyllshire as belonging to the Lower Silurian series. With this view, I have often carefully examined the calcareous beds which are abundant there, but always hitherto without result. Impressed, however, with the specially fossiliferous character, as regards annelids at least, of the sandstone or quartzite deposits in the north, I determined on my return home, in September last, to examine carefully a few beds of that rock which I had long known to occur close to my own residence at Inveraray. Those beds form no noticeable feature in the country. They make no show upon the hills, nor are they conspicuous on the shores. Crystalline quartz is indeed abundant—in veins and in masses. Thin beds or bands among the mica-slates, which are more or less silicious, are also abundant. But true quartzite, well stratified and bedded, is very rare, so rare that I know of it as occurring only at two spots on the shores of Upper Loch Fyne. Close to the town of Inveraray there is a little headland or promontory, called Craig-na-churach, which exhibits a series of true quartzite beds, that may be from 30 to 40 feet thick. It has been a good deal quarried and used for making breast walls against the sea. I had also frequently examined it for metallic ore, since galena is present in detached spots throughout some of the beds. I had also observed some strange cavities in it filled with sand or clay coloured with iron oxide. But nothing had ever suggested itself to me as indicating organic remains. I knew, however, that the same quartzite, although in very

inferior mass, did reappear a few miles to the east of Inveraray, upon the same northern shore of Loch Fyne, and to this bed accordingly my first search was directed. That search led me at first into a mistake as to certain appearances—a mistake not without instruction on the danger of preconceptions in the interpretation of obscure phenomena. I found on the surface, and apparently embedded in the grain of the stone, certain ramifying filaments and minute tubes in red oxide of iron, together with certain knots which seemed at first sight to be the seed capsules of some plant. On breaking up the rock I found interspersed throughout its substance certain discoloured spots, which almost invariably took the form of ovate or spatulate rings, all more or less like the form of a battledore, but without any appearance of a stalk or handle to the battledore. Connecting these internal oval rings with the external forms of vegetable structure preserved in casts of red oxide of iron, I was inclined to believe that they were the same in origin, and that the more distinct character of the forms on the surface was due to that effect of weathering which so often reveals organic remains upon the surface of rocks when no such forms can be detected in the fresh fracture. In pursuit of this idea, I had the rock opened and split at various points of the outcrop of the beds; and everywhere I found, sometimes thickly interspersed, sometimes thinly scattered, oval stains and spots, and some lines, which were all apparently referable to the same ultimate cause and source, whatever that might be. At one place these small ovate stains passed out of the quartzite altogether, and were repeated in some finely laminated overlying beds of pure micaceous schist. This, however, occurred only where such schists lay in immediate contact with the quartzite —an observation which is important, because the total absence of them in the great masses of overlying schists and schistose combinations which constitute the bulk of our hills, is one of the features in the case. The occurrence of the oval rings and other discoloured impressions, which forms the subject of this paper, is, so far as I have yet seen, strictly confined to the quartzite beds, and to a few inches of very fissile mica slate which lies in immediate contact with that rock.

Two specimens, or sets of specimens, and these alone, contributed for some little time longer to carry on my impression that they

might be due to vegetable rather than to animal life. The main objection and difficulty was that the ovate or capsule-like rings were almost always unconnected with any stalk or stem. One specimen, however, had an appearance of being so connected, whilst another suggested the idea of straight linear stalks like those of some kinds of reed. There was yet one other specimen that caught my eye for a moment among a great number of fragments, which I saw at once was in itself the most distinct and definite form of all, but which, as not suiting the interpretation which was then preoccupying my mind, and as not suggesting any other, I did not pick up or preserve at the time. This now turns out to be one of the most instructive of all, and it has fortunately been preserved—only because the recollection of it came back to my memory after some days, and on revisiting the spot I found it, and have brought it here to-day.

At my invitation, the place was visited first by Mr Hill, the officer of the Geological Survey, who is now employed in mapping the strata towards the lower end of Loch Fyne; and at a subsequent date by my distinguished friend, Dr Geikie, the Director-General of the Survey, who very kindly came to Inveraray for the purpose of examining the rock. Both of these gentlemen doubted, as they well might, my idea that they were of vegetable origin; and both, as the alternative, ascribed them to mineral concretions, such as pyrites, or clay balls, drawn out into ovate and linear forms by the effect of shearing or slipping movements of the strata over each other.

Several features in these ring-like and oval stains, as well as some arguments derived from other considerations, impressed me with the belief that this afforded no satisfactory solution of the problem. In the first place, the idea of little balls of clay having been formed in immense numbers on a sandy shore, and of their having been again and again re-formed so as to be buried in every layer of the deposit, seemed to be a purely theoretical idea, and not a little fanciful. In the second place, such balls would not afford, even when sheared and flattened, the tube-like sections in the form of oval-rings which have to be accounted for. In the third place, mineral concretions, and especially sulphides of iron in many forms, are very common throughout all our other rocks, and yet in none of these have similar appearances been produced. In the fourth place, there are certain

very definite features about these bodies which are quite constant, and which it is not easy to conceive could be constant on the hypothesis of a purely mineral origin.

Under these circumstances, I sent a series of the specimens to Mr Etheridge, the skilled palæontologist of the British Museum, for his opinion and determination. He at once pronounced decidedly against my preconception of a vegetable origin. After the most careful and prolonged examination, however, with all the light which could be thrown upon the question by specimens from all parts of the world, Mr Etheridge has come to the conclusion, without the smallest doubt, that the bodies found in the Loch Fyne quartzite are the tubes or burrows of annelids, similar in their nature and origin to those which have been long familar in the same rock as it occurs in Sutherland and Ross. This verdict was supported by arguments, by specimens, and by further investigations on the spot, which have carried complete conviction to my own mind that I was entirely led off the true scent when I thought of vegetable organisms, and that, although in a somewhat new and certainly unexpected form, I had, after all, found what was the original object of my search, namely, the repetition in our south-western beds of quartzite of those remarkable annelid borings which are so abundant in the north. One specimen from the quartzite of Quenaig, in Sutherland, was almost of itself sufficient to satisfy me that Mr Etheridge had recognised the true interpretation. It exhibited a cross section of the well-known annelid tubes of that county in a fresh fracture of the rock; and these cross sections presented precisely the same ovate tubes, with precisely the same defining ring of red oxide of iron. The phenomena are identical. But the interpretation is confirmed by many other facts, and converging considerations. I will now shortly sum up the arguments which seem to me to be tolerably conclusive on this matter. I give them in the order in which they range themselves in my own mind—without laying any stress upon the authority of Mr Etheridge—although, in passing, I may say that in questions such as this, namely, the interpretation of appearances which are of necessity obscure, I attach immense importance to the perceptions of a trained and experienced eye. It acquires an education, and an insight of its own. Interpretation becomes almost instinctive, from that kind of

"correlation," which is the highest of all, namely, the adjustment of our perceptive faculties to the indications of external nature.

In the first place, then, we must bear in mind the nature of the animals whose traces are in question. They are animals absolutely wanting both in external and in internal skeleton. Nothing that constitutes in most other animals the fossilisable elements of structure, has any existence in them. Soft animals, which have a shell, themselves decay, but their shells remain, and thousands of extinct mollusca belonging to the past ages of organic life are in this way preserved to us with the utmost minuteness of detail. In this very rock, as it occurs in Sutherland, the serpulites discovered by Macculloch represent, in a silicified condition, the calcareous shells of some species of annelid, which lived, as some species now do, in an external shelly tube. But large classes of other marine worms or annelids have no such shells. They are entirely composed of tissues extremely soft, liable to rapid corruption after death, and incapable of being fossilised, except in the form of mere casts.

On the other hand, we have to recollect that these naked annelids perform a work in the soft material of future rocks which is altogether peculiar to themselves—a work which arises out of their relation to peculiar sources of food, and for the performance of which they are endowed with very special and very wonderful powers. It is their function to bore into, or perforate, all kinds of marine sediment, both those which are in course of being laid down, and those even, in many cases, which have been long indurated into the hardest rocks. Confining our attention to those species of naked annelids which burrow in soft sands, clays, or mud, we find that there are many different genera which affect different kinds of sediment, and make burrows with some characteristic differences. Some line their tubes or tunnels with calcareous matters which are in the nature of a shell, and might be preserved like any other shell. But this is not the case with those annelids which are most common and abundant upon our existing shores. At every ebb tide upon our coasts, where there is sand or clay or mud, we see the surface covered with worm castings, and the presumption is that these are the living representatives of the primeval annelids which performed the same work in the earliest ages of which we have any record in the rocks.

Looking then to these, as presenting all the facts upon which we must argue in interpreting the past, we find that our foreshore annelids are of many kinds and sizes, from one species which attains the great proportions indicated by a length of 3 feet to other species which are very small. We find that the commonest varieties which leave their marks at low tide on almost every possible spot of shore, bore and tunnel into the wet sands and mud with almost incredible ease and facility of motion. Professor Mackintosh, who, as must be well known to the members of this Society, has made a special study of these organisms, speaks of them as "dashing" through open water and through wet sand with almost equal ease.* He tells us that these perforations are very various in direction and in shape; that the animal secretes from its skin a fine lubricating slime, which must always have a tendency to agglutinate the grains of sand that come into contact with it, and which in some species is employed to construct a tubular case that projects from the surface of the sand, and constitutes a sort of house or fortress from which the creature protrudes its tentacles or its proboscis for the seizure of its food. But the food of the common burrowing sand worms is found in the sand itself, and the agglutinating power of their slime on the surrounding particles of sand may have no other effect than that of so hardening or indurating the inner surface of the tubes as to render them comparatively lasting, or, at all events, to prevent their speedy obliteration. Here we have one obvious explanation of the possibility of fossilisation, or of permanent endurance through the processes which convert soft sediments into hardened rocks. And then, again, we find that another step forward in our investigation of existing facts becomes an important step forward also in our interpretation of the past. We know that the purpose or aim of burrowing worms, in the exercise of their extraordinary muscular energy in boring, is to extract food from the substance, whatever it may be, in which they burrow. We know further, that this food is extracted by the power which the worm has of swallowing the finer particles of its surrounding medium, of passing them through the whole length of its body, of submitting them there to its digestive apparatus, and of discharging behind it all the mere mineral particles

* "On the Boring of certain Annelids," *Ann. and Mag. Nat. Hist.*, Oc. 1868.

which have been stripped of nutriment. In this respect the marine worms perform the same work that on land is done by the earth worm. It would seem to follow from this process that one consequence of it is likely to be that, as the worm passes through the sand, the finer grains within reach of its swallowing power will be segregated from the coarser, and be concentrated in what are so well known as the "castings," most of which are ejected on the surface, but some of which may very probably remain in portions of the burrow. And now we reach a further fact of curious significance and interest. We know that all organic matter has more or less the property of decomposing the mineral compounds of iron, and of reducing them to red oxides. What, then, is the probable effect upon such particles of iron as may be ingested by the worms, and passed through their bodies? Have we any evidence on this point from our living annelids? Yes, we have. It is found, for example, that when a worm burrows through blue or dark coloured shale, the castings of the worm, which are the débris of the mineral, are not of the same colour as the matrix from which they come; from blue or blackish, they are converted into brownish or reddish material.* On the same principle, and by the operation of the same causes, we see that the particles of sand which are agglutinated by the slime of the annelid, and are converted into a thin wall by the coherence thus produced, would probably exhibit the same change of colour due to the peroxidation of any iron present in the surrounding material. Hence, again, we are led to the conclusion that several characteristics may be expected in any marks left in rocks by the boring of annelids—first, that the walls may retain a special character distinguishable from the matrix; secondly, that one indication of these may probably be some visible development of the oxides of iron, more or less conspicuous according to the abundance or scarcity of iron in the matrix; and thirdly, that wherever the castings have been left in the tubes, or have been washed into them again, they would most probably exhibit some difference both of texture and of colour as compared with the matrix. There is another conclusion, which we may draw assuredly from the series of causes thus traced, and that is a conclusion as to the effects which would probably result from the complete breaking up of

* Mackintosh, *Mag. Nat. Hist.*, Oct. 1868, p. 5.

the tube walls, and any dissolving out or excavation of the whole tube and of its contents, by some subsequent exposure to solvent agencies, such as exposure to sea water, or from mechanical violence deforming, and more or less crushing the tubes out of all distinct recognition. One lasting effect, it would seem, is likely to remain, and that is the removal of the finer particles from among the coarser for some little space round the original path of the worm. The effect of this would be to leave a change of texture along that path—a coarser, rougher graining in the sand in any slits or hollows along which the original tubing has been made.

We have not yet done, however, with the lessons to be learned from a careful examination of our living annelids. Hitherto I have been drawing from a few facts reported by others as to the work of annelids, certain conclusions as to consequences which that work would probably or necessarily involve. But as I recollected one day that I was writing this paper within 200 yards of abundant shore worms, I determined to make some experiments and some further observations on what they do. The very first thing I found was that, although the orifice of the tubes as they are opened on the surface are roughly and irregularly cylindrical, yet the slightest pressure on the sand, such as even a sharp knife produces in carefully cutting off the castings, is sufficient to draw or deform the orifice out of its true shape, and to convert it into an oval aperture or ring, or even into a mere slit. Further, I have found that the path of the worms, although generally vertical for a short distance below the surface, does not generally continue to be vertical, but soon becomes sloping, and often almost or altogether horizontal; consequently any section of the sand is sure to cut the tubes at various angles, and therefore to exhibit them variously as ovals, or as circular rings, or as hollow lines. In order to test this, I have used an expedient which has been highly successful. I have had a trench cut deeply round a group of worm castings until the block of sand containing them has been isolated like the stump of a tree. This block has then been undercut horizontally and lifted out of the shore. Then carefully opening the orifices of the burrows, I have injected, with a syringe, liquid plaster-of-paris. The easy and complete passage of this injection proves how completely the path of the worm is kept open by the agglutination of

its walls; whilst, on cutting away the sand, that path becomes visible in a solid plaster-of-paris cast. The combined result of those experiments is to show how worm tubes would be deformed by movements in the bedding of the rocks, sometimes producing simply a change from circular to ovate forms, but in the case of more extensive movements, leading to flattened and broadened shapes and impressions, which must be more or less difficult of interpretation, because more or less widely divergent from original and undistorted tubing. Another most interesting result of this examination has been an ocular demonstration of the fact that the worm path is marked by a distinct development of iron oxide for some little distance on every side of the tube. The sand in this case is much mixed with clay, and is of a dark bluish-black colour. A ferruginous band is produced in this substance round the tube. This ferruginous band varies, as we might expect it to do, in width according to the quantity of iron in the sand, and according to the length of time during which contact with the animal matter has been maintained. Means have been found of hardening the sand so as to preserve these instructive specimens, and I now lay before the Society several of them, one being a specimen in which the worm is exposed lying dead in his burrow-tube, and the ferruginous stain is seen spreading to a very considerable distance on either side. How this chemical effect results from the proximity of the worm is an interesting question. I produce, also, some other specimens showing irregularities in the tubes, swellings and contractions, produced apparently by the worm turning in its burrow, and crossing the path of another worm, or its own, all tending to explain how there must be many obscure marks left in rocks in the original material of which such worms have burrowed. I produce specimens also, which exhibit beautifully the segregation of finer material along the path of the worm, precisely as we should expect from the functions which that animal fulfils in swallowing and in discharging the substances through which it works its way. Every anticipation which can be suggested by reasoning and by inference is found to be fulfilled in the observed results.

Standing then upon this series of facts connected with living annelids, we have solid grounds for defining a whole group of characteristics which must belong to the traces or remains of

annelid working in rocks. In the *first* place, the tubes where exposed in transverse section, in undisturbed material, will be approximately circular. In the *second* place, when they are cut across, not at right angles, but obliquely to the path of the worm, the section will be more or less ovate. In the *third* place, the same effect will be produced when the material has been disturbed by shearing or slipping of the beds or laminæ of deposit. Such movements, if not violent, will produce an elongation or pulling out of the cross section of the tubes, with resulting ovate forms. In the *fourth* place, where the movement of material has been great and effected under great pressure, the tubes will have been more or less completely crushed and flattened and distorted. In the *fifth* place, where the tube has been preserved at all, so as to show its original walls, those walls will very probably be traceable by a deposit or staining of oxide of iron, replacing the animal matter which originally agglutinated the sand. In the *sixth* place, this feature will necessarily be very variable according to the greater or smaller quantity of iron in the sand or clay. In the *seventh* place, where the tube has been so far preserved as to present a cross section of the substance filling up the interior of it, there will be a visible difference between that substance and the material outside the tubes, corresponding to the difference which has resulted from the selection by the worm of the finer from the coarser particles, and of the argillaceous particles from the purely silicious, because of their greater richness in digestible organic matter. In the *eighth* place, this selection of the finer clay particles by the worm, and the voidance of them in the tube behind it, would constitute a definite and probably a lasting difference in the mineral conditions of the whole space occupied by the path of the worm, which difference would be reproduced and perpetuated by the different behaviour of different mineral substances under the processes of metamorphism—whatever these may have been. In the *ninth* place, as in all the surrounding country the silicious strata are in general sharply distinguished from the argillaceous beds, and as these last have been generally metamorphosed into mica slates, with pure mica very highly developed in most of them, we should expect to find, as a necessity of the case, a corresponding development of mica and of micaceous particles in the inside of the worm tubes or along the lines of alteration and

deformation to which these may have been exposed by crushing or slipping movements in the rock. In the *tenth* place, in cases where the tubes have been exposed to the solvent action of sea-water, or, in other words, to weathering on a sea-beach, and where, consequently, all the special products due to the chemical and mechanical and vital action of the worms have been excavated and removed, we should expect to find the sides of the cavity thus left presenting a peculiar coarsely granular surface, from the old abstraction of the finer and the more argillaceous particles which the worm had swallowed and ejected in a comparatively loose and incoherent state. In the *eleventh* place, as our living annelids are various, and produce variously shaped burrows, we should expect to find some worm-paths less simple and less easily recognised than others, —such, for example, as irregularities, bunches or knots, bays, and branches in the path,—or perhaps such definite and characteristic varieties as club-shaped burrows, which are now actually produced by some living species.*

I have placed on the table to-day specimens from the Inveraray quartzite which fulfil and illustrate every one of these anticipations. The prevalence of ovate forms wherever we have cross sections of the old planes of bedding; the narrowing or elongation of these ovate forms in proportion as the section is oblique to the bedding; the reduction of these to mere linear streaks where the squeezing of the rocks seems to have been greatest; the altered character of the matter which occupies the interior of the ovate spaces, where metamorphism has not gone far—all these characters are beautifully marked. The conversion of the same material into shining mica, where metamorphism has been more advanced, is not less striking. In some of the specimens the brilliancy of the silvery mica, especially when seen in sunlight and with a lens, traces out the ancient worm paths with extraordinary distinctness. The oval rings of oxide of iron, which indicate the transverse sections of the tubing, is one of the most marked characteristics, of one at least, of the beds. The coarse granulations of the rock, in cavities out of which the worm-work has been excavated by the sea, is typically exhibited in the only specimen which has been found under exposure to the necessary conditions; whilst there is one specimen

* Mackintosh, *Mag. Nat. Hist.*, 1888, p. 12.

exhibiting the path of the tubes in longitudinal section, in which the annelid origin of the channel explains itself, as it seems to me, unmistakably to the eye. On showing it to an old fisherman at Inveraray, who knows the worms well as bait for his lines, and on asking him if he did not see the burrows there, his only reply was, "Oh aye, but that's in the middle o' the rock!" a difficulty which belongs to what the late Lord Derby called "the pre-scientific age."

There is yet one other specimen to which I must refer, because it was one of those which for a time secured me as most like a vegetable impression. It consists in perfectly straight parallel lines, marked in black upon the white ground of the rock. The lines consist simply of black or dark bronze stains on the grains of the quartzite. That they represent the staining of annelid tubes I have now no doubt. It may be the result of that copious and almost ubiquitous deposit of the black oxide of manganese which Dr Murray has discovered on almost all organic matter long exposed to the penetration of sea-water. Or it may be due to an effect which is noted by Professor Mackintosh in respect to some species of these annelids, that where the animal dies in its tube it leaves upon the walls a black stain like a "a black carbonaceous film."*

I attach no importance whatever to any interpretation of these marks in the quartzite as regards the evidence they may thus afford against a notion which seems to have been lately started, that our western mica-slate rocks are not of sedimentary origin at all, or that their apparent bedding is not truly such, but is the effect of what is called foliation. No geologist, I venture to think, who has lived among these rocks as I have done, and who has become familiar with the innumerable proofs of sedimentary origin which they exhibit, can entertain this new notion even for a moment. The absence of organic remains in the presence of such obvious metamorphism affords no presumption whatever against their sedimentary origin. I have been accustomed sometimes to look at the enormous masses of secondary and tertiary limestones which constitute the maritime Alps in southern France, and to look in vain among these seemingly unaltered beds for the traces of shells or of other organisms. These, indeed, are found; but they are

* Mackintosh, *Mag. Nat. Hist.*, p. 7.

often confined to some one bed, hardly more than a few inches thick —proving that the conditions under which organisms have been preserved in rocks are peculiar and are often absent. The same lesson may be learned from the Cambrian sandstones, underlying this quartzite in the Northern Highlands, in which we find abundant ripple-marks, but no trace of life. In this case, of course, we are not absolutely certain that life existed at all in the area, and at the time, of the deposit of the sandstones. But as regards the mica-slate series in Argyllshire, the indications of marine sedimentation are quite as decisive, and so far as this conclusion is concerned, the discovery of marine organic remains is by no means required. Neither, in my opinion, are they needed to confirm the geological horizon of Silurian age to which our metamorphic rocks in the South-Western Highlands have been long presumably assigned. As regards Argyllshire, at least, this presumption rests upon evidence far more direct than the conjecture of Murchison, that our rocks represent the southward prolongation of the beds which have yielded in Sutherland fossils of ascertained Lower Silurian type, and which are there found above the Cambrians on the west and below the Old Red Sandstones on the east. That conjecture was, I believe, a true one, and the general facts on which it rested remain unshaken. But the evidence for the Silurian horizon of the mica slates, schists, and quartzites of Argyllshire, is much more direct than the evidence in respect to the same rocks as developed farther north or in the central Grampians. All round our Argyllshire coasts we have patches and fragments left of the same Old Red conglomerates and sandstones, which are faulted against the south-eastern flanks of the Grampians, from the Clyde to Stonehaven. These patches and fragments of the Old Red are everywhere seen to lie, or to have lain, over and upon the upturned beds of the metamorphic series. In some places they have remained in considerable mass, and include thick beds of limestone as well as of freestone. This is the case on the coast of Kintyre, from the harbour of Campbelltown to Southend, near the Mull of Kintyre. The limestones of the Old Red in that district have been highly silicified, and so far as yet observed all organic remains have been obliterated. But these limestones with associated beds of freestone have their geological horizon determined in Kintyre by the direct evidence of dipping under the

small local coal-basin of the Laggan, near Campbelltown. This is the only coal-basin remaining in Scotland on the west coast, north of the Firth of Clyde. Richly fossiliferous beds of the Carboniferous limestone crop out from underneath it to the south; and under these beds again the Old Red Sandstones are seen to dip along the same southern boundary of the coal-field. The metamorphic series, therefore, in Kintyre are there exhibited immediately underlying the Old Red, which again is seen immediately underlying the Carboniferous series and the Coal Measures. There is no reason, so far as I know, to separate the metamorphic slates of Kintyre from those of the more northern areas of the county, of which the peninsula of Kintyre seems to be a mere prolongation. Instead, therefore, of seeking to identify the local quartzite of Inveraray as Silurian because of its annelid remains, I should be disposed to identify those remains as due to annelids, because of the clear stratigraphical evidence that these quartzites are Silurian. When we find that in the Northern Silurians these silicious beds have preserved their organic remains far better and far more generally than the calcareous beds, we have good reason to expect as probable the same result in the South-Western Silurian series. It is the obliteration, and not the preservation, of annelid markings, which needs explanation in our Argyllshire quartzites, such as those of Jura and Islay. It is, of course, quite easy to understand that such markings should be squeezed out under enormous pressures. But, on the other hand, it is only the fulfilment of a natural and reasonable expectation when we do find certain beds in the south that have preserved the traces of the particular form of life which seems to have been so abundant when the northern Silurian quartzites were deposited. The comparatively thin beds in which the annelid tubes are found at Inveraray are by no means the lowest in our series. They are underlaid and overlaid by a great number of beds both slaty and silicious. But the great mass is slaty, with a highly-developed micaceous character. The correlated development of mica in the quartzite bed itself, where the annelids have selected and segregated the argillaceous particles, seems to throw an important light on this product of metamorphism.

There is, however, another question connected with geological science which is of high importance, and on which the appearances

now brought under the notice of the Society have a direct and immediate bearing. That question relates to the movements effected in these strata, subsequent to their induration, by subterranean causes. More than thirty-five years ago, before any special controversy had arisen on the comparative effects of atmospheric denudation, and of subterranean movement on the structure of our mountains in the west of Scotland, I had been led by the facts presented in our intrusive rocks, especially by our porphyries, to conclude that these masses had risen along the planes of deposition among the sedimentary series, and that their intrusion among these was contemporaneous with a falling in of the sedimentary beds along certain lines of weakness, due to weight from under which support had been withdrawn. Such a movement of falling in, or tumbling down, along lines of consequent depression, would necessarily be accompanied by the slipping and sliding of the falling beds over each other along the of planes sedimentation. This conclusion I communicated in a paper to the Geological Society of London so long ago as 1853.* It is, therefore, no surprise to me to find that these bodies, whether they be worm-casts, as I now confidently believe, or whether they be purely mineral in their origin, prove conclusively that the rocks in which they are found have been sheared—or have slipped—over each other by movements due to subterranean causes. I am not myself able to conceive how mineral concretions, such as iron pyrites, or clay balls, or any other form of purely mineral aggregation, could have been so formed as, when pulled out and sheared, to present the uniformly ovate, or semi-ovate sections, together with the purely linear tube-like forms presented to us in these specimens. But if this be the explanation of them, the shearing must have been very great. On my own interpretation of an organic origin, some shearing, and squeezing, and elongation of the worm-casts is equally admitted. I see no evidence, indeed, that our hills about Inveraray have been ever exposed to such tremendous earth movements of horizontal displacement as those which have recently been revealed in Sutherland. But I have long been satisfied that the forms of our Highland mountains are due mainly to subterranean subsidences, with contemporaneous issue of plutonic and intrusive rocks; and the appearances in the quartzite now described and exhibited are, on any conceivable

* *Quart. Jour. Geol. Soc. of London*, November 1853.

theory of their origin, a convincing indication that such movements have extensively prevailed, and must have left a corresponding effect on the structure of our mountains. At the same time, it is undeniable that there has been enormous subsequent removal of material, and that in many cases the structure due to subterranean causes has been so smoothed over and obliterated to the eye that it is only in accidental sections they can be traced at all.

I have now only to add, that since this paper was read before the Society, a correspondent, Mr Peter Macnair, Perth, has sent to me some specimens of annelid tubes and castings, from a rock high up Craig-na-Challeich, one of the mountains above Killin, at the western end of Loch Tay, in Perthshire. These markings exhibit all the characteristic features before specified and described. They are on the weathered surface of a rock highly silicious and micaceous, and are more distinctly marked on that surface than any I have found in a similar position. A band of limestone lies below the quartzite near the base of the mountain, and these two beds are separated from each other by many other beds of conformable quartz-schists. These details I state from a very clear section kindly furnished to me by Mr Macnair. Now that we are all put upon the scent, I have no doubt whatever that the Inveraray annelids will be found to have had many countrymen and companions in the central as well as in the south-western Highlands.

On the Virial Equation for Molecular Forces, being Part IV. of a Paper on the Foundations of the Kinetic Theory of Gases. By Prof. Tait.

(Read January 21, 1889.)

(*Abstract.*)

In the preceding part of this paper I considered the consequences of a special assumption as to the nature of the molecular force between two particles, the particles themselves being still treated as hard spheres. My object was to obtain, by means of rigorous calculation, in as simple a form as possible, a general notion of the effects due to the molecular forces. My present object is to apply this general notion to the formation and interpretation of the Virial Equation.

In the extremely ingenious Thesis of Van der Waals, who first succeeded in representing by a simple formula the main results of the researches of Andrews on the Isothermals of Carbonic Acid, the Virial Equation was employed; but, as was remarked by Clerk-Maxwell (*Nature*, Oct. 15, 1874), "where he has borrowed results from Clausius and others, he has applied them in a manner which appears to me to be erroneous." The object of the present paper is to attempt a legitimate (though, of course, only roughly approximate) application of the Virial Equation to the same question. The result cannot differ *much* quantitatively (though it may be widely different qualitatively) from that of Van der Waals, if it also can represent Andrews' experimental data.

The Virial Equation is

$$\tfrac{1}{2}\Sigma(m\bar{v}^2) = \tfrac{3}{2}p\mathrm{V} + \tfrac{1}{2}\Sigma(\mathrm{R}r)\,.$$

From this Van der Waals derives the following expression :—

$$\left(p + \frac{a}{\mathrm{V}^2}\right)(\mathrm{V} - b) = \tfrac{1}{3}\Sigma(m\bar{v}^2)\,,$$

in which the right-hand member is treated as a constant multiple of the absolute temperature. Now it is certain that, when there is no molecular force except that of impact, the term

$$\tfrac{1}{2}\Sigma(\mathrm{R}r)$$

takes, to a first approximation at least, the value

$$-bp,$$

and the whole becomes

$$p(V-b)=\tfrac{1}{3}\Sigma(m\bar{v}^2).$$

The principles on which the Virial equation was itself based forbid us to introduce any change in the meaning of the factor p, which has already been strictly defined as *external* pressure. Van der Waals, however, retains this form of the Virial equation (derived without the assumption of molecular attraction), modifying it simply by the addition to p of a term a/V^2, which is the internal pressure (K of Laplace) due to the molecular forces. Undoubtedly the complete Virial equation, when molecular forces are taken into account, must contain a term somewhat of this nature; but it ought to be obtained (with its proper factor) *directly* as a part of the expression $\frac{1}{2}\Sigma(Rr)$. It seems to me that this must be the main point to which Clerk-Maxwell referred in the passage above quoted. It is curious that Clausius did not raise this objection to Van der Waals' equation, but contented himself with making modifications derived from general considerations, by which it was changed from

$$p=\frac{kt}{V-b}-\frac{a}{V^2}$$

to

$$p=\frac{kt}{V-\alpha}-\frac{c}{t(V+\beta)^2}.$$

The close agreement of Van der Waals' equation, the first of these, with Andrews' experimental results was exceedingly remarkable:—and Clausius' modified form seems to suit them almost exactly!

But Clerk-Maxwell says (*loc. cit.*), "though this agreement would be strong evidence in favour of the accuracy of an empirical formula devised to represent the experimental results, the equation of M. Van der Waals, professing as it does to be derived from the dynamical theory, must be subjected to a much more severe criticism."

Along with the objection just mentioned, there is another and serious one. Van der Waals' equation appears to be unwarrantably simple, as it contains (besides the k of the "perfect gas" formula $pV=kt$) only *two* disposable constants. A little thought shows that (even if the *law* of molecular force were the same for all bodies, which we have no right to assume) we should expect to find *three*

disposable constants at least. For we must provide (1) for the diameters of the particles, on which the "ultimate volume" depends; (2) for the range of molecular force; (3) for the maximum relative potential energy of two particles. It is quite clear that these three must be assumed to be entirely independent of one another; at least unless experiment should, some day, prove the existence of a relation among them.

The general results obtained in Part III. of this paper show that, when molecular attraction is taken into account, the whole kinetic energy must exceed that of particles free from molecular forces by a term proportional to the fraction of the whole particles which are, at any *and every* time, within molecular range of one another. (This raises another point on which I cannot agree with previous writers. It will be discussed later.) Hence, if $\bar{v}^2 = 3/2h$ when the volume is practically infinite, we have approximately

$$\bar{v}^2 = 3/2h + \frac{c}{V - a},$$

where c and a may be treated (within certain limits at least) as constants. The negative sign is given to a because the particles (in consequence of the presumed intensity of the molecular force) move *very* much faster when under mutual influence, and therefore spend relatively less time in that part of their space. It is not easy to see what, if any, change of form this expression must take when V becomes so small that no single particle is ever free from the action of the molecular forces of a number of others. For the present we may employ it as it stands even when the volume is of the order a. It is possible that there may be gases (hydrogen?) in which, on account of the comparative feebleness of the molecular force, a may be positive: the effect of the increased length of the relative path not being overcome by the increase of relative speed. But, in this abstract, I merely mention the question.

The term $\frac{1}{2}\Sigma(Rr)$ gives the negative part $-p\beta$ due to the impacts, where β also may be regarded (at first, at least) as constant, though it certainly increases with diminution of volume. This is, at least, the result obtained when the only molecular forces considered are due to the elastic resilience. It evidently should be considerably modified by the introduction of the molecular attraction; but we will not, for the present, insist on this point further than to say

that the main effect is merely to alter the value of the disposable quantity A, below. But, if $\psi(r)$ be the molecular force (attractive) at distance r, there is also a positive term proportional to

$$\int_s^\mu \frac{er^3 dr\psi(r)}{V-g} = A/(V-\gamma) \text{ suppose,}$$

where s is the diameter of a particle, μ the range of the molecular forces, e and g functions of r. The factor $e/(V-g)$ represents the density, taken as (on the time average) uniform throughout the shell $r, r+dr$ round any one particle. $er^2dr/(V-g)$ is thus proportional to the average mass distant r to $r+dr$ from any particle; and the remaining factor, r, is in the virial expression.

With these preliminaries it is obvious that we may now write the virial equation approximately as follows:—

$$E+\frac{C}{V-\alpha}=p(V-\beta)+\frac{A}{V-\gamma}$$

where E is proportional to the whole kinetic energy when the volume is indefinitely great—*i.e.*, when the number of particles at any time within molecular range of one another is an infinitely small fraction of the whole.

From the mode in which they are obtained, it is clear that the quasi-constants α and γ can be treated as equal *only* in the very special case in which the molecular force is supposed to act (impulsively) at *one* definite distance. If it increase rapidly as particles approach one another, γ must apparently be less than α, and may be negative. Both depend upon temperature. But, since β depends directly on the distance at impact, while γ and α depend indirectly on the range of the molecular force, it is not possible to rank them all in order of magnitude.

This equation is, at first sight, quite different in form from that of Van der Waals, even as modified by Clausius, and I think it undoubtedly represents more accurately than do these the true result of the application of the Virial. It introduces five constants, of which three at least are entirely independent, so that (when expressed as a cubic in V) it can necessarily (in an infinite number of ways) be made to have three equal roots, corresponding to the critical point.

But we have a difficulty of a new and serious kind which requires

special consideration. *What is now to be taken as the measure of temperature?* Van der Waals and Clausius alike take the whole kinetic energy as proportional to the absolute temperature. And Clerk-Maxwell says (*l.c.*) "The assumption that the kinetic energy is determined by the absolute temperature is true for perfect gases, and we have no evidence that any other law holds for gases, even near their liquefying point."

On this question I differ completely from these great authorities, and may err entirely. For it appears to me that *only* if E (with a *constant* added, when required, as shown below) is regarded as proportional to the absolute temperature, is the equation that of an *Isothermal.* But if the whole kinetic energy, $E+\frac{C}{V-a}$, is to be considered as proportional to the absolute temperature, the equation would seem to be that of a species of *Adiabatic.* Van der Waals and Clausius do not call attention to the fact that the whole kinetic energy necessarily varies with the volume (when there are intermolecular forces, and the mean-square speed of a *free* particle is regarded as constant), and therefore cannot be regarded as constant in the Virial Equation unless heat is given to (or taken from) the particles.

The answer would seem to be:—difference of temperature, as measured experimentally, depends upon the gross rate of heat transactions between two bodies in contact; so that, as change of *relative* speed of pairs of particles will not affect their heat-transactions with the walls of the containing vessel, or with a thermometer, the part E, alone, is entitled to be interpreted as proportional to the absolute temperature. This will, apparently, be the case so long as, at every instant, the majority of the particles are free from molecular force. But a different result may be expected when none (or a small minority only) are free from molecular force. And here I may mention, in passing, another connected complication. There must be molecular force between the particles and the walls of the vessel; so that p, *as defined in the Virial equation*, is necessarily greater than the value calculated from h, even when the volume is so great that the effect of the molecular force between the particles is insensible. [It is possible that this consideration may help to explain the serious differences between

the results of the ablest experimenters, such as Mendeléef and Amagat, regarding the relation between p and V in air, hydrogen, &c., at pressures considerably less than an atmosphere.]

Consider, for a moment, the case of a gas, in which the kinetic energy is not much greater than that due to the molecular forces alone, these being so intense as to aggregate its particles into a group, so that scarcely any of them ever escape from the thick of the encounter. Its external pressure would be practically *nil*, and its temperature (as measured by a thermometer) close to absolute zero, although the mean kinetic energy per particle may be very high. Such a group would no more communicate heat to a thermometer plunged in it than would water (in consequence of Laplace's K) squeeze a finger dipped into it. Next, consider the case of a liquid in contact with its saturated vapour, at a temperature so low that there is great difference of density between the states. On the hypothesis which underlies the whole of my work (viz., that the particles are hard spheres, with unit coefficient of restitution) *permanently* double or multiple particles cannot occur in the vapour. Here the average kinetic energy per particle, in the liquid, should apparently be much greater than in the vapour, *yet their temperature and their (external) pressure are the same.* On the other hand, the condensation of part of the vapour produces a rise of temperature. It seems to follow that E (defined as above) becomes less in the liquid than in the vapour state, if the temperature be maintained constant. In other words, no formation of liquid is possible isothermally unless heat be abstracted; not even if the walls of the containing vessel could be made to shrink in, bit by bit where no impact is impending, without doing work on the gas. And, conversely, a liquid cannot, without supply of heat, be dissipated into vapour even *in vacuo*. The effect on the above equation would be to make E + L, *and not* E, proportional to the absolute temperature, L being a quantity which becomes rapidly less as the temperature rises towards the critical point. The only noteworthy effects of this, on the graphical representation of the Isothermals, would be to shift them parallel to the pressure axis, by amounts which increase from the critical point downwards; and to (slightly) modify their form in the neighbourhood of the minimum ordinate of each. Their general appearance will be unchanged, while our hypothetical

measure of temperature accounts for the so-called latent heat of vapours. And it is clear that similar reasoning may be extended to the passage from the liquid to the solid state. A great number of very curious results follow as immediate consequences of this mode of regarding the temperature of a group of particles, but this is not the place for them.

If, with Van der Waals and Clausius, I had taken the whole kinetic energy as the measure of the absolute temperature, my equation would have been of the form

$$p(\mathrm{V}-\beta)=kt-\frac{\mathrm{A}}{\mathrm{V}-\gamma},$$

which does not seem fitted to represent Andrews' results.

For the reasons given above, I think we ought to write

$$p(\mathrm{V}-\beta)=kt-\frac{\mathrm{A}}{\mathrm{V}-\gamma}+\frac{\mathrm{C}}{\mathrm{V}-\alpha}.$$

This may be made to resemble closely Van der Waals' equation if we take $\beta=(\mathrm{A}\alpha-\mathrm{C}\gamma)/(\mathrm{A}-\mathrm{C})$; but I see no physical reason for this assumption.

It is clear, from the point of view which I have taken, that the equation as given cannot be more than a rough approximation to the truth. The calculation of the proper values of the constants, to adapt it with great accuracy to Andrews' results for carbonic acid, would involve considerable labour:—more perhaps than the results to be obtained are worth.

I have therefore, guided by the indications given above as to the relative magnitudes of α, β, γ, tentatively found values of the constants, which (while eminently simple for calculation) are on the face of them obviously provisional, but which sufficiently show how well-fitted the formula is to express the main facts. The resulting equation which I first got in this way is

$$p(\mathrm{V}-2)=1000\frac{273+\mathrm{T}}{273}-\frac{5000}{\mathrm{V}-4}+\frac{2000}{\mathrm{V}-5},$$

where p is in atmospheres, and T is temperature Centigrade. The carbonic acid has volume 1000 at 0° C. and 1 atm.

This formula gives three equal values of V (7·8), for a pressure of 91 atm., and temperature about 35°·5 C. These are only in rough

agreement with the experimental data for carbonic acid. The critical temperature is too high, the pressure and volume both too large.

The isothermals which it gives, for temperatures at various intervals above and below the critical point, are very similar in their general form to those originally given by Andrews, with the modification suggested by J. Thomson. In one respect, however, they show a considerable divergence of character:—viz., in the position of the point of inflexion in the vapour region; which appears to be somewhat too far outside the region of saturated vapour in presence of liquid.

If we write next

$$p(V-3)=1000\frac{273+T}{273}-\frac{15{,}984}{V-1}+\frac{9250}{V-2}$$

we find, at the critical point, 6·87, 74 atm., 30° C. This presents a pretty fair agreement with Andrews' data, and is not liable to the objection raised against the former tentative formula, probably because β has been taken greater than either α or γ. As the same may be said for very different sets of constants, such as

$$p(V-1)=1000\frac{273+T}{273}-\frac{62{,}390}{V+0{\cdot}6}+\frac{55{,}829}{V+0{\cdot}4}$$

it is clear that nothing definite can be asserted as to the true values of the constants until the labour of deducing them directly from the experimental data has been successfully undertaken. The result *might* give us some real information as to the range and intensity of the molecular force.

If we assume the critical values V_o, E_o, p_o, from experiment, the general equation of the Isothermals takes the form

$$p(V-\beta)=E-\frac{p_o}{\alpha-\gamma}\left\{\frac{(V_o-\gamma)^3}{V-\gamma}-\frac{(V^o-\alpha)^3}{V-\alpha}\right\}$$

in which the disposable constants are reduced to two. For there is a single relation among α, β, and γ, viz.,

$$3V^o=\alpha+\beta+\gamma+\frac{E_o}{p^o}.$$

This, like the values of A and C, is given by the condition that the three values of V are equal at the critical point.

Strophanthus hispidus: its Natural History, Chemistry, and Pharmacology. By Dr Thomas R. Fraser.

(*Abstract.*)

(Read February 4, 1889.)

A. *Natural History.*

In February 1870, the author made a communication to this Society on the Kombe Arrow-Poison of Africa, a product of the *Strophanthus hispidus* plant. In that communication the nature of its action on the various structures of the body, and the chemical composition of the seeds of the plant, which are the most active part, were described. It was pointed out that the action is chiefly exerted upon the heart and upon the muscles of the body, and that the seeds contain a crystalline active principle of the nature of a glucoside, to which the name Strophanthin was given.

From the examination then made of the action of the seeds of this *Strophanthus*, as well as of its active principle, strophanthin, it was anticipated that Strophanthus would prove to be of great value in the treatment of disease, and especially of disease of the heart; and a few years later the author employed it for this purpose in a small number of cases.

Various circumstances, such as the difficulty in procuring sufficient supplies of the seeds, prevented the author from making the number of observations that appeared to be necessary before the value of Strophanthus in the treatment of disease could be properly estimated; and it was not until 1885 that sufficient evidence had been obtained to authorise any public announcement on the subject.

In the interval of fifteen years which elapsed between the first communication to this Society and the communication of 1885 to the British Medical Association, the subject attracted so little attention that only two papers were published on it.

One of these papers dealt with the physiological action, and confirmed the statements made in the communication to this Society. The second paper dealt only with the chemical composition of the Strophanthus seeds, but the chief statements it contained, such as that the active principle is not a glucoside, have since been amply shown to be erroneous.

Subsequently, however, to the communication of 1885, upon the therapeutical applications of the substance, the literature of the subject has rapidly increased, and it now embraces about a hundred separate papers, the greater number of which deal with its uses in the treatment of disease.

Until 1885, also, Strophanthus, elsewhere than in Africa, was a mere curiosity, represented in a few museums in Europe by specimens of its flowers and fruit. Since that time, it has become a not inconsiderable article of commerce, several tons of the seeds having been exported from Africa by London merchants alone, in order to supply the requirements of medical practice.

In the present paper it is proposed to give an account of the observations that have been made by the author on the natural history, chemistry, and pharmacology (or physiological action) of Strophanthus, but to-night only the first of the above subdivisions of the subject would be dealt with.

In nearly every narrative of exploration in uncivilised tropical regions, accounts are given of poisonous substances, which in many instances are stated to possess remarkable properties. Usually these poisons are of vegetable origin, and nearly all of them may be included in the two great divisions of Ordeal and of Arrow poisons. Among the most interesting of the Ordeal poisons are the *Physostigma venenosum* and the Akazga or Akaja of West Africa, the Sassy or Muave of wide distribution over Africa, and the *Tanghinia venifera* of Madagascar; and of the Arrow poisons, the *Antiaris toxicaria* and *Strychnos Tieuté* of Java, the *Aconitum ferox* of China, and the famous Wourali or Carare poison of South America.

It is to the enterprise and observation of explorers and travellers that we are indebted for the first knowledge of the Strophanthus Kombe-poison. The first specimens of the plant that reached Edinburgh appear to have been a few ripe follicles sent to Sir Robert Christison early in 1869, by the Rev. Horace Waller, who had been a member of the Oxford and Cambridge Universities' Mission of 1861–62, superintended by the late Bishop Mackenzie, with whom had been associated, during the operations of the Mission in the country between the river Shire and the lake Shirwa, the famous traveller Livingstone and the enterprising botanist Kirk, at that time H.M. Consul at Zanzibar. Mr Waller informs me that,

at his suggestion, the follicles were brought to this country by Mr E. D. Young, R.N., when he went to Africa, in 1867, to clear up the story of Livingstone's murder.

Sir John Kirk had previously discovered that the Kombe poison is prepared from the seeds contained in these follicles. In a letter received from him (31st October 1888), he thus describes the discovery:—"I had long sought for it (the source of the Kombe poison), but the natives invariably gave me some false plant, until one day at Chibisa's village, on the river Shire, I saw the 'Kombe,' then new to me as an Eastern African plant (I had known an allied species at Sierra Leone (1858), where it is used as a poison). There, climbing on a tall tree, it was in pod, and I could get no one to go up and collect specimens. On mounting the tree myself to reach the Kombe pods, the natives, afraid that I might poison myself if I handled the plant roughly or got the juice in a cut or in my mouth, warned me to be careful, and admitted that this was the 'Kombe' or poison plant. In this way the poison was identified."

Livingstone, in his *Narrative of an Expedition to the Zambesi and its Tributaries* (1858–1864), states that the tribes inhabiting the Mikuru-Madse, a tributary of the Shire river, use this poison for arrows, with which they kill buffaloes and other game. "Poisoned arrows are made in two pieces. An iron barb is fastened to one end of a small wand of wood, ten inches or a foot in length, the other end of which, fined down to a long point, is nicely fitted, though not otherwise secured, in the hollow of the reed, which forms the arrow shaft. The wood immediately below the iron head is smeared with the poison. When an arrow is shot into the animal, the reed falls to the ground at once, or is very soon brushed off by the branches, but the iron barb and poisoned part of the wood remain in the wound. If made in one piece, the arrow would often be torn out, head and all, by the long shaft catching in the underwood, or striking against trees."

Mr John Buchanan thus describes the method followed in preparing the poison:—"A man breaks a follicle and puts the seeds with the wool attached into a pot. He then takes a small piece of bamboo, which has two thin splints inserted crosswise in the end, and he revolves this speedily by rubbing it between his hands. The seeds are thus put into motion, and fall to the bottom of the

pot, and the wool rises and comes out at the top, and is carried away by the least breath of wind. The seeds are then put into a small mortar and pounded into a paste, which is then ready for use. It is common to mix the milky juice of a Euphorbia with it to make it stick to the arrow."

This arrow poison has also been found at the western side of Africa, where it is known as the "Inée" or "Onage," and the poison has been traced to a *Strophanthus*, which is probably the species *hispidus*, although the flowers do not appear to have been yet obtained.

Only a few poisoned arrows have reached this country from Africa, owing, no doubt to some extent, to the difficulties of carriage, but certainly much more to the reluctance of the natives to place poisoned arrows in the possession of Europeans. The author had, however, been able to examine arrows of eight different forms obtained from various parts of Africa. Two of them were arrows known to be poisoned with Strophanthus. Of the others, either no knowledge of the poison existed, or it was believed to be derived from plants other than Strophanthus.

Microscopical, chemical, and physiological examination showed that the poison of six of the eight arrows consists principally, if not entirely, of a substance made with the seeds of *Strophanthus;* and it is an illustration of the extensive use of this poison that these arrows should have been obtained from districts so widely separated from each other as the river Gambir, the Tanganyika lake, and the Zambesi river.

Of the other two arrows, one, originally poisoned on the arrow-head, was found to be inert; and the other, obtained in the Wanika country, was found to be poisoned with a substance acting like Strophanthus, but not giving its chemical reactions, nor exhibiting, on microscopic examination, any structure that could be identified with the structures in the seeds of Strophanthus. It is probable that the poison of the last arrow has been derived from a wood or root.

Decandolle, in 1802, first described the genus *Strophanthus*, and gave it this name because of the twisted, thong-like prolongations of the lobes of the corolla (στροφος, a cord, and ανθος, a flower). About twenty species are at present known, eight of which are found in Africa, and the others in India China, Malacca, and Burmah.

Strophanthus Kombe is not included in this enumeration, as Professor Oliver, after an examination of further and more complete specimens of the flowers and leaves, now regards it as "a variety, a geographical race, of *Strophanthus hispidus*." The species *hispidus* has been found only in Africa, and is widely distributed over its tropical and subtropical regions.

Mr Buchanan has at various times sent specimens of the root, stem, branches, leaves, flowers, and fruit, and has thus supplied materials for a description of the different parts of the plant.

These parts were exhibited and described, and also a young growing plant, reared from seed by Mr Lindsay, of the Royal Botanic Garden of Edinburgh.

In reference to the fruit, it was pointed out that it consists of two follicles, united at the ventral surfaces in the young state, but gradually separated, as maturity advances, by a hinge-like movement at their bases, until each separated follicle projects from the fruit stalk, almost at right angles with it. When fully mature, the two follicles form together a nearly straight line, whose extremities are the apices of the follicles.

Each ripe follicle contains three separate structures—the placenta, the seeds, and a large quantity of hairs interposed between the seeds and the endocarp.

As the follicle matures, its ventral or placental surface enlarges by the inverted edges of the carpels, which project united together into the interior of the follicle in its immature condition, splitting up more and more, and so expanding this surface. At the same time, the dorsal surface of the follicle, consisting of the thick and strong pericarp, becomes less rounded, and the placenta, with its still attached seeds, is brought close to the expanded ventral surface. By and by, as maturity advances, and the funiculus of the seeds becomes weakened by drying, the seeds break off from the funiculus, and lie loose in the interior of the follicle.

The follicle ruptures at the expanded ventral surface, which is its weakest portion, and through this rupture the seeds are extruded. The actual extrusion of the seeds seems to be produced by the separation from each other of the hairs of the comose appendages, and especially of the hairs separating the seeds from the endocarp. These hairs, in the green and moist state of the follicles, are in close contact

with each other; but in the mature dry state they acquire elasticity, and tend to become separated from each other, and they thus press the seeds through the split ventral surface of the follicle.

The hairs separating the seeds from the endocarp seem to possess the additional function of preventing fracture of the long and brittle stalks of the comose appendages, by forming a soft and yielding bed for the seeds during their changes of position before they escape from the follicle; and they thus insure that the seeds shall be disseminated with the comose appendages attached to them.

Drawings and microscopic preparations were exhibited to illustrate the histology of the different parts of the plant.

A description of the results that had been obtained in the chemical and pharmacological examination of *Strophanthus* was deferred to a future meeting of the Society.

A New Type of Dimorphism found in certain Antipatharia.

By **George Brook**, *Lecturer on Comparative Embryology in the University of Edinburgh.*

(Read January 21, 1889.)

A more or less elaborate system of polymorphism is of frequent occurrence in certain groups of colonial Cœlenterata. For example, in many Hydroids certain individuals perform the nutritive functions for the colony, others are specialised for reproductive purposes, and so on. The variously modified individuals are connected together by a general cœnenchyma, which enables the nutriment elaborated by the gastrozooids to be utilised by other members of the colony. Perhaps some of the most interesting and complex cases of polymorphism are to be found amongst the *Siphonophora*.

Amongst the Anthozoa dimorphism frequently occurs in certain groups of *Alcyonaria*, but in these cases apparently the specialisation never results in the formation of reproductive zooids. The modified individuals (*Siphonozooids*) differ from those of typical structure in the absence of tentacles, the great development of the siphonoglyphe, and in other points. They are usually but not invariably sexless, and in certain cases are stated to develop into typical zooids. In the *Actiniaria* the animal is usually solitary;

colonial forms, however, do occur, but so far as is known none of them are dimorphic. The *Madreporaria* include both colonial and solitary forms. Our knowledge of the structure of the zooids is as yet confined to a few species, and so far as I am aware the only known case of dimorphism is that described by Fowler in *Madrepora Durvillei*. In this case the specialisation affects certain of the mesenteries in the modified individuals. In the normal zooids all the mesenteries are similar in structure. In the modified zooids six out of twelve mesenteries (alternate ones) become thickened, and contain a ciliated ectodermal canal running through their substance, and opening at both ends into the stomodæum. Apparently both types are nutritive and both are reproductive, but Fowler is of opinion that the normal zooid is more reproductive, and the modified zooid more nutritive. Thus in this, the only case of dimorphism previously described amongst the Zoantharia, there is no *complete* specialisation into nutritive and reproductive zooids, and it is of interest to note that in another species of Madrepora (*M. aspera*), examined by Fowler, a similar dimorphism does not exist.

A study of the "Challenger" collection of *Antipatharia* has shown that some of the genera are dimorphic, whilst others are not. Nearly all the species obtained at great depths are dimorphic, but others come under the same category which occur in shallower seas. The nature of the dimorphism and the manner in which it has probably been produced will be best understood by a study of the general morphology of a few typical forms.

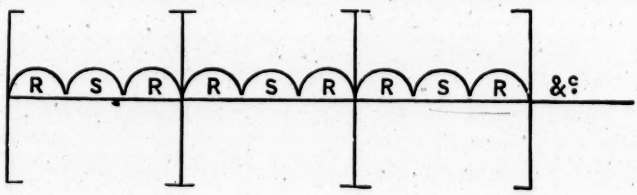
Most *Antipathidæ* have the horny axial skeleton more or less branched, and the zooids are usually situated in a single linear series on the branches, and are connected together by a cœnenchyma which contains an axial prolongation of their cœlentera, neighbouring zooids being thus brought into communication with one another. A vertical mesoglœal partition is usually present between adjoining zooids, but is never so complete as to prevent intercommunication.

In a typical zooid there are six tentacles and three pairs of well-developed mesenteries, and, in addition, there may be two or three other pairs, which are always more or less rudimentary. The stomodæum is elongated in the sagittal axis, and usually occupies a position at right angles to the axis of the branch on which the

zooid is placed. Only one pair of mesenteries ever bear reproductive organs; these occupy the transverse axis, and are situated one on each side of the stomodæum. The transverse pair of mesenteries, on account of their position, have a greater horizontal breadth than any of the others, and they are also usually somewhat longer than those at each end of the stomodæum ("directives"). In the species already examined there is a gradual tendency for the zooid to become elongated in the transverse axis, as a result of which the transverse mesenteries show a corresponding increase in breadth. In *Cirripathes* the outline of the zooid is more or less rounded, but the insertion of the sagittal tentacles into the body wall instead of into the peristome interferes with the regularity. The diameter of the zooid in the sagittal and transverse axes is, however, practically the same, and the tentacles have a radiate arrangement. In *Antipathella* a slight elongation of the zooid in the transverse axis has the effect of making the tentacles appear in two rows of three each, parallel with the axis of the branch. On account of the oval outline, there is in this genus a greater disproportion between the breadth of the transverse mesenteries, as compared with the "directives," than is the case in *Cirripathes*. The increase in the length of the transverse axis is, however, not great, and the tentacles do not become removed far apart. In *Parantipathes*, however, the transverse mesenteries become enormously elongated, so that the length in the transverse axis is three or four times that in the sagittal. The elongation has the effect of carrying the "lateral" tentacles further away from the stomodæum, so that they now appear clearly as three pairs some distance apart. The middle pair, as in other genera, are situated one at each end of the stomodæum, and the "directive" mesenteries are very narrow. In *Parantipathes larix* (Esp.) the peristome becomes somewhat depressed on each side of the oral prominence, so that the zooid presents indications of a division into three lobes, a central one containing the stomodæum and the proximal ends of all the mesenteries, and two lateral ones containing the greater part of the transverse mesenteries. It will be remembered that the reproductive elements are borne on the transverse mesenteries only, and in *Parantipathes* they are confined to those portions of them which are situated within the lateral lobes. It may be mentioned that in this genus the greatest diameter of the

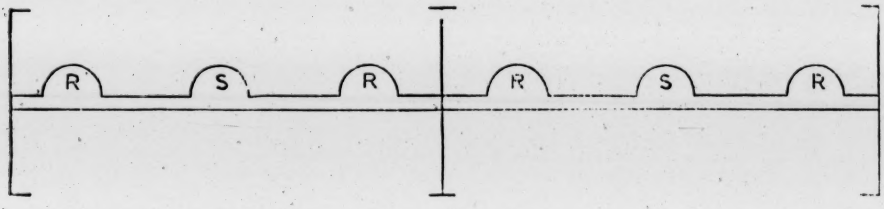
stomodæum frequently corresponds with the transverse instead of with the sagittal axis, as is also the case in *Amphianthidæ* amongst the Actiniaria.

In *Schizopathes* the three lobes of the zooid in *Parantipathes* become separated from each other by a further depression in the peristome, and also by the formation of a mesoglœal septum, which projects downwards for some distance from the base of the depression towards the skeletal sheath. By these means each lobe of the primitively simple zooid becomes separated from its neighbour in the same manner as the simple zooids of *Antipathella*, *Parantipathes*, &c., are separated from each other. The three lobes of the zooid in *Parantipathes*, having become separated from each other in *Schizopathes* by the formation of vertical mesoglœal partitions, may now be considered dimorphic forms. The middle one containing the stomodæum, which opens by the mouth at the apex of an elongated tubular projection, may be termed the *gastrozooid*, whilst the two lateral ones, containing the reproductive organs, may be distinguished as *gonozooids*. Each of the three dimorphic zooids bears a pair of tentacles, but the *gastrozooid* is the only one possessing a permanent opening to the exterior. In *Schizopathes* the dimorphic individuals are arranged in a single linear series along one aspect of a branch; all are in communication with one another through the bases of their cœlentera, and there is typically no isolation of the zooids into triplets, but all are pressed closely together. In specimens in which the reproductive elements are well developed, the gonozooids become much distended, and the sequence of the dimorphic forms along a branch is then readily recognised. Using the letter R to represent the gonozooid and S the gastrozooid, the arrangement may be indicated in the following manner, the derivatives of a primitive zooid being included within brackets:—



In *Bathypathes* the differentiation is carried a step further, on account of the fact that the individual zooids are separated from

each other by a considerable interval, but are still connected together by axial prolongations of their cœlentera. This arrangement may be indicated in the following manner:—



In *Bathypathes* the isolation of the dimorphic zooids, each bearing a pair of tentacles, might lead one to suggest quite a different interpretation if the intermediate steps in the differentiation were not known. It will, however, be evident from the points already indicated in outline, that the dimorphism in Antipathidæ is brought about—firstly, by an elongation of the zooid in the transverse axis; and, secondly, by the formation of two vertical constrictions and mesoglœal partitions by which the elongated zooid is divided into three portions, one nutritive and two reproductive. These in *Bathypathes* are frequently more isolated than are the unmodified zooids of normal types, and have as much claim to rank as individuals.

I have thus been led to divide the family Antipathidæ into two sub-families, of which the following short diagnoses will serve our immediate purpose:—

ANTIPATHIDÆ.

1. *Antipathinæ.*—Zooids of the normal hextentaculate type, showing a tendency to become elongated in the *transverse* axis, which corresponds to the axis of the supporting skeleton. Examples—*Leiopathes*, *Cirripathes*, *Antipathes*, *Parantipathes*, &c.
2. *Schizopathinæ.*—Zooids dimorphic and bitentaculate, of which three—viz., two gonozooids and one gastrozooid—are morphologically equivalent to one unmodified zooid of the normal type. Examples—*Schizopathes*, *Bathypathes*, &c.

In this connection the genus *Parantipathes* forms an interesting link between the two sub-families, and shows clearly the mode by which the dimorphism has been produced.

In conclusion, attention may be called to the twofold bearing of these observations. First, there is the interest attaching to the fact of the occurrence of a dimorphism resulting in the formation of specialised nutritive and sexual individuals amongst the Antipatharia, such a condition having been hitherto unknown, not only in that order, but in the Zoantharia generally. Secondly, the specialisation resulting in dimorphism takes a peculiar course, and is probably connected with the extension of a colony in the direction of its branches. The dimorphism of the *Schizopathinæ*, it is to be remembered, is brought about by the division of one primitive zooid into three, and not as in many other cases by a specialisation of different individuals. It thus differs essentially from the dimorphism of *Madrepora Durvillei* as well as from that of the modified individuals which perform similar functions amongst the Hydroids.

The Change in the Thermoelectric Properties of Wood's Fusible Metal at its Melting Point. By Albert Campbell, B.A.

(Read February 18, 1889.)

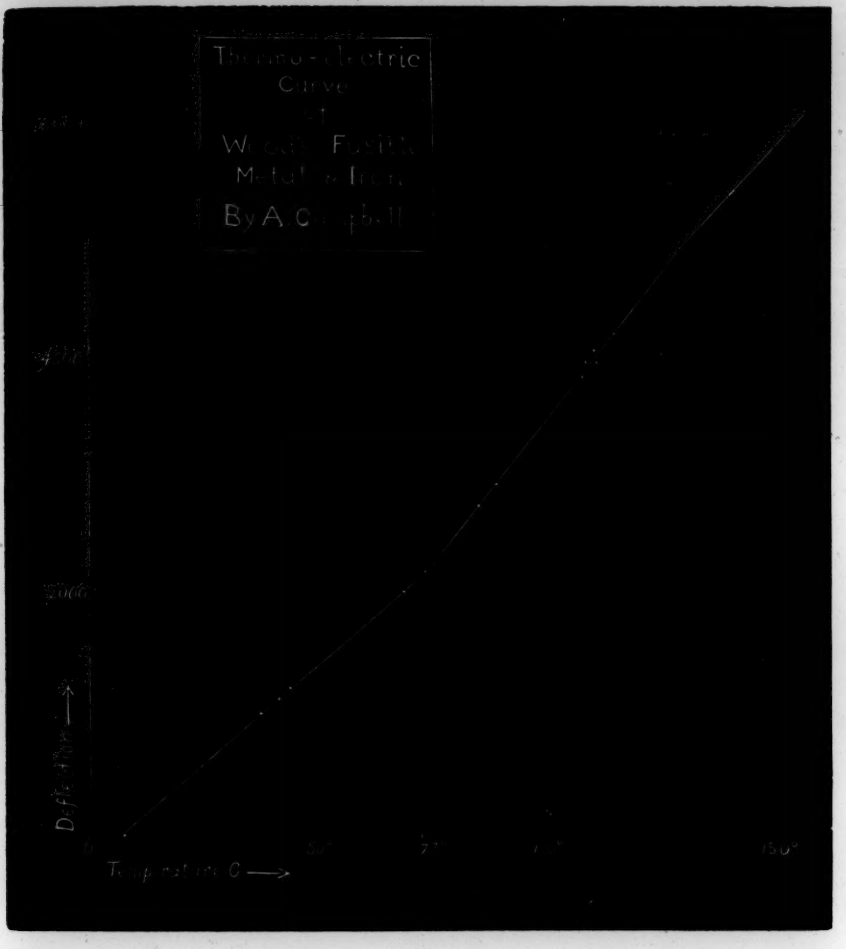
The thermoelectric properties of tin, at temperatures below and above its melting point, were investigated by the writer about a year ago, and the results of the experiments were given in a paper read before this Society.* In those experiments, however, the height of the melting point of tin made it impossible, with mercury thermometers, to obtain anything but an approximate indication of the change in the thermoelectric properties in passing the melting point. In the present experiments, therefore, Wood's fusible metal was chosen as having a conveniently low melting point. The composition of the alloy used was approximately the following:—

Bismuth, . . .	50 per cent.
Lead, ...	26 „
Tin, ...	13 „
Cadmium, . . .	11 „

The metals used were not pure. The melting point was found to be 73° C.

* January 16, 1888.

The chief difficulty with this alloy was its expansion after solidifying, which invariably shatters any glass vessel in which it is allowed to cool down. This difficulty was avoided by using, instead of a glass tube, a thin indiarubber one, which was filled with the alloy. One end of this tube was bent up at right angles, and into the fusible metal at that end of the tube a thin strip of iron (tin-plate) was inserted. This junction was placed, beside the bulb of a



thermometer, in asbestos wool contained in copper cylinders separated from each other by layers of asbestos, and heated by a small spirit-lamp underneath. The other ends of the iron strip and the fusible metal were joined to copper wires leading through a commutator to a sensitive mirror galvanometer with a scale at 12 feet distance, and the junctions, well varnished, were immersed in a large can of cold water. During the sets of observations (which lasted

many hours) the temperature of these cold junctions rose gradually from 7°·2 C. to 8°·1 C. The readings were all reduced to 7°·2 C. by a suitable correction.

For each reading the temperature of the hot junction was allowed to rise till almost perfectly steady, and then four deflections taken alternately to opposite sides of the scale. The mean of the four was taken as the reading. To keep the deflections well on the scale the sensitiveness of the galvanometer was altered when the deflections became large. Thus three sets of readings [(A), (B), and (C) in the Table] were obtained. These were pieced together by making *one* point in set (B) lie on the curve got from set (A), and then reducing all the other numbers in set (B) in the same proportion. Then set (C) was pieced on to (B) in a similar manner. The points marked * in the table were those by which the joining was effected. Of course, they are omitted in the diagram.

The table gives in the second column the temperature of the hot junction, and in the third column the observed deflections reduced to cold-junction-temperature 7°·2 C. It was found that up to at least 68°·5 the deflections (D) agreed very nearly with the formula

$$D = 33{\cdot}62(t - 7{\cdot}2),$$

t being the temperature of the hot junction; so that the curve is a straight line. From 74°·4 C. up to 150° C. the readings agree very fairly with the formula

$$D = -{\cdot}1348t^2 + 79{\cdot}22t - 2879,$$

i.e., a parabola whose vertex is at

$$t = 293°{\cdot}8.$$

The last two columns in the table give the values of D calculated from the two formulæ above. The curve in the diagram is from the second and third columns. At 8° C. the position of the line of this alloy in the thermoelectric diagram is between the iron and copper lines.

In conclusion, we see from the above formulæ that from 7° C. up to about 73° C. (the melting point), the line of this specimen of Wood's metal runs very nearly parallel to the iron line, and that at the melting point it takes a sudden bend away from the iron line, but almost immediately bends towards it again, keeping straight till

at least 150° C., and probably meeting the iron line at the neutral point 294° C.

Set.	t (Hot Temp.).	D Observed and Corrected.	D Calculated (1).	D Calculated (2).
A	37°·1 C.	1010	1006	
A	41°·6	1142	1157	
A	44°·2	1238	1244	
A	51°·1	1476	1476	
B	* 63°·2	(1882)	(1882)	
A	68°·5	2066	2061	
A	74°·4	2284	2259	2268
B	84°·0	2812	...	2825
A	86°·8	3009	...	2982
B	105°·8	3948	...	3993
B	105°·9	4029	...	3999
A	107°·4	4041	...	4075
A	108°·7	4155	...	4140
B	113°·2	4324	...	4362
C	*114°·0	(4400)	...	(4400)
C	127°·5	5018	...	5030
C	138°·4	5485	...	5499
C	151°·0	6025	...	6008

Note on the Relation between the Mutual Distances of Five Points in Space. By Thomas Muir, LL.D.

(Read March 4, 1889.)

Lagrange, in his paper "Solutions analytiques de quelques problèmes sur les pyramides triangulaires," *Nouv. Mém. de l'Acad. Roy.* . . . (de Berlin), Ann. 1773, pp. 149–176, gives unintentionally the following expression of the relationship between the mutual distances of five points in space, viz.—

$$\begin{aligned}4\Delta^2 f = {} & \alpha(a+f-g)^2+\alpha'(a'+f-g')^2+\alpha''(a''+f-g'')^2 \\ & +2\beta(a'+f-g')(a''+f-g'')+2\beta'(a+f-g)(a''+f-g'') \\ & +2\beta''(a+f-g)(a'+f-g'),\end{aligned}$$

where, if the points be called 1, 2, 3, 4, 5, the letters

$$\left.\begin{matrix} c'' & c' & a & g \\ & c & a' & g' \\ & & a'' & g'' \\ & & & f \end{matrix}\right\} \text{ represent the squares of the distances } \left\{\begin{matrix} 12 & 13, & 14, & 15 \\ & 23, & 24, & 25 \\ & & 34, & 35 \\ & & & 45, \end{matrix}\right.$$

and Δ, α, α', α'', β, β', β'', are certain functions of the said distances.

In 1841 Cayley published a paper "On a Theorem in the Geometry of Position," *Camb. Math. Jour.*, ii. pp. 267–271, in which he expressed the relation in question by equating to zero the determinant—

$$\begin{vmatrix} . & c'' & c' & a & g & 1 \\ c'' & . & c & a' & g' & 1 \\ c' & c & . & a'' & g'' & 1 \\ a & a' & a'' & . & f & 1 \\ g & g' & g'' & f & . & 1 \\ 1 & 1 & 1 & 1 & 1 & . \end{vmatrix}.$$

These two equations, Lagrange's and Cayley's, ought of course to agree; and Cayley, in a note just published in the *Messenger of Mathematics*, shows that not only is this the case, but that if the term $4\Delta^2 f$ in Lagrange's equation be taken to the other side, the expression put equal to zero in the one equation is really identical with the corresponding expression in the other. This conclusion is reached by examining only the coefficient of f^2, and showing that in both cases it is

$$= -(c^2 + c'^2 + c''^2 - 2c'c'' - 2c''c - 2cc').$$

A still more interesting question, it seemed to me, was the possibility of direct transformation of the one into the other. On trial I was surprised to find that this could be settled in a few lines, and that considerable interest attached to the transformation, because it brought to light a third expression different from either Lagrange's or Cayley's, and useful as a link not only between these two, but between either of them and a well-known fourth.

Starting with Cayley's determinant, and subtracting the 4th column from the 1st, 2nd, 3rd, 5th columns, and thereafter the 4th row from the 1st, 2nd, 3rd, 5th rows, we obtain the form

$$\begin{vmatrix} -2a & c''-a-a' & c-a-a'' & a & g-a-f & . \\ c''-a'-a & -2a' & c-a'-a'' & a' & g'-a'-f & . \\ c'-a''-a & c-a''-a' & -2a'' & a'' & g''-a''-f & . \\ a & a' & a'' & . & f & 1 \\ g-f-a & g'-f-a' & g''-f-a'' & f & -2f & . \\ . & . & . & 1 & . & . \end{vmatrix},$$

which, on account of the zero elements, manifestly reduces to

$$\begin{vmatrix} -2a & c''-a-a' & c'-a-a'' & g-a-f \\ c''-a'-a & -2a' & c-a'-a'' & g'-a'-f \\ c'-a''-a & c-a''-a' & -2a'' & g''-a''-f \\ g-f-a & g'-f-a' & g''-f-a'' & -2f \end{vmatrix}. \quad \text{(A).}$$

Now this, strange to say, is the whole matter; for if the form thus reached be expanded according to binary products of the last row and column, we obtain Lagrange's expression

$$-4\Delta^2 f + a(a+f-g)^2 + \ldots\ldots$$

The form (A) is seen to be axisymmetric like Cayley's, but the full regularity of its presentment is not apparent until we do away with the letters and denote the squared distances by 12^2, 13^2, ..., 45^2. Making these changes we have, as our expression of the relationship between the mutual distances of five points 1, 2, 3, 4, 5 in space, the equation

$$\begin{vmatrix} 15^2+15^2-11^2 & 25^2+15^2-21^2 & 35^2+15^2-31^2 & 45^2+15^2-41^2 \\ 15^2+25^2-12^2 & 25^2+25^2-22^2 & 35^2+25^2-32^2 & 45^2+25^2-42^2 \\ 15^2+35^2-13^2 & 25^2+35^2-23^2 & 35^2+35^2-33^2 & 45^2+35^2-43^2 \\ 15^2+45^2-14^2 & 25^2+45^2-24^2 & 35^2+45^2-34^2 & 45^2+45^2-44^2 \end{vmatrix} = 0,$$

from which the fourth identity above referred to is got by dividing in every case the sth row and s^{th} column by $s5$.

The Relation among Four Vectors. Note on Dr Muir's Paper. By Prof. Tait.

(Read March 4, 1889.)

A system of five points is completely determined by the vectors joining one of them with the other four. If α, β, γ be three of these, the fourth is necessarily $\delta = x\alpha + y\beta + z\gamma$. Hence any property characteristic of a group of five points will remain when x, y, z are eliminated. But we have

$$\begin{aligned}
S\alpha\delta &= xS\alpha\alpha + yS\alpha\beta + zS\alpha\gamma\,,\\
S\beta\delta &= xS\beta\alpha + yS\beta\beta + zS\beta\gamma\,,\\
S\gamma\delta &= xS\gamma\alpha + yS\gamma\beta + zS\gamma\gamma\,,\\
S\delta\delta &= xS\delta\alpha + yS\delta\beta + zS\delta\gamma\,.
\end{aligned}$$

Hence, at once, a determinant of the 4th order.

If we note that each term, as $S\beta\gamma$ for instance, can be written either as $\frac{1}{2}(\beta^2+\gamma^2-\overline{\beta-\gamma}^2)$ or as $-T\beta T\gamma\cos\hat{\beta\gamma}$, we see that the determinant may be written either in Dr Muir's form or as

$$0=\begin{vmatrix} 1 & \cos\hat{\alpha\beta} & \cos\hat{\alpha\gamma} & \cos\hat{\alpha\delta} \\ \cos\hat{\beta\alpha} & 1 & \cos\hat{\beta\gamma} & \cos\hat{\beta\delta} \\ \cos\hat{\gamma\alpha} & \cos\hat{\gamma\beta} & 1 & \cos\hat{\gamma\delta} \\ \cos\hat{\delta\alpha} & \cos\hat{\delta\beta} & \cos\hat{\delta\gamma} & 1 \end{vmatrix}$$

which is the relation among the sides and the diagonals of a spherical quadrilateral. The method above can, of course, be extended to any number of points. One additional point introduces *three* new scalars to be eliminated, and *six* new scalar equations for the purpose.

(*Addition*—Read March 18.)

If we operate, as above, with any other four vectors, we have

$$\begin{vmatrix} S\alpha_1\alpha & S\alpha_1\beta & S\alpha_1\gamma & S\alpha_1\delta \\ S\beta_1\alpha & S\beta_1\beta & S\beta_1\gamma & S\beta_1\delta \\ S\gamma_1\alpha & S\gamma_1\beta & S\gamma_1\gamma & S\gamma_1\delta \\ S\delta_1\alpha & S\delta_1\beta & S\delta_1\gamma & S\delta_1\delta \end{vmatrix} = 0\,,$$

and the tensors are again factors of rows or columns. Thus, if ABCD, *abcd*, be any two spherical quadrilaterals

$$\begin{vmatrix} \cos Aa & \cos Ab & \cos Ac & \cos Ad \\ \cos Ba & \cos Bb & \cos Bc & \cos Bd \\ \cos Ca & \cos Cb & \cos Cc & \cos Cd \\ \cos Da & \cos Db & \cos Dc & \cos Dd \end{vmatrix} = 0\,.$$

This has many curious particular forms; one, of course, being the former result, when the two quadrilaterals coincide. Another is

when the quadrilaterals are "polar." Let a be the pole of AB, b of BC, &c., then

$$\cos Ab \cos Bc \cos Cd \cos Da - \cos Ac \cos Bd \cos Ca \cos Db = 0.$$

And numerous other relations can be obtained, with equal ease, by the same simple process.

Cayley's form of the expression connecting the distances, two and two, among five points in space is an immediate consequence of the *identity*

$$\Sigma x(\alpha - \theta)^2 = \Sigma x\alpha^2 - 2S\theta\Sigma x\alpha + \theta^2\Sigma x\,,$$

where α_1, α_2, &c., are n given vectors, θ any vector whatever, and x_1, x_2, &c., n undetermined scalars.

For, *provided that n is greater than* 4, we may always assume

$$\Sigma x = 0\,, \quad \Sigma x\alpha = 0\,,$$

which are equivalent to four *homogeneous* linear relations among the xs.

Let, then, $n = 5$, and write the above identity separately for each α, put in place of θ. Thus we have

$$\begin{aligned}
\Sigma x(\alpha - \alpha_1)^2 &= \Sigma x\alpha^2,\\
\Sigma x(\alpha - \alpha_2)^2 &= \Sigma x\alpha^2,\\
\cdot\ \cdot\ \cdot\ \cdot &= \cdot\ \cdot\\
\Sigma x(\alpha - \alpha_5)^2 &= \Sigma x\alpha^2.
\end{aligned}$$

Take, with these, $\Sigma x = 0$,

and we obtain six linear equations from which to eliminate the five values of x. The result is, at once, A, B, C, D, E being the points,

$$\begin{vmatrix}
AA^2 & BA^2 & CA^2 & DA^2 & EA^2 & 1\\
AB^2 & BB^2 & CB^2 & DB^2 & EB^2 & 1\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
AE^2 & BE^2 & CE^2 & DE^2 & EE^2 & 1\\
1 & 1 & 1 & 1 & 1 & 0
\end{vmatrix} \quad \Sigma x\alpha^2 = 0\,.$$

As $\Sigma x\alpha^2$ may have *any* value, this is Cayley's expression. An interesting variation of it is supplied by taking $\Sigma(x\alpha) = 0$, instead of $\Sigma(x) = 0$, as the sixth equation.

The History and Theory of Heredity. By J. Arthur Thomson, Esq., M.A.

(Read January 21, 1889.)

The present paper has a three-fold object:—(1) to give a history of the theories of heredity which have been proposed by so many naturalists, with an appreciation of these in the light of recent advances; (2) to gather together the various contributions which have made the modern restatement possible; (3) to enter a protest against what the author believes to be the extreme position so firmly maintained by Weismann, to whom, however, much of the recent progress has been due. An historical review is necessary (in Britain at least) both as a contribution to the general history of the science, and as a basis for further construction; the accumulating mass of recent literature makes a collation desirable; the importance of Weismann's position in relation to the Theory of Evolution makes criticism imperative.

I. The Facts of Inheritance.

It is necessary, at the outset, to summarise what are usually regarded as the principal facts of inheritance. (1) The general likeness between parent and offspring is a commonplace of observation, condensed in the familiar saying, "Like begets like." (2) But besides the general resemblance, which expresses the relative constancy of the species, a particular similarity is often demonstrable. The offspring reproduces not only the general features, but often minute characteristics of the parent. (3) In very many instances the offspring exhibits, not parental, so much as grandfatherly characteristics, *e.g.*, in the familiar phenomena of atavism. (4) Without entering debatable ground, we must further note the fact of the frequent *recurrence* of the pathological characters of the parent in the constitution of the offspring. (5) But the fact in regard to the explanation of which most debate at present obtains, is that characters individually acquired by the parent as the results of environmental or of functional influence, certainly reappear, whether they be transmitted or not, in the offspring.

Denials of these Facts.—Now, while the resemblance between

offspring and parent, both in general and in particular, alike in normal and in pathological characteristics, cannot be denied as a fact, it has often been denied as the result of transmission. Such denials have varied greatly in degree and motive, and some of the most important may thus be classified.

1. Denial of *any* transmission, on philosophical grounds; *e.g.*, Wollaston.

2. Denial of the transmission of *individual* characteristics; *e.g.*, Bonnet.

3. Denial of the transmission of *psychical* individual characteristics; *e.g.*, Buckle.

4. Denial of the transmission of characters *individually acquired* by the "body" of the parent in course of function, or impressed upon the same by the environment; His and others, but notably Weismann.

Of those denials the only one of serious importance is that of Weismann, which will be separately considered later on. The truth which the others more or less clearly suggest is the influence of similar conditions of function and environment in evolving resemblances, without there being any real transmission of the same. This all naturalists will allow, while at the same time refusing to believe that similar conditions are sufficient to account for the greater part of the result. Of course, if we push back far enough and include in the similar conditions the specific constants in the history of the germ-cells, in the mechanics of development, and in the life of the embryo, the denial becomes more reasonable, though more truistic.

II. Problems of Heredity.

There are three general problems of heredity, which must be carefully distinguished, as has not always been done.

1. In what consists the unique character of the germ-cells?
2. Granted the unique character of the germ, what are the conditions of its reconstruction of the parent?
3. In what way is the reappearance of individual peculiarities, as opposed to that of the general features, to be interpreted?

In other words (1) it is necessary to understand in what way the germ-cell of plant or animal comes to be such a marvellous unit

that its multiplication or development will result in a new organism. (2) Understanding the uniqueness of the germ-cell as distinct from other units in the body, why should its division or development follow the old path? (3) Granting the general resemblance of offspring to parent, what is the truth in regard to the reappearance of peculiar characters? In what is the germ-cell peculiar, what are the rails which rule its course of development, what brings back idiosyncrasies?

III. Theories of the Uniqueness of the Germ-Cells.

1. *Mystical Hypotheses.*—The theories of heredity, like so many others, exhibit three phases—theological, metaphysical, and more or less scientific. We need not discuss the possession of the germs by spirits, nor yet the postulates of vires formativæ, nisus formativus, principle of heredity, Vererbungskraft or Bildungstrieb, but begin with the gradual emergence of the theories of heredity into fuller scientific daylight. It is necessary, however, to linger for a little over the so-called mystical hypotheses of the uniqueness of the germ-cells. They only require to be stated. According to Haller, Bonnet, and others, the seed, egg, or male element contained an excessively minute micro-organism, a complete though miniature model of the adult. This is stimulated out of potential life in fertilisation, and with the absorption of nutriment in its interstices, unfolds or expands into the adult organism. The animalculists found this miniature model in the male element, which was nourished by the ovum, while the ovists held that the model lay in nuce within the egg, and was, so to speak, awakened by the sperm. This hypothesis was further backed up by that of "emboitement," according to which the germ was not only itself a marvellous micro-organism, but contained in ever smaller proportions, after the manner of an infinite juggler's box, the miniature models of the generations to follow. The germ was thus, as they said, like a bud to be unfolded with every part ready made or preformed, and all in perfect transparency. "Es gibt kein Werden," said Haller, and his preformation theory certainly disposed at once of development and of the problems of heredity. But, at the same time, the germ was far more than a bud—it had imprisoned within it the buds of all its descendants.

It is of course evident, to be quite fair, that in their general proposition that the germ was a potential organism, the preformationists were quite correct. Their theory was metaphysical, but it was at least an advance on the more archaic theological theory of possession by spirits. The objection of to-day is simply that a model, in the strict sense of the term, does not exist in the germ. The early researches of Wolff alone were quite sufficient to show that neither the hypothesis of preformation nor its consequent hypothesis of "emboitement" had any basis of fact. From this point, therefore, a new start was made.

2. *Special Pangenetic Theories.*—Passing from the mystical hypotheses, we come to a whole series of theories, which are in varying degrees scientific, and may be fairly enough described by the general designation *pangenetic.* They all have this in common that they seek to explain the uniqueness of the germ-cell by regarding it as a centre of contributions from different parts of the organism.

Early Forms.—I shall briefly pass over the earlier and vaguer forms of this supposition. At such different epochs as are suggested by the names of Democritus and Hippocrates, Paracelsus and Maupertius, incipient theories of pangenesis—prophecies of Darwin's—were suggested. Thus Democritus maintained that the "seed" of animals was elaborated by contributions from all parts of the body, and that the constituent parts reproduced their several origins. Two millennia later Buffon, of whose speculation Darwin appears at first to have been unaware, again regards the germs as mingled extracts from all parts of the body, or as collections of samples from the various organs. If such were indeed the case, Buffon and his predecessors saw no further difficulty, for each contributed sample produced in the embryo a structure like its parental origin.

Spencer's Theory.—In 1864, in his *Principles of Biology*, Herbert Spencer suggested the existence of "physiological units," derived from and capable of development into cells,' and supposed their accumulation in the reproductive elements, which thus become, in some conceivable sense, micro-organisms.

Darwin's Theory of Pangenesis.—The best known theory of this class is, of course, the "provisional hypothesis of pangenesis" sug-

gested by Darwin in his *Variation of Animals and Plants under Domestication.* The chief suggestions of this theory are well known to be as follows:—

(1) Every cell of the body, not too highly differentiated, throws off characteristic gemmules;
(2) These multiply by fission, retaining their characteristics;
(3) They become specially concentrated in the reproductive elements;
(4) In development the gemmules unite with others like themselves, and grow into cells like those from which they were originally given off.

The applications of this, in one sense very satisfactory theory, to the phenomena of atavism, and reappearance of similar characters at similar times, do not concern us in this general survey.

Jäger's Theory.—The next theory is somewhat difficult to summarise, partly because of its technical character, partly because the author does not appear to be quite consistent in his statement of it at different times. The main points, under the present section, are as follows:—

(1) Each organ and tissue contains, along with the molecules of its albumen, a specific "scent-stuff" (Duft-und-Würzestoff).
(2) In hunger and similar experience the albumen liberates the "scent-stuff," which penetrates through the body as fatty acids, ethers, &c.
(3) These are specially attracted to the reproductive cells, which, when mature, are thus specialised by the reception of scent-stuff, and have in their protoplasm vires formativæ enough to reproduce a new organism like the parent.

It will be seen later on that this hypothesis of chemical pangenesis is not the most important contribution made by the author to the theory of heredity.

Galton's modified Theory of Pangenesis.—From experiments on the transfusion of blood, Galton was led to conclude that "the doctrine of pangenesis, pure and simple, is incorrect." But he did more than urge serious objections against Darwin's theory; he formulated one of his own, to which, with the exception of Professor Herdman, subsequent investigators do not appear to me to have

attached sufficient importance. The very important part of Galton's theory will be discussed in its proper place; it is not included in the series of pangenetic hypotheses. Galton is, in fact, one of the numerous biologists who have suggested the continuity of the germinal protoplasm. He is included at this stage, however, because he admitted as a subsidiary hypothesis a limited amount of pangenesis. To account for those cases which suggest that characters acquired by the individual parent are "faintly heritable," Galton supposed that "each cell may throw off a few germs that find their way into the circulation, and have thereby a chance of occasionally finding their way to the sexual elements, and of becoming naturalised among them." This part of his theory is obviously a cautious admission of limited pangenesis to account for a limited number of doubtful cases.

Brooks' Theory.—In 1883, in his valuable work entitled *The Law of Heredity*, Professor W. K. Brooks gave full expression to a modification of Darwin's view of pangenesis. The main positions, which are here relevant, may be summarised as follows, almost in the author's words:—

(1) The male and female cells are specialised in different directions; their union gives variability.

(2) The ovum is a cell which has gradually acquired a complicated organisation, and which contains material particles of some kind to correspond to each of the hereditary characteristics of the species.

(3) The ovum reproducing its like, as other cells, gives rise not only to the divergent cells of the organism, but also to cells like itself.

(4) Each cell of the body has the power to throw off minute germs. When, through a change in its environment, its functions are disturbed, and its conditions of life become unfavourable, it throws off small particles which are the germs or gemmules of this particular cell.

(5) These germs may be carried to all parts of the body. They may penetrate to an ovarian ovum or to a bud, but the male cell has gradually acquired, as its especial and distinctive function, a peculiar power to gather and store up germs.

(6) In fertilisation each gemmule unites with that particle of the

ovum which is destined to give rise in the offspring to the cell which corresponds to the one which produced the gemmule, or else it unites with a closely-related particle, destined to give rise to a closely-related cell. Such a cell will be a hybrid, tending to vary.

(7) As the ovarian ova of the offspring share by direct inheritance all the properties of the fertilised ovum, the organisms to which they give rise will tend to vary in the same way.

(8) A cell which has thus varied will continue to throw off gemmules, and thus to transmit variability to the corresponding part in the bodies of successive generations of descendants until a favourable variation is seized upon by natural selection.

(9) As the ovum which produced this selected organism will transmit the same variation to its ovarian ova by direct inheritance, the characteristic will be established as specific, and transmitted henceforth without gemmules.

The above theory, being important, has been stated at some length. Apart from the suggestion of variation as due to sexual intermingling, with which Weismann has made us more familiar; apart, too, from the suggestion of germinal continuity, the credit of which Brooks shares, there are several important points to be emphasised in the modification proposed. It is in *unwonted and abnormal* conditions that the cells of the body throw off gemmules. The *male* elements are the special centres of their accumulation; the female it is that keeps up the *general* resemblance between offspring and parent.

It is not proposed to enter into criticism of pangenesis theories. The best criticism is found in that abandonment of special hypotheses which recent advances have rendered possible. It has often been urged that the hypothesis of pangenesis involves not one but many suppositions—that it is just as difficult to understand why a gemmule should reproduce a cell like its own origin as to understand the entire problem, and so on. Detailed criticism will be found in the works of Galton, Ribot, Brooks, Herdman, Plarre, and others. It is enough for us to emphasise the comparative gratuitousness of any special theory whatever, a paradox which is explained in the succeeding section.

IV. Abandonment of Special Hypothesis—The Doctrine of Germinal Continuity.

As far back as 1849, Owen pointed out in his paper on Parthenogenesis that in the developing germ it was possible to distinguish between cells which became much changed to form the body, and cells which remained little changed and formed the reproductive organs. This was probably the earliest distinct suggestion of the modern theory of germinal continuity.

In 1866, in his classic *Generelle Morphologie*, Haeckel emphasised the simple and yet fundamental fact of the material continuity of offspring and parent. In a historical note upon the distinction between the "personal" and "germinal" parts of an organism, Rauber states that the distinction was proposed by Haeckel in 1874, and by himself in 1879.

Jäger stated the doctrine of germinal continuity very clearly and concisely at an early date:—"Through a great series of generations the germinal protoplasm retains its specific properties, dividing in every reproduction into an ontogenetic portion, out of which the individual is built up, and a phylogenetic portion which is reserved to form the reproductive material of the mature offspring. This reservation of the phylogenetic material I described as *the continuity of the germ protoplasm.*" . . . . . "Encapsuled in the ontogenetic material, the phylogenetic protoplasm is sheltered from external influences, and retains its specific and embryonic characters."

Brooks notes that, in papers published in 1876 and 1877, he had also suggested the notion of germinal continuity, and the conception is clearly expressed in his work already quoted:—"The ovum gives rise to the divergent cells of the organism, but also to cells like itself. The ovarian ova of the offspring are these latter cells, or their direct unmodified descendants. The ovarian ova of the offspring share by direct inheritance all the properties of the fertilised ovum."

The important theory of Galton now requires notice. Two preliminary notes are requisite. Galton is extremely doubtful in regard to the genuine *transmission* of acquired characters. It is to account for the possible faint inheritance of some of these that he still admits, as a subsidiary hypothesis, a limited amount of pangenesis.

In the second place, it is needful to notice at the outset Galton's term "stirp," which he uses to express the sum total of the germs, gemmules, or organic units of some kind, which are to be found in the newly-fertilised ovum.

(1) Only some of the germs within the stirp attain development in the cells of the "body." It is the dominant germs which so develop.

(2) The residual germs and their progeny form the sexual elements or buds. The part of the stirp developed into the "body" is almost sterile. The continuity is kept up by the undeveloped residual portion.

(3) The direct descent is not between body and body, but between stirp and stirp. "The stirp of the child may be considered to have descended directly from a part of the stirps of each of its parents, but then the personal structure of the child is no more than an imperfect representation of his own stirp, and the personal structure of each of the parents is no more than an imperfect representation of each of their own stirps."

Here it will be seen that there is a very definite expression of the notion that the germinal cells of the offspring are in very direct continuity with those of the parents. The antithesis between the "soma" and the chain of sex-cells is emphasised.

The history must also include Nussbaum, who called emphatic attention to the very early differentiation and isolation of the sex-elements to be observed in some cases. The theory both of Jäger and of Nussbaum is that of a continuity of germinal *cells*. The theory of Weismann is more strictly that of the continuity of germinal *protoplasm*. The position of Jäger and Nussbaum may first be summarised more definitely:—

(1) At an early stage in the embryo, the future reproductive cells of the organism are distinguishable from those which are forming the body.

(2) The latter develop in manifold variety, and lose almost all likeness to the mother germ.

(3) The former—the reproductive rudiments—are not implicated in the differentiation of the "soma," remain virtually unchanged, continue the protoplasmic tradition unaltered.

(4) The sex-cells of the offspring being thus continuous with the parental sex-cells which give rise to itself, they will in turn develop into similar products.

Now this fact of continuity of reproductive elements is obviously most satisfactory. If a fertilised egg-cell has certain characters, x, y, z, it develops into an organism in which these characters x, y, z are expressed; but, at the same time, the future reproductive cells are early set apart, retaining the characters x, y, z in all their entirety, to start a new organism again with the same capital. Balbiani, who was not influenced by theoretical considerations, observed in Chironomus that the future reproductive cells were isolated before even the blastoderm was completed; that is to say, before any differentiation almost had occurred, a portion of the unadulterated ovum was insulated to continue the constancy of the species.

In this aspect the reproductive cells form a continuous chain, and the reproduction of like is as natural and necessary as it was in the Protozoa. No special theory is required. Similar conditions produce similar results. Unfortunately, however, a serious difficulty besets this easy theory. Such an early appearance and insulation of the reproductive cells, continuous with the very ovum itself, does indeed occur, and where it does the problem of heredity is simple. Early origin of special germ-cells, distinguished from those of the general "body," has been observed in some "worm-types" (leeches, Sagitta, threadworms, many Polyzoa) and in some Arthropods (Moina among crustaceans, not a few insects, Phalangidæ among spiders), while indications of the same early separation are not wanting in a number of other organisms. But it must be distinctly allowed that in most cases it is only after differentiation is relatively advanced that the future reproductive cells make their appearance. Thus we have to pass from the few cases as yet known of the continuity of the germinal *cells*, to the more general, but less luminous, fact of the "continuity of the germ-plasma."

Weismann's Theory.—Weismann, like the previous investigators, had reached his conclusion independently. In the fact of continuity between the reproductive elements of generations, the solution of likeness must be found. But a direct chain of cellular continuity can only be said to exist in a few cases. The solution which is proposed for the majority of cases is as follows:—

(1) "In each development a portion of the specific germinal plasma ('*keimplasma*'), which the parental ovum contains, is not used up in the formation of the offspring, but is reserved unchanged for the formation of the germinal cells of the following generation."

(2) What is actually continuous is the germinal protoplasm—the "keimplasma"—"of definite chemical and special molecular constitution." A continuity of germinal cells is now rare; a continuity of intact germinal plasma is constant.

(3) This keimplasma has its seat in the nucleus, is extremely complex in structure, but has nevertheless an extreme power of persistence (von ungemein grossem Beharrungsvermögen), and enormous powers of growth.

V. Elaborations of Doctrine of Continuity.

It may now be concluded that in the more or less strict continuity of the successive sets of reproductive elements lies the solution of the main problem of heredity. This appears the most convenient place to notice the various suggestions made as to what it is exactly that is continuous. The earlier of these suggestions were brought forward indeed before the notion of continuity had its present definite form, but I have deemed it better to introduce them here than to mix them up with the pangenetic series.

In the simplest animals, organism A buds and hands on a fraction of its living matter to A′, surely A′ being so really continuous with A, must grow into a form like its origin. But while emphasising that the explanation of the similarity lies in the continuity, we may probe further into the continuity itself, and express it in chemical, physical, or even psychical terms.

So with higher organisms. A germ x develops into a body, and the reproductive cells thereof. The latter arise in such a way that they are virtually continuous with the undifferentiated original germ x. They retain its constitution intact, become the starting points of new organisms, which from similar origins are naturally similar in result. The whole emphasis is laid on the notion of continuity, but it is necessary to consider the attempted analyses made at different levels.

The Memory Theories.—Hering in Prag and Samuel Butler in

England suggested about the same time a psychical aspect of the hereditary continuity. The two suggestions may be so far summed up together. Memory is a general function of organised matter and the reproduction of parental likeness is the result of unconscious recollection of the past. What are ordinarily called memory, habit, instinct, and embryonic reconstruction are all referable to the memory of living matter. Hering finds the basis of this unconscious memory in the persistence of the undulatory movements supposed to be characteristic of the molecules. These undulations are sensitive to change, and room is thus left for variability, but their tendency to persist in their established harmony is the basis of heredity.

"*The Perigenesis of the Plastidules.*"—Haeckel also emphasised the luminous metaphor of "organic memory," and sought to analyse it in terms of molecular motion. His theory is summed up in the characteristic phrase "perigenesis of the plastidules." Comparing the course of historic development to a complex, ramified series of wave lines, in which a single life is represented by a single wave, he imagines a similar ontogenetic wave-motion in the development of the individual. The invisible activity of the organic molecules is, he believes, a branched wave-motion, continuous with that of the history—such is "the perigenesis of the plastidules." "The developing impulse which in the one case is transferred from the ancestral species to the whole group of species, and in the other case from the ancestral cell to the entire group of cells, assumes in both cases the same form of a branching wave-motion." "The true and ultimate *causa efficiens* of the biogenetic process, I propose to designate by a single word, Perigenesis—the periodic wave-generation of the organic molecules or plastidules." The tendency that this periodic motion has to persist, preserving as it were a characteristic rhythm, explains the relative constancy of ordinary inheritance, while at the same time the results of new experience may be added on to the dominant molecular movement. In very simple organisms, as he says, the plastidules have, so to speak, learned little and forgotten nothing, while in highly perfected types the plastidules have both learned and forgotten much. Haeckel thus emphasises on the one hand the psychical, on the other the physical or molecular aspect of the real continuity.

Jäger regards the continuity in cellular terms. It is a protoplasmic continuity effected after the ordinary fashion of cell-division. To this there has to be added his chemical conception of pangenesis, which when expressed in more modern phraseology is reasonable enough—being simply the supposition that characteristic anastates and katastates find their way to the reproductive elements, and make these to some limited extent still sharers in the general life of the organism.

Galton does not make the continuity much more precise than is implied in the general statement that a residue of the germs, gemmules, or organic units in the "stirp," remaining latent in the construction of the body, are passed on into the reproductive elements, and keep up a continuity between "stirp" and "stirp." In regard to the future history of the gemmules, Galton supposes that they form groups in the ovum, and become directly associated with its division, while at later stages they wander and give rise to new cells. To obviate histological difficulties, Herdman proposes the following reasonable amendment, "that the body of the new individual is formed, not by the development of gemmules alone and independently into cells, but by the gemmules in the cells causing, by their affinities and repulsions, these cells so to divide and redivide as to give rise to new cells, tissues, and organs." All this admits of more direct expression in terms of "chemical pangenesis." Brooks and Nussbaum rest satisfied in maintaining a cellular continuity.

What keeps up the continuity, according to Weismann, is the *keimplasma*, *i.e.*, a special portion of the nuclei of the reproductive cells, which with great morphological stability keeps itself intact, and is sooner or later once more established in the reproductive cells of the growing organism. Nägeli finds sufficient explanation of the constancy of inheritance in the individuality and persistence (Beharrungsgesetz) of what he calls the "idioplasma."

Kölliker, O. Hertwig, Strasburger, and Bambeke may be noted for the emphasis which they have laid upon the nuclei as transmitting or rather continuing the essential characteristics from generation to generation. Thanks to the researches of such investigators as Van Beneden and Boveri, it is now certain that the male and female nuclei contribute an equal share in forming the segmentation nucleus of the ovum. Nay more, each of the first two daughter-cells has in

its nucleus half of the male and half of the female nuclear elements, and it is possible that this marvellously exact dualism persists throughout.

Most hopefully, perhaps, has the continuity been expressed by several, *e.g.*, Berthold, Gautier, and Geddes, in chemical terms. In the paper by the last mentioned on "Growth, Sex, Reproduction, and Heredity," the following weighty sentence occurs:—"If the reproductive elements start with a specific protoplasm continuous with that of the combined mother ovum and fertilising sperm—that is, with a concentrated accumulation of characteristic anastates and katastates —the simple fact that the products of protoplasmic change must be fixed, definite, and continuous, as in all chemical processes, gives us at once a protoplasmic basis from which to explain the constant and necessary symmetry of segmentation and development." The views of Berthold are closely similar. Inheritance is possible only on the basis of the fundamental fact that in the chemical processes of the organism "the same substances and mixtures of substances are reproduced in quantity and quality with regular periodicity." Gautier discusses both variation and heredity from a chemical point of view. "The force which maintains the species, and gives it the character of constancy and resistance, is nothing more than the resultant of the forces which maintain the *chemical species* of which the organism is composed."

VI. The Problem of Reconstruction.

The doctrine of the continuity of the reproductive protoplasm obviously answers not only the first problem of the uniqueness of the germ-cell, but it casts a new light upon the problem of reconstruction. The problem is simplified, and, to a certain extent, disappears. Why should the germ-cell divide, redivide, and build up an embryo in the precise way in which it does? Because it is virtually continuous with the parent germ, which behaved in a precisely similar fashion. Thus the question ceases to be particular, and becomes general—ceases, in fact, to be a problem in heredity, and becomes a subject for investigation under the mechanics of development.

This, it need hardly be said, is to refer to a field of investigation which has been but little worked. In spite of the luminous sugges-

tions of His, Rauber, and others, there are few general facts on which one can find foothold for further construction. Yet the task has been more than begun in the experimental investigations of O. Hertwig, Fol, Pflüger, Born, Roux, Schultze, Gerlach, and others. Observations as to the actual dynamics of cell division—such, for instance, as those of Van Beneden and Boveri,—are beginning to appear; while the title of Berthold's book on "Protoplasmic Mechanics" shows how the biologist begins to seek the aid of the student of physics in explaining the architecture of the living organism. But we are at present only concerned in emphasising that the conception of development as what Pflüger called "organic crystallisation" must become dominant. The laws of growth which express the mode in which each fertilised egg-cell must undergo segmentation, gastrulation, and the like have to be expressed in terms of the internal and external physico-chemical conditions.

"To think that heredity will build organic beings without mechanical means is a piece of unscientific mysticism," as Professor His noted in his valuable paper recently submitted to this Society, and yet the tendency does not rapidly disappear from even scientific literature. It is impossible, for instance, to exaggerate the importance of the general conclusion variously expressed by Von Baer, Spencer, and Haeckel, that the life-history of the individual is a recapitulation of the evolution of the race. To say that ontogeny recapitulates phylogeny, or that "the microcosm of the ontogenetic tree is a reflection of the macrocosm of the genealogical tree," is to express a marvellous generalisation. But what we now wish to understand is, as Hallez expresses it, how the protoplasm is at each stage the architect as well as the material of its own development. The metaphors suggest that the developing organism has somehow a feeling for history, or that the dead hand of the past is literally upon the present, while our aim must be to get beyond any mere phrase of organic memory, and to understand the chemical and physical conditions which, more or less modified in the course of history, must still be present to rule the development. There can be no doubt that, in the modern theory of continuity, there is found the reconciliation between those who maintain that the likeness of offspring to parent is due to the presence of similar conditions and those who are satisfied in referring the resemblance simply to

"heredity." That there is similar material to start with is one half of the truth; that there are similar conditions throughout the development is the other.

VII. Inheritance of Acquired Characters.

The third problem, which we stated at the outset, concerned the inheritance of acquired characters. It is well known that many organisms in the course of their individual life are affected by environmental influences, or by use and disuse of their organs. Environmental and functional variations of the body of the individual organism thus result. The question is, whether these may be transmitted to the offspring by the parent which acquires them. Two cautions may be noted in starting—(1) No naturalist doubts the inheritance of *constitutional or organismal* variations. These may be reasonably traced back to the fertilised egg-cell. But what is involved in the fertilised egg-cell is also, by hypothesis, involved in the germ-cells which give rise to the next generation. There is no argument on this fact; the present scepticism relates to functional and environmental variations. (2) No one doubts that functional and environmental variations often *reappear*. Many doubt, however, that they reappear because they have been transmitted. Another alternative is obviously open. The conditions which originally brought about a given change may still persist, and may hammer the same effect upon the offspring which they wrought upon the parent.

Doubt as to the transmission of acquired characters is not novel. It has, however, become precise in Weismann's statement. Brock has noticed that even the editor, whoever he was, of Aristotle's *Historia Animalium* seems to have differed from his master on this subject. Aristotle naïvely refers to the inheritance of the exact shape of a certain cautery; but the editor seems to doubt whether apparent instances of the inheritance of acquired characters are not, after all, exceptional. The same writer notes Kant's vigorous opinion against the transmission of such individual features, and also Blumenbach's cautious inclination to the same position. In more recent times, His expressed his strong conviction against such inheritance; and Galton is at once cautious and emphatic. Pflüger is also among the earlier sceptics. A few sentences from Galton,

whose merit has not been sufficiently emphasised, may be quoted. The inheritance of characters acquired during the lifetime of the parents "includes much questionable evidence, usually difficult of verification. We might almost reserve our belief that the structural cells can react on the sexual elements at all, and we may be confident that at the most they do so in a very faint degree—in other words, that acquired modifications are barely, if at all, *inherited* in the correct sense of that word."

(1) In regard to climatic variations, Galton doubts any reaction of the "body" upon the germs, but believes that the germs are themselves directly affected.

(2) The same is true in many constitutional diseases that have been acquired by long-continued irregular habits.

(3) The cases of the apparent inheritance of mutilations are outnumbered by the overpowering negative evidence of their non-inheritance.

(4) The case of Brown-Séquard's hereditarily epileptic guinea-pigs, in consequence of an operation performed upon the parents, is *perhaps* interpretable as the result of imitative influence.

(5) It is hard to find evidence of the power of the personal structure to react upon sexual elements, that is not open to serious objection. That which appears the most trustworthy lies almost wholly in the direction of nerve changes, as shown by the inherited habits of tameness, pointing in dogs, and the results of Dr Brown-Séquard.

Weismann, however, has brought the scepticism to a climax. He denies all inheritance of acquired characters, a denial which at the present day should be welcome to optimists. Weismann finds no convincing evidence that characters impressed upon the parental organism by the surroundings, or acquired as the result of use and disuse, can be transmitted. The case is not proven. More than that, however, Weismann's whole theory of variation, adaptation, and heredity raises, he believes, strong probabilities against the inheritance of acquired characters. It is necessary to quote a few of his sentences.

(1) "Acquired characters are those which result from external influence upon the organism, in contrast to such as spring from the constitution of the germ."

(2) "Characters can only be inherited in so far as their rudiments (anlagen) are already given in the germinal protoplasm (keim-plasma)."

(3) "Modifications which are wrought upon the formed body, in consequence of external influences, must remain limited to the organism in which they arose."

(4) "So must it be with mutilations, and with the results of use or disuse of parts of the body."

(5) "No such modifications of the soma (affected by environment or by use and disuse) can be transmitted to the germ-cells, from which the next generation springs. They are, therefore, of no account in the modification of the species."

(6) "The only principle that remains for the explanation of the modification of the species, is direct germinal variation." The intermingling of the sex elements is the origin of the variations, on which natural selection in the usual way operates.

Weismann's position is thus clear and definite. The sole fountain of specific change is found in the intermingled nuclear plasma of the sex-cells. The environment does make dints upon the organism, but only upon its soma; the reproductive cells, through which alone the variation could be transmitted, are unaffected. The effects of use and disuse may be marked enough, and important for the individual, but they are not transmitted, and therefore of no account in the history of the species. The ground is taken from under the feet of Lamarckians and Buffonians, and the whole burden of progress is laid upon germinal variation in sexual reproduction, and upon natural selection.

And as to the alleged cases of the inheritance of acquired characters in which we and our fathers have believed, they usually admit of one of three rebuffs. They may be entirely fictitious or in some cases mere coincidences. The inheritance of a letter branded upon the arm, which Aristotle notes, is an extreme type of what His calls a handful of anecdotes. Or the apparent inheritance of acquired characters may be explained in this way; similar surroundings hammer the same change upon successive generations, and we mistake reappearance for transmission. The case of Nägeli's Alpine plants which seemed to have been thoroughly changed, but lost their characters when the influencing conditions were removed, is often

quoted as an illustration. Or, in the third place, apparent exceptions to Weismann's conclusion have been shown by him to be rather corroborations, by tracing them back to an internal not external origin, and interpreting them as primary or secondary results of original germinal variations.

Not only has this conclusion as to the non-inheritance of acquired characters run counter to the presuppositions of many naturalists, not to speak of the laity, but if it is true, it literally takes the ground from under the feet of those who have based their theory of evolution upon the postulate that organisms could hand on as a legacy their individually acquired gains. Weismann's theory, though accepted by authorities like Ray Lankester, has given rise to much criticism, part of which must be noticed. That the case is not yet surrendered may be seen from the very title of Professor Eimer's recent important work, *The Origin of Species, on the Basis of the Inheritance of acquired Characters, according to the Laws of Organic Growth.*

Criticism of Weismann.

(1) Various naturalists have brought forward what appear to them to be examples of the genuine transmission of individually-acquired characters. Thus Detmer and Hoffmann among botanists, and Eimer among zoologists, may be quoted. The latter especially gives numerous examples to prove the untenability of Weismann's position. To some of the instances urged against him, Weismann has replied; but as each case has to be carefully tried on its own merits, and as sufficient decisive experiments are still awanting, the matter lies beyond the sphere of the present paper.

(2) Virchow has urged against Weismann what appear to him to be cases of the direct inheritance of climatic changes and pathological variations. But he appears to differ from Weismann in his definition of acquired characters, which for the latter do not include anything that can reasonably be traced back to a germinal variation. Ziegler has discussed the whole question of the inheritance of pathological characters, and comes to a conclusion harmonious with that of Weismann. Nor are the slow results of acclimatisation good

cases in the present discussion, since Weismann expressly allows that in long-continued conditions affecting the whole system the germinal cells may be directly affected along with, though not exactly by, the other elements of the organism.

(3) A criticism of a different nature has been suggested by several, but is well stated by Eimer. If the source of variation be restricted by hypothesis to the keimplasma intermingled in sexual reproduction, is this sufficient to account for the facts? "In what way, one must ask, have *new* characters first been introduced into the series? The sexual mixture could produce nothing; it could only work with what was already given." Professor M'Kendrick has forcibly emphasised a similar objection. There is no doubt, at any rate, that Weismann's theory, which excludes the direct assistance of environmental and functional variations, throws a still heavier burden than Darwin did on the shoulders of Natural Selection, which many believe to be already somewhat overweighted.

Without urging concrete cases, in regard to which one cannot but allow the necessity of fresh observation; without venturing on the pathological field where authorities like Virchow and Ziegler so much differ; without urging difficulties from the general theory of evolution, I wish to emphasise what appears to me to be the physiological compromise at present tenable. Does the doctrine of the continuity of the reproductive protoplasm really force one to deny the transmission of acquired characters? Stable and persistent as the keimplasma may be, can one believe that it leads a charmed life in the general symbiosis of the organism? Are the reproductive cells in so rigid a degree insulated from the general life?

(1) Every one allows the general conception of the various organs as symbions in a common life. We constantly speak of correlated variations, and though these generally work from the centre or germinal plasma outwards, there is no *a priori* improbability against an environmental influence of some strength saturating through the entire organism, affecting one system by another, till eventually the reproductive cells share in the change.

(2) Apart from the general connectedness and the common medium of the blood, it seems worth while to refer to the frequent occurrence of protoplasmic continuity within the system. In plants the intracellular connections by means of protoplasmic bridges are widespread; this is true in a more limited degree of animals. How open in such cases is the organism to an influence from the body to the reproductive cells? Take such a case, not altogether unique, as the embryo *Peripatus*, where, according to Sedgwick, an actual syncytium of cells obtains for a considerable time, how difficult it is there to conceive of the keimplasma keeping quite intact, in spite of somatic influences which may play upon it.

(3) Even in the comparatively few cases where a continuity of reproductive cells is demonstrable, it does not seem justifiable to reject the notion that these may be affected by the somatic sheath which has grown up around them. They are surely reachable by the anastates and katastates of the body, which is simply a more exact way of expressing the penetration of "scent-stuff" which Jäger maintained, or the limited liberation of gemmules which Galton allowed. But in most cases, as we have seen, there is not even a continuity of reproductive cells, but only a continuity of a specific reproductive protoplasm or nuclear plasma. How much more difficult, then, is it to conceive of this as remaining practically untouched by the physico-chemical conditions of the "soma"? Even Weismann at times admits that nutritive vacillations may produce "ever so little modifications in the molecular structure of the keimplasma." Proof of the degree of modification possible may be as legitimately demanded on the one side as on the other. A continuance of many "ever so littles" may amount to much. Nor is it possible to draw any hard line between modification of the germ-plasma along with and through the general "body."

(4) It is useful, also, to allude to the numerous experiments which have been made on the determination of sex. Take only one example, the familiar case of Yung's tadpoles, where, by altering the quantity and quality of the food, he

was able, for instance, to raise the percentage of females from the normal of about fifty to the abnormal of about ninety. Here, then, an environmental influence, playing in the first place on the nutritive system, saturated throughout the organism, and affected the reproductive system so as to swing the balance emphatically to the female side. General hypertrophy brought out of the primitive indifference an emphatic predominance of females. In this case the reproductive system was unquestionably reached, and though the change that resulted was not, of course, one that was not in a sense implicit in the reproductive cells, it was none the less an alteration of the natural bias. Some of the forms which turned out females would with less nourishment in their natural environment have become males. Now the difference between a quantitative change such as the above and a qualitative change such as the modification of a given structure, is only one of degree. Admit the one, and there is no logical objection against admitting the possibility of any other modification which can be interpreted in terms of anabolic or katabolic preponderance.

My general conclusion, then, is, that while Weismann's position in regard to the non-inheritance of acquired characters suggests the advisability of a cautious re-criticism of all apparent cases of the reverse, Galton's position of the limited inheritance of the same features is at present more tenable. Apart from the three arguments—(1) from alleged cases, (2) from pathological inheritance, (3) from the general theory of evolution—it seems to me that the physiological probabilities are strongly in favour of Galton's view. Nor is it at all necessary, in allowing the limited inheritance of acquired characters, to depart from some form of the theory of continuity on which Weismann has so well insisted. It is not necessary to revert to any literal pangenesis. It is only necessary to admit that decisive functional and environmental variations may send their roots deep into the system, and may affect the reproductive cells along with and even through the others. The sex-cells will share in the altered nutriment and waste products, and become infected to a varying degree by the anastates and katastates, which are the chemical results of the individually acquired characters.

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N.B.—For the older literature, see especially the works of P. Lucas and E. Roth.

On the Anatomy and Histology of Phreoryctes.

By **Frank E. Beddard, M.A.**, *Prosector to the Zoological Society of London, Lecturer on Biology at the Medical School of Guy's Hospital.*

(Read February 18, 1889.)

(*Abstract.*)

The genus *Phreoryctes* has been known to zoologists since the year 1843, but there is no published account of the reproductive system sufficiently detailed to permit of comparison with other Oligochæta. The *gonads* (testes and ovaries) and *spermathecæ* were discovered by Leydig,* who did not distinguish between ovaries and testes, owing to the immature condition of the specimens studied. This writer considered that the genital products were evacuated through the nephridia of their segment. The supposition is, however, incorrect, as genital ducts exist. Mr W. W. Smith of Ashburton, New Zealand, forwarded to the author in the spring of 1888 a single specimen of a new species of *Phreoryctes*, which was described in the *Annals and Magazine of Natural History* for June 1888 as *Phreoryctes Smithii.* In that paper the gonads and their *ducts* were briefly described and figured.

The present memoir is based upon a further supply of material from Mr Smith, consisting of a large number of sexually mature worms. It has, therefore, been found possible to prepare a *complete* account of the genitalia of *Phreoryctes.*

The following is a brief description of the genus *Phreoryctes*, Hoffmeister:—

1. The *body* is extremely elongated, sometimes reaching to the length of a foot, while the diameter is very small.

2. The *prostomium* is divided into two by a transverse furrow.

3. The *setæ* are simple, *not* bifid; they are disposed in four rows of single setæ, or of pairs of setæ.

*4. There are *no genital* or *penial setæ.*†

* References to the literature of the subject will be found in the detailed memoir.

† The asterisk refers to statements of fact which are made for the first time in the present memoir.

*5. The *clitellum* occupies 3–4 segments from the 10th to the 13th; its epidermis is formed by a single layer of cells, differing from the epidermis of the general body surface by their greater length and glandular character.

6. The structure of the *longitudinal muscles* is more complicated than is usually the case with aquatic Oligochæta; the longitudinal muscular coat more resembles that of certain Earthworms.

*7. The nephridia commence in the sexually mature worm in the 16th segment.

8. There are *two pairs of testes*, which are digitate organs attached to the anterior septum of segments 10 and 11.

9. There are *two pairs of vasa deferentia* opening into the interior of these segments by a wide funnel-shaped orifice closely attached to the posterior septum. Each vas deferens opens independently on to the exterior; the first pair open to one side of the ventral setæ on the 11th segment, the second pair open on to the 12th segment *in front of* the ventral pair of setæ.

10. There are *two pairs of ovaries*, occupying a position corresponding to that of the testes in the 12th and 13th segments.

11. There are *two pairs of oviducts* opening into the interior of those segments by a wide funnel closely attached to the posterior mesentery; the external orifices are in the intersegmental grooves between segments 12–13, 13–14, on a line with the ventral setæ.

*12. In both vasa deferentia and oviducts the distal section is lined with a chitinous membrane continuous with that covering the body; they are in other respects closely similar, and the position of the external orifice of the 2nd pair of vasa deferentia is intermediate between that of the 1st pair of vasa deferentia and of the oviducts.

13. The developing spermatozoa are lodged in *sperm sacs*, which extend from the 9th to the 13th segment, and resemble more closely the corresponding structures in many aquatic Oligochæta (*e.g.*, Tubificidæ) than those of Earthworms.

*14. The *ova*, which are, when fully mature, of large size (one-half the diameter of the body), and loaded with yolk granules, undergo their development in *egg sacs* contained in segments 14–16. The ova and egg sacs are more like those of the Tubificidæ than those of Earthworms.

15. The *spermathecæ* are present to the number of from two to three pairs in segments 7, 8 (and 9).

The above characters show that *Phreoryctes* occupies an intermediate position between Earthworms and the lower Oligochæta. It should form the type of a distinct family.*

The paper concludes with a discussion of the morphology of the genital ducts and the classification of the Oligochæta.

Note on the Transformation of Ciliated into Stratified Squamous Epithelium as a result of the Application of Friction. By Dr John Berry Haycraft and E. W. Carlier, M.B., B.Sc.

(Read January 21, 1889.)

(*Abstract.*)

In man, in the rabbit, and some other animals the trachea is built up of a series of cartilaginous rings incomplete behind; the rings being completed in this position by the trachealis muscle.

The mucous membrane forms a smooth cylindrical lining for the whole tube, and is covered by a ciliated epithelium.

In the cat and dog the cartilage rings completely encircle the trachea, and overlap posteriorly, and the trachealis muscle, which is well developed, is placed outside the cartilage, and has a powerful action in varying the diameter of the tube.

When this muscle contracts the overlapping ends of the cartilage ride one upon another, and the projecting ends form a vertical ridge down the inside of the trachea, which can be readily seen on slitting the organ open. This ridge is separated by a deep groove from the other end of the plate.

The mucous membrane which lines the trachea is reflected in between the overlapping ends of the cartilage plates, and therefore will be subject to friction during their movements.

That portion of mucous membrane which lines the tip of the projecting ridge will be subject to the greatest amount of friction, and here we find that the ciliated epithelium which lines the general cavity of the trachea is replaced by stratified squamous epithelium.

* It is so placed by Vejdovsky.

Those portions of the groove where less friction occurs are lined by a form of epithelium transitional between the ciliated and the stratified squamous varieties.

This led us to believe that friction was the determining agent in forming the squamous epithelium found in these situations.

In order to determine whether or not the squamous epithelium is produced from ciliated epithelium present in those situations where friction eventually occurs, we made sections of the trachea of a fœtal kitten, 5 cm. long.

The cartilage rings were found to surround the trachea for only three-fourths of its circumference, the trachealis muscle being well developed posteriorly, and the mucous membrane being thrown into folds beneath it. The mucous membrane was lined by a stratified epithelium, the surface cells of which were columnar, and devoid of cilia.

We then cut sections from a kitten two weeks old, and found that the cartilage now completely encircled the trachea, the ends of the plates, however, not overriding as yet, though there were indications of one end projecting inwards into a fold of the mucous membrane. The epithelium lining the mucous membrane was seen to be stratified and ciliated in all positions.

It appears, therefore, that the stratified squamous and transitional forms of epithelia present in the adult trachea result from a modification of ciliated epithelium of the ordinary type, due to the influence of friction exerted during the animal's life.

Still more remarkable changes have been described by biologists. Of Hunters' Gull there is no histological record. But in other cases there is abundant testimony to the mutability of one form of cell into another; as, for example, the transformation of osteoblasts into osteoclasts, and *vice versa.*

In such cases the cause is unknown, but is probably environmental.

But in the case which we have described there can be no doubt that the change is brought about by the agency of friction occurring as a physiological phenomenon during the life of the animal. This induces the cells in the deeper layers of the epithelium to take on a new line of development, producing flat keratinised cells in place of ciliated ones.

Observations on the Metabolism of Man during Starvation.

By **D. Noël Paton**, M.D., and **Ralph Stockman**, M.D.

(Read March 4, 1889.)

Although the metabolism during starvation has been investigated in the most elaborate and exhaustive manner in the lower animals by many different observers, as yet few observations have been accomplished in man.

In 1880 Tanner undertook a fast of forty days. We have been able to procure only a few fragmentary observations upon his case (*British Medical Journal*, vol. ii., 1880). At the commencement of his fast he weighed 71·600 kilos., and at the end of twenty-five days his weight had fallen to 60·000 kilos., indicating a loss of about 11·600 kilos., or ·162 kilos. per kilo. of his original weight.

During the first sixteen days he pretended to take no water, merely gargling his mouth with it. Under these conditions he became seriously ill, and lost weight with great rapidity. After the sixteenth day he took water *ad libitum*, and in the course of the next four days he gained 4½ lbs. After this he again commenced to lose weight.

On the 1st day of his fast he passed 29 grms. urea.
„ 5th „ „ 16 „
„ 18th „ „ 14 „

Throughout the period the weather was excessively warm. Tanner rarely walked, but daily drove for some time. He, however, spent much of his time receiving people and talking to them.

A more satisfactory series of observations was made upon an Italian named Cetti, who in 1887 commenced in Berlin a fast of thirty days. His parents interfered, and his fast was stopped on the tenth day.

A large number of the leading scientific men in Berlin interested themselves in the case, and together undertook a most careful and elaborate series of observations, which are recorded in the *Berliner klinische Wochenschrift* for 1887.

Cetti's age was 26. He was lean, and at the beginning of the fast weighed 57 kilos.; at the end, 50·650 kilos., so that in the

ten days he lost 6350 grms., or ·111 kilos. per kilo. of his original weight. This loss was by no means regular, there being somewhat large variations. The whole fast might be divided into three periods:—1st period of five days, during which there was considerable waste, the loss amounting to 4400 grms., or 880 grms. per diem. In the 2nd period of two days the loss was very slight—only 250 grms., or 125 grms. per diem. During this period he drank more water than usual. During the last three days there was a nearly equal loss of from 500 to 600 grms. per diem—in all, 1700 grms.

The urea fell slowly and regularly from 29 grms. per diem to 20 grms. The chlorine of the urine fell from 5·5 grms. to 0·6 grms.; while sodium and potassium also diminished in amount and changed their relative proportions. The amount of urine passed fell gradually from the beginning, was always below the normal, and scanty in proportion to the fluid consumed.

It was always acid, and became more strongly so towards the end of the fast, being then passed thick and turbid, with large quantities of urate of ammonia crystals. Phosphoric acid and calcium were, although absolutely diminished, increased proportionately to the other urinary constituents, a result attributed by the observers to a waste of bony tissue.

Indican disappeared after the first day, showing a cessation of intestinal digestion, while acetone was present in very large amount. This is considered by Senator to be an inanition symptom, and its occurrence in diabetes and in some cases of cancer is by him supposed to be due to profound interference with the metabolic processes.

It may be mentioned that Cetti spent most of his time in a somewhat large and cold room.

The individual upon whom the following observations were made is a Frenchman named Alexandre Jacques, who in October 1888 voluntarily undertook, as a public performance, a fast of thirty days.

Considering the scantiness of our information in regard to the metabolism of man during starvation, we thought it advisable to make what use we could of the case.

On making inquiries we found that arrangements had been made with a number of medical students and others, who had constituted

themselves into a committee, taking watch in rotation, so that Jacques should never be left for a moment unattended by one of them. (*I may say that one or two members of my class acted upon this committee, and that I have not the slightest doubt that the surveillance was complete.*—D. N. P.)

We have to thank the gentlemen of this committee for much valuable assistance, without which our observations could not have been carried out. More especially are we indebted to Mr Griffiths for the endless trouble he took in preparing for us notes of weights, of amount of exercise taken, and of other points of interest.

Jacques is a somewhat slightly built individual, of 47 years of age. He is a block-printer, and was born at St Amant-les-Eaux, Nord. He has no great muscular development, nor has he any excess of subcutaneous fat. He has always enjoyed good health, and states that he has already successfully carried out five different fasts of varying duration.

Four or five years ago he fasted in private for one week. A month later he fasted for fourteen days. In 1886 he fasted in London for twenty-one days, being watched during the period. We have ascertained that the surveillance was by no means so thorough as could be desired.

In the spring of 1887 he fasted in private for thirty days, and in the spring of 1888 he underwent in private a fast of forty days.

During all these fasts he believes that he has been sustained by taking in very small quantities a mixture of certain herbs, which he in his first three experiments used in the form of a decoction, but in all subsequent experiments as a powder.

The composition of this preparation he keeps a profound secret. But before commencing the present fast, he made a sworn declaration that it was entirely composed of herbs growing in Kent. Of this powder he took only the smallest quantities, a mere pinch, generally once, more rarely twice daily. On several days he did not take any.

During the course of his fast he rubbed into his thighs and trunk small quantities of camphorated olive oil. But in the course of the thirty days, only 306 grms. were so rubbed in. He also used small quantities of a lotion which he applied to the head, as he said, because it refreshed him greatly. On examination this was found

to be composed of water with a little alcohol, and a small quantity of carbonate of ammonia.

On the third day of his fast he developed the habit of drinking his urine in quantities of from 2 to 9 oz. This he did daily, usually on rising in the morning. The disturbing influence of this upon the metabolism is practically *nil*, since all the urinary constituents are simply excreted unchanged. This somewhat disgusting habit is by no means unknown, as Jacques himself informed us that it has long been the custom among French miners to drink their urine when deprived of food, owing to mining accidents. Certain savage peoples, such as the Patagonians, also do so during starvation.

He was permitted to drink ærated mineral waters *ad libitum*, a careful note being always made of the amount consumed. He smoked a good many cigarettes daily, and occupied himself in reading newspapers, in playing cards, in talking, and in resting on the sofa. He almost daily took a walk, sometimes during the earlier part of his fast two. The time occupied in this way was noted.

During the whole period of the fast his health was never much disturbed. He was usually cheerful, though rather irritable. During the first few days his tongue became somewhat coated, and his breath very offensive. On the third and fourth day he complained of dull epigastric pain, which he said was relieved by taking a pinch of his powder. His bowels were not moved during the whole period of his fast, except on the first day, when a few scybalous masses were passed, but were not kept. A few hours after his first meal on the completion of his fast, he had a copious formed evacuation, which was, however, unfortunately lost.

During the whole period, but more especially towards the latter part, he was very sensitive to cold and draughts, and, although he kept his room at a temperature usually of 75° F., his temperature was invariably subnormal, ranging from 96° to 93°·4 F. His pulse averaged between 50 and 60. His respirations were usually from 23 to 30. His skin felt moist and warm during the whole period. His expression, naturally somewhat anxious, became rather haggard towards the end.

On the 9th of November he complained of pain and tenderness in the ball of the great toe, which was observed to be red and

inflamed. His pulse was 84 at 12 noon, the only occasion on which it rose so high; but his temperature was only 96°·8–97°·8 F. Since a few days after the completion of his fast he had a well-marked attack of gout, it is fair to conclude that this also was a slight attack of the same disease.

One very important point, upon which we hoped to gain valuable information, was the influence of exercise on the metabolism during starvation. Unfortunately, it was not possible to get results at all satisfactory. In the first place, the amount of exercise taken was exceedingly small; in the second place, it was not easy to estimate the precise amount of exercise taken daily. Inasmuch as during the hours spent indoors practically no exercise was indulged in, we came to the conclusion that the most satisfactory results might be arrived at by taking, as a measure of the muscular exercise, the duration of his daily walks.

It is to be regretted that the conditions of the fast and the pressure of other work rendered it impossible for us to undertake anything like the complete series of observations carried out in Cetti's case.

The following observations only could be accomplished:—

The weight was daily taken before Jacques had dressed. The amount of fluid consumed was carefully measured. The urine passed was all collected and measured. Its reaction was taken, and the amount of urea estimated by the hypobromite method—Dupré's apparatus, previously tested on standard solutions of urea, being used. A record of the pulse, temperature, and general condition was also kept.

From the urea the daily waste of flesh was calculated. By this term is, of course, meant not merely the muscle substance, but all the various tissues of the body containing nitrogen in the proportion in which this occurs in muscle.

The difference between this and the total loss of weight gave the loss of non-nitrogenous substances. These non-nitrogenous wasting substances are the fats and carbohydrates; but inasmuch as the latter occur only in small quantities, and are rapidly used up, they may be neglected, and we may consider these non-nitrogenous matters as practically entirely composed of fat.

Of course, the possibility of variations in the percentage amount of water in the body had to be considered, and in all probability this

played a certain part in the variations of loss of weight from day to day. But its influence over longer periods need not be considered, as is clearly indicated by some recent observations of L. Hermann (Pflüger's *Arch.*, Bd. xliii. p. 239).

The first table gives the results of our daily observations.

TABLE I.—*Alexandre Jacques. Thirty Days Fast.*

	Date.	Weight in grms.	Fluid taken. cub. cms.	Urine passed. cub. cms.	Difference between Fluid taken and Urine passed.	Urea in grms.	Loss of Weight in grms.	Loss of Flesh in grms.	Exercise in Minutes (Walking).
	1888.								
I.	Oct. 25,	62,008	566?	...	...	...	1333	...	0
	,, 26,	60,675	850?	...	...	...	935	...	105
	,, 27,	59,740	1672	917	755	21·37	510	292·77	120
	,, 28,	59,230	1672	1037	635	29·70	+7	406·89	90
	,, 29,	59,237	1501	1073	428	21·40	262	293·18	45
II.	,, 30,	58,975	2124	986	1138	13·60	623	186·32	45
	,, 31,	58,352	1388	1230	158	9·40	+198	128·78	75
	Nov. 1,	58,550	1275	1145	130	13·70	1870	187·69	90
	,, 2,	56,680	1218	1030	188	12·40	454	169·88	30
	,, 3,	56,226	1417	893	524	11·10	0	152·07	105
III.	,, 4	56,226	594	1040	+446	13·50	425	182·95	0
	,, 5,	55,801	779	765	14	12·60	198	172·62	60
	,, 6,	55,603	1317	486	831	5·67	936	77·68	60
	,, 7,	54,667	1374	1100	274	16·67	0	228·38	20
	,, 8,	54,667	1118	683	435	6·10	+369	83·57	10
IV.	,, 9,	55,036	1246	770	476	12·32	57	168·51	20
	,, 10,	54,979	1133	628	505	7·89	+ 85	117·09	25
	,, 11,	55,064	990	725	265	5·30	255	72·61	0
	,, 12,	54,809	1331	778	553	11·00	878	150·70	0
	,, 13,	53,931	1331	873	458	10·00	0	137·00	0
V.	,, 14,	53,931	1473	812	661	10·52	114	143·85	20
	,, 15,	53,817	842	835	7	9·87	455	135·22	20
	,, 16,	53,362	850	575	275	2·78	367	38·08	20
	,, 17,	52,995	1183	515	668	8·55	255	117·13	0
	,, 18,	52,740	1162	892	270	14·30	284	195·91	0
VI.	,, 19,	52,456	609	529	80	4·20	57	57·54	0
	,, 20,	52,399	1020	560	460	8·12	509	111·24	0
	,, 21,	51,890	921	585	336	8·93	+339	112·34	drive.
	,, 22,	52,229	991	450	541	6·55	367	89·73	0
	,, 23,	51,862	878	440	438	8·80	170	120·56	20
	,, 24,	51,692	...	...	...	...	...	...	...
Totals, .		...	33,409	22,052	11,357	316	8813	4330	...

It will be observed that on some occasions the loss of flesh, as calculated from the urea, is greater than the total loss of weight. This is to be explained by the fact that Jacques was by no means regular in the hours at which he emptied his bladder, so that the weighing of one day is with a full bladder, on another with the viscus empty.

During the thirty days he lost 10·316 grms. or ·166 kilos. per kilo. of his original weight; in all about $\frac{1}{6}$ of his original weight. On an average he lost ·34 kilos. per diem.

During the first five days of the fast the urea excretion was high and irregular—a fact which has been so frequently observed in starving animals, but which was not manifested in Cetti's case.

Dividing the fast up into six periods of five days, we see the gradual fall in the daily excretion of urea till the very low figure of 7·3 grms. per diem is reached.

TABLE Ia.—*Urea Excretion in Grms., Average per Diem during Six Periods of Five Days.*

I. 25·7	IV. 9·3
II. 11·6	V. 9·2
III. 10·9	VI. 7·3

On account of his irregular habits in regard to the emptying of his bladder, whereby the night urine is sometimes counted with the past day, sometimes with the succeeding day, the urea excretion manifests on one or two occasions somewhat large variations.

Table II. gives a general summary of the results worked out for six periods of five days each.

The results for Period I. are not given, because our observations during this period were incomplete.

It will be observed that in Periods II., III., IV., V., and VI. there is a slow and steady fall in the flesh waste. On the other hand, the loss of non-nitrogenous matter is by no means so constant, its relationship to the flesh waste being in Periods III., IV., and VI. as about ·6 to 1, while in Periods II. and V. the proportion is very much raised. That this is not due to a retention of water during the former three periods is indicated by column 8 of the table. Period III. may, perhaps, be accounted for by this, but in the other periods the proportion between the fluid taken and the fluid excreted by the urine remains constant. Of course, it is

possible that a diminished loss of water from the skin and air passages may have occurred during these periods. On this subject we have no evidence. We do not consider that the variations in the exercise taken during these different periods will account for these variations in the non-nitrogenous waste, although it is highly probable that the large non-nitrogenous waste during the second period was associated with the large amount of exercise taken. We are entirely at a loss to explain the rise in the non-nitrogenous waste during Period V.

TABLE II.

Period.	Loss in Grms.	Total in Period.	Per Diem.	Per Kilo. of Body Weight per Diem.	Proportion of Flesh to Fat.	Average Exercise in Min. per Diem.	Relationship of Water not excreted by Kidneys to Water so excreted.
II.	Total,	2749	549	9·4	...	...	...
30th to 3rd,	Flesh,	825	165	2·8	1:2·3	69	1:1
inclusive, 5 days.	Non-nitrogenous,	1924	384	6·6	...	...	...
III.	Total,	1190	233	4·3	...	...	...
4th to 8th,	Flesh,	745	149	2·7	1:0·6	30	1:3·6
inclusive, 5 days.	Non-nitrogenous,	445	89	1·6	...	...	...
IV.	Total,	1105	221	4·0	...	...	...
9th to 13th,	Flesh,	646	129	2·3	1:0·7	9	1:1·8
inclusive, 5 days.	Non-nitrogenous,	559	112	2·7	...	...	...
V.	Total,	1475	295	5·5	...	...	...
14th to 18th,	Flesh,	630	126	2·3	1:1·3	12	1:1·9
inclusive, 5 days.	Non-nitrogenous,	845	169	3·2	...	...	...
VI.	Total,	764	152	2·9	...	...	...
19th to 23rd,	Flesh,	491	98	1·8	1:0·5	4	1:1·6
inclusive, 5 days.	Non-nitrogenous,	273	54	1·1	...	...	...

But we may analyse still further these results.

During the twenty-eight days on which Jacques was under observation, he passed 316 grms. of urea, corresponding to about 147 grms. of nitrogen. This represented a waste of flesh amounting to 4330 grms., which would contain 500 grms. of carbon. Of this, 63 grms. were excreted as urea. The remaining 437 must have been passed out as carbonic acid.

But in addition to this, 4046 grms. of non-proteid matter were used up. Now this, as already indicated, we may practically regard as fat, since the cabohydrates need not be considered. Now fat contains 30 per cent. of water, and hence this amount would represent 2832 grms. of solid fat, and in this would be contained about 2174 grms. of carbon.

Thus we see that in all 2230 grms. of carbon were lost from the body in twenty-eight days; that is, during the fast there was an average daily excretion of 77·6 grms. of carbon. Now in a man on ordinary diet and work about 280 grms. of carbon are daily excreted, so that the carbon excretion fell to about $\frac{1}{3}$ of its normal amount.

On the other hand, 147 grms. nitrogen being excreted in these twenty-eight days, we have a daily excretion of 5·2 grms. instead of the normal 15 or 16 grms., so that the nitrogenous excretion also fell to about $\frac{1}{3}$ of its normal amount.

In this case, then, apparently the proportion of the proteid to the non-proteid waste was undisturbed during starvation.

Is it possible, from these observations, to come to any general conclusions in regard to the probable composition of the body at the end of the fasting period as compared with the beginning? Taking as our basis the composition of the body of a man aged 33 years, as given by Bischoff (*Zitsch. f. rat. Med.*, 3d Reihe, Bd. xx. p. 75), we may conclude that at the beginning of the fast Jacques' body had something of the following composition:—

Total weight on 26th October, . .	60,675 grms.
"Flesh," *i.e.*, muscle, liver, lung, &c., .	35,550 „
Fat, ...	10,834 „

His condition at the end of the experiment may be calculated by subtracting from these figures the loss in total weight, in flesh, and in other substances, or fat:—

	Loss.	Wt. at end of Fast.
Total, ...	8813	51·862
Flesh, ...	4330	31·220
Other substances—Fat, . .	4046	6·788

The oil rubbed in amounted to only 306 grms., an amount which need not be considered. Hence we see that even at the end of a fast of thirty days, there still was a very considerable quantity of fat in the body.

Hofmann (*Ztsch. f. Biolgie*, Bd. viii. p. 153) found that dogs had to be starved for a period of about thirty days before all the fat had disappeared from the body, and that the final disappearance of fat was indicated by a sudden rise in the excretion of nitrogen, after which the dogs, unless fed, rapidly died. This did not occur in Jacques' case, and we should consider the appearance of such a rise in the urea excretion as the important indication for instantly stopping any voluntary fast.

From the accompanying table it will be seen that the metabolism in the present case was much slower than in either Tanner's or Cetti's. Nor was there any evidence of the same profound disturbance in the metabolism which was indicated in Cetti's case by the appearance of acetone in large quantities in the urine. At one period Jacques' urine was observed to have a peculiar smell, suggestive of liquorice, but not in the least resembling acetone. Some of this urine was distilled, but the distillate was free of any unusual smell.

Altogether the metabolism here is much more like that observed by Voit in old fat dogs, while Cetti's case rather resembles the processes as seen in young lean animals.

Comparison of Tanner, Cetti, and Jacques.

	Tanner.	Cetti.	Jacques.
		Weights.	
1st Day.	71·600	57·000	62·008
10th Day.	...	50·650	56·226
25th Day.	60·000	...	52·740
30th Day.	...	...	51·692
	Loss of Weight per Kilo.	*Original Body Weight, in Kilos.*	
10th Day.	...	·111	·093
25th Day.	·162	...	·149
30th Day.	...	...	·166
	Loss of Flesh per Diem (in Jacques' Case, in Average of 3 Days).		
5th Day.	219	315	295
10th Day.	...	270	168
18th Day.	192	...	113
28th Day.	...	...	107

While Cetti spent most of his time in a large somewhat cold room, Jacques, on the contrary, inhabited a small highly-heated chamber.

In every way the conditions of his case were peculiarly favourable to the maintenance of life with the smallest possible metabolism. Exercise was only taken in most moderate amount, while the katabolic changes necessary to maintain the temperature of the body were reduced to a minimum by the high temperature at which his room was constantly kept.

Whether his powder had any influence in diminishing metabolism we are unable to say. But even without it we see no reason why a man should not undergo with impunity such a period of starvation under like favourable circumstances.

A Method of Demonstrating the Presence of Uric Acid in the Contractile Vacuoles of some Lower Organisms.

By **Dr A. B. Griffiths**, F.R.S. (Edin.), F.C.S. (Lond. and Paris), *Member of the Physico-Chemical Society of St Petersburg, &c.*

(Read January 21, 1889.)

After some years of patient observation and research, I have found, from direct experiment, that at certain times the contractile or pulsating vacuole of the Protozoa performs the function of a true kidney, or, in other words, its secretion is capable of yielding microscopic crystals of uric acid.

Three organisms were used in these experiments, namely:—Amœba, Vorticella, and Paramœcium.

I. Amœba.

We will consider, in the first place, the small Protozoon which Haeckel called *Amœba sphærococcus*. By observing a number of these organisms under the high powers of the microscope, there is seen, within the structure of each, a small cavity or vacuole filled at certain times with a transparent fluid. There is little doubt that the fluid which gathers in the vacuole is drawn from the surrounding protoplasmic substance, and is returned to it, or forced out to the exterior on the contraction of the walls of the vacuole.

I have shown in my paper, "Further Researches on the Physio-

logy of the Invertebrata" (*Proc. Roy. Soc. Lond.*, vol. xliv. p. 325), that the five pouches of the *stomach* of the Asteridea also perform the function of kidneys (*i.e.*, the digestive apparatus performs a dual function). And whatever may be the multitudinous functions of the Protozoan contractile vacuoles, one thing is certain, that they secrete periodically a waste nitrogenous substance. This nitrogenous substance was proved to be *uric acid.*

A number of amœbæ were placed on a microscopic slide and covered by a thin glass slip. Alcohol was run in between the slide and cover-slip, so as to kill the organisms. It was found that in many cases moderately weak alcohol caused no contraction of the vacuole. The alcohol was followed by nitric acid; the slide gently warmed, and, finally, ammonia introduced between the slide and cover-slip. In a few minutes, prismatic crystals of *murexide*,* having a beautiful reddish-purple colour, made their appearance. After the addition of alcohol (as already stated), minute flakes could be distinctly seen floating in the fluid of certain contractile vacuoles. Bearing in mind the murexide reaction, there is every reason to believe that these flakes are nothing more or less than minute crystals of uric acid.

These reactions have been constantly repeated during the past few years, and always with the same results.

It appears, from close microscopic observations, there are times when the fluid of the contractile vacuoles does not contain the least trace of uric acid. Most likely the Protozoan vacuoles perform more than one function. It is possible that they represent "an internal respiratory apparatus" as well as an excretory organ or "kidney." There is little doubt that we have in the contractile vacuole of the amœba a primitive kidney or renal system. The protoplasmic matter of this naked little cell (as well as the waste albuminous substances of its food) frequently undergoes chemical "disintegration."

II. Vorticella.

The contractile vacuole of *Vorticella* exhibits, during life, fairly regular diastolic and systolic movements. The fluid which it con-

* The crystals had a splendid green metallic lustre when seen by reflected, and a reddish-purple colour by transmitted light.

tains is drawn from the surrounding protoplasmic matter, and is ultimately forced by the contraction of its walls towards the periphery of the bell, and finally ejected into the water in which the organism lives its life-history. The *contractile vacuole* of Vorticella performs the function of a true kidney. Its secretion yields microscopic crystals of murexide and uric acid when submitted to the same chemico-microscopical reactions as those already described.

III. Paramœcium.

Paramœcium bursaria belongs, like Vorticella, to the *Infusoria.* We have in this organism the beginning of a true alimentary canal, —with its mouth and slender œsophagus. The *contractile vacuoles* of Paramœcium are situated in the ectosarc almost at each end of the long axis of the "body." These cavities are filled with a transparent fluid. During the systole fine radiating canals are produced which probably communicate with the exterior.

The contractile vacuoles of *Paramœcium bursaria* are physiologically the "kidneys." By the same reactions as those already described (in connection with the Amœba), the secretion of these vacuoles yields crystals of murexide and uric acid. There is little doubt that these vacuoles get rid of the waste nitrogenous products during the systoles which take place periodically.

In these three primitive forms of the animal kingdom, we have the rudiments of a true renal system. The contractile vacuoles perform the same function as the kidney of higher forms, by yielding the same nitrogenous substance which is found in the renal organs of the highest vertebrates. By the agency of living protoplasm (that all-important life substance), even these insignificant microscopic cells bring about chemical metamorphoses in albuminoid molecules, with the production of *uric acid* and possibly other substances. Therefore, in these primitive cells, there lies the same power of chemical metamorphosis as we find in the more complex cells of the highest vertebrate.

Through all the multitudinous changes which have taken place during the lapse of ages, in the development of the mammalian kidney, we find that the physiological functions are the same as occur in its original or primitive form (represented by the Protozoa). Surely it is not going too far to say that within these lower forms

The Invertebrate Kidney.

(Compiled from the Author's Researches.)

Classes, Natural Orders, &c.	Kidney Represented by	Products Formed.
Protozoa, (a) *Protoplasta*, (b) *Infusoria*,	Contractile Vacuoles,	Uric Acid.
Porifera or Spongidia,	Contractile Vacuoles,	(?)
Cœlenterata,	Water-vascular System,	(?)
Echinodermata, (a) *Asteridea*,	The five pouches of the Stomach, (*Stomach has a dual function*),*	Uric Acid.
Annelida, (a) *Hirudinea*,	Cellular tubes ("the Nephridia"),	Uric Acid (*as Sodium urate*).
(b) *Oligochæta*,	,, ("Segmental organs"),	Uric Acid.
(c) *Polychæta*,	,, (,, ,,),	(?)
Arthropoda, (a) *Orthoptera*,	Cellular Tubules ("Malpighian tubes"),	Uric Acid.
(b) *Neuroptera*,	,, (,,),	,,
(c) *Coleoptera*,	,, (,,),	,,
(d) *Araneina*,	,, (,,),	Uric Acid (*as Sodium urate*).‡
(e) *Brachyura*,	Cellular "glands,"	Uric Acid and Guanin.
(f) *Macroura*,	Cellular "green glands,"	,, ,,
Brachiopoda, (a) *Clistenterata*,	"Pseudo-hearts" (tubular organs),	(?)
Mollusca, (a) *Lamellibranchiata*,	Cellular "Organs of Bojanus,"	Uric Acid, Urea, Calcium, Phosphate.
(b) *Gasteropoda*, (1) *Pulmogasteropoda*,	Cellular "Gland" (with a well-developed urinary duct),	Uric Acid.
(2) *Branchiogasteropoda*,	A large cellular "Gland" (Nephridia")	Uric Acid.
(c) *Cephalopoda*, (1) *Dibranchiata*,	Pair of sacculated and glandular organs ("Nephridia"),	Uric Acid and Calcium Phosphate.
(2) *Tetrabranchiata*,†	,, ,, ,, ,,	Uric Acid (?) ,,

* In some forms an organ has a dual, triple, &c., function (see Darwin's *Origin of Species*, chapter vi.).
† *Memoir on the Pearly Nautilus*, by Sir Richard Owen, K.C.B., F.R.S.
‡ Weinland has found guanin in the excrements of spiders (*Zeit. Biol.*, vol. xxv. p. 390).

of animal life we have all the necessary "mechanism" for the organisms to breathe, digest, and excrete. The only difference is that in the Protozoa the cell performs numerous functions, whereas in the Vertebrata these functions are localised in special organs.

Having in my studies on the physiology of the Invertebrata proved the *renal* functions of many doubtful organs, it would be interesting at this point to give, by means of a table, the physiological development of the renal system from the Protozoa to Mollusca.

It will be seen from the above table that the primitive "kidney" was a simple cell performing other functions besides that of a renal organ. In the next progressive stage of its development, we find the kidney, a *cellular* tubule secreting only waste nitrogenous matters and assuming a definite function. And, finally, in the higher stages of the *Invertebrata* the kidney becomes a sacculated and glandular organ resembling more in detail the vertebrate kidney.

To conclude in the words of Professor Huxley:—"In the *Vertebrata*, the renal apparatus is constructed on the same principle [as the renal organs of the *Mollusca*]. . . . . The vertebrate kidney is an extreme modification of an organ of the Annelid; and, to go still lower, in the water-vascular system of the Turbellarian. And this, in its lowest form, is so similar to the more complex conditions of the contractile vacuole of a Protozoon, that it is hardly straining analogy too far to regard the latter as the primary form of uropoietic as well as of internal respiratory apparatus" (*The Anatomy of Invertebrated Animals*, p. 62).

On Improvements in the Apparatus for Counting the Dust Particles in the Atmosphere. By John Aitken, Esq., Darroch. (With four Plates.)

Part I.

(Read February 4, 1889.)

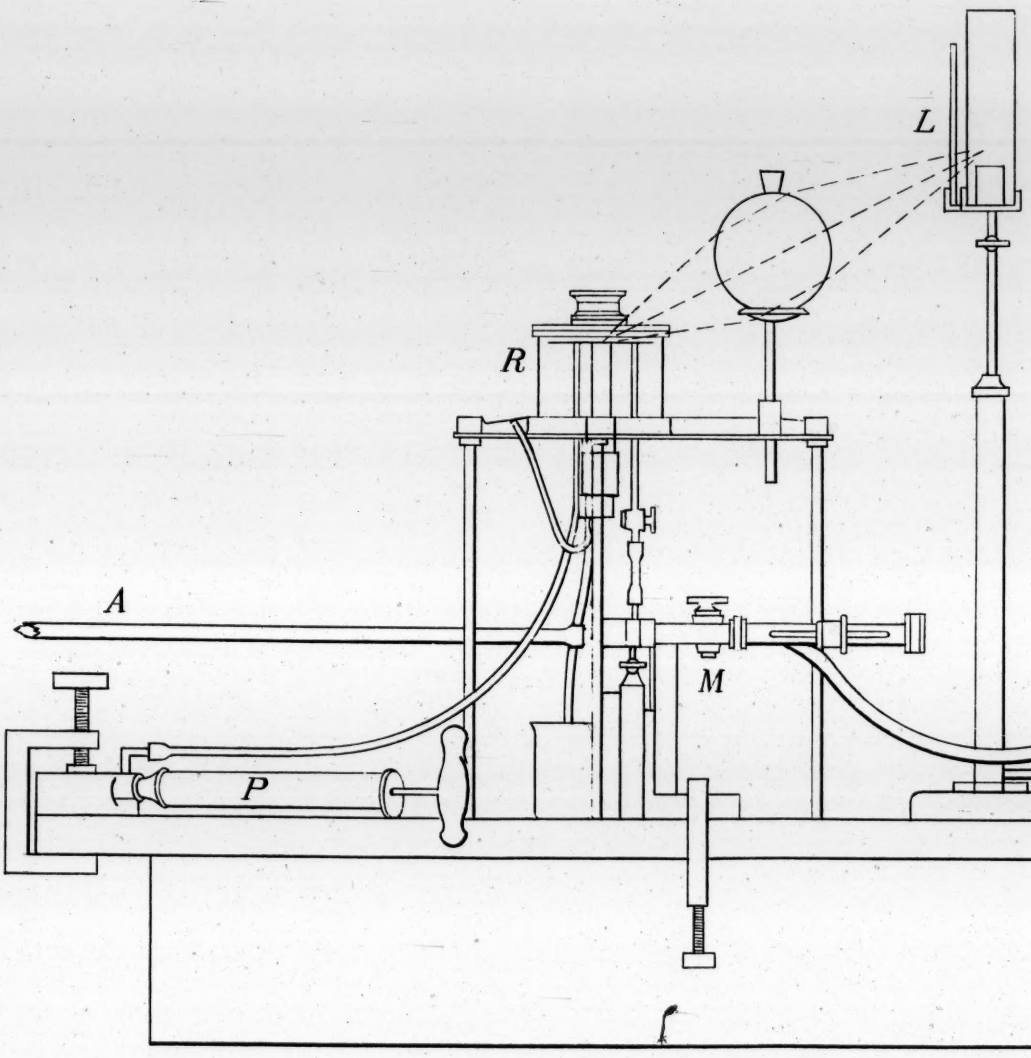
In a previous communication I described the apparatus first used for counting the dust particles in the atmosphere. That apparatus was constructed of such materials as could be easily obtained ready made, and was fitted together in such a way that any one acquainted with laboratory work could easily repeat the

experiments. Though that apparatus is satisfactory enough for preliminary work, and gives fairly good results, yet it is evidently not suited for regular everyday use; and, besides that, there are certain defects in it which can be avoided in apparatus specially constructed. If a regular examination of the dust in the air was to be made from day to day, it seemed advisable that I should devote some time to devising a more practical form of apparatus, one which could be more easily and quickly worked, and which could be managed by any one not having a special knowledge of the subject.

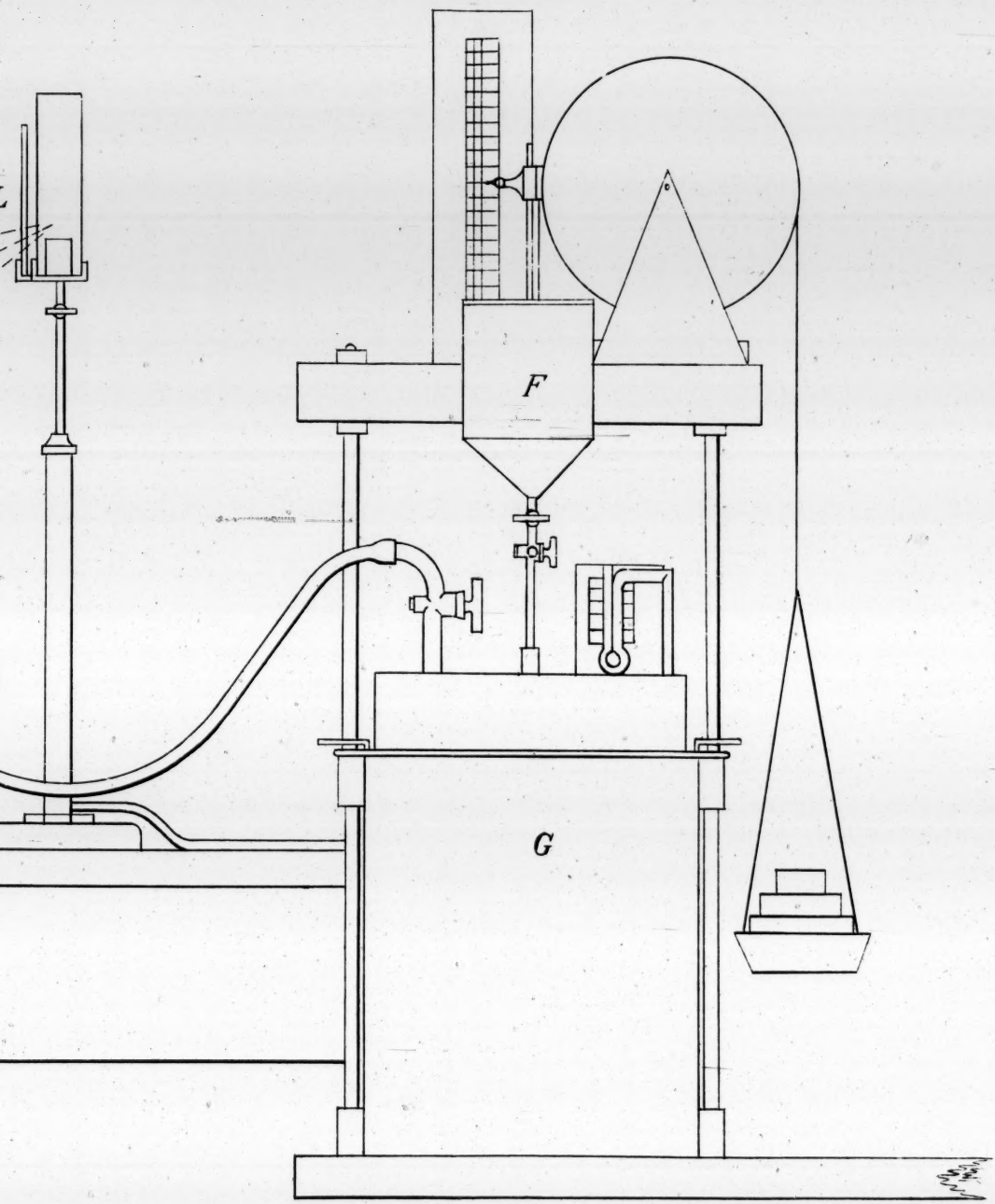
With this object in view I have, during this summer, devoted a considerable time to the construction and testing of the new apparatus; and the object of this paper is to describe the improvements I have been able to make, and to give detailed working drawings of the different parts of the apparatus, so that any instrument-maker may be able to reproduce it. I may as well give notice here that the general reader is warned off, as what follows is a dreary desert of mechanical details, which however necessary for those who are going to assist in developing the investigation, yet contains nothing new of scientific interest. Anything new in that direction will be found near the end of the paper.

The alterations and improvements will be best understood by the drawings given with this paper. Plate I. shows the general arrangement of the different parts as fitted up for work; Plate II. gives detailed drawings of the test-receiver; while Plate III. shows the apparatus for measuring the air to be tested. Most of the drawings on Plates II. and III. are to a scale of half size.

Returning to Plate I. showing the general arrangement, there are five distinct parts in the apparatus—(1) the test-receiver R; (2) the air-pump P; (3) the measuring apparatus M; (4) the illuminating arrangements L; and (5) the gasometer G. The air to be tested is drawn through the pipe A by means of the gasometer and its connecting pipes. The air on its way passes through the measuring apparatus M, where a measured quantity of it is taken and passed into the receiver R, where it is mixed with a certain quantity of dustless air, and saturated with water. The air in R is then expanded by the pump P, a shower of rain produced, and the number of drops which fall on a measured area are counted. Such



J. Aitken, del.



McFarlane & Erskine Lith^rs Edin^r

Proc. Roy. Soc. Edin^r Vol. XVI, Plate I

Dust Particles.

is a general outline of the apparatus and the method of working it; but, before going further, it will be better to describe in detail the different parts.

Test-Receiver.

In the apparatus described in the previous communication, the test-receiver was made of an ordinary flat-bottomed flask. When working with this apparatus it was noticed that the raindrops did not always fall vertically on the counting stage, but from time to time they were seen to fall obliquely. Through the magnifying glass, it looked as if a miniature storm was raging inside the receiver, and driving the raindrops before it. When this happened the reading had to be rejected, as no reliance could be placed on the number obtained under these conditions. To investigate the cause of these irregular currents, the interior of the receiver was illuminated by a strong light concentrated in it by means of the water lens, and the movements of the air at the different points, when expansion was made, was examined by means of a lens, the condensed particles indicating the direction of the currents.

Examination showed that when expansion was made, a vertical circulation of the air in the flask took place—the air next the sides of the flask forming the ascending current, while the descending one occupied the centre. The cause of these currents is evident. When expansion is made the air is cooled, but the air in contact with the sides of the flask rapidly absorbs heat from the glass, and an upward rush of air takes place all round the sides. If the flask has the same temperature all round, then these ascending currents meet near the top, curve over, and descend in the centre of the flask, leaving the part over the stage almost unaffected; but if one side of the flask is hotter than the other, then the current from that side is stronger than the current opposed to it at the top, and overpowers it, driving the air across the stage in a horizontal direction.

The cause of the disturbance having been found out, attempts were made to remedy it, by placing screens round the stage to prevent the currents passing across it. The result of this arrangement was however unsatisfactory, because these screens, though they were less than a centimetre in height, gave rise to currents, by heating the air in contact with them; so that, while they checked the larger currents, they caused small ones, which, owing to their nearness to the

stage, interfered with the correctness of the results. After this another plan was tried, and it has been found to work satisfactorily. It consists in reducing the height of the receiver as much as possible, and increasing the horizontal dimensions to such an extent that the vertical currents do not move across the receiver and disturb the air over the counting stage; in addition to this, the stage is placed on a floor in the receiver, because when supported at a height, as in the first receiver, currents are formed on the support and rise round it, disturbing the air, and interfering with the equal distribution of the drops. The new arrangement gives most satisfactory results. When expansion is made, the condensed particles are seen falling vertically, and the distribution of the particles is even all over the stage, which would not be the case unless there was an entire absence of currents while the drops were falling.

The manner in which this plan has been practically carried out will be best understood by a reference to Plate II., where fig. 1 is a vertical section of the receiver; fig. 2 is another vertical section at right angles to fig. 1; fig. 3 is a horizontal section, while fig. 4 is a plan of the top of the receiver. A is a glass cylinder, such as is used for pneumatic experiments, with its ends melted and ground. B is a circular disc of plate glass, ground and polished, which closes the upper end of the cylinder A. The bottom of the cylinder is closed by the metal disc C, through which pass the different tubes presently to be referred to. The bottom may be cemented to the cylinder, or a ground joint may be employed. The counting stage E is supported inside the receiver by means of the tube D—the upper surface of the stage being at a distance of exactly 1 cm. from the glass top B. The tube D is supported inside the receiver, and kept vertically in its place by means of the tube F, which is part of the metal bottom C, or fixed firmly to it. The tube D has a shoulder-piece G worked on it in such a position that when D is in its place, and the shoulder G pushed up into contact with the lower end of F, the counting stage is in its correct position, namely, 1 cm. below the glass cover. H is a piece of indiarubber tubing, to prevent air entering between D and F. I is a thin disc of metal slightly smaller than the inside of the receiver, and pierced with three holes to allow tubes to pass through it. The diaphragm I is used as a false bottom and as a stirrer for mixing the air in the receiver. K is a short

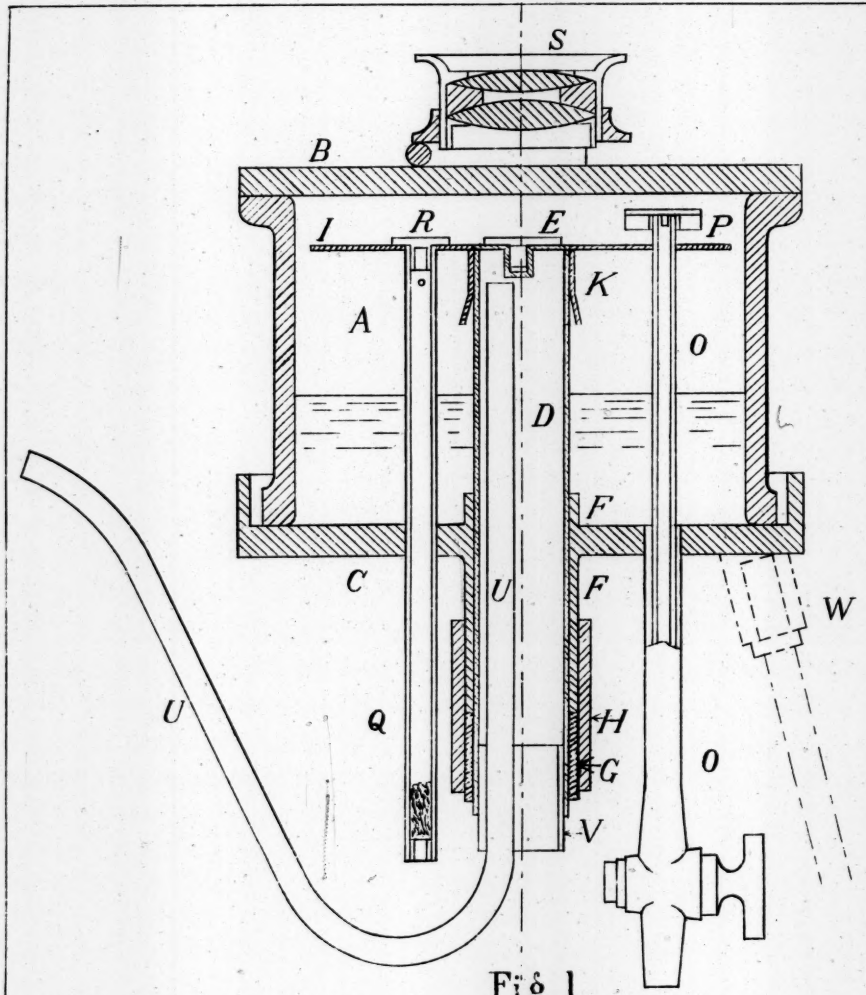


Fig. 1.

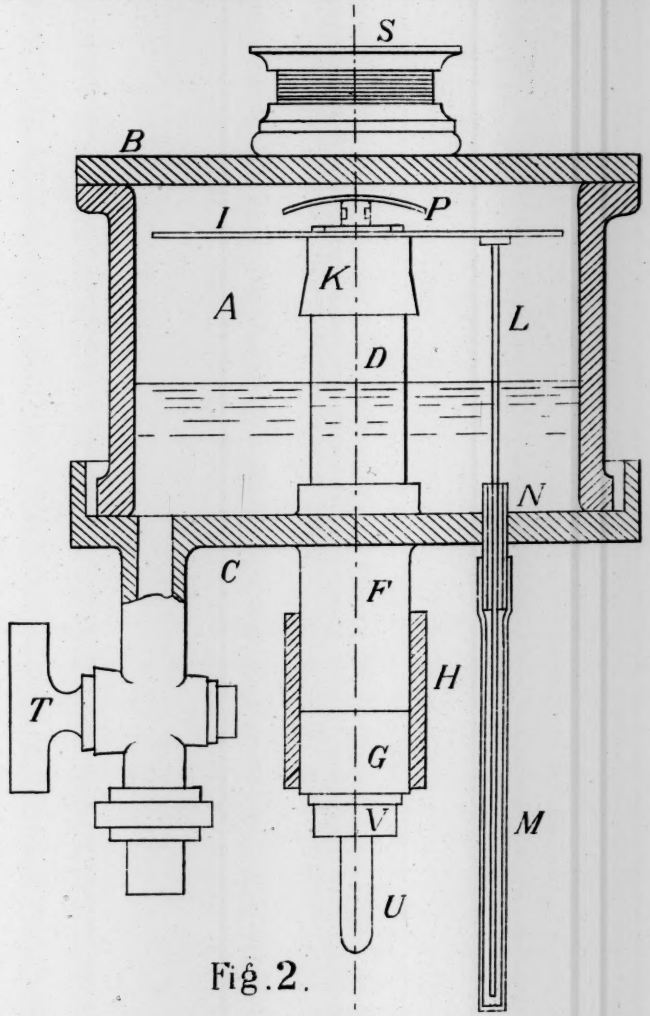


Fig. 2.

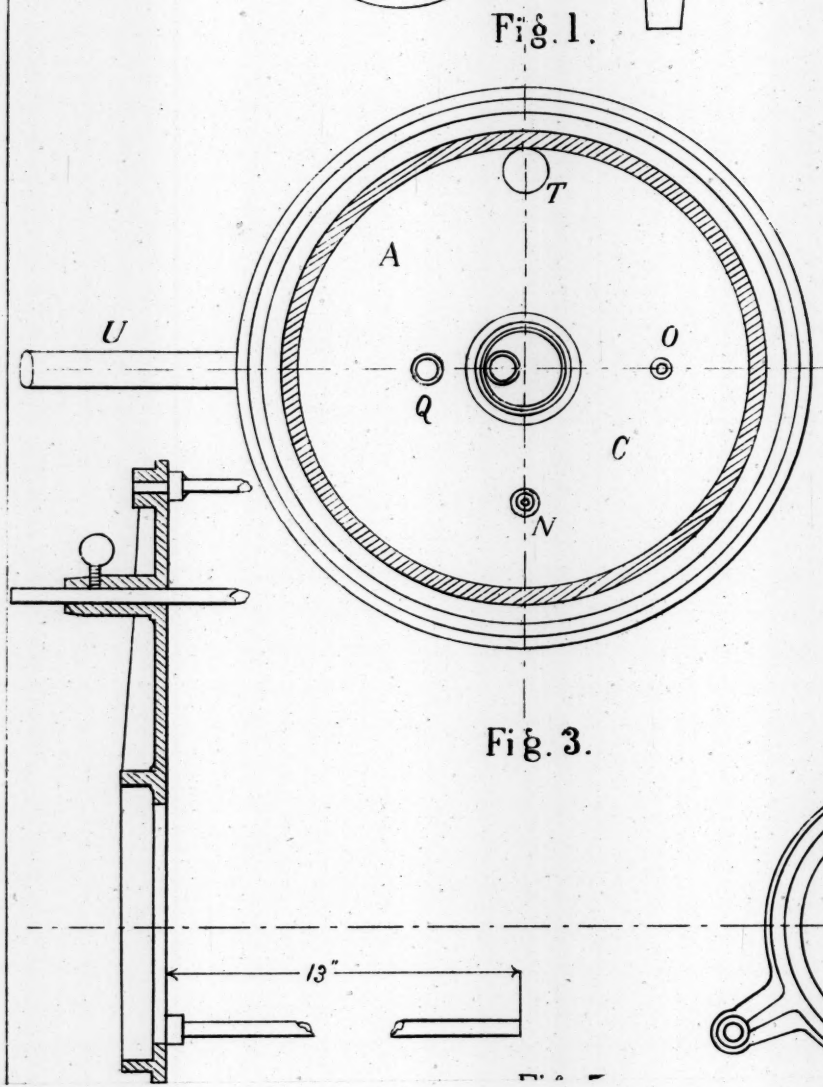


Fig. 1.

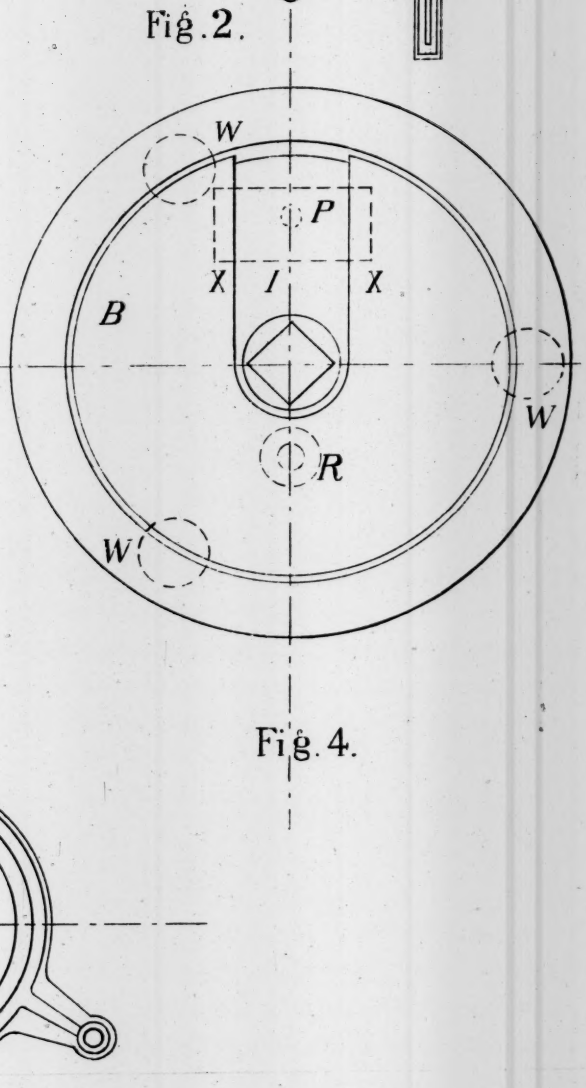


Fig. 2.

Fig. 3.

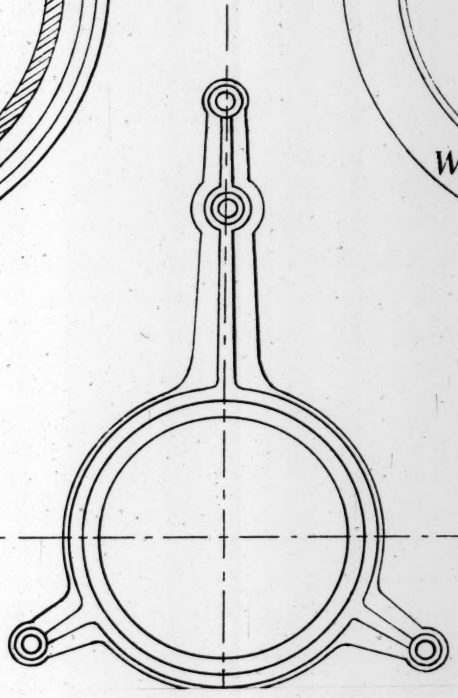


Fig. 4.

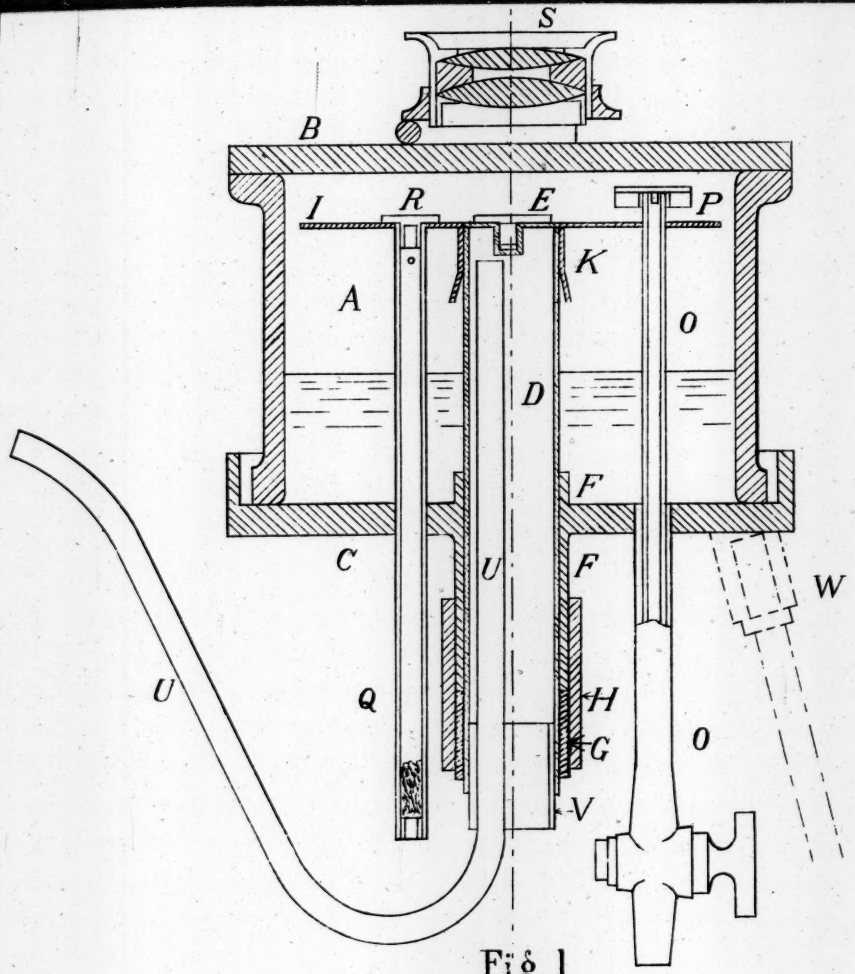


Fig. 1.

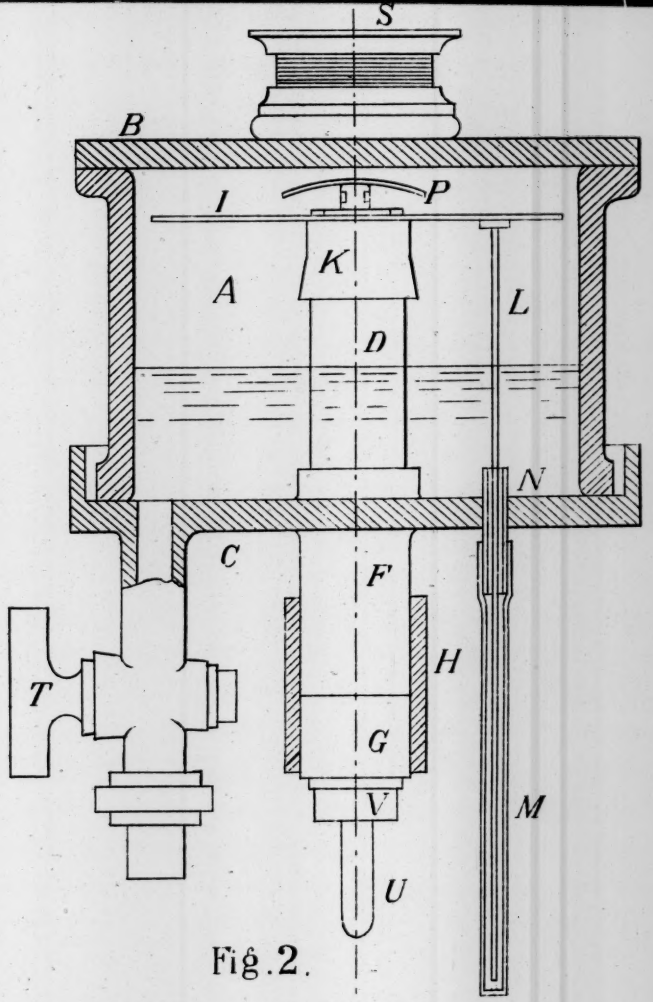
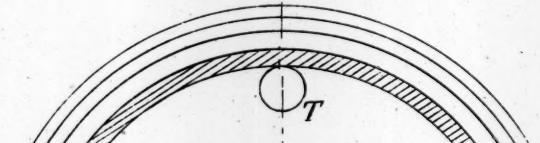


Fig. 2.





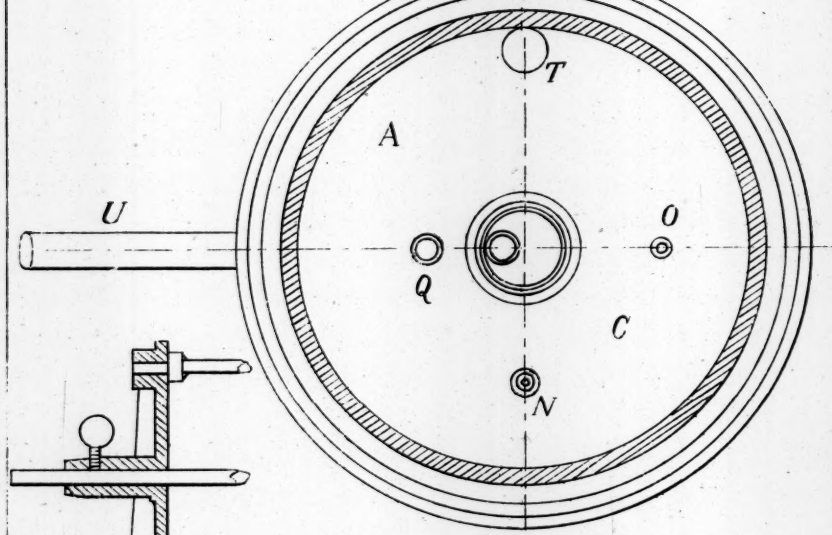


Fig. 3.

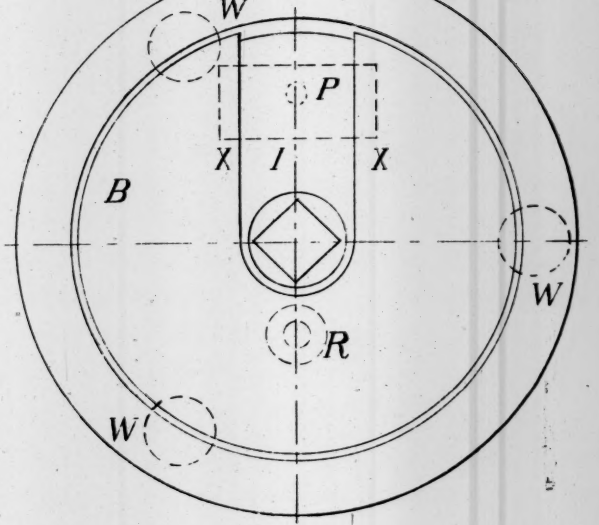
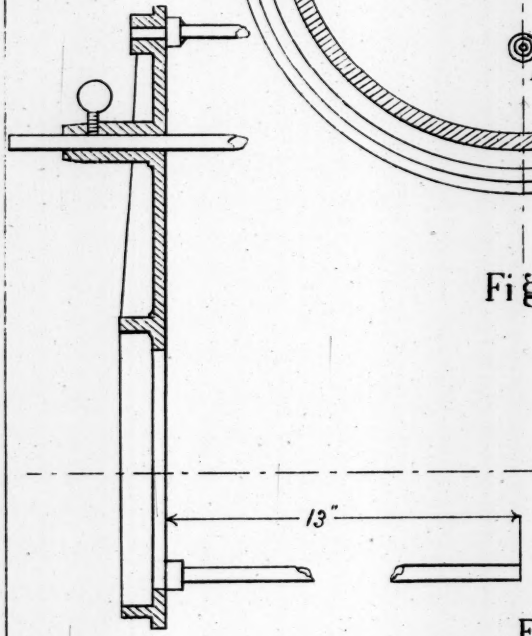


Fig. 4.



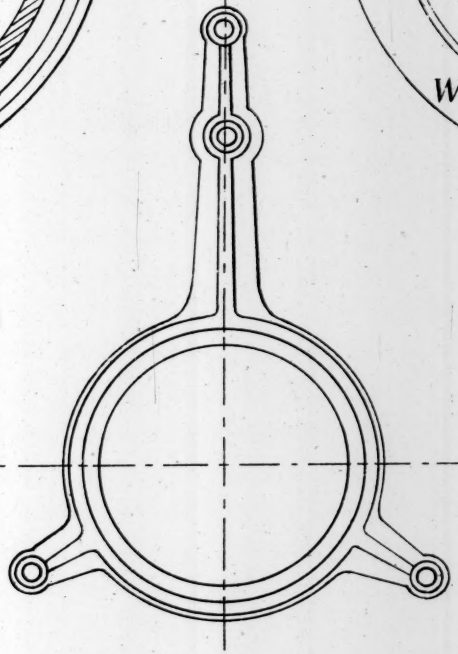


Fig. 5

J. Aitken, del.

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J. AITKEN ON DUST PARTICLES.

length of tube fixed to an opening in the centre of I. This tube is of such a size that the tube D slides easily inside it, and its lower end is slightly widened, as shown. The disc I is attached to a wire L, which passes through the bottom of the receiver, and is used for moving the diaphragm I up and down. In order to avoid the use of a stuffing box, with its inconvenient tightness, the wire L projects to some distance outside the bottom of the receiver, and is there surrounded by an indiarubber tube M, slightly larger in internal diameter than the wire. The tube M is closed at the bottom, and firmly attached to the receiver by the pipe N. O is the pipe by which the air enters the receiver. It is provided at the lower end with a stopcock, while the top is closed, and two side openings close to the top are provided for the escape of the air. Over the top of this pipe is fixed the deflector P. The pipe O should be small, say 2 mm., or the expansion of the air in it will give rise to a slight disturbance of the air in the receiver when condensation is taking place. Q is the outlet pipe, and its lower end is connected by means of an indiarubber tube with the air-pump. At the upper end of Q is a cap R screwed into the end of the pipe. This cap has a large cover, which prevents the disc I from rising above its proper position. At the lower end of Q is a tuft of closely-packed cotton wool, kept in its place by means of a ring. The air enters Q through a small hole near the top of the tube; just under the screw of the cap S is a magnifying glass for viewing the stage. To the under side of its ring is attached a piece of indiarubber cord, which prevents the glass getting scratched and the lens from slipping. T is a pipe provided with a stopcock for supplying the receiver with water. It is connected by means of an indiarubber tube with a vessel of water. U is a bent tube, one leg of which passes up the tube D, and terminates just under the closed end of D. Both ends of tube U are open. It is soldered to the short tube V, which fits tightly inside D; it is thus kept firmly in its position, but can be removed when required.

The two sides of the diaphragm I are covered with blotting-paper fastened on with indiarubber solution. The under side of the top plate B is also covered in the same way, but an opening is cut in this piece for viewing the counting stage and for admitting light (*see* XX. fig. 4).

After the receiver is completed, with its pipes, &c., it is graduated. This is done by filling it with water, which is easily accomplished by inverting it and pouring in water through O, while the air is allowed to escape by T. The receiver is then put in its correct position and levelled; after which 350 c.c. of water is run out, and three marks are made at equal distances round the cylinder at the water-level. In using the receiver afterwards the water is kept at this level.

The receiver is supported at a height of 13 inches above the table. For this purpose three sockets, W, W, W, figs. 1 and 4, may be cast on the bottom plate C, into which three legs may be fixed; or the tripod stand shown in section and plan in fig. 5 may be used. In the tripod sketched, in addition to the three sockets for the legs, there is a fourth for carrying a rod to support the lens used for condensing the light on the stage. This saves a special stand for the lens. The tripod is drawn to a scale of $\frac{1}{4}$ size.

On Plate III. fig. 7, is given a full size sketch of the counting stage, and the top end of the tube D. This stage is made of silver and highly polished. It is ruled with lines at right angles to each other, and at exactly 1 mm. apart. These lines should be extremely fine. To the under side of the stage is fixed a rod which is used for holding it when polishing. This rod also fits into the socket in the top of the tube D and keeps the stage in its place.

As the silver counting stage requires to be kept well polished, it is necessary to remove it frequently from the receiver. In actual practice I generally take it out after the day's work. To enable this to be done easily, there are two plans, either of which may be adopted. The plate B, forming the top of the receiver, may be fitted on with grease. As the two surfaces are ground, it makes an easily opened air-tight joint. If this plan is adopted, then the tube D may be firmly fixed to the bottom of the receiver; but it should be retained, as it has other duties besides that of supporting the stage. The other plan is the one shown in the drawings. The top of the receiver is cemented on, and the stage is withdrawn from the receiver by taking out the tube D to which it is attached, the indiarubber tube H making the joint between the tubes D and F air-tight. The principal objection to the first plan is that it is very

dirty, the grease sticking to everything with which it comes in contact, so that I prefer to use the second.

The plate B is fastened on with the ordinary strong solution of indiarubber, which makes an air-tight joint, and is sufficiently strong for the purpose, while at the same time the joint can be forced when necessary to get access to the interior for cleaning, &c. Should this joint leak at any time, all that is necessary is to make a stroke of the pump, and rub the place where the fault is with a little of the indiarubber solution, which soon closes it. At first some difficulty was found in making quite tight the joint where the tube D enters the receiver. The leakage was due to imperfections on the interior surface of the indiarubber tube, and the joint had always to be tied. This difficulty has, however, been overcome by improving the inside of the indiarubber tube. To do this the tube is drawn over a piece of brass tube slightly larger than itself, half an inch or so of it is turned back so as to expose the interior. This surface is then carefully rubbed over where defective, with the solution of indiarubber, till good contact is made; a quantity of the solution is put on sufficient to fill up all imperfections; the surface of the brass tube is then wetted with water, and the indiarubber tube turned to its natural position, and left a day or two to dry. By this means all inequalities in the interior of the tube are levelled up, and it is always found to make a perfectly tight joint when put in its place without being tied. For convenience, the indiarubber tube H is cemented to the shoulder piece G on the tube D.

If this indiarubber joint is objected to, then a metal one held close by means of a screw may be used. The joint would require to be made quite true and air-tight. If a leather washer is used, care must be taken, as any variation in its thickness will alter the distance between the stage and the top of the receiver. The arrangement sketched has been found to work satisfactorily and easily. When the stage is to be taken out, the tube D is simply drawn down with a slight twisting movement, and it comes away easily; in putting it in its place, the process is simply reversed, twisting till one feels the shoulder is resting against F. But for general use, perhaps the screw joint is the best.

Before taking the stage out, the stopcock T is opened and all the water is allowed to run out of the receiver. In withdrawing the

stage it sometimes gets wet in its passage through F. This is caused by air rushing in between D and F, and blowing the water over it. This wetting can be easily avoided by keeping the stopcock on O open, and drawing the stage out slowly; or, better still, by blowing air into the receiver through T before removing the stage. The pressure inside drives the water down and keeps the stage dry. Perhaps the stage might be more easily taken out dry if the tube F were carried up to the height of the stage, so that its interior might be always dry. But as this would give rise to disturbing currents round the stage every time expansion was made, on account of the air in the space between the tubes expanding and rushing upwards, the advantage therefore seems doubtful. As the plan shown works well with care, I have not thought it worth while testing the other.

It will be noticed that the tube K in the centre of the diaphragm I, which slides on the centre tube D, has its lower end widened. The object of this is, that when D is introduced through the bottom of the receiver, its top end shall be guided into its place without trouble, and the stage prevented from touching the lower edge of K. The diaphragm I forms a floor at the level of the counting stage, and is kept up to its correct level by the indiarubber tube M pressing it against the stop R.

Measuring Apparatus.

In the measuring apparatus described in the previous paper there are defects which evidently make it unsuitable for everyday work. In devising the new apparatus, besides making it a more permanent form, one point specially kept in view was to make the arrangements such that the time required for measuring should be short, in order that the air might lose as few of its particles as possible while it was being measured. Two different plans have been tried, and both found to work well. Each of these plans possesses certain advantages over the other, and it is for the operator to determine which will best suit the conditions under which his tests are to be made. The two plans are shown in Plate III. Figs. 1 to 5 show one arrangement, the other is shown at fig. 6. In the first, a cylinder and piston is used as the measure; in the other, the bore in the plug of a stopcock.

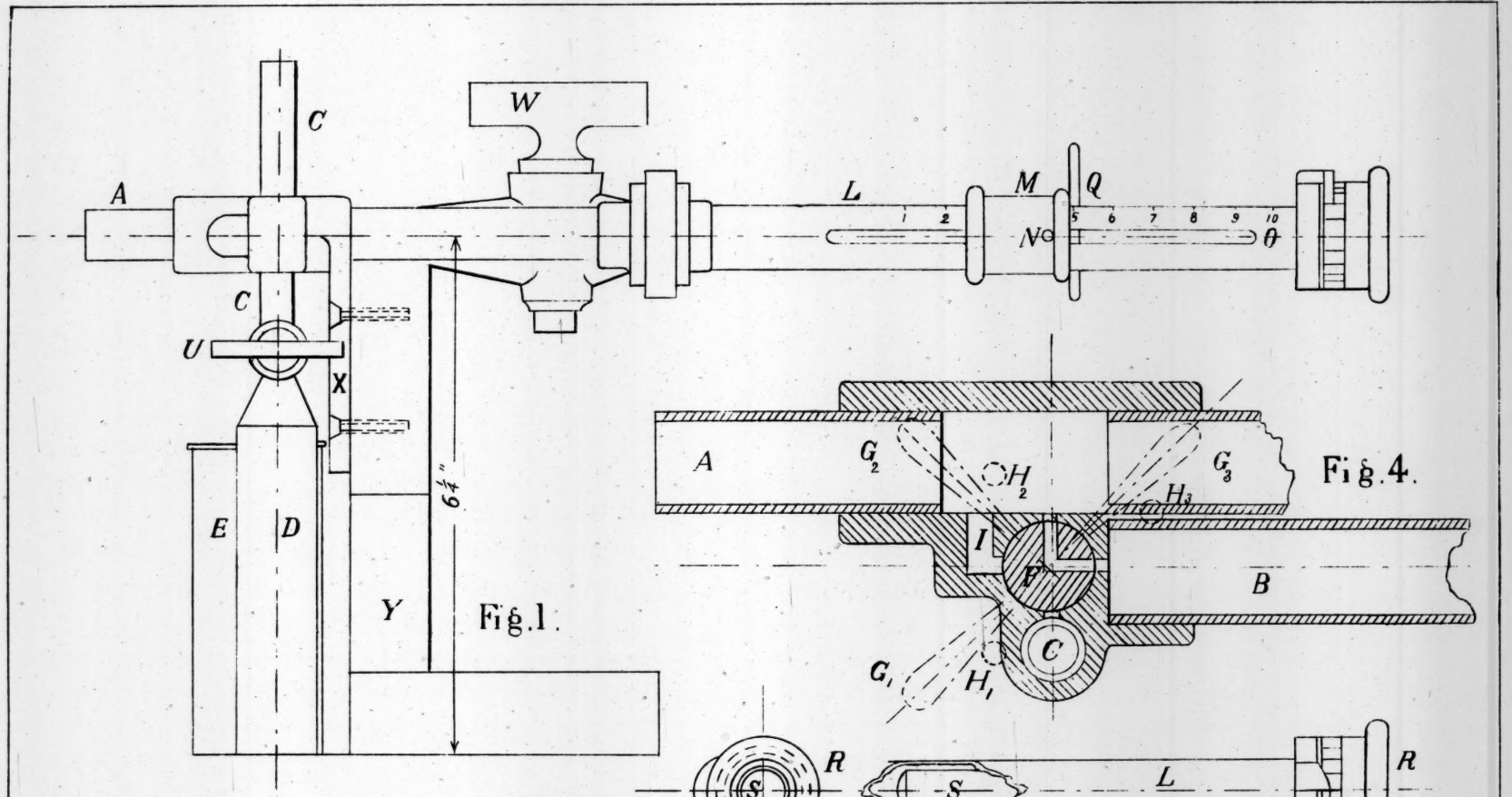


Fig. 1.

Fig. 4.

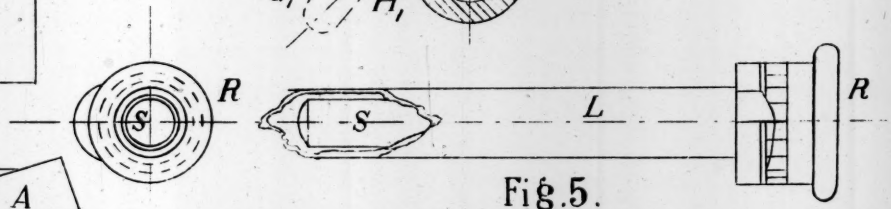


Fig. 5.



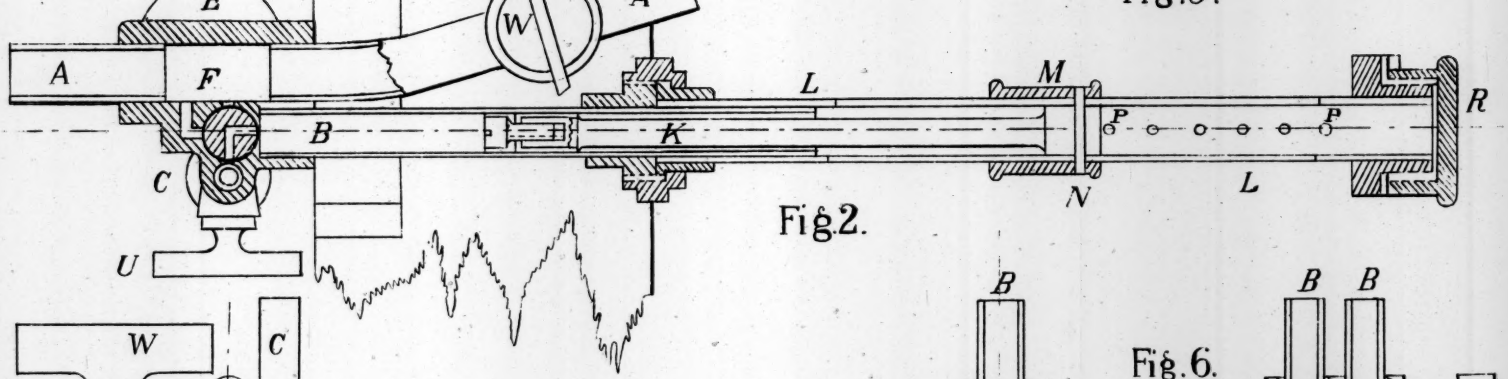


Fig.2.

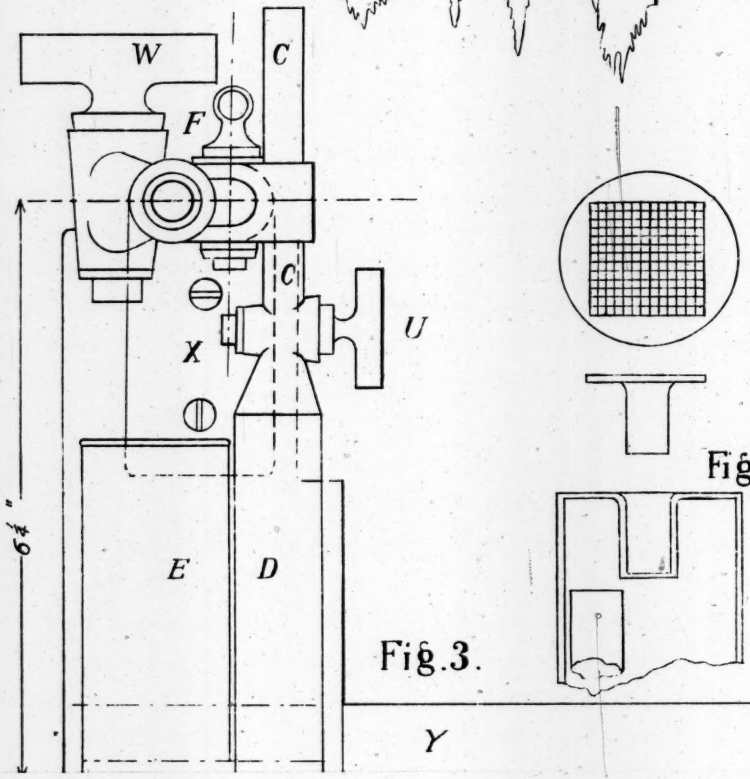


Fig.3.

Fig.7.

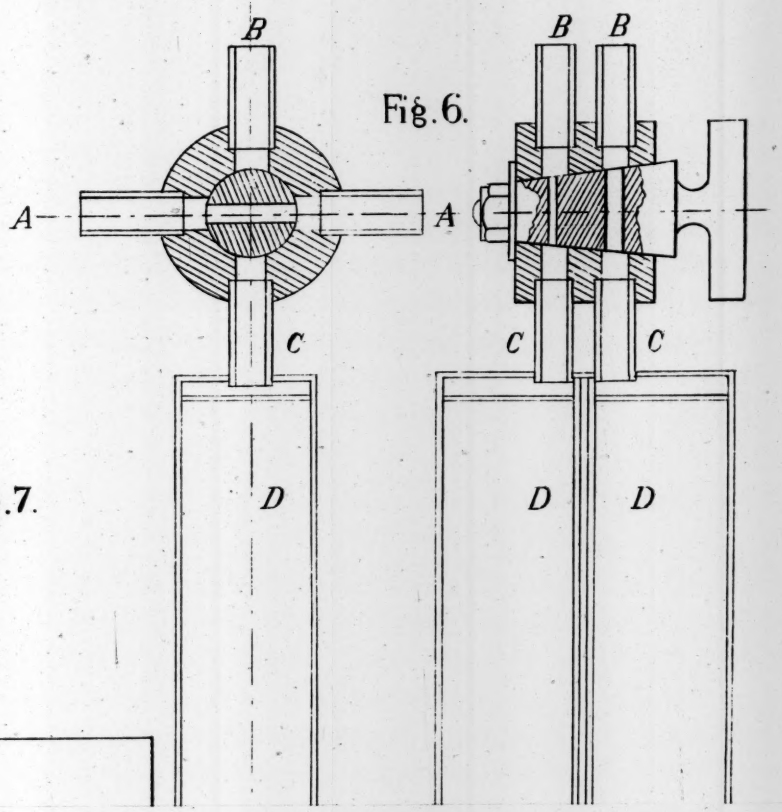


Fig.6.

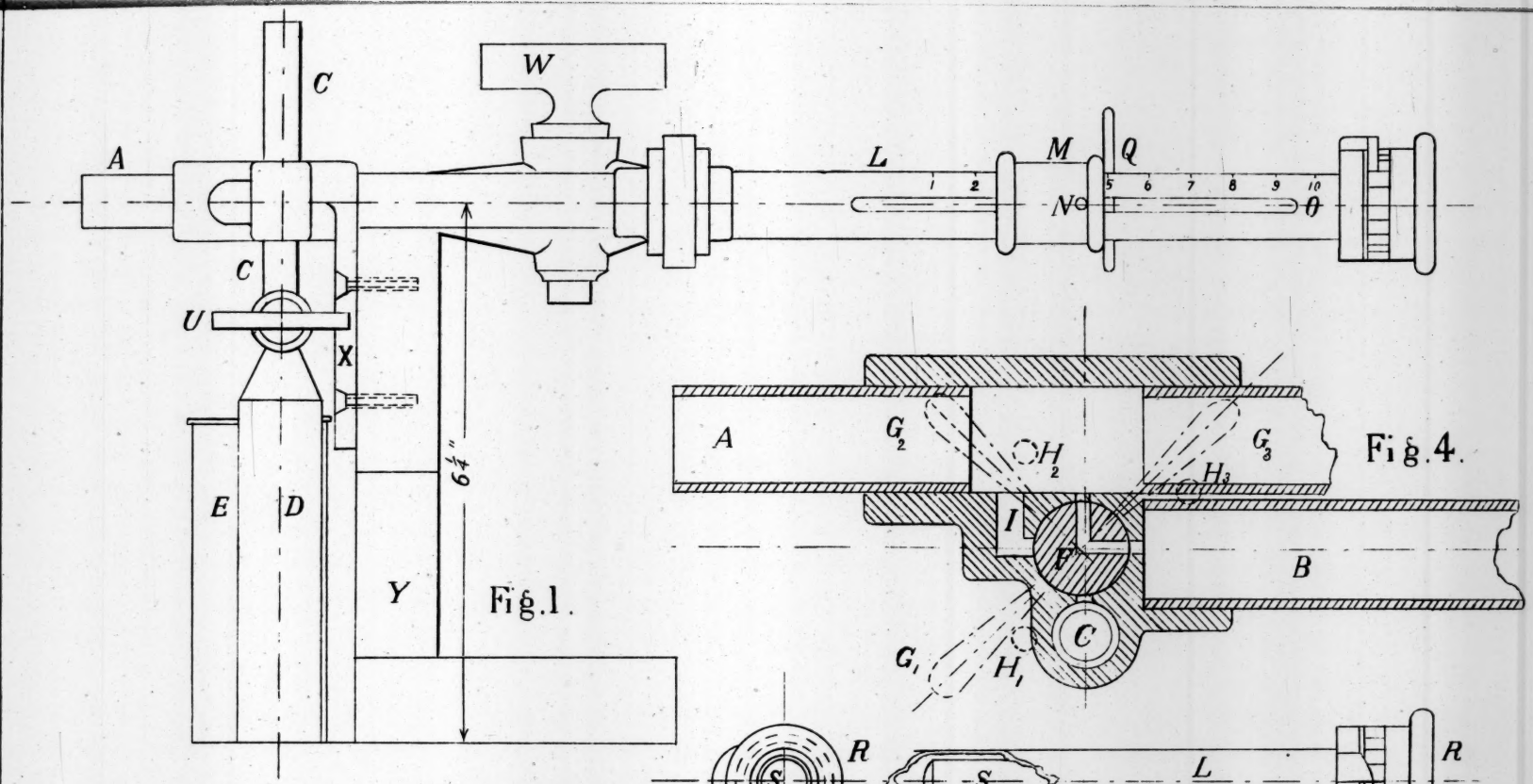


Fig.1.

Fig.4.

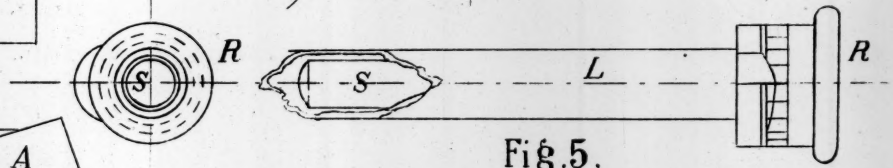
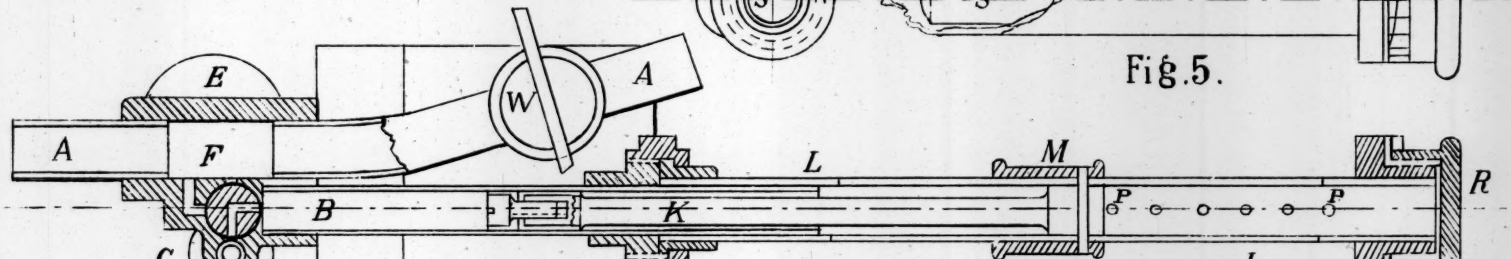
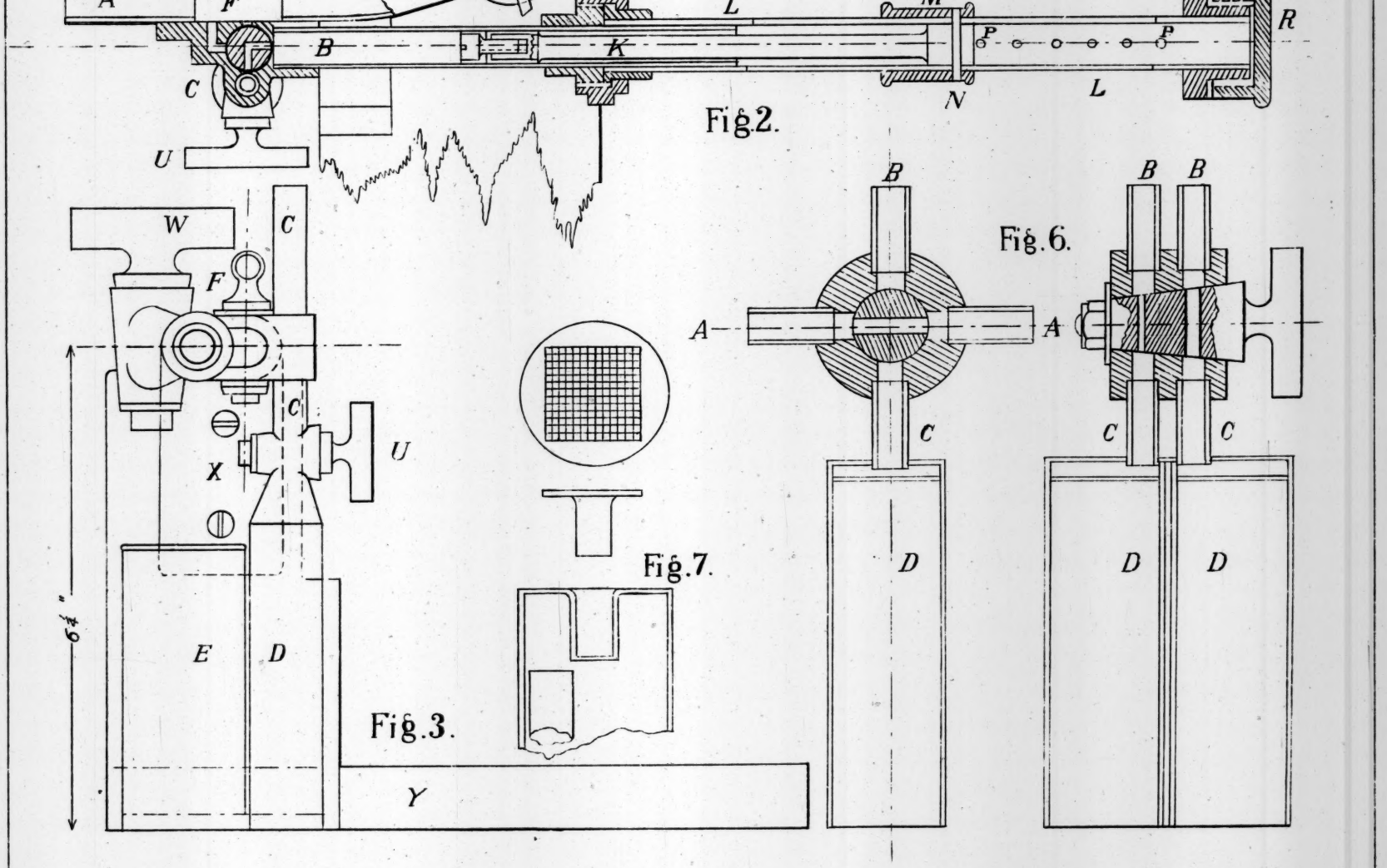


Fig.5.





J. Aitken, del.

McFarlane & Erskine, Lith.rs Edin.r

J. AITKEN ON DUST PARTICLES.

Turning now to the first plan, fig. 1 is an elevation of the apparatus, fig. 2 a plan in section, fig. 3 an end elevation. These are all drawn to a scale of half size. Fig. 4 is a full size section of an important part of the apparatus. The letters in the different figures all correspond. AA is a pipe through which the air to be tested is caused to circulate, B is the measuring cylinder, CC is a pipe connected at its lower end with the chamber D and a filter E. When in use CC is connected at its upper end with the test-receiver by means of a short length of indiarubber tube (*see* Plate I.). F is a stopcock situated, as shown, between A, B, and C. The plug of this stopcock has the passage through it bored at right angles as shown, and not straight through, as is usually the case. The working of this arrangement will be most easily understood from an examination of the full size drawing, fig. 4. It will be seen that when the plug is in the position drawn, the air-pump B is in connection with the pipe carrying the air to be tested, and if the piston be drawn out the cylinder will get filled with this air. If the stopcock be now turned one quarter round, the pump will be put into connection through the stopcock with the pipe C; and if the piston be now pushed down, the air in the cylinder will be sent by the pipe C into the chamber D, its passage upwards being checked by the stopcock on the receiver. From D the measured quantity of dusty air is drawn up through C and carried into the receiver, its place being supplied by filtered air from E.

The cylinder B has a capacity of 10 c.c., and by means of it we can measure any quantity of air from $\frac{1}{10}$ c.c. to 10 c.c. When more than 10 c.c. of dusty air requires to be sent into the receiver at one time, then the plug of the stopcock is turned one quarter farther round. By this means the pipe C communicating with the receiver is put into direct connection with the pipe A, and the air to be tested can be drawn direct from A into the receiver. The method of measuring when the plug is in this position, and when quantities larger than 10 c.c. are required, will be explained further on.

The points to be aimed at in the construction of the part of the apparatus shown at fig. 4 are—first, to have the plug of the stopcock as small as possible with the requisite tightness. As there is never much pressure on it, a great amount of "cover" is not necessary. The plug sketched is large enough. The object of having a small

plug is in order that the passage through it may be short, so that as little dust as possible may be lost by settling in the narrow passage. The second point to be attended to is to have the passages in the body of the stopcock joining A, B, and C as short as possible. The passage joining I is of no importance. The object of having these passages short is to ensure that the air in them shall be in the same condition as the air in the spaces into which they open.

In order that the stopcock F may be worked with ease and precision, three stops are provided for checking the movement of the handle when the ports are in the correct position. These stops are shown by the dotted lines at H_1, H_2, H_3, fig. 4. The first and last of these stops are fixed; the middle one is arranged so that it can be removed. When working with the cylinder measure, the handle G moves between the stops H_1 and H_2. When at H_1 the cylinder is in connection with A, and the air to be measured and tested is then taken in. When the handle is pushed up against the stop H_2, the passage between the cylinder and the pipe C is opened, and the measured quantity of dusty air can be driven into D. When we wish to allow the air to pass direct from A to the receiver, the stop H_2 is removed and the handle brought up against H_3, when the passage between A through I to C is opened.

Turning now to the method of measuring by means of the cylinder. It will be seen that it is provided with a piston which is fitted with two cupped leathers. Two leathers have been used, because the piston must be quite tight, and as pressures act both ways, it was thought best to use a cup looking each way. The tube L is securely fixed to the end of the cylinder by means of a movable screw joint, in the manner shown. This tube is slightly larger in diameter than the cylinder, and acts as a guide to the piston rod; it also carries the scale and the stops used in measuring. To the piston is fixed the piston rod K, which is moved by means of the collar M, which slides on L. The piston and collar are connected by means of a pin N passing through them. The pin N slides in the slots OO cut in the pipe L. The collar M is grasped by the finger and thumb, and when moved back and forward carries the piston with it. In the pipe L, at right angles to the slots OO, are a series of holes P . . . P drilled at certain equal distances apart. In the drawing the cylinder is supposed to have an area of 1 square

cm. If this be so, then the holes P . . . P are drilled at 1 cm. intervals, the first being at 1 cm. from lowest position of the piston. In the instrument I have constructed the cylinder has an area slightly less than 1 square cm. In graduating it, the length of cylinder required to hold 10 c.c. of water was carefully ascertained. This length was then marked on L, and divided into 10, and the holes bored. By this arrangement, when the collar is moved up one hole, 1 c.c. of air is taken into the cylinder; if moved up two holes, 2 c.c. are taken in, and so on. To enable a number of tests to be made quickly with exactly the same quantity of dusty air, a pin, Q, is put into one of the holes, P, at the desired capacity. This pin stops the outward movement of the collar at the correct position, and enables the successive measurements to be made accurately as well as quickly.

The whole measuring apparatus is supported by the wooden stand Y, to which it is fixed by screws through the flange X; and Y is securely clamped to the table under the test-receiver.

In working with this apparatus, one or two trials are first made to find out the amount of dusty air that will be most suitable for testing. Suppose we find 1 c.c. gives too few drops and 5 c.c. gives too many, we may select to work with, say, 2 c.c. The pin Q is accordingly put into the second hole, and testing begun. The handle of the stopcock F is put up against the stop H_1; the piston is then drawn out till stopped by the pin Q at 2 c.c.; the stopcock turned till it is stopped by H_2, the piston pushed down, and the 2 c.c. of air sent into C. From this it will be seen that very little time is required to measure the air; practically only the time required to pull out and push back the piston, and for each succeeding test, of which perhaps ten may be made to get a good average, exactly the same amount of air is measured and sent into D. After the air is in D it loses but little of its dust, as it has not time to fall down through the long vertical column of dustless air underneath it before it is carried into the receiver.

By working in the manner above described, we can by means of the cylinder and piston measure accurately any quantity of air from 10 c.c. down to 1 c.c., and even to $\frac{1}{2}$ c.c., if a hole is drilled in the right place. But if we wish to measure smaller quantities, then we must use the part of the apparatus shown at R, which is

simply an arrangement for moving the piston accurately to short distances by means of a screw. The male screw is fixed to the end of the tube L, while the nut closes the end of the tube, as shown. This screw should have a very wide pitch. One turn of the screw should advance the piston at least 2 mm.—that is, according to size of cylinder, should push the piston forwards sufficient to displace $\frac{1}{5}$ c.c. The object of the wide pitch is to enable the movement to be made quickly. The nut is provided with a projection which comes against a stop on the male screw at the point furthest in, as shown in fig. 5. The circumference of the nut has a scale engraved on it. The rod S fits easily inside the tube L, and presses against the closed end of the nut. This rod should be of such a length as to prevent the piston being drawn out more than $\frac{1}{2}$ or 1 cm. The reason for using the rod S, and not allowing the piston rod to be drawn out to the end of its stroke, and there come into contact with the screw, is, that the great amount of air in the long length of the cylinder might be so much influenced by change of temperature as to make it impossible accurately to measure very small quantities. When using this part of the apparatus the rod S is put in its place, and this reduces the stroke of the pump to less than 1 cm. Before drawing out the pump, the screw R is turned back from its stop till the index points at the quantity we wish to measure, say 1, on the scale. The piston is then drawn out till stopped by the rod S, and after the stopcock is turned the screw R is turned till its movement is arrested by its stop. By these movements the desired quantity is rapidly and accurately measured, and little time is given for the dust to settle in the apparatus. After the quantity of air is measured in this way, the stopcock F should be turned before the receiver stopcock is opened; and before making the next test the cylinder B should be emptied and a fresh charge taken in.

The tightness of the piston in the measuring cylinder should be tested from time to time, because it is evident that any leakage will give rise to errors in the number of dust particles counted. The manner of testing is as follows:—The stopcock F is put up against the stop H_1, and the air in the receiver is thoroughly purified. The stopcock is now turned against the stop H_2, and two or three strokes of the measuring piston made. By these movements filtered air is taken out of C, and again returned. If the piston is not quite tight,

some dust will have got into the air in C, which will show when this air is sent into the receiver and expanded.

Having explained the action of the apparatus when measuring quantities from a fraction of a cubic centimetre to 10 cubic centimetres, I shall now explain its action when larger quantities have to be measured. In this case the pump connected with the receiver is used as the measure. When dealing with large quantities of dusty air, the first thing to be done is, as before, to purify the air in the receiver. To do this the stopcock U is opened, and F has its handle against the stop H_1. After the air is purified the stopcock U is closed, and F brought up against H_3. The receiver stopcock is closed, a stroke of the pump made, the piston pushed back, and the stopcock W closed. On now opening the receiver stopcock, air from A rushes in, and from the diameter of the air-pump and the length of stroke we have made, we can calculate the number of cubic centimetres of air that have entered the receiver. We shall, however, refer more particularly to this method of working later on.

We now come to the second plan of measuring the dusty air. This is shown in Plate III. fig. 6. As will be seen, it consists simply in the use of a four-way stopcock as a measure, and the quantity measured depends on the capacity of the hole in the plug. The stopcock shown in the figure is the smallest of the set belonging to my apparatus. As it was to be used for small quantities, two measures of different capacities were put into the one plug, but the larger measures have only one. The air to be tested is drawn through AA. The port B is connected with the test-receiver, while the port C is attached to the filter D. In using this apparatus one of the ports A is connected by means of a pipe with the source of the air we wish to test; the other is attached to an aspirator. When the plug is turned so that the bore is horizontal, as shown in left-hand figure, the aspirator draws the air to be tested through the stopcock, and the bore gets full of the air we wish to test. While this is taking place, one stroke of the receiver air-pump is made. When the measuring stopcock is now turned with the bore vertical, as in right-hand figure, the air rushing up from the filter carries the plug-ful of dusty air along with it into the receiver. Knowing the capacity of the plug, we know the amount of dusty air that has entered the receiver. In the measuring stopcock sketched, the larger

bore has a capacity of $\frac{1}{5}$ c.c., the other $\frac{1}{20}$ c.c. The others of the set have capacities of 1 c.c. and 5 c.c.

There is a pipe, not shown in sketch, connecting A to A, outside the stopcock. This pipe is of smaller bore than A, and its object is to allow a constant current of air to be kept flowing through the pipe while the measuring stopcock is closed in that direction. This pipe is most easily connected a little on each side of AA.

As to the relative advantages of the two methods of measuring the air, the cylinder plan has the advantage that the one apparatus measures all quantities without any change in the arrangements; and further, it does not confine us to certain fixed quantities. It also enables us to check our results, by using twice or any suitable multiple of the dusty air, and for experimental purposes it seems to be the best. On the other hand, the stopcock plan seems to be the most accurate, as, practically, no time is given for the dust to settle in the measure. If the air to be tested is very impure, so that the quantities to be measured are very small, such as $\frac{1}{10}$ c.c. and under, and if the impurity of the air does not vary much, then the stopcock measure is the best. It is also much more easily worked than the other, fewer movements being required for each test. On the other hand, if the quantity of dust varies much, it necessitates a change of measure to meet the changing conditions, a large stopcock being necessary when the air is pure, and a small one when dusty.

In gauging the bore of these stopcocks, for the large sizes I have generally used measurement and checking by gauging with water. The small ones have been gauged by means of wires. The bore was first drilled a little smaller than it was ultimately to be. From this bore the length was got which enabled the proper diameter to be determined. The following plan of determining the diameter was found of easy application:—Take the case of the stopcock sketched in Plate III. The length of the larger of the two bores was 2 cm., and it was required to have a capacity of $\frac{1}{5}$ c.c. From a bundle of steel wires of different sizes one was selected, the volume of which was 1 c.c. per 10 cm. of its length. This was ascertained by selecting a wire which, when dipped into a burette nearly full of water, displaced 1 c.c. per 10 cm. of its length. The bore in the plug of the stopcock was then made to fit this wire exactly, and, as the bore is 2 cm. long, its capacity will be $\frac{1}{5}$ c.c.

The Air-Pump.

Turning now to the air-pump used for producing an expansion in the test-receiver, and shown at P, Plate I. The one used has a capacity of 150 c.c. There is no necessity for the pump being exactly of that size, but its capacity should bear a certain proportion to the air-contents of the receiver, and be such as to make the necessary calculations as simple as possible. It may be thought that the pump is rather large, and that we might with advantage reduce the size of both pump and receiver. But it must be remembered that if we reduce them, then the measuring apparatus must be reduced also; and as the measure is reduced to as small dimensions as is thought advisable, there would require to be some good reason for change in this direction. No doubt, with a pump large compared with the receiver, the degree of expansion is more trying on the joints and stop-cocks than if a smaller pump were used; and there is also a tendency to spontaneous condensation when the expansion produced by a large pump is employed. Yet, in spite of these disadvantages, it is thought desirable to use a large pump, as the high expansion gives a great fall in temperature, and thus causes the raindrops which descend on the stage to be larger and more easily counted than if a smaller pump were used.

With the pump of a capacity of 150 c.c., and the receiver a capacity of 350 c.c., the calculations are easily made. Whatever number is counted on the stage per square mm. requires to be multiplied by 100 to get the number per c.c. in the air of the receiver, and this multiplied by 500—that is, by the contents of the receiver and pump—gives the total number in the measured quantity of air sent into the apparatus. Suppose, for instance, that 1 c.c. of dusty air was measured and sent into the receiver, and if we counted 2 drops per square mm., then as there was 1 cm. of air above the stage, there would be 200 drops per c.c. of the air in the receiver; but as the original 1 c.c. of air was mixed with 350 c.c. of pure air, and expanded to 500 c.c., there will have been $200 \times 500 = 100{,}000$ dust particles in the original 1 c.c.

If we had mixed only $\frac{1}{10}$ c.c. of the dusty air, and got the same number of drops, then the number of particles would have been ten times greater, or 1,000,000; but if we had used 10 c.c. and got the

same number of drops, then the number of particles would have been one-tenth, or 10,000.

We can test air having any of these quantities in it with the small cylinder measure. But suppose that, with 10 c.c. of the dusty air, the particles are too few for easy counting, then we will require to send more than 10 c.c. into the receiver. To do this, the stopcock F is turned against the stop H_3, the stopcock of the receiver is closed and a stroke of the pump is made; this takes a certain quantity of air out of the receiver, and, on opening the receiver stopcock, air from the source to be tested rushes in to supply its place. This is mixed with the air in the receiver, expansion is made, and the drops are counted, all as usual. If the number of particles has not been too great, all of them will have been thrown down, and the receiver stopcock may be again opened, and another quantity taken in, mixed, expanded, and counted, and so on till the required number of tests has been made to give a good average.

We must now know the quantity of dusty air taken in at each of these tests before we can calculate the number of particles in the air. The capacity of the receiver is 350 c.c. and of the pump 150 c.c. The pump therefore takes out $\frac{1}{3{\cdot}33}$ of the whole air of the receiver—

$$\frac{350}{3{\cdot}33} = 105 \text{ c.c.}$$

Practically only 100 c.c. of air is required to replace that taken out by the pump. The reason of this is, the theoretical quantity does not really come out with each stroke of the pump, owing to the pressure required to open the pump valve preventing the escape of the last 5 c.c., and probably partly owing to the expansion producing a greater cooling effect on the larger mass of air in the receiver than on the smaller one in the pump,—the smaller mass more quickly recovering its heat from the enclosure.

When working with the apparatus in this way, at each test there is taken into the receiver 100 c.c. of the air to be tested; this is mixed with 250 c.c. of pure air, and the whole is then expanded to 500 c.c., so that the number counted on the stage, in this case, requires to be multiplied by only 5 to get the number of particles in the air tested. It may be as well to note that, after making the stroke of the pump, the piston must be pushed back again; other-

wise more than 100 c.c. of air will be taken in, and an unknown quantity will pass out into the pump.

But suppose that the particles are still too wide apart, or that we wish to check our results, and we desire to get a greater quantity of dusty air into the receiver, then all that is necessary, after making the expansion and counting the drops, is to open the receiver stopcock, as before, to allow the air to enter, and then work the pump for some time, say making fifteen or twenty strokes; in this way we can clear the purified air out, and fill the receiver entirely with which we wish to test. When working in this way, the numbers counted on the stage require to be multiplied by 1·43, to allow for the effect of the expansion.

We have seen, then, how we can, by means of the small cylinder, measure any quantity of the air to be tested from $\frac{1}{10}$ c.c. to 10 c.c.; and also how, by means of the air-pump, we can measure 100 c.c., or cause the receiver to be filled entirely with the dusty air. There is evidently, however, too great a step between the largest quantity measured by the small cylinder measure and the contents of the large pump—that is, from 10 c.c. up to 100 c.c. To bridge over this interval, the air-pump has a scale attached to the piston-rod, and sliding with it. On this scale is marked, not the quantity displaced by the piston, but the quantity taken out of the receiver by the pump, when the receiver stopcock is closed. When provided with this scale, we need not always take in 100 c.c.; but, by making a partial stroke, we may take in, say 50 c.c. of dusty air, or any other quantity we may desire. In this way a perfectly graduated scale is obtained, and we can send into the receiver any measured quantities of dusty air we may desire from $\frac{1}{10}$ c.c. to 100 c.c., or even fill the receiver full of the air to be tested.

It may be asked, Why not graduate the pump to actual displacement, and open the receiver stopcock before making the stroke? The objection to this plan is, that some of the dusty air would escape with the pure air while the stroke was being made, and we should not have any means of knowing how much was lost.

Another plan for measuring large quantities of air with the apparatus described has occasionally been used for check experiments, but it is more troublesome to work. In working by this plan, the pipe T, for supplying water to the receiver, is connected with a graduated vessel full of water. By the use of the air-pump

the water is made to flow into the receiver; when the desired number of c.c. have left the vessel and entered the receiver, the receiver stopcock is opened, and the water flows out and draws in the dusty air, the stopcock T being closed when the water returns to its original level. Which of these plans is the best I leave to the operator; but it is always well to vary one's way of working, as it tends to keep us free from errors, and for this reason alone this method may be occasionally used.

The Gasometer.

The gasometer sketched on Plate I. is of the usual construction. The only points specially attended to are a nice balance, ease of movement, and an accurately graduated scale. The scale is in litres and percentages. The gasometer has a capacity of 20 litres. Attached to the gasometer is the filter F, and inside the gasometer is a stirrer for mixing the filtered and unfiltered airs. The gasometer has a number of duties to perform. It is used as an aspirator for circulating the air through the measuring apparatus. It is used for testing very dusty air, such as that from flames, when the smallest quantity measurable by the apparatus described would give a cloud of particles far too dense for counting. The impure air is drawn into the gasometer mixed with a known quantity of filtered air, and sent through the measuring apparatus to be tested in the usual way in the receiver. The gasometer has also been found very useful for making check experiments, and for trying the working of the different parts of the apparatus. For instance, in comparing the results given by different ways of working, it was soon found that the number of particles in the outside air was far too variable to make it suitable for experiments of this kind. Air with the desired amount of dust was, therefore, put into the gasometer and thoroughly mixed, and the tests were made with it. Allowance, of course, had to be made for the gradual diminution of the dust during the experiments, and errors from this cause checked by working the different plans alternately a number of times.

Illuminating the Stage.

The stage may be illuminated, as shown in Plate I., by means of a gas flame; when gas cannot be had, a paraffin lamp does very well. A screen, with an opening opposite the flame, should be

used to shield the observer as much as possible when a strong light is employed. For concentrating the light on the stage a globular flask full of water may be used, or a short focussed glass lens, such as the condenser of a magic lantern, may be employed.

General Remarks.

In beginning work with the apparatus, the first thing to be attended to is the counting stage. If bright, it is carefully dusted only, but it is generally polished every time before use. This is done either with a buff wheel with rouge, and finished on a woollen wheel, or it is rubbed on a piece of chamois leather stretched on a flat surface, a very little rouge being used, and the polishing being finished with a clean piece of leather. In polishing the stage, the rubbing should be all done in straight lines along the engraved lines on the plate, as previously directed; it need not, however, be always along the same lines. When properly polished, the plate should have a brilliant black appearance, if one may use the expression; this enables the counting to be easily done. The rubbings must on no account be done circularly, as is usual in polishing silver, nor must it be done in straight lines at any considerable angle to the engraved lines, or the rubbing marks will be distinctly visible, destroying the perfect blackness of the mirror, and making counting difficult. To save the frequent polishing required by silver, platinum and other metals are now under trial.

Having polished the mirror, and been careful to keep its surface free from dust specks, we put it into its socket in the top of the supporting pipe D, and D is put into its place in the receiver. The stage is then adjusted so that the corners of the squares point to the light. In that position it should look quite black, and the lines be distinctly visible on it. It is probable that all the little squares will not be perfect. Owing to imperfections in the metal, &c., little specks will be visible in different places; but as there are more than one hundred squares, the most perfect are selected for observing and counting the drops. The stopcock on the entering pipe O is now connected with the measuring apparatus and closed. The stopcock T is opened, and the pump worked till the water entering by T rises to the mark in the receiver; T is then closed. A slight circular shaking is given to the receiver to cause the water inside to

wet the sides of the receiver and the paper covering the top along its outer edges. If the outer edges get wet, the water soon spreads all over it. The necessity for an occasional wetting inside explains why the receiver is connected with the measuring apparatus by means of a piece of indiarubber tubing. The disc I can be easily wetted from time to time by drawing it down into the water beneath it. The air in the receiver is now purified by pumping in air through the filter, and finally by expansion and consequent throwing down of particles by showers of rain, filtered air being admitted after each shower. If all the joints are air-tight, condensation rapidly ceases, and after a few showers not a drop will be seen falling when expansion is made. The counting stage should now be attended to. If it is dewed, then the mouth of the operator should be applied to the tube U, and warm air blown through it. This rapidly warms the stage and clears it. If the stage should be troublesome to keep clear of dew, it is an indication that the water in the receiver is too cold; that is, too cold for the temperature of the upper part of the receiver. Some water ought therefore to be run out, and some slightly heated water added to that in the receiver. Too much hot water must not be added, or the drops rapidly evaporate before they can be counted. The best condition seems to be when the stage tends slightly to get dewed, and requires an occasional blow through the tube U to clear it. In that condition the drops remain visible some time, and are easily counted.

Supposing the air to be perfectly pure, and the stage in good working order, we may then proceed to testing the air. The first thing done, after seeing that the air is circulating through the apparatus, is to close the receiver stopcock, and then make one stroke of the air-pump, and push the piston back to the bottom position. A quantity of the air to be tested is then measured, say by the small cylinder measure, and sent into D. The receiver stopcock is then opened, and the air rushing in from the filter, carries the dusty air with it into the receiver. After this is done the receiver stopcock is closed, and the stirrer I is rapidly moved up and down two or three times. This is done by grasping the lower end of the indiarubber tube M and moving it up and down; the indiarubber tube expands and contracts, and allows the stirrer to be worked as easily as if it were not inside an air-tight receiver. The

length of the tube M is such that its elasticity keeps the diaphragm I at its top position, and against the stop R. The pure and impure airs having been thoroughly mixed, expansion is made, a shower of rain produced, and the drops counted.

We must now consider what is the best quantity of dusty air to be sent into the receiver for making a test. When beginning a new test, we have to be guided by experience as to what is likely to be the correct quantity. Suppose we think that 2 c.c. will be enough, that quantity is accordingly measured and tested, and from the density of the condensation produced by that quantity, we get an idea as to whether it is too little or too much. But what is to too little and what too much? A little experience soon settles this point; but I may state that if more than 5 drops fall per square mm., there is too much dust, not only because when the number is much above 5 there is a difficulty in counting them before they evaporate, if the stage be slightly hot, but also because, with so large a number, we cannot be quite sure that all the particles have been thrown down. Suppose, for instance, that 10 drops fell per square mm.; if we now admit only filtered air, and again make an expansion, we shall find that some drops will make their appearance, showing that some particles have escaped the first condensation. It has, therefore, been the practice to limit the maximum number of drops to 5 per square mm. With that number no drops appear on a second expansion being made. The lower limit, however, is not so definite; there is nothing in the conditions limiting us here; it is simply a question of convenience; 1 per square mm. makes a fairly good lower limit. When working with that number, I generally use 4 squares instead of 1. By taking the number that falls on the 4 squares, we get a better average. The number so obtained is multiplied by 25 to get the number per c.c. We are not, however, limited to 1 or even 4 squares, occasionally 9 squares have been used, and the number that fell on these 9 observed. Working with so large a surface requires some care. We have first to select a part of the stage where there are 9 squares all perfect and spotless. The eye is steadily kept on the square of 9 small ones, and a little practice enables us to count the number that fall on that area. I need not say that 9 squares are only used when the drops are very few, say 6 or 7, over the whole area; if more fall,

then we had better restrict our attention to 4 squares. For general work, however, 5 per square mm. is the top, and 1 per square mm. the lower limit. It is for this reason that in the set of stopcock measures described, their capacities have been fixed at something like this proportion. The sizes of these measures are 5 c.c., 1 c.c., $\frac{1}{4}$ c.c., and $\frac{1}{20}$ c.c. The second last one is not $\frac{1}{5}$ of the next largest, but it is of a size that makes the calculations easy. The largest measure requires the number per c.c. counted on the stage to be multiplied by 100, the next by 500, the next by 2000, and the last by 10,000.

It may be as well here to call attention to the necessity of making all joints in the apparatus perfectly air-tight, and all stopcocks must be of first-class workmanship, as it is found that any leak that allows air to pass, also allows dust to get in along with it. No hole between metal surfaces seems to be too small for dust to pass through, or is it that no hole that cannot pass dust will pass air, owing to the dust particles sticking and closing the passage to the air?

I shall now refer to a precaution necessary in working this apparatus. If the air in the place where we are working is damp and nearly saturated, care must be taken in opening the receiver stopcock. It must be done very gradually, so that the air may enter slowly. If opened to the full extent suddenly, the dusty air in the entering pipe is expanded, condensation takes place on the dust, the drops formed in the pipe are driven violently by the rush of air, and many adhere to the surfaces with which they come in contact. This effect only takes place when the air is very damp, otherwise the expanded air is not cooled to its condensing point before entering the receiver. This condensation in the entering pipe is greatly due to the resistance of the cotton-wool filter preventing the air rushing in to supply the partial vacuum. A filter of large area is therefore desirable as a prevention of errors of this kind; but as even a large filter will not entirely prevent condensation, a little care must always be taken to open the receiver stopcock slowly when the air is damp.

The difficulty above referred to was met on the first occasion on which I made tests away from the laboratory. These tests were made in winter in an outhouse, and when the atmosphere was very damp. In a great number of tests the numbers counted on the stage varied to an amount far exceeding any previous experience.

At first it was difficult to say whether the difference in the numbers counted was due to a real difference in the impurities of the air. A greenhouse fire, at a considerable distance to the windward, suggested it might be so, and on some occasions, no doubt, there was a real difference due to this cause; but as the great variation continued after the wind changed, another explanation had to be found. It was then noticed that the manner of admitting the air to the receiver had an influence. After repeated trials, it was found that if the air was admitted with a rush far fewer drops were counted than when it was allowed to flow in slowly; but which, if either, of the numbers counted under these two conditions was the correct one? It had previously been observed that if the air was allowed to rush in, and in its passage strike the surface of water, nuclei were manufactured, and generally that violent currents of air have a tendency to produce centres of condensation. But in the case above referred to, the slowest moving air gave the greatest number of drops. Were nuclei formed under some unknown conditions? or was it possible that when the air enters with a rush the expansion in the entrance tube is sufficient to cause condensation before entering the receiver, if the air is moist? To settle this point, the method of working was altered. The air was not allowed to go into the receiver immediately after expansion was made, but water was allowed to enter by the pipe T. The stopcock by which the air enters was then opened full, and the outflowing water drew the air slowly into the receiver, through wide open passages and without a rush. By this plan we could trust that no particles were formed by the movement of the air, and no expansion produced in the entrance pipe sufficient to cause condensation in even nearly saturated air. When tried, it was found that the numbers obtained by this plan of working corresponded with those got when the air was admitted slowly. The numbers, therefore, obtained when the air was allowed to rush in were too small, and the smallness must have been due to the air losing some of its dust in the process by forming raindrops while still in the entrance pipe.

The time required to make a test need not be great; indeed, considering the numbers we have to deal with, we may say it is very short. To put the apparatus together and ready for work need not take more than five minutes, more or less, according to the expertness of the operator. After it is in working order, half a minute is quite

enough for one test; so we can make ten tests to get a good average in five minutes, or, let us say, a quarter of an hour for everything.

Results Obtained with the New Apparatus.

In my first communication on this subject, there was given a table in which is entered the number of particles in the atmosphere of this district, and the number in air polluted with the products of combustion. These numbers have been checked by means of the new apparatus, and found to be practically correct. For air from a Bunsen flame, different numbers have been obtained, depending on the manner of collecting the products; but the one entered in the table is not the highest that has been observed. For the air of the laboratory the results are very variable, but those given in the table are not a bad average, and the same may be said of the numbers given for the outside air.

Confining our attention now to the outside air, a great number of tests have been made here, and the condition of the air has been found to vary greatly, the smallest number observed per cubic centimetre is 11,000; frequently there are 50,000 to 70,000 particles per c.c., and as many as 140,000 per c.c. have been counted.

These numbers are very great, and one naturally asks the question, How many of these particles are what we might call natural, and what proportion is due to artificial causes? The situation, where these tests were made, is what might be called "in the country." It is quite outside of the town of Falkirk. It is, however, surrounded on many sides by public works, and in most directions there are villages at greater or less distances. It is, therefore, very evident that the air at this situation must be very much polluted by artificial causes. I was constantly reminded of this while making my tests. At no great distance there is a railway, and when an engine passed, if the wind was from that direction, the particles would suddenly increase in number, and for a short time become so great it was impossible to count them.

Now that the apparatus was in a satisfactory working condition, it seemed desirable that the investigation should be extended, and tests be made of the air at other places. The first point to which attention was given was to find out what the number is in air under natural conditions—that is, free from all artificial pollution. For

this purpose a situation was sought which, while it could be easily got at, would yet be as far removed from human habitations as possible. Attention was afterwards directed to the condition of the air in densely populated areas.

The best set of observations I have yet obtained, in what may be called pure air, were taken at Colmonell, in January of this year. During my visit of a few days there were examples of most types of weather. I may mention that Colmonell is a small village in the south of Ayrshire, and is situated in a pastoral district, which is very thinly peopled. During most of the time of my visit the wind was southerly, and the air tested must have been fairly free from artificial pollution, as it had travelled from 20 to 40 miles —according to the direction—over the bare and uncultivated hills of Wigtown and Kirkcudbright before it arrived at the testing place.

Along with this paper is given a table showing the results of recent observations. In the table is entered the place of the observation, the date, and the hour when the tests were made. The results are given in numbers per cubic centimetre, and also per cubic inch of the air—the state of the weather at the time being entered in the last column of the table. By a reference to the table, it will be seen that the Colmonell observations show the air of that situation to be very free from dust, and that the air near Falkirk is very highly polluted by artificial causes.

We shall now discuss the Colmonell observations in detail. It will be seen from the table that the morning of the 7th, when the observations began, was foggy. This fog was general all over the country. The number of particles counted at 10 A.M. on that day was 5350 per c.c. As the day advanced the fog cleared away, and the number of particles fell to 2500, or about one-half of what it was in the morning. Next morning (the 8th) was dull and wet, and the numbers went up to about what they were on the morning of the previous day. The day continuing very stormy and wet, the air became very thick. On testing at 5 P.M. the dust particles were found to have increased, and the number now was 9500. On the 9th but few observations could be made, owing to the direction of the wind bringing the smoke of the village in the direction of the testing place. On the morning of the 10th there was a slight shower; the air was clear, and the sun bright. By 12 o'clock the great impurity

of the previous day had greatly decreased, and the number now counted was 1650. The day continued fine, with transparent air and bright sun; and when tested at 1 P.M. the number was only 500 per c.c. After this hour the tests were unreliable, owing to change of wind bringing the smoke towards the place of observation.

The night of the 10th was frosty; but the next morning opened dull, and the air was thick. The number of particles was found to be 4600 per c.c. The day got stormy and duller, the thickness in the atmosphere increased a good deal, and when tested at 3 P.M. it was found to contain 9250 particles per c.c. The air got clearer towards night, and at 11 P.M. the number was reduced to less than one-half. This clearing continued, and next morning the air was quite clear, and the number of particles was down to 625 per c.c.

From an examination of these Colmonell observations, it will be noticed that on these occasions there was a direct connection between the transparency of the air and the number of dust particles in it. When the air was clear and transparent, the number fell to 500 or 600 per c.c., when it was thick the number rose to 5000 per c.c.; and when it was very thick and hazy, the number was nearly double that. It will also be seen from the table that rain had little or no effect on the occasions these tests were made. The 8th was a wet, stormy day, with thick air; and the number was as high as on the 11th, when there was no rain, and the air not quite saturated.

We cannot draw any very satisfactory conclusions from these Colmonell observations, as they are far too few for anything like a stable foundation. It was, however, distinctly observed while the tests were being made, that there was a direct relation between the numbers counted and the transparency of the air. Before making each succeeding test, it was found possible to say whether the number would be greater or less than the preceding, by simply observing the clearness of the distant hills.* It would, however, be rash to

* Though there was a close relation between the transparency of the air and the number of dust particles in it, during these Colmonell observations, yet it is evident that we will not be entitled to expect this relation to hold good under all conditions. The amount of vapour in the atmosphere, or rather the degree of saturation of the air, will have an effect on the *size* of the particles, more especially when the air is nearly saturated. We are, therefore, only entitled to expect this relation to hold good while the degree of saturation remains constant, which it probably did during these observations, the air being nearly saturated during most of the time.

conclude that all atmospheric haze is due to dust; yet these indications are worth keeping in view for future consideration. It seems possible that the "alpine haze," about which so much correspondence has lately appeared in *Nature*, may be due to dust. This, however, can be determined only by actual observation. There is another point of vast importance in the economy of nature, to which attention ought to be directed, which is this. If further investigation should prove that the transparency of the air is due to its freedom from dust, then we would be inclined to think that the diathermancy of the air will also have a close connection with the quantity of dust in it. These two points seem worthy of future investigation.

Our interest in the matter, however, does not end here. If dust should really prove to be a good absorber of the sun's heat, then it will also be a most important factor in the formation of fogs in another way than has been already pointed out. If the atmosphere was perfectly diathermanous, then probably we should have no fogs. Because the cooling of the air at night would be done entirely by the cold radiating bodies at the earth's surface; the air passing over these cold surfaces would be robbed of its moisture as well as of its heat, and would thus be prevented from becoming saturated. As fogs are formed during calm weather, they cannot at all situations be caused by the mixture of hot and cold saturated airs, but must frequently be produced by radiation from the atmosphere itself. Now, if dust is a good absorber, it will also be a good radiator; and, as a consequence, an abundance of dust in the atmosphere will at night cause it to be rapidly cooled to the dew-point, when a fog will begin to form, and by its formation increase the radiating power of the air. The radiating power of dust is probably one of the causes of the greater frequency of fogs in towns than in the country, there being far more dust in town than in country air. After condensation begins, there is plenty of evidence of the radiating power of the particles. A good example of this is given in a letter by Mr E. J. Lowe, in *Nature*, vol. xxxvii. p. 319. In this letter is described a remarkable rime. The peculiarity was that the thickness of the rime was greater high up than low down. At 5 feet the length of the crystals was $\frac{5}{8}$ inch, and they gradually increased to $1\frac{1}{2}$ inch at 25 feet. This increase in the deposit upwards was due to the fog

particles doing all the radiation, and the upper layers protecting the lower ones. The temperature on the grass was warmer than at 4 feet up.

On the 5th of January observations were made on the number of dust particles in the air at Ballantrae, a small village on the coast of Ayrshire. This place was selected with a view of testing the condition of the air resting on the sea. Though the day was one in every way suited to the purpose, the results are of little value. The wind was very slight, moving with but a gentle motion from the south-west—that is, towards shore—so that the air that had been resting on the sea was brought direct to the apparatus; yet the number counted was as high as 5000 per c.c., though the air that day was clear and bright. This large number was not caused by any artificial impurities, as the place of observation was close to the shore and within 50 yards of the water. The great number counted was evidently in part due to the action of the small waves breaking on the shore. Though very small, little more than a ripple, yet these waves manufactured a vast quantity of water particles, which, though they might dry, would yet leave a salt particle as residue.

One would scarcely have expected this result from such small waves, and we might have felt inclined to accept the number as correct for sea air. Owing, however, to the atmospheric conditions, I was able to observe that the very small waves were manufacturing vast numbers of particles. The day was very clear, with bright sunshine; and on looking along the shore-line close to the sea, the bright sunshine disclosed the existence of a perfectly distinct mist all along the shore, where the little waves were breaking. With a stronger wind, or a more rapid evaporation, these manufactured particles might not have been visible. For observations on sea air it will be necessary to select calm weather, and make the tests in a boat at some distance from shore. The numbers obtained at Ballantrae are therefore far too high for sea air at the time and place, and the impurity in the air tested there was probably salt particles. At sea, if there is as much wind as will break the surface, there will be manufactured vast quantities of salt dust, and the number of these particles will probably vary with the force of the wind.

During the past year a great number of tests have been made of

the air here with the new apparatus, but it is unnecessary to refer in detail to the result of these tests, because the air here is so contaminated by artificial causes, that the numbers, though indicating the condition of the air in this neighbourhood, are yet of little general interest. I have, however, entered in the table the least and the greatest numbers observed here.

A few tests were made of the air in Edinburgh, and the results are entered in the table. Those tests were made at the rooms of the Royal Society on the Mound. The pipe by which the air was taken in to the apparatus was carried to the outer air, through one of the windows, on the west side of the building facing Princes Street Gardens. The wind at the time was blowing towards the west side of the building. The result of these tests, as might be expected, show that the air in Edinburgh has a considerably greater number of particles in it than the air of this district. The first tests were made on 2nd February, and the last on the 5th. The numbers obtained on these two days are likely to be small for Edinburgh, because the air was clear and cold, with a strongish west wind, and heavy snow showers on the 2nd. On the 4th there was less wind, the air was not so clear, and the numbers observed rose considerably above what it was on the other days.

The apparatus was then removed to Glasgow, and tests made of the air there. The situation selected for these observations was in Bothwell Street, which is situated near the Central Station. There is a large open space in front of the selected position, and the air in this street will be a fair average for Glasgow. The first of the tests was made on the 8th of February. This was a remarkably stormy day. A strong north-west wind was blowing all day, with heavy snow showers. The lowest number obtained on that day was 170,000 per c.c. It is probable, owing to the high wind, low temperature, and dry air, that the air on this day was as free from dust as it is ever likely to be. Tests were again made on the 12th. The day was frosty, with westerly wind, sky clear, and air dry. The number counted on the 12th was double that obtained on the 8th. On the 16th of the month the air was again tested, and found to have even a larger number of particles in it than on the 12th. A much stronger wind was blowing on the 16th than on the 12th, and yet the numbers were highest on the windy day. This might possibly be due to

slight difference in the direction of the wind, but more probably to the dampness of the air loading the particles, and preventing them from rising so quickly on the 16th. The relative numbers for Edinburgh and Glasgow cannot be estimated from the figures in the table. The day when a large number was counted in Edinburgh was dull, with little wind; whereas all the Glasgow observations were made when there was more wind and the air clear. In a still damp day, or a foggy one, the number for Glasgow will probably be many times greater than any yet obtained.

The next set of observations entered in the table refer to some tests made in the Meeting-Room of the Royal Society on the evening this paper was read. The first observations were taken shortly before the meeting began at 8 P.M. The air was tested at a height of about 4 feet from the floor, and also near the ceiling, the air being drawn down through a pipe by means of the gasometer. Near the floor the air before the meeting began had 275,000 particles per c.c., or very little above that of the air outside on that day. Near the ceiling the number was 3,000,000. After an hour and three quarters the numbers were found to have increased to 400,000 near the floor, and 3,500,000 near the ceiling. These tests were checked the following day, and the numbers are entered in the table. It will be seen that the results were somewhat similar. The increase in the numbers was very much the same, though in the latter case the gas had been burning a little longer than in the former. It is, of course, the increase in number, and not the numbers themselves, that have to be noted here.

It may be as well to state that the Meeting-Room of the Royal Society is lighted by means of two sets of open lights, and each set is provided with a ventilator immediately over it. This accounts for the small increase in the number of particles while the gas was burning. Most of the increase which took place was probably due to two side lights which have no ventilators over them. It will be noticed that there was a large number of particles in the air near the ceiling of the Meeting-Room, not only while the gas was burning, but also on the morning after the meeting. The cause of these large numbers is greatly due to the ventilators projecting downwards to some distance from the ceiling; the air near the ceiling does not therefore get the benefit of the circulation, and the particles rising

from the two side lights get collected in the upper space. From the tests it would appear that this hot polluted air had remained all night, as on testing next morning it had lost only about one half of its particles.

By way of illustration, I have made some tests of the air in an ordinary room while gas was burning, but where there are no ventilators over the gas. As will be seen from the next numbers in the table, the contrast is very marked, and show that the ventilators in the Meeting-Room of the Royal Society do their work efficiently. The results of this series of tests are given in the table, and show in a marked way the effect of burning gas on the purity of the air of an ordinary room such as we are accustomed to live in. These tests were made here in a room 24 feet long by 17 broad and 13 high. There is a fireplace, in which a good fire was burning during these experiments; the room has two windows and two doors, and the polluting effect of four jets of gas was tested.

The air outside was tested at one o'clock, and also at three o'clock, when the tests began, and the numbers are entered in the table. Before the gas was lighted, the air in the room was tested at a height of 4 feet from the floor, and near the ceiling, and was found to have very nearly the same number of particles as the outside air, as will be seen from the figures in the table. The air near the ceiling seemed to have slightly less than that near the floor. This point I checked twice, and always found the higher air the purer. The lower number in the air from near the ceiling would partly be due to the air losing some of its particles in the pipe by which it was drawn down to the apparatus.

After these tests were made the gas was lighted at 3 P.M. The rapidity with which the products spread through the room was very remarkable. By the time I had lighted all the jets and turned to the apparatus, the number in the air from near the ceiling had increased to many times its original amount. Lower down, also, the number rose very rapidly. After an hour the air was again tested, and found to be greatly polluted by the products of combustion, as will be seen from the numbers given in the table taken at 4 P.M. The air was again tested at 5 P.M., and, as will be seen, the impurity in the air near the ceiling had again greatly increased. From the table it might be thought that the air at 4 feet from the floor, when tested at

5 P.M., had less dust in it than it had an hour previously. This decrease in the number was, however, caused by the two tests not being made at the same part of the room. The 4 o'clock test was made near the window, while the later one was made near the fire. The quantity of dust in different parts of the room near the floor varied greatly; the apparatus showed this very clearly, and by its means it was possible to trace the circulation of the air in the room. For instance, at the beginning of the tests, while the apparatus was near the window, the quantity of dust in the air, at 4 feet from the floor, rose at once when the gas was lighted, and became about as high as it was later on. This was due to the cold window producing a down current, and drawing the products from near the ceiling towards the apparatus. Again, near the fire, the number was frequently much below that entered in the table, owing to the supply of air to that part of the room being drawn from the lower, colder, and uncontaminated air of the apartment.

One point which impressed me greatly in making these last tests was the short time required to change the air near the ceiling of this room, compared with the slow circulation which we have referred to, near the ceiling of the Meeting-Room of the Royal Society. When the apparatus was tested before beginning the test last recorded, the number of particles of the air in the room was counted, and the gas was lighted to see how quickly the products would diffuse through the air of the room. The gas was not allowed to burn long, but on beginning the test about an hour after it was extinguished, not a vestige of the products of combustion could be detected. This was different from what might have been expected, after the experience in the Meeting-Room. It will be remembered that, on making the tests next day, the air near the ceiling was found still to contain a very large number of particles. Why this difference in the two rooms? A short time sufficed to clear all the products of combustion out of the air near the ceiling of the room here, while a whole night seemed to have reduced the number in the other room to only about one half. The windows in the room of the Royal Society do not go nearly up to the ceiling, whereas in the room here they rise to within a short distance of it. Again, some of the difference will probably be due to a difference in the porosity of the walls and ceilings of the two rooms.

TABLE *showing the Number of Dust Particles in the Atmosphere of Different Places.*

Place.	Date.	Hour.	Number of Particles per c.c.	Number of Particles per c.in.	Wind.	Remarks.
Colmonell,	January 7	10 A.M.	5,350	87,740	S.E. 1	Foggy morning.
,,	,, 7	2 P.M.	2,500	41,000	,,	Air thick, but clearing.
,,	,, 7	5 P.M.	2,050	33,620	,,	Been raining for two hours.
,,	,, 8	11 A.M.	5,500	90,200	S. 5	Been wet night, air thick.
,,	,, 8	5 P.M.	9,500	155,800	,,	Air extremely thick, raining all day.
,,	,, 10	12 A.M.	1,650	27,060	S.E. 1	Been slight shower, air much clearer.
,,	,, 10	1 P.M.	500	8,200	,,	Air clear, day bright and fine.
,,	,, 11	10 A.M.	4,600	75,440	S.E. 2	Frosty night, dull thick morning.
,,	,, 11	3 P.M.	9,250	151,700	S.E. 3	Air become very thick.
,,	,, 11	11 P.M.	4,250	69,700	,,	Air getting clearer.
,,	,, 12	10 A.M.	625	10,250	E. 1	Fine clear, bright morning.
Ballantrae,	,, 5	11 A.M.	5,000	82,000	S.W.	Clear and bright, slight wind.
Falkirk,	October 8	10.30 A.M.	140,000	2,296,000	...	Greatest number observed here.
,,	,, 12	1.30 P.M.	11,000	180,400	...	Smallest number observed here.
Edinburgh,	February 2	3 P.M.	75,000	1,230,000	W. 3.	Air clear, passing snow showers.
,,	,, 4	1 P.M.	250,000	4,100,000	N.W. 1	Fair, but air thick.
,,	,, 5	11 A.M.	45,000	738,000	W. 2	Day fair, with clear air.
Glasgow,	,, 8	11 A.M.	228,000	3,739,200	N.W. 7	Strong north-west gale, air clear.
,,	,, 8	2 P.M.	170,000	2,788,000	,,	Strong north-west gale, air clear.
,,	,, 12	3 P.M.	332,500	5,453,000	W. 1	Frosty, sky clear.
,,	,, 16	1 P.M.	466,000	7,642,400	S.W. 3	Wet morning, windy and wet day.
Meeting-Room,	,, 4	7.45 P.M.	275,000	4,510,000	...	4 feet from floor, before meeting.
,,	,, 4	9.30 P.M.	400,000	6,560,000	...	4 feet from floor, near end of meeting.
,,	,, 4	7.45 P.M.	3,000,000	49,200,000	...	Near ceiling, before meeting.
,,	,, 4	9.30 P.M.	3,500,000	57,400,000	...	Near ceiling, near end of meeting.
,,	,, 5	10.45 A.M.	125,000	2,050,000	...	4 feet from floor, beginning of experiment.
,,	,, 5	1 P.M.	275,000	4,510,000	...	4 feet from floor, after gas burning two hours.
,,	,, 5	10.45 A.M.	1,800,000	29,520,000	...	Near ceiling, at beginning of experiment.
,,	,, 5	1 P.M.	2,300,000	37,720,000	...	Near ceiling, after gas burning two hours.
Falkirk,	,, 19	1 P.M.	27,500	451,000	W. 1	Air clear, day fine.
,,	,, 19	3 P.M.	39,000	639,600	,,	Air clear, day fine.
Room,	,, 19	3 P.M.	28,700	470,680	...	At 4 feet from floor.
,,	,, 19	3 P.M.	26,000	426,400	...	Near ceiling.
,,	,, 19	4 P.M.	800,000	13,120,000	...	At 4 feet from floor.
,,	,, 19	4 P.M.	1,750,000	28,700,000	...	Near ceiling.
,,	,, 19	5 P.M.	500,000	8,200,000	...	At 4 feet from floor.
,,	,, 19	5 P.M.	2,800,000	45,920,000	...	Near ceiling.

The rapidity with which the products of combustion are cleared out of a room is a subject well worth investigation, and the dust-counting apparatus promises to be of considerable assistance in the investigation. Apart altogether from the question of ventilation by means of openings, there will be in most rooms a certain amount of circulation through the plaster of the walls and ceilings, and it might not be a bad plan to make the plaster of our houses as porous as possible. If this were done, and the space behind the plaster were connected with the open air, an insensible circulation of pure air would be secured.

The air of our smoking-rooms will probably contain a very great number of particles. As yet, however, I have made no actual determination of the number. I may, however, mention that I find a cigarette smoker sends 4,000,000,000 particles, more or less, into the air with every puff he makes. As this smoke is not very hot, it does not tend to keep near the ceiling like the products of combustion, and must therefore make the air breathed by the smokers very full of particles.

It will be observed that the smallest number of particles entered in the table is 500 per c.c. This is about the lowest yet observed in nature, though in some tests made in an agricultural part of Dumfriesshire a number slightly less was obtained. When testing such pure air as this, it does not require to be mixed with filtered air; the receiver is filled entirely with the air to be tested. When the particles are so few they are just sufficiently far apart for easy counting, and all of them fall with one expansion. Though one expansion is sufficient to bring down all the dust particles in pure country air, we must not imagine that therefore a fog could not be formed in air with so few particles. We must not suppose that, because they are so few, they would form rain, and all of them fall as in the test-receiver. The conditions are very different in the two cases. The high expansion used in the testing apparatus gives rise to a considerable cooling, and consequent supersaturation; whereas in nature the load of vapour tending to condense might be small, and each particle only get enough to make it visible. Five hundred fog particles per c.c. are quite enough to make a fog, and as these particles may not be heavy, they may float and fog the air.

Part II.—Portable Apparatus.

(Read March 18, 1889.)

When working with the apparatus described in the beginning of this paper, it soon became evident that, though it was suitable for laboratory work, it was yet very inconvenient when observations had to be made on the air of places at a distance—first, because of its size; and second, because it required a house of some kind in which it could be fitted up and worked. Further, in making tests in a house, there is always the difficulty of local contamination. We may select the place of observation as carefully as possible, yet the house we select can almost never be in a situation towards which the products of combustion from some neighbouring house, or even from the selected house itself, may not be driven by some direction of wind. As a result of this, we may not be able to make any tests of the air when the wind is from certain directions.

It therefore became desirable, for testing the pure air of the country, and for examining the air at places at a distance, that some portable form of the apparatus be constructed—one which could be easily carried by one person, and which could be worked in the open air, so that the observer might carry it to a situation somewhere to the windward of human habitations or other sources of pollution. With this object in view, I have prepared designs of a portable instrument which can be packed into a small space, and be easily carried.

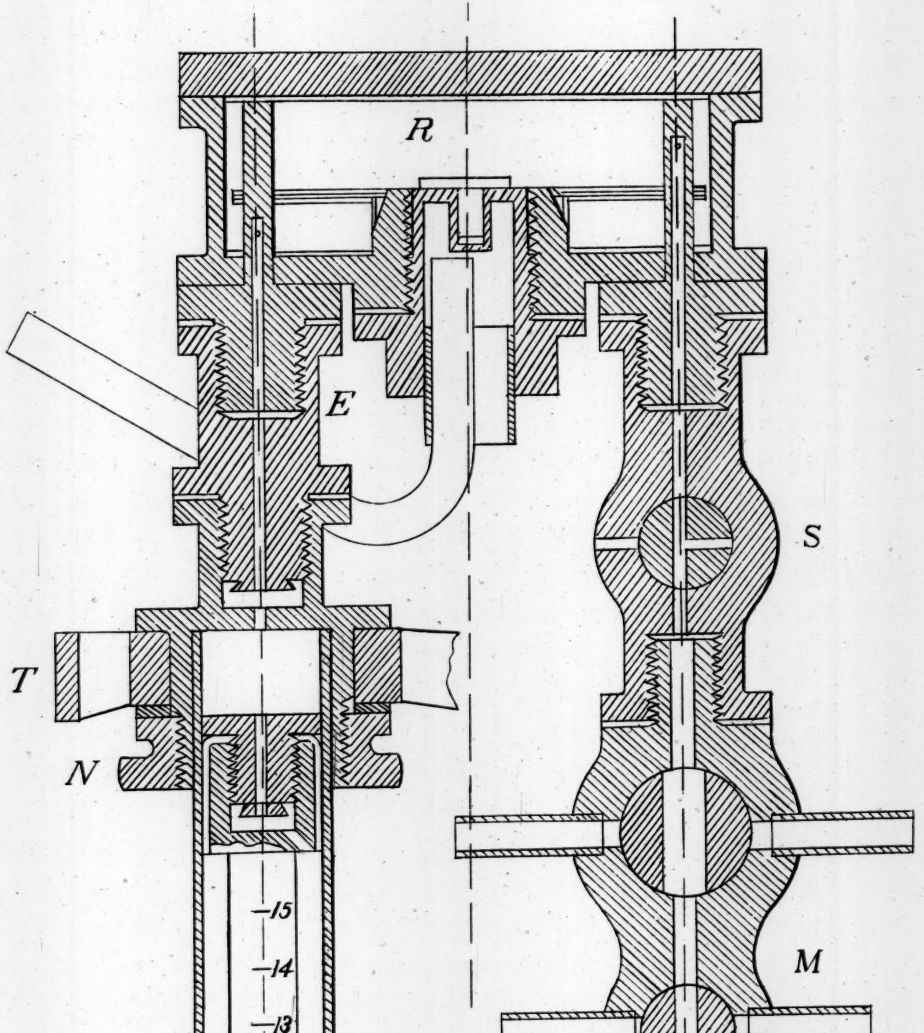
The portable apparatus is shown in Plate IV., which is drawn full size. As will be seen, the instrument is simply a rearrangement of the apparatus already described, but on a smaller scale, and with some slight modifications. All the sizes in this apparatus are reduced to $\frac{1}{10}$th the size of the one previously described. In Plate IV., R is the receiver, in this case made of brass with a glass cover. Part of the glass cover, the two sides of the diaphragm, and the bottom are covered with blotting-paper cemented on them. This paper is used for holding the water required for saturating the air tested in the receiver. The receiver has a capacity of 35 c.c., exclusive of the space occupied by the wet blotting-paper. The stage, the inlet and the outlet pipes, all occupy the same positions as before; the diaphragm, however, in this case, when stirring the air, has to be moved in the space over the counting stage, and not below it.

The diaphragm, as in the other instrument, is moved by a rod, surrounded by an indiarubber tube. This rod is not shown in the drawing, owing to its position being at right angles to the section; A is the air-pump, which has a capacity of 15 c.c.; the piston-rod is graduated in this case to its displacement, as shown; S is a stopcock, bored as shown; M is the apparatus for measuring the air to be tested, while F is the filter.

It will be noticed, that in this instrument we have adopted the stopcock plan, for measuring small quantities of air, which was described in the first part of this paper. The method of working has, however, been so far improved that it is not necessary by the new arrangement to change the measuring apparatus when we require to use a larger or smaller quantity of dusty air. As will be seen, the different measures are arranged in series, and we can use any of them without making any alteration in the apparatus.

In this portable apparatus, as in the other one, the pump is used when we require to mix large quantities of dusty air with the air in the receiver, and the measure M is used for small quantities. If 1 c.c. of dusty air is required to be sent into the receiver, then, after sufficient time has been allowed for the filtered air to enter the receiver, and when the pressure inside the receiver is the same as outside, but not before, the stopcock is turned quarter a turn from the position shown, by this means the receiver is put in direct connection with the outer air. The piston of the air-pump is now drawn down to the figure 1 on the scale; by this means 1 c.c. of the air is taken into the receiver, where it is mixed with pure air and tested. Two, or perhaps three, cubic centimetres may be measured in this way; but if more is required, then the stopcock S must be closed before the piston is drawn down, otherwise some of the dusty air might be taken out of the receiver by the pump, the piston of the pump should be pushed back to the top before the stopcock is opened to admit the air. When the stopcock is closed while the pump is used as a measure, allowance must be made for the expansion of the air, as already explained.*

* After a considerable amount of practice with this apparatus, I find it better always to close the stopcock before drawing down the piston, even when only 1 c.c. of air is required. The graduation of the pump ought therefore to be made to show the amount of air taken out of the receiver while the stopcock is closed.



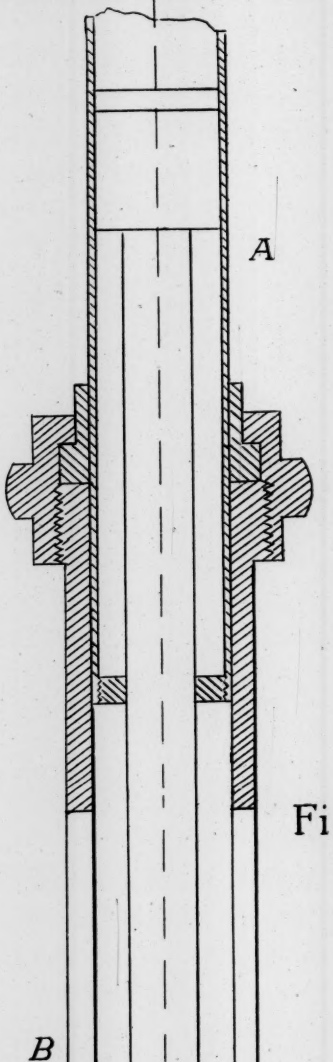


Fig. 2.

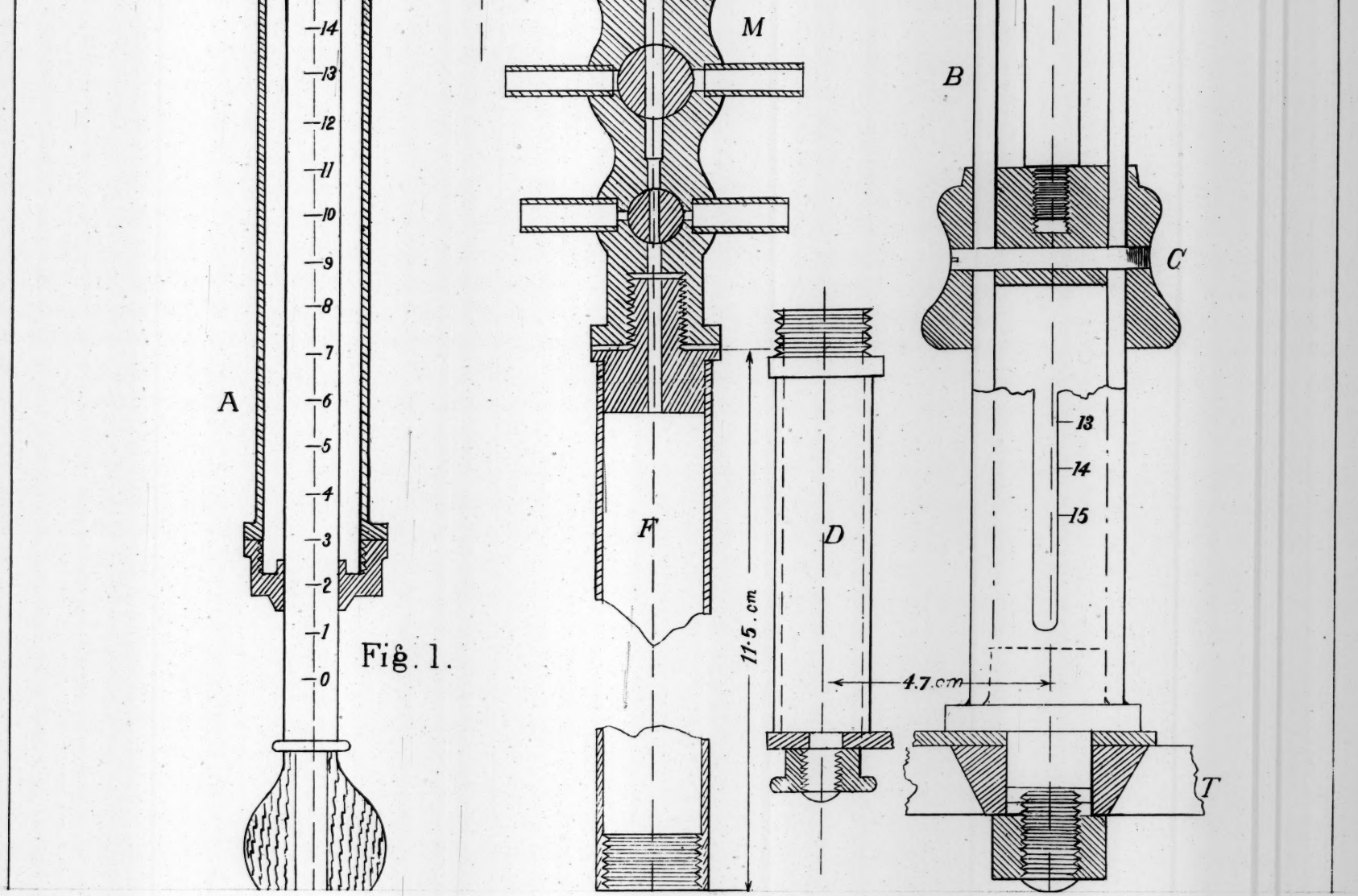
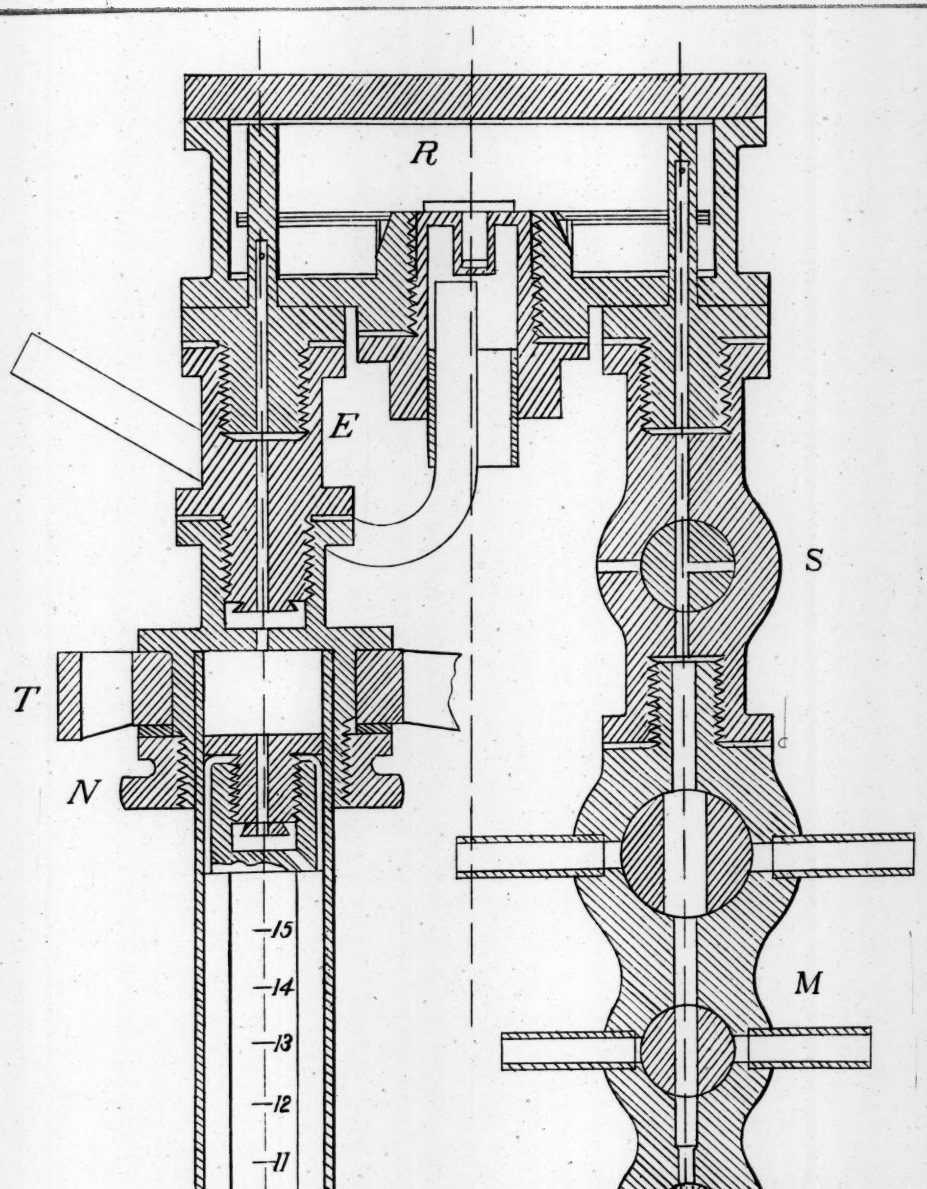


Fig. 1.



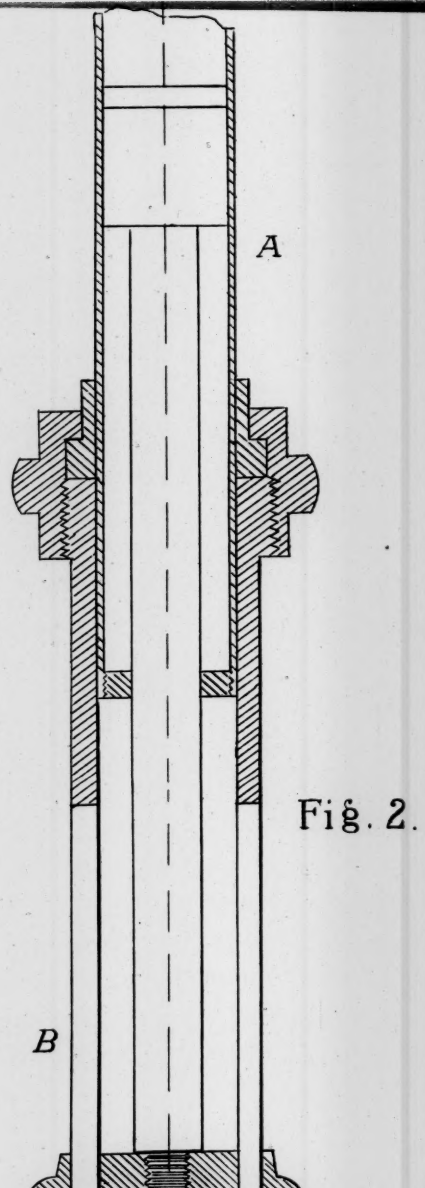


Fig. 2.

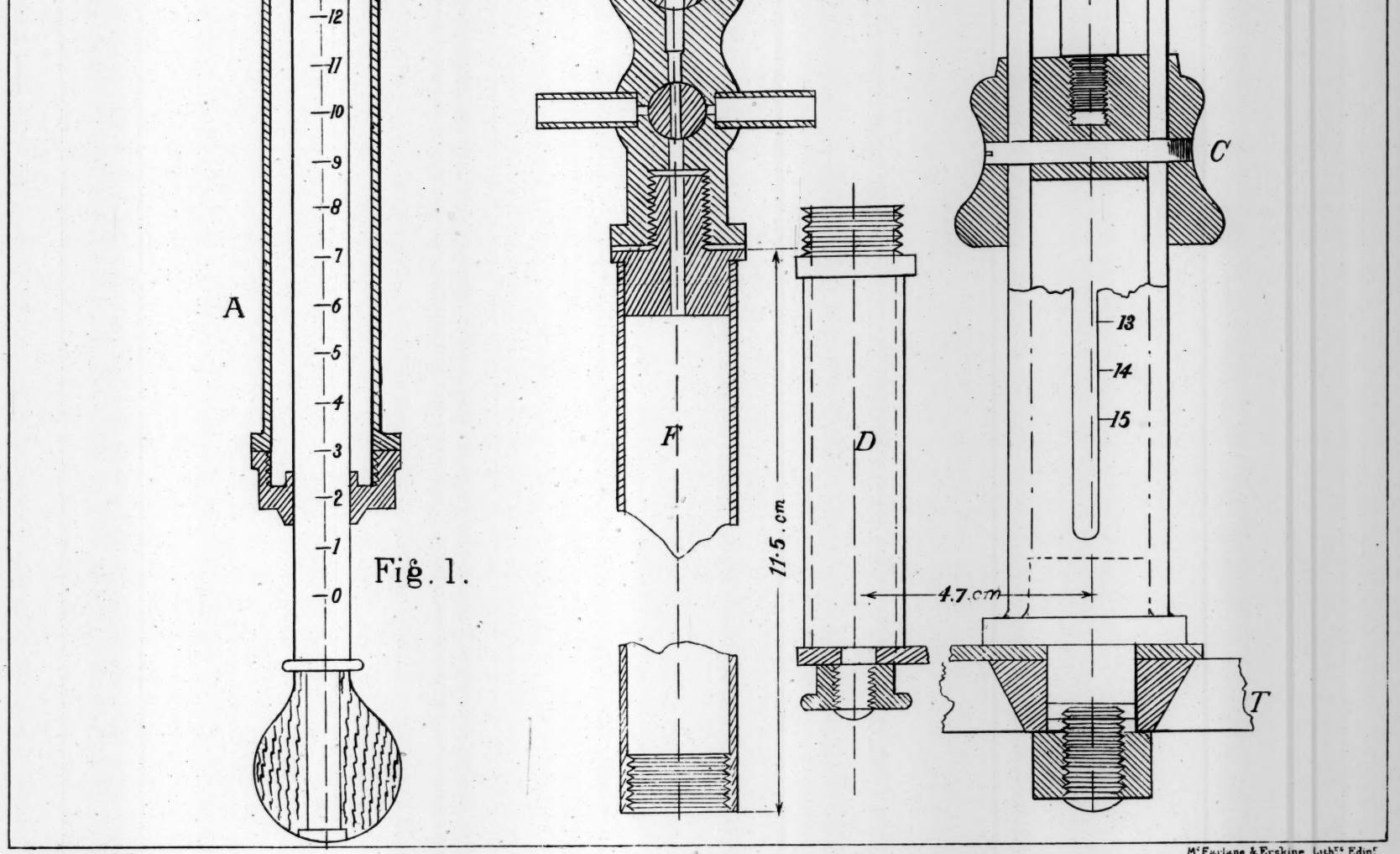


Fig. 1.

J. Aitken, del.

Mc Farlane & Erskine Lith^rs Edin^r

J. AITKEN ON DUST PARTICLES.

For measuring quantities less than 1 c.c., the "battery" of stop-cock M is used. The following are the sizes of the bores in the plugs of the different measures:—

	Diameter.	Length.	Capacity.
Large plug, .	4·61 mm.	15 mm.	250 c.mm.
Medium plug, .	2·53 mm.	10 mm.	50 c.mm.
Small plug, .	1·27 mm.	8 mm.	10 c.mm.

That is, their capacities are respectively $\frac{1}{4}$, $\frac{1}{20}$, and $\frac{1}{100}$ of a cubic centimetre, so that, in working with these measures, the number counted per c.c. of the air in the receiver requires to be multiplied by 200 for the large measure, by 1000 for the medium, and by 5000 for the smallest measure.

When using the portable apparatus, it is supported on a tripod stand. Two plans are shown in Plate IV. for doing this. Fig. 1 shows the design first made, and fig. 2 an alteration afterwards adopted. In fig. 1, T is the "head" of the tripod. The upper end of the pump is provided with a flange and a nut N, by means of which the apparatus is securely attached to the tripod when in use, but can be separated for easy carriage. The objection to this plan is, that the tripod legs require to be at least 4 feet long. Under most conditions this is no disadvantage, but, when much coach or railway travelling has to be done, a bundle of rods that length is a considerable inconvenience. No doubt, a folding tripod might be adopted, but it was thought better to make the stand like a walking-stick 3 feet long, so that it might be an assistance in walking, and at the same time easily packed for carriage along with umbrellas and sticks. In order to carry out this idea, the plan shown in fig. 2 was designed. A is the air-pump as before; B is a metal support, to the top of which the air-pump is secured by a movable screw-joint, while its lower end is securely fixed to the tripod "head" T. The pump is separated from the support B when packed for travelling. The piston of the pump is moved by means of the collar C, which slides on B. The scale for the pump is placed on B, as shown at lower end, fig. 2. In order to bind the whole apparatus firmly together, one end of the tube D, fig. 2, is screwed into the lower end of the filter F, fig. 1, and the other end is fixed to the tripod. When the length of the tripod legs is no objection, the plan shown in fig. 1 seems to be the steadiest, whilst the other is the most convenient for travelling.

When the plan shown on fig. 1 is adopted, it may be advisable to lengthen the part E, so as to bring the lower end of the stopcock S in a line with the top of the tripod, and to bind the apparatus together by means of a support attached to S, and fixed to the tripod by the nut and screw N.

When working this instrument daylight is used for illuminating the stage, and is found to work well in the open air. A good deal of the glass top of the receiver is left uncovered with blotting-paper to let in as much light as possible; and a magnifying glass, not shown in drawing, is used for counting the drops on the stage. This glass should have as little brass mounting as possible below the lens, so as not to interfere with the illumination of the stage. When working in pure air, the measuring apparatus M is not necessary, and may be removed, and the filter screwed direct into the stopcock S.

The whole apparatus weighs with tripod-head only 3 lb. $0\frac{1}{4}$ oz., and is packed into a tin-lined leather case $8 \times 5 \times 3$ inches. The apparatus with case weighs a little over $5\frac{1}{2}$ lbs., but this might be reduced by omitting the metal case. It is, however, easily carried by means of a shoulder strap. The legs of the tripod weigh only 15 oz., and when fitted together form a round staff, which makes a good walking stick when provided with top and bottom caps of indiarubber.*

The Prolonged Action of Sea-Water on Pure Natural Magnesium Silicates. By Alexander Johnstone, F.G.S.

(Read February 4, 1889.)

Pure mineral magnesium silicates are amongst the most difficult substances to decompose by naturally occurring agents. Pure water exerts no chemical action on them, neither does water containing carbonic acid gas,† even although the latter body be present to the

* After a considerable experience with this apparatus, I have never found it necessary to use the smallest of the measures, even when testing the air of cities. It may, therefore, be omitted. But if retained for special reasons, the centre stopcock should be placed at an angle with the others, so that the handles may be more easily worked, than when they are all crowded in one line.

† As far as I have been able to ascertain by experiment, carbonic acid water cannot decompose a *pure* magnesium silicate, such as white talc.

point of saturation. *Fresh* spring, river, or lake waters containing alkaline carbonates in solution are also, as my experiments prove, totally unable to decompose pure natural silicate of magnesia. Of course, it must be remembered that the amount of alkaline carbonate in natural waters is very small. In the Loire, near Orleans, Deville was able to find only 1·46 parts of carbonate of soda in 100,000 parts of water; while in the Garonne, near Toulouse, the same chemist could only detect 0·65 part of alkaline carbonate in 100,000 of water.

Sea-water, however, and also the waters of salt lakes and brinel springs, *i.e.*, all waters which contain a considerable amount of sodium chloride in solution, act chemically on pure mineral magnesium silicates. This fact I have clearly ascertained by the following experiment.

I allowed a litre of a nearly saturated *pure aqueous* solution of *pure* sodium chloride to act for two months on a thin fragment (presenting a fairly large surface) of pure and thoroughly clean white steatite ($4Mg''SiO_3.SiO_2$). On the expiry of that period I removed the mineral from its bath, filtered the liquid in which it had lain very carefully, and proceeded to test it chemically, with the object of discovering whether magnesia had been dissolved out of the steatite by the action of the salt solution or not.

I acidulated a portion of the clear filtered liquid with nitric acid, and then added excess of ammonium hydrate, and after mixing the whole properly, filtered at once. Into the filtrate I now poured some ammonium phosphate solution, agitated the mixture violently for several minutes, and then put it aside to allow it to settle for about a quarter of an hour. By the end of that time a little, but a very distinct, white crystalline precipitate, indicative of the presence of magnesia, had fallen to the bottom of the vessel. This white crystalline powder was afterwards examined under the microscope, and was found to consist mainly of translucent prisms belonging to the trimetric system. Thus the microscope confirmed the chemical test. The remainder of the clear solution in which the steatite had lain was evaporated to dryness, and the sodium chloride and magnesium salt of the residue were completely dissolved out by means of *dilute* hydrochloric acid. After this had been accomplished there still remained a *trace* of residue which

could not be removed in solution, and which I subsequently identified as silica.

This then I consider is a clearly proved fact, that magnesia and silica are removed in solution from mineral magnesium silicates by the action of water containing sodium chloride. A considerable amount therefore of the magnesia present in the ocean, must, I believe, have been brought into a state of solution by the chemical action of the sea-water on the abundant magnesium silicate minerals of the earth's crust.

I have strengthened my belief in this matter also by experiment. A piece of pure steatite was permitted to remain completely immersed in water, brought from the North Sea, for several months. It was weighed before being placed in the liquid, and after being carefully dried, weighed after its removal. It was found to have decreased *slightly* in weight; its second weight was less than its first, not much it is true, but still decidedly less.

(1) Weight of pure steatite before being placed in sea-water, ...	12·421 grammes.
(2) Weight of (1) after removal from the sea-water in which it had lain for several months, ...	12·416 „
Difference, showing amount of magnesia and silica removed in solution by the sea-water, ...	0·005 „

In an impure magnesium silicate decomposition by sea-water goes on of course more rapidly.

It is, I consider, unnecessary to bring forward more evidence to prove that pure magnesium silicates are decomposed by sea-water; but this question has yet to be answered, *How* were the magnesium and silica removed? In what conditions? What chemical changes took place? I am daring enough to attempt to answer this question, and although I may be wrong, I am unable to find at present any other possible explanation than the one I now give. It seems to me that the magnesium combined with the chlorine of the common salt, and that the discarded sodium united with a portion of the silica freed from the magnesium, the other part of the

liberated silica being dropped in the insoluble or almost insoluble silicon dioxide condition.

Perhaps the following equation states the changes correctly :—

Pure Steatite,	and	Water,	and	Sodium Chloride.
$4Mg''SiO_3.SiO_2$	+	$8H_2O$	+	$8NaCl$

gives

Sodium Silicate,	and	Magnesium Chloride,	and	Silica.
$= 4Na_2SiO_3.8H_2O$	+	$4MgCl_2$	+	SiO_2

Magnesium, as all are aware, exists chiefly in sea-water in the form of chloride.

White pure talc is decomposed at about the same rate as pure steatite. The common green variety, however, owing to the presence of ferrous silicate in fair quantity, is more readily altered. One thing must certainly be remembered distinctly, and that is, that the process of decomposition which is undoubtedly promoted in pure magnesium silicates by sodium chloride waters, does not progress at a rate which from a human standpoint can be considered at all rapid.

Deductive Evidence of a Uterine Nerve Centre, and of the Location of such in the Medulla Oblongata.

By **James Oliver**, M.D., F.R.S.E.

(Read February 18, 1889.)

At no time does Nature furnish us with any proof of bodies existing in a state of absolute rest. The whole molecular world, organic as well as inorganic, is, as far as we can ascertain, in constant motion. In consequence of this well-established principle, every function of the body may rightly be considered as resulting from a change in the molecular state of the organ manifesting such, and as being the expression of a correlative variation occurring in its representative nerve centre. Through the agency of long continuance the visceral disturbances are now carried on in a somewhat automatic manner, and fail to excite any feeling of their existence, although they may at one time, in the evolution of higher organisation, have produced a conscious sensation. In no organ do we find these revelations so well depicted as in the uterus. In every

typically healthy woman this organ, so long as it is free from the influences of gestation and lactation, is periodically for a greater or less length of time the seat of a regularly recurring functional variation in its molecular state, evidenced by the emission of a more or less marked hæmorrhagic discharge, and which to all intents is its sole manifestation. The disturbance is evolved quite independently of the will, and apart altogether from any definable excitation. It appears to be induced spontaneously through the agency of an automatic nerve centre, and fails in consequence to produce any conscious sensation. When, however, the uterus becomes the habitat of a developing ovum, or prior to this occurrence, and whilst segmentation is as yet progressing in the Fallopian tube, the waves of motion radiated by and from the germinal mass affect in a very decided manner the molecular state of the uterus, and determine a cessation of its routine function, and consequently of its regularly recurring manifestation of activity. Impregnation having resulted, other well-defined symptoms, in addition to that of the cessation of menstruation, are engendered, and help not only to guide the woman in arriving at a definite conclusion regarding her state, but aid us very materially in approximately estimating the duration of pregnancy.

The symptoms associated with pregnancy to which I wish more especially to draw attention, as evidence of the existence of a generative centre, and of its location in the medulla oblongata, are two—sickness and cough. It has been alleged, through the agency of experiments, that the sexual centre is located in the lumbar region of the cord. This opinion, however, appears to me to be founded on no very substantial basis. The mere fact that all, or nearly all, the sexual phenomena may be witnessed in animals after the lumbar portion of the cord has as far as possible been isolated by section is no very special criterion. The respiratory centre is located in the medulla, yet under special circumstances all the movements associated with respiration may be carried on after the medulla has been entirely removed.

From the earliest period of existence every organism has been endowed with two distinct qualifications—1st, that of maintaining self; 2nd, that of perpetuating the species. At first, in the most primitive state, the double function was performed by a uniform

mass, free from any semblance of structural differentiation. Habitual localisation of function, however, produces eventually a specialisation of structure, and with it the evolution of a nerve tract, whereby interdependence is maintained. It is therefore feasible to suppose that the nerve centre, which regulates the process of assimilation (the pneumogastric nerve centre), is either in close apposition, or at least in more or less direct communication, with that centre which presides over the organs of generation. All the visceral functions are now performed automatically, and appear to be regulated by nerve centres located in the medulla oblongata. It is feasible, therefore, to surmise that the uterine functions are governed by an automatic centre —a centre which, because of some innate quality, is thrown into a state of trepidation, and produces thereby evidence of associated disturbance in the uterus itself.

When the uterus becomes the nidus for a developing germinal mass, the molecular state of the organ is altered, and certain new impulses are generated. The waves of motion, resulting from segmentation of the ovum and further evolution of the chick, are radiated through the agency of the uterus and its afferent nerve fibres to the uterine or reproductive centre. These disturbances occurring in the uterus are in excess of those commonly generated, so too the disturbances correlatively produced in the uterine centre are greatly in excess of those usually developed. The extra amount of motion must discharge itself in some other direction until time has accustomed the uterine centre to the augmentation. The direction the overflow of energy shall take, is determined according to existing nerve communications, and that centre is likely to be first affected, by such radiations, which is in closest proximity. Considering, therefore, the intimate relationship that exists throughout life between the process of assimilation and the process of generation, it is not astonishing that the excess of molecular motion transmitted to the uterine centre should be radiated to and expend itself upon that governing the nutritive processes generally. In consequence of this there occurs early in pregnancy, and for a greater or less length of time, sickness, and in some women cough. At present we know but little of the nervous mechanism of vomiting. The respiratory centre seems, however, to participate in the act. Both phenomena, however (sickness and cough), in the pregnant woman may with some amount of assurance

be referred to molecular radiations from the uterine to the pneumogastric centre. Usually, however, in the course of a few months, through the agency of habit, the pneumogastric centre becomes tolerant, and the symptoms evidencing disturbance in this centre coetaneously disappear. It is difficult to understand why the sickness should be experienced, more especially, although not solely, in the morning. It is quite possible that the change from the recumbent to the erect position may after sleep render the whole nervous system more liable to explosive disturbances. The state of the stomach, too, may also aid in determining this somewhat anomalous phenomenon.

Frequently we find epileptic patients who suffer only from their disturbances on assuming the erect position after sleep. The anomalous phenomenon, therefore, as it occurs in the pregnant woman is not without a parallel. The more highly unstable the nervous system is generally, the more likely is a woman when she becomes pregnant to suffer markedly, and for a lengthened time, from sickness, whether matutinal simply or more or less constant. It is well, however, to remember that the inherent tone of the uterus itself will affect materially the molecular radiations engendered by the developing germinal mass, and transmitted through the agency of the uterine tissue to the nerve centre. The resulting disturbances will be correlatively augmented or diminished according as the tone of the uterus is high or low.

On the so-called "Liver" of Carcinus mœnas.

By **Dr A. B. Griffiths**, F.R.S. (Edin.), F.C.S. (Lond. and Paris), Member of the Physico-Chemical Society of St Petersburg, &c.

(Read February 4, 1889.)

"A true knowledge of biology must be based on a knowledge of chemistry and physics."—*M. M. P. Muir.*

This memoir details a continuation of the author's investigations on the physiology of the Invertebrata. At this point we consider the physiological functions of the so-called "liver" of the *Brachyura.*

Was it not M. Letourneau, in his *La Biologie*, who said, "Does the *pancreas* exist in the invertebrates? This is a question of comparative physiology which still waits for a reply. We have seen that we do not begin clearly to recognise the pancreas except

in fishes, and then only in a rudimentary state." From the recent researches of Krukenberg, Fredericq, Jousset de Bellesme, Plateau, Hoppe-Seyler, as well as those of the author, the problem now requiring solution is the following:—Does a true *liver* exist in the Invertebrata? The pancreas appears to be the chief digestive organ (other than a true stomach) of the earlier forms of animal life.

The Liver of Carcinus mœnas.

The "liver" of *Carcinus mœnas* consists of two large glands on each side of the stomach, and extends the whole length of the cephalo-thorax. These organs are of a yellow colour, and consist of numerous cœcal tubes arranged in tufts, which are easily seen in a dissection beneath the surface of water. The secretion of the so-called "liver" of *Carcinus mœnas*, when freshly killed, gives an acid reaction.

1. The secretion of the organs forms an emulsion with stearin, yielding subsequently fatty acids and glycerol:—

$$C_{57}H_{110}O_6 + 3H_2O = 3C_{18}H_{36}O_2 + C_3H_8O_3.$$

2. The secretion acts upon starch paste. The starch granules disappear, with the exception of their celluloïd covering; and on treating with water, and then adding Fehling's solution, a deposit of cuprous oxide was obtained. This reaction shows that there exists in the secretion a substance capable of converting starch into glucose.

3. The secretion dissolves coagulated albumin.

4. Tannic acid gives a white precipitate with the secretion.

5. The action of the secretion upon milk was to render it transparent.

6. When a few drops of the secretion of these organs were examined with chemical reagents under the microscope, the following reactions were observed:—

On running in a solution of iodine in potassium iodide between the slide and cover-slip, a brown deposit was obtained; and, on running in concentrated nitric acid upon another slide containing the secretion, a yellowish coloration was produced, due to the formation of xanthoproteic acid. These reactions show the presence of albumin in the secretion of the organs in question.

The presence of albumins in the secretion was also confirmed by the reactions recommended by Dr R. Palm (*Zeitschrift für Analytische Chemie*, vol. xxvi. part i.).

7. The secretion contains leucin and tyrosin, no doubt produced by the metamorphoses of certain albuminous substances.

We know from Professor Foster's *Physiology* (4th ed. p. 438), that "one result of the action of the *pancreatic juice* is the formation of considerable quantities of leucin and tyrosin." Leucin and tyrosin are "dehydrated in a *true liver*, forming a series of cyanhydrins or cyan-alcohols attached to a benzene nucleus, which then pass into the circulation"—(*Latham*).

8. The principal mineral ingredient found in the ashes (incinerated at a low temperature) of the so-called "liver" of *Carcinus* was sodium carbonate. In the ash of a vertebrate liver the chief mineral constituents are potassium and phosphoric acid.

9. The soluble zymase (ferment) secreted by the organ in question was extracted according to the Wittich-Kistiakowsky method (Pflüger's *Archiv für Physiologie*, vol. ix. pp. 438–459). The isolated ferment converts fibrin into leucin (*a*-amidocaproic acid, $C_6H_{13}NO_2$) and tyrosin (paraoxyphenylamidopropionic acid, $C_9H_{11}NO_3$).

10. No glycocholic and taurocholic acids could be detected by the Pettenkofer and other tests.

11. No glycogen was found in the organ or its secretion.

12. The secretion has no action upon cellulose.

13. By using the methods adopted by M. Zaleski (*Zeitschrift für Physiologische Chemie*, vol. x. pp. 453–502) for ascertaining the presence of ferrous, ferric, and ferrosoferric compounds in a true liver, the author could *not* detect the presence of iron in the organ or its secretion.

From these reactions the conclusion to be drawn is, that the so-called "liver" of *Carcinus mœnas* is pancreatic in function, *i.e.*, its secretion is more like the secretion of the pancreas of the Vertebrata than those of a true liver.

Some biologists look upon the vertebrate liver, pancreas, and salivary glands as differentiated bodies of an original pancreas of the Invertebrata. But have not many forms of the lower animals

similar salivary glands to those found in the Vertebrata?* And is not the so-called "liver" of the Invertebrata a true pancreas, capable of producing the same chemical and physiological reactions as the pancreas of higher forms?

On the Air's Resistance to an Oscillating Body (its Influence on Time-Keepers). By Edward Sang, LL.D.

(Read April 15, 1889.)

The influence of the air on the going of time-keepers has naturally been the subject of much discussion, particularly in reference to time-keepers used by astronomers. On pendulums the air acts in two ways:—by its buoyancy it lessens the downward tendency of the parts, and so lengthens the time of the oscillation; and it opposes resistance to the motion. On the chronometer balance, the latter action alone is felt; to this action we shall confine our remarks.

The incitement to motion is thus composed of two parts: the one due to the inclination of the curve or to the flexure of the balance spring, proportional to the distance from the point of rest; the other proportional to the square of the velocity; so that, if x represent the distance from the point of rest and v the velocity, the differential coefficient of that velocity, $\frac{dv}{dt}$ or ${}_{1t}v$, is represented by an expression of the general form

$$ {}_{1t}v = \mathrm{P}x + \mathrm{Q}v^2 ,$$

or, in Leibnitz's notation,

$$\frac{d^2x}{dt^2} = \mathrm{P}x + \mathrm{Q}\left(\frac{dx}{dt}\right) .$$

The resolution of this equation, which expresses a relation among the function, its first and its second derivative, belongs to what I have called the Third co-ordinate branch of the Higher Calculus, or the Calculus of Primaries. Here we seek the relation of the primary variable t, to some one of the three functions.

The only useful application of this inquiry is to the doctrine of

* See the author's papers in the *Proc. Roy. Soc. Edin.*, and the *Proc. Roy. Soc. Lond.*, 1885–88.

time-keeping, and we may, therefore, first in a general way, consider what may be the relative values of the coefficients P and Q, and also the signs which these may take.

In my mean-time clock, the bob is a cylinder of lead cased in brass, having its diameter 3 inches, its length 5, its weight, by estimate, being 101 200 grains; and therefore for each inch of deflection at the mean distance the redressing tendency is 2586 grains.

The air's resistance on a square inch of surface in grains for a velocity v measured in inches per second, varies, as deduced from several formulæ in use, from $\frac{v^2}{1200}$ to $\frac{v^2}{1600}$; now the bob offers a surface of 15 square inches, and thus we may assume $\frac{v^2}{100}$ in grains as the air's resistance in our case.

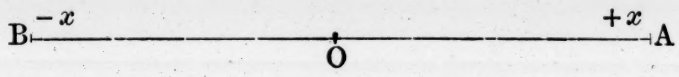
Now the oscillation extends to 1·3 on each side, wherefore the greatest incitement amounts to $PX = 3362$ grains; the maximum velocity is $V = 1{\cdot}3 \times \pi = 4{\cdot}08$ inches per second, and causes a maximum retardation of ·166, or the sixth part of a grain at the lowest point; and thus it appears that the maximum resistance by the air is only the twenty-thousandth part of the greatest redressing tendency.

Because of the complexity of the parts, it is not so easy to form an estimate in the case of the chronometer balance. In my two-day chronometer the compensating weights have a diameter of ·3, or just the tenth part of the pendulum bob; and they oscillate to a distance, measured along the arc of 1·2 or 1·3, almost the same as that of the pendulum. Hence, if the times of oscillation had been alike, the ratio of disparity would have been 2000 instead of 20000. But the balance makes two complete oscillations per second, while the pendulum makes only one-half. Now, in order to quadruple the number of oscillations per second, we must augment P sixteen times, Q being unchanged; however, since the velocity is now quadrupled, the product QV^2 is made sixteen times greater, and thus the ratio of PX to QV^2 remains 2000 to 1. The rim of the balance meets with direct resistance only on its ends; its motion is impeded by what may be called friction, on the sides, and in all likelihood the ratio of disparity is higher than for the weights;

we may, then, regard the ratio of 2000 to 1 as not too high. Thus, in all cases of any importance to us, we may regard Q as a very small fraction whose higher powers need not be taken into account; it may be more than enough for us to retain the second power.

We have now to examine the signs of P and Q; let us consider first that of P.

When the oscillating body is moving from A on the side to which we have agreed to attach the sign $+x$, towards the middle point O, $+x$ is decreasing and its derivative v must take the sign $-$, at the same time that velocity is increasing or becoming more $-$, wherefore $_{11}v$ is $-$, and so therefore must be P. When the body has



passed O, x is increasing in the $-$ direction, v therefore is still $-$; it is, however, now decreasing, its derivative is therefore $+$, so that Px must have the sign $+$, but then x is $-$, wherefore P must be $-$ in this quarter of the oscillation. When the body has reached the extreme limit B and is returning along BO, its $-x$ is decreasing more and more rapidly; that is to say, both its velocity and the increment of its velocity take the sign $+$, Px then is $+$, but x is $-$, wherefore P must be $-$ in this third quarter also. Lastly, when the body has passed O into the quarter OA, its $+$ velocity is decreasing, and both Px and P take the sign $-$. Thus it seems that, in all the four quarters of the oscillation, P must have the sign $-$; we shall then write it as $-a^2$.

The air's resistance, denoted by Qv^2, tends to lessen the velocity, to hinder acceleration, to help retardation irrespective of signs. When the body is moving from A towards O, the $-$ velocity is being increased in the $-$ direction by $-a^2x$, and this increase is opposed by Qv^2, wherefore Q here takes the sign $+$, and the total influence is $-a^2x+Qv^2$. In the second quarter, that is from O to B, the $-$ velocity is being lessened, and the air's resistance helps this, wherefore Qv^2 takes here also the sign $+$. Again, in the third quarter, from B towards O, the $+$ velocity is being augmented, and the resistance opposing this augmentation must take the sign $-$. Also, in the fourth quarter, the $+$ velocity is being retarded, the retarda-

tion is helped, and Qv^2 takes the sign $-$. Thus we see that Q must have the sign $+$ during the half oscillation from A to B, and the opposite sign $-$ during the other half from B to A. We shall therefore replace Q of the general formula by $+\beta$, observing that the conclusions thence drawn only hold good from the limit $v=0$ until the next recurrence of the same condition $v=0$. This is one of the many instances in which generalisation by the accepted law of signs is inadmissible.

Setting out from the equation

$$_{1t}v = -a^2x + \beta v^2,$$

and taking the differential coefficient in regard to the primary variable t, we get

$$_{2t}v = -a^2v + 2\beta v_{1t}v,$$

and substituting for $_{1t}v$ its value as above,

$$_{2t}v = -a^2v - 2a^2\beta xv + 2\beta^2v^3.$$

Repeating this operation, but rejecting all terms containing β^3 and the higher powers, we find

$$\begin{aligned}
_{3t}v &= +a^4x - 3a^2\beta v^2 + 2a^4\beta x^2 - 8a^2\beta^2xv^2 \\
_{4t}v &= +a^4v + 10a^4\beta xv - 14a^2\beta^2v^3 + 16a^4\beta^2x^2v \\
_{5t}v &= -a^6x + 11a^4\beta v^2 - 10a^6\beta x^2 + 84a^4\beta^2xv^2 - 16a^4\beta^2x^3,
\end{aligned}$$

and so on.

If now we place the zero of time at the instant when the oscillating body is at the limit A where $v=0$, and if we write X for the extreme value OA of x, the theorem of Taylor gives us

$$\begin{aligned}
v = 0 - a^2x\frac{t}{1}x + (a^4x + 2a^4\beta x^2)\frac{t^3}{1.2.3} \\
- (a^6x + 10a^6\beta x^2)\frac{t^5}{1\ldots5} + \&c.,
\end{aligned}$$

and we see that only derivatives of odd orders enter into the result. We shall therefore save ourselves much labour in writing by tracing the order of formation of these odd derivatives alone.

Let us then assume

$$_{(2n-1)t}v = +a^{2n}x - Aa^{2n-2}\beta v^2 + Ba^{2n}\beta x^2 - Ca^{2n-2}\beta^2xv^2 + Da^{2n}\beta^2x^3,$$

and operate twice thereon; the result is

$$_{(2n+1)t}v = -a^{2n+2}x + (4B+3)a^{2n}\beta v^2 - (4B+2)a^{2n+2}\beta x^2$$
$$+ (10B+7C+6D+8)a^{2n}\beta^2 xv^2 - (2C+D)a^{2n+2}\beta^2 x^3 ;$$

and by help of this scheme we form the following table of the numerical coefficients :—

	x	βv^2	βx^2	$\beta^2 xv^2$	$\beta^2 x^3$
$2n-1$	$+1$	$-$ A	$+$ B	$-$ C	$+$ D
$2n+1$	-1	$+(4B+3)$	$-(4B+2)$	$+(10B+7C+6D+8)$	$-(2C+3D)$
1	-1	+ 1	0	0	0
3	$+1$	− 3	+ 2	− 8	0
5	-1	+ 11	− 10	+ 84	− 16
7	$+1$	− 43	+ 42	− 792	+ 216
9	-1	+ 171	− 170	+ 7268	− 2232
11	$+1$	− 683	+ 682	− 65976	+ 21232
13	-1	+ 2731	− 2730	+ 696052	− 195648
&c.	&c.	&c.	&c.	&c.	&c.

Whence

$$v = -aX \left\{ \frac{at}{1} - \frac{a^3t^3}{1.2.3} + \frac{a^5t^5}{1\ldots5} - \frac{a^7t^7}{1\ldots7} + \&c. \right\}$$
$$+ a\beta X^2 \left\{ 2\frac{a^3t^3}{1.2.3} - 10\frac{a^5t^5}{1\ldots5} + 42\frac{a^7t^7}{1\ldots7} - 170\frac{a^9t^9}{1\ldots9} + \&c. \right\}$$
$$+ a\beta^2 X^3 \left\{ -16\frac{a^5t^5}{1\ldots5} + 216\frac{a^7t^7}{1\ldots7} - 2232\frac{a^9t^9}{1\ldots9} + \&c. \right\} .$$

The series by which $-aX$ is multiplied is that for $\sin at$. The series of coefficients B may be written

$$2 = 2\{1\};\ 10 = 2\{1+4\};\ 42 = 2\{1+4+16\}, \text{ and so on};$$

or in the more concise form,

$$2 = \frac{2}{3}(2^2-1);\ 10 = \frac{2}{3}(2^4-1);\ 42 = \frac{2}{3}(2^6-1), \text{ and so on};$$

wherefore the term involving β may be written

$$\frac{1}{3}a\beta X^2 \left\{ \frac{2^3a^3t^3}{1.2.3} - \frac{2^5a^5t^5}{1\ldots5} + \&c. - 2\frac{a^3t^3}{1.2.3} + 2\frac{a^5t^5}{\ldots5} - \&c. \right\},$$

or supplying the defective terms by adding zero in the form

$$-\frac{2at}{1} + 2\frac{at}{1}$$

$$\frac{1}{3}a\beta X^2\left\{\begin{matrix} -\frac{2at}{1} + \frac{(2at)^3}{1.2.3} - \frac{(2at)^5}{1\ldots5} + \&c. \\ +2\frac{at}{1} - 2\frac{(at)^3}{1.2.3} + 2\frac{(at)^5}{1\ldots5} - \&c. \end{matrix}\right\} \text{ or}$$

$$\frac{1}{3}a\beta X^2\{2\sin(at) - \sin(2at)\}\,.$$

The coefficients D succeed each other in a very complex manner; I have not succeeded in separating them into series of powers, the nearest approach giving the formula

$$\frac{2}{15}\{5\sin(at) - 4\sin(2at) + \sin(3at)\},$$

and thus we must be content to represent v by the formula

$$v = -aX.\sin at + \frac{1}{3}a\beta X^2\{2\sin at - \sin 2at\}$$

$$+a\beta^2X^3\left\{-16\frac{a^5t^5}{\ldots5} + 216\frac{a^7t^7}{\ldots7} - 2232\frac{a^9t^9}{\ldots9} + \&c.\right\},$$

and, for the reasons already given, we may neglect this term containing β^2.

Now when $at = \pi$, its sine is zero, but so also is the sine of $2at = 2\pi$, wherefore v is zero when $t = \frac{\pi}{a}$; in other words, the air's resistance does not influence the time of oscillation.

The value of x may be found in the same way, observing that the first derivative of v is the second derivative of x, and so on; or it may be found by integration from the value of v. Either process gives

$$x = X\cos at + \beta X^2\left\{\frac{1}{6}\cos 2at - \frac{2}{3}\cos at\right\},$$

which when $at = 180°$ gives

$$X' = -X + \frac{5}{6}\beta X^2,$$

that is to say, the length of the oscillation is shortened by $\frac{5}{6}\beta X^2$.

Thus we see that, while the air's resistance lessens the extent of the oscillation, it also lessens the velocity, and in such a way that the body is brought to rest in the same time as if there had been

no resistance. The diminution of the space compensates for that of the distance; but this compensation is not equally distributed. Thus, in the quarter of the time of an oscillation, the position is got by making $\alpha t = 90°$, which gives

$$X'' = +\frac{1}{6}\beta X^2,$$

so that the distance passed over has been

$$X - \frac{1}{6}\beta X^2,$$

whereas the distance described in the second quarter of the oscillation is only $X - \frac{4}{6}\beta X^2$.

In order to get the actual effect on the clock above mentioned, we observe that, since the half oscillation is performed in one second, we must have $\alpha = \pi$. Now the extreme distance is 1·3, wherefore the greatest incitement to motion is $1{\cdot}3 \times \pi^2$. But the maximum velocity is $1{\cdot}3 \times \pi$, and the maximum air's resistance $\beta \times 1{\cdot}3^2 \times \pi^2$, so that $1{\cdot}3 \times \pi^2 = 20000 \times 1{\cdot}3^2 \times \pi^2.\beta$, whence $\beta = \frac{1}{26000}$. In this way we find the shortcoming at the end of a half oscillation to be $\frac{5}{6}\beta \times 1{\cdot}3^2 = \frac{1}{18460}$th part of an inch, a quantity very small even in comparison with that due to the imperfect resistance of the suspending spring.

In the case of the chronometer, the half oscillation is performed in the quarter of a second, wherefore $\alpha = 4\pi$, and the effect is, therefore, sixteen times as great as in the case of the clock, on this account alone; but the ratio of disparity is only 2000; wherefore we must augment the above computed shortcoming 160 times, giving the 115th part of an inch.

These two examples may serve to give a general idea of the magnitude of the disturbance due to the air's resistance. In neither case is the time of the oscillation changed; in clock-work the extent of the arc is slightly lessened, so slightly that the diminution is scarcely worthy of attention. In watch-work it is so considerable as to require perceptible greater maintaining force.

A Contribution to the Chromatology of the Bile.

By **John Berry Haycraft, M.D., D.Sc.,** and **Harold Scofield, M.B.**

(Read March 4, 1889.)

One of the chief biliary pigments is bilirubin. It has a red-orange colour, and is derived from the decomposition of hæmoglobin. It can be oxidised first into a green, then into a blue, then into a red, and finally into a yellow-brown pigment. Between the blue and red a violet substance is produced, but it is uncertain whether or not this is only a mixture of the blue and red pigment. The formation of the green pigment, according to Städeler,* is due to oxidation, together with the addition of a molecule of water. These pigments do not always present exactly the same characters. Thus, according to Dr MacMunn, the green pigment present in the ox-bile differs from what is artificially produced from the oxidation of bilirubin, say, from human bile, in that it is soluble in chloroform.† This pigment can, however, be oxidised up into the blue pigment, and so on; and it belongs, therefore, to what we may term the bilirubin series. Its colour indicates its position in the scale of oxidised products. We shall in this paper use the term biliverdin as designating a green pigment, which is more oxidised than the bilirubin, and bilicyanin as designating a blue pigment still more oxidised. The violet substance (if such exist) we shall term the violet pigment, the red oxidation product the red pigment, and, finally, the yellow-brown pigment, the most highly oxidised product of all, choletelin.

Although, by the action of an oxidising agent, such as impure nitric acid, it is easy to pass from a lower to a higher member of the bilirubin series, it is frequently, and we believe truly stated, that no successful attempt has been made to reduce the higher back again to the lower ones. By the reducing action of, say, sodium amalgam, hydrobilirubin ($C_{32}H_{40}N_4O_7$) has been produced both from bilirubin and from biliverdin. When, however, ox-bile is placed in a tall vessel, and allowed to remain for some hours, we have observed that a change in colour takes place. If blue pigment is present

* Gorup-Besanez, *Physiol. Chemie*, p. 207.

† *Jour. of Phys.*, vol. vi. p. 2.

it changes to green, and this finally to orange-brown. If some of the brown bile be then treated with a few drops of nitric acid, it is oxidised back into biliverdin, and further additions of acid develop the blue, violet, red, and yellow pigments. Here, then, without going any further, we have an instance of the reduction of the pigment. The reduction does not produce hydrobilirubin alone, as that substance cannot be reoxidised in the way we have described.

If a bladder, fresh from the slaughter-house, be opened, one seldom or never fails to see signs of reduction within it. Blue or blue-green bile fills the cavity, but thick orange-brown bile is seen next the mucous membrane, which is itself brown in colour. It is not improbable, therefore, that during life, reduction of the biliverdin takes place, due perhaps to the action of the mucous membrane of the gall-bladder or to the mucus secreted by it. This is, indeed, almost certainly the case, and one can recall the fact that the pigment present in gall-stones from the ox consists, not of biliverdin, but of bilirubin.

It was obviously a matter of some importance to investigate the reduction processes we have just described, in order, if possible, to ascertain their cause, and the influence of modifying conditions upon them. We have been greatly assisted in this inquiry by Dr MacMunn. We sent to him on several occasions solutions in which we had difficulty in determining whether definite absorption-bands were present, and we have very warmly to thank him for the courtesy with which he was ever willing to help us.

Experiment I.—Temperature of the laboratory, 60°. Two test-tubes were filled with fresh green ox-bile, and watched from day to day. After three days the bile in the lower part of one of the tubes had become of a brown-green colour. The change in colour spread upwards, and in twelve hours the whole tube contained bile of a uniform brown tint. Putrefaction, indicated by an unpleasant odour, had set in by this time. In the other tube the bile did not change in colour, nor was there any sign of putrefaction until the following day, when both appeared simultaneously. Later on the bile became by degrees of a light amber colour, giving, however, the play of colours for nearly four months, after which Gmelin's test failed. The fluid was then examined with the spectroscope. There was slight shading at the violet end of the spectrum, and the

bands of cholohæmatin * were present. There was no distinct hydrobilirubin band. It is seen, therefore, that on exposure to the atmosphere the biliverdin is reduced; the bilirubin disappears without forming hydrobilirubin.

Experiment II.—Test-tubes were filled with ox-bile, and pieces of the mucous membrane of the gall-bladder were added. The changes observed were similar to those detailed in Experiment I. The mucous membrane seemed to hasten the reduction of the bile in its neighbourhood. The reduction of the pigment seemed remarkably to coincide in point of time with the establishment of putrefaction. The following experiments were conducted in order to eliminate putrefactive changes from the other conditions present.

Experiment III.—Ox-bile was boiled in test-tubes which had previously been plugged with wool. No alteration in colour was produced by boiling, nor did any take place for seven days. It then became of a light brown colour, rapidly fading, until after a fortnight it was nearly colourless. Gmelin's test failed after three weeks, and two weeks afterwards the fluid was examined with the spectroscope. Cholohæmatin bands were absent; there was possibly a trace of hydrobilirubin. This experiment indicates that reduction of the biliverdin takes place, and the bilirubin disappears in bile which has been prevented from putrefying. The reduction of the biliverdin seems to be hastened by putrefaction.

Experiment IV.—Ox-bile was evaporated to dryness at 50°. It very slowly changed its colour, and only failed to give Gmelin's test after several months' exposure.

Experiment V.—Some exhausted and sterilised glass tubes were drawn out into capillary points. The points were thrust through the wall of a fresh gall-bladder, and broken off by the fingers which grasped them from without the bladder. They instantly filled with bile, and after their withdrawal from the bladder their ends were at once sealed up in the blow-flame. No change of colour occurred for fourteen days; the weather was cold and dull. By that time, however, they had become brown in colour. No further change occurred, and after a year they gave a distinct play of colours with nitric acid.

* Cholohæmatin is a pigment which gives absorption bands. It is present and often partially replaces biliverdin, in ox-bile.—MacMunn, *loc. cit.*

Changes, not very dissimilar in their nature, occur when blood is received directly into a sterilised tube from an artery. In this case the oxyhæmoglobin is reduced, and remains for years without undergoing further change. In the sterilised tubes filled with bile the change in colour seemed to be influenced in the thick mucus of the bladder, for those that contained most mucus invariably became brown before those that contained less mucus.

It will be seen, however, by the results of the next two experiments, that reduction occurs quite readily in the absence of bladder mucus. The two experiments were unfortunately performed, the first in dull cold weather, and the second in warm bright weather, so that they cannot be very rigidly compared. One can conclude, however, from the general results obtained, that thick bladder mucus assists the reduction, although its presence is not essential.

Experiment VI.—Exhausted and sterilised tubes were filled with bile drawn directly from the hepatic duct of a freshly killed ox. Reduction took place in three days.

Experiment VII.—In order to ascertain whether reduction would take place in bile absolutely free from mucus, this substance was precipitated by means of alcohol. The filtrate subsequent to the separation of the mucus was evaporated over a water-bath until the alcohol was expelled. The reduction took place, however, in two days; the weather was hot and bright.

The last experiments were made during hot midsummer weather, and it was noticed that, while in all cases reduction took place with rapidity, this was especially the case in the tubes most exposed to light. In the following experiments the action of light was more fully investigated.

Experiment VIII.—(*a*) Bile within test-tubes exposed to light was reduced in twenty-four hours, and after a week failed to give Gmelin's test; weather hot and bright. Other portions of the same bile in a dark metal chamber were reduced in three days.

(*b*) Bile boiled in plugged tubes and exposed to light was reduced in twenty-eight hours. A portion treated in a similar way, but placed within the dark chamber, still preserved a trace of the original green colour for seventeen days.

(*c*) Bile within the exhausted and sterilised tubes was reduced

by the light in twenty hours. In the dark chamber they began slowly to undergo change at the bottom of the tube after twenty-four hours.

(*d*) A film of bile about 1 millimetre thick was dried at a low temperature. On the fourth day it had completely changed its colour to brown; while, on the other hand, a film, similarly prepared, but kept in the dark, remained blue-green, the original colour of the bile, for a year and a-half—the time when it was finally examined.

It would seem, therefore, that biliverdin readily parts with its oxygen like oxyhæmoglobin. It is reduced in the sterilised tubes to bilirubin, but no further. This reduction is hastened by light, and putrefaction, and the presence of thick mucus. It is only prevented by drying the bile and keeping it in a dark chamber.

When bile putrefies, or when, without putrefaction, the bile has been altered by boiling it, the bilirubin finally disappears, and no play of colour is obtained by the application of Gmelin's test.

This residue contained a brown pigment giving no absorption bands, and differing from hydrobilirubin in its solubilities. It was insoluble in ether, but readily soluble in alcohol.

Appendix.

On the Reduction and Oxygenation of Pigments in the Bilirubin Series.

Bilirubin, when exposed to the air, under ordinary circumstances, never becomes converted into biliverdin. This takes place, however, if the solution has previously been rendered strongly alkaline by the addition of caustic soda solution. Both nascent oxygen and ozone, we find, are capable of effecting this change in bile of normal reaction. To do this experiment, pieces of blotting-paper soaked in bile should be exposed to the vapour of ozone; ozonic ether, liberating nascent oxygen, can also oxidise these papers. If human bile is poured into a small beaker, and a stream of electricity passed through it, obtained from some five or six Grove cells—the terminals should be of platinum—the oxygen given off from the positive terminal, in the course of three or four minutes, changes the bile in its vacinity, first to a green, and finally to a blue-green

colour. The experiment may be performed in, perhaps, a more satisfactory manner by dipping a piece of blotting-paper in the bile, and laying it directly on the terminals. The brown colour changes near the positive terminal, first to green, then to blue, and finally to violet. At this stage, however, much of the pigment becomes bleached, so that the violet is not so distinct as either the blue or the green.

The lower oxidation products of bilirubin can be reduced artificially. If some bile be acidulated with impure nitric acid, so as to oxidise its bilirubin to biliverdin or bilicyanin, and if pieces of blotting-paper be then dipped in these, the vapour of ammonium sulphide can reduce them. The blue colour is reduced through green to brown, and it can then be oxidised by nitric acid. If blotting-paper is soaked in bile and reoxidised by the positive pole of the battery, the pigment is again reduced on reversing the poles. The brown can be oxidised to green and then to blue, and afterwards reduced to brown, passing through green. If oxidised to red, reducing reagents then change the colour to a yellow-brown, but not through any intermediate stages. Oxidising agents restore the original red colour.

It is much easier to perform these experiments with bile than with pure bilirubin. Bilirubin dissolved in caustic soda is difficult either to oxidise or to reduce. Pure bilirubin, powdered on blotting-paper, moistened with normal saline solution, can be oxidised and reduced without difficulty by the current given by three or four Groves.

On the Identity of Hofmann's "Dibenzyl-Phosphine" with Oxide of Tribenzyl-Phosphine, and on some other Points connected with the Phosphorised Derivations of Benzyl.

By **Professor Letts** and **R. F. Blake**, Queen's College, Belfast.

(Read May 20, 1889.)

In his well-known researches on the phosphines, Hofmann has shown (or believes that he has shown) that when an alkyl iodide (or other haloid derivative) is heated with phosphonium iodide and oxide of zinc, primary and secondary phosphines alone result; whereas, when an alcohol is heated with iodide of phosphonium, tertiary and quaternary phosphines are formed exclusively. Thus

the two reactions are complementary to each other. Among the series to which he extended his investigations was that of benzyl, and in a paper published in the *Berichte* of the Berlin Chemical Society* he describes the preparation of mono- and dibenzyl-phosphine, and gives their properties. Dibenzyl-phosphine he isolated as a crystalline substance perfectly tasteless and odourless, insoluble in ether, but soluble in alcohol. Its melting-point he found to be 205° C.

In a paper read before this Society (19th December 1887), one of us, in conjunction with Mr W. Wheeler, describes further investigations on this body, and shows that it forms a series of compounds of a somewhat remarkable nature for a secondary phosphine. This fact, and some other properties of the substance, led to the suspicion that it was not dibenzyl-phosphine at all, but the oxide of tribenzyl-phosphine. Accordingly the investigation was re-opened with Mr R. F. Blake, and its course has been as follows:—

1. On carefully re-crystallising the substance from alcohol, its corrected melting-point was found to be 216–216·5° C., while that of two specimens of oxide of tribenzyl-phosphine (prepared by two different methods) was found to be the same.

2. Very little difference exists in the percentage amount of carbon and hydrogen in dibenzyl-phosphine and oxide of tribenzyl-phosphine, as the following numbers show:—

	$(C_7H_7)_2HP$	$(C_7H_7)_3PO$
Carbon	= 78·50	78·75
Hydrogen	= 7·01	6·56

Consequently it would not be possible to decide with absolute precision between the two substances by a mere combustion. On the other hand, there is a considerable difference between the two bodies in their percentage of phosphorus:—

	$(C_7H_7)_2HP$	$(C_7H_7)_3PO$
Phosphorus	= 14·48	9·69

Unfortunately, however, as we have again and again found, the processes for phosphorus determinations in ordinary organic substances are absolutely untrustworthy when applied to phosphines. A new method was therefore necessary, and after many trials we believe we have found one which is perfectly accurate, trustworthy, and capable

* *Berichte* (1872), v. 100.

of general application. It is extremely simple, though somewhat tedious in carrying out.* It consists in making an ordinary combustion of the substance with *pure* oxide of copper, and afterwards dissolving the contents of the combustion tube in nitric acid, and determining the phosphorus with molybdate of ammonia, &c. Applying this method to the analysis of the supposed dibenzyl-phosphine, we obtained the following results (IV. and V.).

We give at the same time the determinations of phosphorus made both by Hofmann (I.) (by a method not described), and by one of us and W. Wheeler (II. and III.), in the same substance (by burning with lime in a stream of oxygen):—

	I.	II.	III.	IV.	V.
Phosphorus:—	13·6	14·35	15·00	9·86	9·98

I. Hofmann.

II. and III. Letts and Wheeler.

IV. and V. Letts and Blake.

3. The following compounds of Hofmann's body were prepared and analysed†:—

Bromide.—Obtained by adding bromine to a solution of the body in glacial acetic acid. It crystallises usually in yellow needles. It is unstable, and loses bromine when boiled with water or glacial acetic acid, and possibly on drying also.

	Prepared from		
	Hofmann's Dibenzyl-Phosphine. Obtained:	Oxide of Tribenzyl-Phosphine.	
Bromine	mean of 10 determinations, 26·7	28·4	
Carbon,	56·4, 56·5, 56·5	56·9	
Hydrogen,	5·31, 5·57, 5·04	4·9	
Calculated:	$\{(C_7H_7)_2HP\}_2Br_2$	$7(C_7H_7)_3PO.5Br_2$	$5(C_7H_7)_3PO.4Br_2$
Bromine,	27·2	26·31	28·5
Carbon,	57·1	58·00	56·3
Hydrogen,	5·1	4·83	4·7

* Details of this method will be given in another paper.

† It will be seen that a formula can be devised in every case, both for a compound of $(C_7H_7)_2HP$ and $(C_7H_7)_3PO$, which corresponds with the results obtained; and it is remarkable how closely most of the results obtained agree with those required for a compound of the former. We also give in some cases the analyses of compounds prepared in a similar manner with what was known to be the oxide of tribenzyl-phosphine. Most of the compounds are unstable, and their composition often varies according to the method or conditions employed in their preparation.

Platinum Salt.—Prepared by mixing alcoholic solutions of chloride of platinum and of the substance. The compound crystallises out in minute leaflets. The composition of the substance varies with the conditions under which it is prepared.

	Obtained from				
	Hofmann's Dibenzyl-Phosphine.			Oxide of Tribenzyl-Phosphine.	
	I.	II.	III.	IV.	V.
Carbon,	59·5	56·6	58·5	58·5	59·4
Hydrogen,	5·8	5·3	5·9	5·3	5·4
Chlorine,	...	...	...	...	...
Platinum,	12·8	13·1	13·1	12·4 12·6	...

I., II., III., IV., and V. were all separate preparations, obtained under slightly different conditions.

	Calculated for	
	$5(C_7H_7)_2HP.PtCl_4$	$4(C_7H_7)_3PO.2HCl.PtCl_4$
Carbon,	59·5	59·5
Hydrogen,	5·3	5·0
Chlorine,	...	11·7
Platinum,	14·0	12·5

Iodide.—Prepared like the bromide. It crystallises in minute red crystals of the same colour as ferricyanide of potassium.

Obtained.	Calculated for		
Iodine, 36·86	$(C_7H_7)_2HP,I_2$ 37·24	$7(C_7H_7)_3PO.5I_2$ 36·18	$5(C_7H_7)_3PO.4I_2$ 38·84

Chloride.—Obtained by passing chlorine into a solution of the body dissolved to saturation in warm acetic acid. It crystallises when the solution cools in pale yellow crystals, much like pentachloride of phosphorus in appearance. The compound is most unstable, and loses chlorine rapidly *in vacuo*, and probably also when air-dried.

Obtained.	Calculated for	
Chlorine, 12·00	$(C_7H_7)_2HPCl$ 14·23	$7(C_7H_7)_3PO.5Cl_2$ 14·68

Hydriodate. Obtained by saturating a solution of the body in glacial acetic acid with hydriodic acid gas, and separated as the solution cooled in colourless crystals.

Obtained.	Calculated for	
Iodine, 21·0 21·5	$2(C_7H_7)_2PH.HI$ 22·8	$3(C_7H_7)_3PO.2HI.$ 21·05

Hydro-bromate.—Obtained as the hydriodate :—

Obtained.	Calculated for	
	$2(C_7H_7)_2HP.HBr$	$3(C_7H_7)_3PO.2HBr$
Bromine (1) 19·5		
(2) 20·5	15·9	14·44
(3) 16·3		

Nitro-Compound.—Obtained by dissolving the body in cold fuming nitric acid, and then precipitating with water. Amorphous.

Obtained. (With Hofmann's Dibenzyl-Phosphine.)	Calculated for	
	$(C_7H_6NO_2)_2HP$	$(C_7H_6NO_2)_3PO$
Carbon, 55·23	55·26	55·38
Hydrogen, 4·30	4·27	3·95

Double Salt with Iodide of Zinc.—This compound separates out when fairly strong alcoholic solutions of the body and iodide of zinc are mixed, in tufts of characteristic needles.

Obtained from		
Hofmann's Dibenzyl-Phosphine.	Oxide of Tribenzyl-Phosphine.	
	I.	II.
Iodine, 26·57	26·0	25·9

Calculated for	
$3(C_7H_7)_2HP.ZnI_2$	$2(C_7H_7)_3PO.ZnI_2$
Iodine, 26·43	26·48

4. *Action of fused Potash on Hofmann's "Dibenzyl-Phosphine."*—In the paper by one of us and W. Wheeler already alluded to, the statement is made that when Hofmann's dibenzyl-phosphine is heated with caustic, potash, or soda, "it fuses and floats on the surface of the melted alkali. No violent action occurs, but on cooling the mixture and treating it with water the greater portion dissolves, and acids then precipitate a flocky crystalline substance, which is dibenzyl-phosphinic acid." In corroboration of this statement, the melting-point of the acid and analyses of its lead and barium salts were given, all in accordance with the required numbers. After we had satisfied ourselves that Hofmann's dibenzyl-phosphine was oxide of tribenzyl-phosphine, and nothing else, this reaction recurred to our minds as a further and very striking excuse for the mistake we (and Hofmann) had fallen into, and we thought it of importance to verify the previous observation. This we have accordingly done, both with Hofmann's dibenzyl-phosphine and with a specimen of oxide of tribenzyl-phosphine prepared by a different

method. The phenomena observed were exactly the same as those previously described, and the melting-point of the acid obtained after precipitation with hydrochloric acid and two recrystallisations from alcohol was found to be 192° C., which is the melting-point of pure dibenzyl-phosphinic acid. Our previous observation is thus fully confirmed. The reaction in all probability occurs as follows:—

$$(C_7H_7)_3PO + KHO = C_7H_8 + (C_7H_7)_2KPO_2.$$

The occurrence of oxide of tribenzyl-phosphine among the products of Hofmann's sealed tube reaction led us to suspect that tribenzyl-phosphine had been formed in the first instance, but was subsequently oxidised by atmospheric oxygen. It became necessary, therefore, to search for the tertiary phosphine in the original product. The investigation was attended with considerable difficulties, as we had already proved that the products of the sealed tube reaction are a highly complex mixture, and contain, among other substances, resinous bodies which are exceedingly difficult to get rid of. We succeeded at last in isolating a liquid which grows hot on exposure to air, with production both of oxide of tribenzyl-phosphine, and of dibenzyl-phosphinic acid; which is precipitated by hydriodic acid, forming solid compounds; and which in contact with sulphur gives rise to a crystalline compound, which we believe to be tribenzyl-phosphine sulphide. Moreover, the liquid acts energetically upon crystallised iodide of benzyl to give iodide of tetrabenzyl-phosphonium. The liquid appears, in fact, to be a mixture of the secondary and tertiary phosphines. Continuing our investigations, we have succeeded in obtaining from the liquid two solid substances which are easily separated from each other. The first is almost insoluble in ether, and has either the formula $(C_7H_7)_3PO_2$ or $(C_7H_7)_3PS$.* The second is undoubtedly tribenzyl-phosphine itself. It crystallises easily from alcohol, and unites with sulphur at ordinary temperature and with oxygen also. We have not as yet obtained it in sufficient quantity to thoroughly investigate its properties.

* We give this last formula partly because the first is improbable, and partly because the substance in question has the same melting-point and properties as the sulphide of tribenzyl-phosphine. But, on the other hand, we have not as yet detected sulphur in it, and unless that element was present in the crude products used, its occurrence is incomprehensible.

We have, we consider, all the necessary evidence to show that in Hofmann's sealed tube reaction phosphuretted hydrogen acts upon chloride of benzyl as ammonia does on an alkyl iodide; that is to say, that all of the following reactions occur:—

(1) $C_7H_7Cl + PH_3 = (C_7H_7)PH_2.HCl$
(2) $2C_7H_7Cl + PH_3 = (C_7H_7)_2PH.HCl + HCl$
(3) $3C_7H_7Cl + PH_3 = (C_7H_7)_3P.HCl + 2HCl$
(4) $4C_7H_7Cl + PH_3 = (C_7H_7)_4PCl + 3HCl$

With regard to the reaction (4) we are disposed to think that the oxide of tribenzyl-phosphine found by Hofmann, and mistaken by him for dibenzyl-phosphine (and by ourselves as well), owes its origin to the action of the potash on tetrabenzyl-phosphonium chloride or iodide, an action which N. Collie and one of us has already proved to occur easily in the following manner:—

$$(C_7H_7)_4PCl + KHO = (C_7H_7)_3PO + KCl + C_7H_8$$

We may here mention that we have also isolated from the products of Hofmann's reaction *all* the possible oxidised derivatives which the whole series of benzyl-phosphines can give rise to, viz.:—

$(C_7H_7)H_2PO_2$—Benzyl phosphinous acid.
$(C_7H_7)H_2PO_3$—Benzyl phosphinic acid.
$(C_7H_7)_2HPO_2$—Dibenzyl-phosphinic acid.
$(C_7H_7)_3PO$.—Tribenzyl-phosphine oxide.

5. We have also investigated the action of monobenzyl-phosphine on iodide of benzyl, with the view to obtaining pure dibenzyl-phosphine.

The reaction occurs readily at ordinary temperatures, though no sensible heat is evolved, and a solid crystalline product results. Although some of this was found to contain the correct percentage of iodine for the formula $(C_7H_7)_2HP.HI$, it is either a mixture of three substances at least,—viz., dibenzyl-phosphine hydriodate, tribenzyl-phosphine hydriodate, and tetrabenzyl-phosphonium iodide,—or readily decomposes and give rise to them. On decomposing it with potash a colourless liquid results, together with a solid—the latter being undoubtedly oxide of tribenzyl-phosphine. The liquid when exposed to the air oxidises readily, and yields both dibenzyl-phosphinic acid and oxide of tribenzyl-phosphine.

When it is distilled under diminished pressure with great care, a liquid passes over which contains the *primary* phosphine, as well as some of the secondary body, and possibly a little tribenzyl-phosphine also.

Our investigations on this very interesting and apparently extraordinary reaction are proceeding — the chief difficulty which we have to contend with being to obtain sufficient of the primary phosphine in a pure condition for the experiment.

So far the results of our investigations show that—

1. Hofmann's "dibenzyl-phosphine" is undoubtedly the oxide of tribenzyl-phosphine.

2. Dibenzyl-phosphine is probably a liquid combining easily with hydracids to give solid products of normal composition; oxidising in contact with air to form dibenzyl-phosphinic acid.

3. Tribenzyl-phosphine is a solid crystalline substance combining with hydracids, and both with sulphur and oxygen at ordinary temperatures to form solid products.

4. In Hofmann's sealed tube reaction (*i.e.*, action of phosphuretted hydrogen on benzyl chloride) *all* the phosphines (as well as the quaternary compound) are obtained, and also *all* their possible products of oxidation (some of the latter may possibly not pre-exist in the crude product of the reaction, but be formed by subsequent treatment). In addition, other substances are obtained. The reaction is in fact highly complex, and is, we venture to think, extremely interesting, as it is certainly different from all similar reactions observed by Hofmann with fatty derivatives.

5. Monobenzyl-phosphine is acted upon by crystallised iodide of benzyl at ordinary temperatures, and probably gives rise to free hydriodic acid, and to the secondary, tertiary, and quaternary compounds. If this be the case, the action is also comparable with that which occurs between ammonia and an alkyl iodide.

Our researches on the above subjects have involved a large expenditure of time, energy, and material, and have been tedious and troublesome in the extreme. We have, however, the satisfaction of believing that the work, which has extended over several years, is nearly at an end, and we trust in a few months to be able to give a detailed account of the whole of our experiments to the Society.

Differentiation of any (Scalar) Power of a Quaternion.

By **Alexander M'Aulay**, Ormond College, Melbourne.
Communicated by **Professor Tait.**

(Read February 18, 1889.)

Nowhere, I think, does Hamilton, or any other author, attempt the very fundamental problem of finding the differential of q^n where q is a quaternion and n any scalar. It was by noticing an oversight in Tait's *Quaternions*, § 182, where he considers $d.q^{\frac{1}{3}}$, that I was led to a consideration of the subject.

In that section Tait says that the equation

$$dq.r - rdq = dr.q - qdr \quad . \quad . \quad . \quad . \quad . \quad (1),$$

where

$$r = q^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

is sufficient to determine dq as a function of dr, but this will be found not to be the case. Equation (1) will be found to be equivalent to but two equations among scalars, whereas the equation from which it is derived, viz.,

$$q^2dq + qdq.q + dq.q^2 = dr \quad . \quad . \quad . \quad . \quad . \quad (3),$$

is equivalent to four such equations. Equation (1) may be written

$$\mathrm{VV}r\mathrm{V}dq = \mathrm{VV}q\mathrm{V}dr,$$

and thus it only gives the component of $\mathrm{V}dq$, viz., $\mathrm{V}^{-1}r.\mathrm{VV}r\mathrm{V}dq$, perpendicular to the axis of r. There are thus two scalars involved in dq (viz., $\mathrm{S}dq$ and the resolved part of $\mathrm{V}dq$ parallel to the axis of r), which, so far as equation (1) is concerned, are left perfectly arbitrary.

In fact, a, b being given quaternions and q a sought one, the equation

$$aq - qa = b,$$

in which the conditions

$$\mathrm{S}b = 0, \quad \mathrm{S}ab = 0,$$

must be satisfied, gives as solution

$$q = x + \tfrac{1}{2}(y + b)\mathrm{V}^{-1}a,$$

where x and y are arbitrary scalars. If we use the method and result Tait suggests, we are led to $dq = \infty$.*

[* See Note appended.—P. G. T.]

Before considering the problem in hand—that of finding $d.q^n$ explicitly—we must consider the properties of a certain linear quaternion function of any quaternion. The form of the function depends on q and n, and we shall denote it by (q, n). Suppose a is any quaternion. Split it up into two parts, a' a quaternion coplanar with q, and a'' a vector perpendicular to the axis of q. There are several useful forms of a' and a''. Notice that we have

$$a' = \mathrm{S}a + \text{component of } \mathrm{V}a \text{ parallel to } \mathrm{V}q.$$
$$a'' = \text{component of } \mathrm{V}a \text{ perpendicular to } \mathrm{V}q.$$

This gives

$$a' + a'' = a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$
$$a' = \mathrm{S}a + \mathrm{V}q\mathrm{S}.\mathrm{V}^{-1}qa \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$
$$a'' = \mathrm{V}q\mathrm{V}\mathrm{V}^{-1}q\mathrm{V}a = \mathrm{V}^{-1}q^n\mathrm{V}\mathrm{V}q^n\mathrm{V}a \quad . \quad . \quad . \quad . \quad (6),$$
or $$2a'' = \mathrm{V}^{-1}q(qa - aq) = \mathrm{V}^{-1}q^n(q^n a - aq^n) \quad . \quad . \quad . \quad (7).$$

These are some of the simple forms of a' and a'', and we shall employ more than one of them.

The quaternion function (q, n) is defined by the equation

$$(q, n)a = nq^{n-1}a' + \frac{\mathrm{V}q^n}{\mathrm{V}q}a'' \quad . \quad . \quad . \quad . \quad . \quad (8).$$

We will give some of the simpler forms of (q, n) in full, though equation (8) is what we shall use in the present investigation. Putting $a' = a - a''$, and substituting for a'' the last value in equation (7), we get

$$(q, n)a = nq^{n-1}a + \tfrac{1}{2}\left(\frac{1}{\mathrm{V}q} - \frac{nq^{n-1}}{\mathrm{V}q^n}\right)(q^n a - aq^n) \quad . \quad . \quad . \quad (9).$$

Again, substituting the first value in equation (6),

$$(q, n)a = nq^{n-1}a + (\mathrm{V}q^n - nq^{n-1}\mathrm{V}q)\mathrm{V}\mathrm{V}^{-1}q\mathrm{V}a \quad . \quad . \quad (10).$$

Another important modification is obtained from the fact that if λ is a vector perpendicular to the axis of r,

$$r\lambda r = \mathrm{T}^2 r.\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

(for by Tait's *Quaternions*, § 354, $r^{-1}\lambda r = \mathrm{T}^2 r.r^{-2}\lambda$).

$$\therefore \quad q^{\frac{n-1}{2}} a'' q^{\frac{n-1}{2}} = \mathrm{T}^{n-1}q.a''.$$

Again, $\because$ a' and q are coplanar $q^{\frac{n-1}{2}} a' q^{\frac{n-1}{2}} = q^{n-1} a'$. $\therefore$ from eq. (8)

$$(q, n)a = q^{\frac{n-1}{2}} \left(na' + \frac{\mathrm{VU}q^n}{\mathrm{VU}q} a'' \right) q^{\frac{n-1}{2}} .$$

Let us here substitute $a - a''$ for a'; and for a''

$$\mathrm{V}^{-1}\mathrm{U}q^n \mathrm{V}\mathrm{V}\mathrm{U}q^n \mathrm{V}a = \tfrac{1}{2}\mathrm{V}^{-1}\mathrm{U}q^n (\mathrm{U}q^n . a - a\mathrm{U}q^n) .$$

Thus

$$(q, n)a = q^{\frac{n-1}{2}} \{ na + \tfrac{1}{2} \left(\frac{1}{\mathrm{VU}q} - \frac{n}{\mathrm{VU}q^n} \right) (\mathrm{U}q^n . a - a\mathrm{U}q^n) \} q^{\frac{n-1}{2}} \quad (12).$$

This again may be written

$$(q, n)a = nq^{\frac{n-1}{2}} a q^{\frac{n-1}{2}} + \frac{1}{2\mathrm{T}q} \left(\frac{1}{\mathrm{VU}q} - \frac{n}{\mathrm{VU}q^n} \right) (q^{\frac{n}{2}} a \mathrm{K}q^{\frac{n}{2}} - \mathrm{K}q^{\frac{n}{2}} a q^{\frac{n}{2}}) \quad (13).$$

We might with ease write a number of other forms.

It is to be observed that in equations (9), (10), (12), (13) the long second term is in every case a vector, for it consists of a vector perpendicular to the axis of q operated upon by some quaternion which is coplanar with q. Hence

$$\mathrm{S}\{(q, n)a\} = n\mathrm{S}q^{n-1}a \quad . \quad . \quad . \quad . \quad . \quad (14).$$

We now proceed to those properties of (q, n) which we require. First notice that (q, n) is commutative with any quaternion coplanar with q; and also with (r, m) (which if not sufficiently obvious will appear incidentally immediately) if r is a quaternion coplanar with q. Now we have

$$\begin{aligned} (q, n)(r, m)a &= (q, n)(r, m)a' + (q, n)(r, m)a'' \\ &= mnq^{n-1}r^{m-1}a' + \frac{\mathrm{V}q^n \mathrm{V}r^m}{\mathrm{V}q\mathrm{V}r} a'' \text{ by equation (8)} \\ &\quad [= (r, m)(q, n)a]. \end{aligned}$$

Putting then $r = q^n$ we get

$$(q, n)(q^n, m) = (q, mn) = (q, m)(q^m, n) \quad . \quad . \quad . \quad (15).$$

Putting $m = \dfrac{1}{n}$

$$(q, n)\left(q^n, \frac{1}{n}\right) = (q, 1) = 1 \text{ by equations (8) and (4)},$$

i.e.,

$$(q, n)^{-1} = \left(q^n, \frac{1}{n}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

[This means that if $(q, n)a = b$, then $a = \left(q^n, \frac{1}{n}\right)b$].

Remembering the meaning of equation (11), we see that

$$(q, n)q^{-n}(\quad)q^{-n} = -(q, -n) \quad . \quad . \quad . \quad . \quad (17).$$

The last property we propose to prove is that when n is a positive integer

$$(q, n) = (\quad)q^{n-1} + q(\quad)q^{n-2} + q^2(\quad)q^{n-3} + \ldots + q^{n-1}(\quad) \quad (18).$$

Calling, for brevity, the linear function on the right Q, we see that $\because$ a' and q are coplanar

$$Qa' = nq^{n-1}a'.$$

Again

$$2Qa'' = QV^{-1}q(qa - aq)[\text{equation (7)}] = V^{-1}qQ(qa - aq)$$

$$= V^{-1}q(q^n a - aq^n) = 2\frac{Vq^n}{Vq}a'' \text{ [equation (7)]}.$$

$$\therefore \qquad Qa = Qa' + Qa'' = nq^{n-1}a' + \frac{Vq^n}{Vq}a'' = (q, n)a,$$

which proves the proposition. [Notice that equation (16) combined with (18) solves the equation $aq^{n-1} + qaq^{n-2} + \ldots + q^{n-1}a = b$].

We can now prove that for all scalar values of n

$$d.q^n = (q, n)dq \quad . \quad . \quad . \quad . \quad . \quad . \quad (19).$$

Equation (18) proves this for n a positive integer. Next suppose $n = l/m$ where l and m are positive integers. Let

$$q^n = r, \text{ i.e. } q^l = r^m.$$

Differentiating, and using the first case,

$$(q, l)dq = (r, m)dr$$

$$d.q^n = dr = (r, m)^{-1}(q, l)dq$$

$$= \left(r^m, \frac{1}{m}\right)(q, l)dq \quad \text{[equation (16)]}$$

$$= \left(q^l, \frac{1}{m}\right)(q, l)dq$$

$$= (q, n)dq \qquad \text{[equation (15)]}.$$

Lastly, suppose n is negative, and $= -m$. Thus

$$\begin{aligned} d.q^n &= -q^n d.q^m.q^n \\ &= -(q,-n)q^n dq q^n \\ &= (q,n)dq \qquad \text{[equation (17)]}, \end{aligned}$$

which proves the proposition for all cases.

The various forms given above for (q, n) thus give so many forms for $d.q^n$. Notice the meaning of equation (14). This gives

$$\mathrm{S}(d.q^n) = n\mathrm{S}q^{n-1}dq \quad . \quad . \quad . \quad . \quad . \quad (20),$$

which gives the ordinary form for the differential of a power of a scalar. If we put $q = a$ vector ρ, $a = d\rho$, we have

$$\alpha' = \rho\mathrm{S}\rho^{-1}d\rho \qquad \alpha'' = \rho\mathrm{V}\rho^{-1}d\rho,$$

and $\therefore$ equation (8) gives

$$d.\rho^n = n\rho^n\mathrm{S}\rho^{-1}d\rho + \mathrm{V}\rho^n.\mathrm{V}\rho^{-1}d\rho \quad . \quad . \quad . \quad (21),$$

and this, be it remembered, is true not merely for integral values of n.

Note on Mr M'Aulay's Paper. By Professor Tait.

(Read February 18, 1889.)

There is, undoubtedly, an omission in § 182 of my *Quaternions* (2nd ed.), but it is by no means so serious as Mr M'Aulay asserts. In fact the solution there given is merely an unfinished one, not in any sense erroneous. I sketch briefly the completion of it, as prepared for the new edition of my book, which is now being printed.

The equation

$$q^{n-1}dq + q^{n-2}dqq + \ldots + dqq^{n-1} = \phi(dq) = dr \quad . \quad . \quad (1)$$

gives, as in my book,

$$q^n dq - dqq^n = qdr - drq,$$

but this does *not* make dq infinite. In fact it gives

$$2\mathrm{V}.\mathrm{V}q^n\mathrm{V}dq = 2\mathrm{V}.\mathrm{V}q\mathrm{V}dr \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Now it is easy to see that

$$\mathrm{V}q^n = \mathrm{Q}_n\mathrm{V}q,$$

where

$$\mathrm{Q}_n = n(\mathrm{S}q)^{n-1} - \frac{n.n-1.n-2}{1\ .\ 2\ .\ 3}(\mathrm{S}q)^{n-3}(\mathrm{TV}q)^2 + \&c.$$

Thus (2) gives

$$Q_n \nabla dq = \nabla dr + x \nabla q ,$$

so that

$$Q_n dq = (y + x\nabla q) + dr \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

x and y being undetermined scalars.

Substitute in (1), and again use (3), and we have

$$Q_n dq = dr + \frac{1}{nq^{n-1}}(Q_n dr - \phi dr) ,$$

which is the complete solution.

Note that this gives, by means of (1), for an equation satisfied by the linear function ϕ,

$$(\phi - nq^{n-1})(\phi - Q_n) = 0 .$$

The fact that this equation is of the second, instead of the fourth degree, is of course due to the very special form of ϕ as shown in (1) above. In fact the first factor kills any scalar, or any vector in the plane of q; while the second kills a vector parallel to the axis of q.

Additional Remarks on the Virial of Molecular Force.

By **Prof. Tait.**

(Read March 18, 1889.)

(*Abstract.*)

In my paper, read Jan. 21, I stated that I would not "for the present, insist on this point [the value of β] further than to say that the main effect is merely to alter the value of the disposable quantity A, below."

The present paper contains the more complete investigation here promised, and shows that the Virial equation takes the form

$$p(V - \beta) = kt - \frac{A(V - \beta)}{V(V - \gamma)} + \frac{C}{V - \alpha}$$

which, *as* α *and* γ *are now at least nearly identical*, is practically the same form as that previously given.

The Theory of Determinants in the Historical Order of its Development. By Thomas Muir, M.A., LL.D.

PART I. *Determinants in General* (1829–35).

(Continued from p. 544 of vol. xv.)

REISS (1829).

[Mémoire sur les fonctions semblables de plusieurs groupes d'un certain nombre de fonctions ou élémens. *Correspondance math. et phys.*, v. pp. 201–215.]

In Reiss we have an author who starts to his subject as if it were entirely new, the only preceding mathematician whom he mentions being Lagrange. Like Cauchy he opens by explaining a mode of forming functions more general than those of which he afterwards treats, the essence of it being that an expression involving several of the $n.\nu$ quantities,

$$\begin{matrix} a^\alpha & a^\beta & a^\gamma & \ldots & a^\rho \\ b^\alpha & b^\beta & b^\gamma & \ldots & b^\rho \\ c^\alpha & c^\beta & c^\gamma & \ldots & c^\rho \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r^\alpha & r^\beta & r^\gamma & \ldots & r^\rho \end{matrix},$$

is taken, and each *exponent* ("exposant") changed successively with all the other exponents, $\alpha, \beta, \ldots$, or each *base* changed with all the other bases, $a, b, \ldots$. Only a line or two, however, is given to this, the special class known to us as determinants being taken up at once.

His notation for

$$a^1b^2c^3 - a^1b^3c^2 - a^2b^1c^3 + a^2b^3c^1 + a^3b^1c^2 - a^3b^2c^1$$

is

$$(abc, \overline{123}), \qquad \text{(VII. 7)}$$

a line being drawn above the exponents to indicate permutation. His rule of formation of the terms and rule of signs are combined after the manner of Hindenburg. Like Hindenburg, he arranges the permutations as one arranges numbers in increasing order of magnitude; but, unlike Hindenburg, after the arrangement has

been made he determines the sign of any *particular* term. On this point his words are (p. 202)

> "Cela fait, déterminons généralement le signe du Mme produit (soit $\dot{M}$) de la manière suivante. Le nombre M sera renfermé entre les produits $1.2.3 \ldots l$ et $1.2.3 \ldots l(l+1)$; soit $M = m + \lambda \times 1.2.3 \ldots l$, de sorte que $\lambda < l+1$, et $m > 0$ et $< 1 + 1.2.3 \ldots l$. Cela étant, faisons $\dot{M} = \dot{m}(-1)^{\lambda}$." (III. 24)

This apparently means that if the sign of the 23rd term in the expansion of

$$(abcd, \overline{1234})^*$$

be wanted, we divide 23 by 1.2.3, getting the quotient 3 and the remainder 5, and thence conclude that the sign wanted is got from the sign of the 5th term by multiplying the latter by $(-1)^3$. Of course 5 has then to be dealt with after the manner of 23, the quotient and remainder this time being 2 and 1, so that we conclude that the sign of the 5th term is got from the sign of the 1st term by multiplying by $(-1)^2$. And the sign of the 1st term being +, the sign of the 23rd is thus seen to be

$$(-1)^{3+2} \quad i.e. \quad -.$$

It would seem at first as if the case where M is itself a factorial were neglected. This however, is not so, the condition $m < 1 + 1.2.3 \ldots l$ being corrective of the opening statement that M must lie between $1.2.3 \ldots l$ and $1.2.3 \ldots l(l+1)$. For example, the term being the 24th, we put 24 in the form $3 \times 1.2.3 + 6$, and thus learn that the sign required is different from the sign of the 6th term: then we put 6 in the form $2 \times 1.2 + 2$, and thus learn that the sign of the 6th term is the same as the sign of the 2nd term; finally, we put 2 in the form $1 \times 1 + 1$, which shows that the sign of the 2nd term differs from the sign of the 1st: the conclusion of the whole being that the signs of the 24th and 1st terms are the same, or that they are connected by the factor $(-1)^{3+2+1}$.

Though interesting in itself, a more troublesome form of the rule of signs for the purposes of demonstration it is scarcely possible to conceive, and, as might therefore be expected, it is on the score of logical development that Reiss' paper is weak. Through

* Or $(abcde, \overline{12345})$, or indeed $(a_1a_2 \ldots a_n, \overline{123 \ldots n})$.

inability to use the rule later in the demonstration of the so-called Laplace's expansion-theorem, he is forced to supplement it by another convention. His words are (p. 203)—

"Avant d'aller plus loin, faisons encore la détermination suivante. Soit ω une fonction quelconque dans laquelle les k quantités A,B,C,... A^k entrent d'une manière quelconque. Supposons que ces dernières soient les k premières de l'échelle $\begin{pmatrix} A\ B\ C\ \ldots\ A^k\ \ldots\ S \\ 1\ 2\ 3\ \ldots\ k\ \ldots\ s \end{pmatrix}$. Qu'on fasse avec ces s élémens toutes les combinaisons sans répétition de la classe k, et qu'on les substitue successivement au lieu de A,B, . . . A^k dans la fonction ω; c'est-à-dire le premier élément de chaque combinaison à A, le second à B, etc. Nous obtiendrons par là autant de fonctions semblables à ω qu'il y a de combinaisons de la classe k de s élémens. Or, entre toutes les combinaisons qui en précèdent une quelconque, il s'en trouvera une qui aura $k-1$ élémens communs avec elle, tandis que les deux élémens qui restent isolés dans l'une et l'autre se suivent immédiatement dans l'échelle. Donnons à la fonction qui contient la dernière de ces combinaisons le signe opposé à celui de l'autre fonction; par conséquent les signes de toutes les fonctions semblables à ω seront parfaitement déterminés, et dépendront du signe de la première fonction (f (A,B,C, . . . A^k)). Soit, par exemple, $s=5$, $k=3$; nous aurons successivement, en remplaçant A,B,C, . . . S par 1, 2, 3, 4, 5, et en donnant le signe (+) à f (123),

$$+f(123), -f(124), +f(125), +f(134), -f(135)$$
$$+f(145), -f(234), +f(235), -f(245), +f(345).$$

Voici comment on déterminera le signe de chaque fonction semblable à ω d'après celui d'une antre quelconque. Qu'on cherche les nombres qui se trouvent dans l'échelle $\begin{pmatrix} A\ B\ C\ \ldots\ A^k\ \ldots\ S \\ 1\ 2\ 3\ \ldots\ k\ \ldots\ s \end{pmatrix}$ sous les élémens de l'une et de l'autre de ces fonctions. Si l'on nomme h et h' leurs sommes respectives, on trouvera le signe de l'une des fonctions = $(-1)^{h'-h}\times$ le signe de l'autre."

Four theorems he considers fundamental, viz., those known to us as (1) Bézout's recurrent law of formation, in all its generality; (2) Vandermonde's proposition that permutation of bases leads to the same result as permutation of exponents; (3) Laplace's expansion-theorem; (4) Vandermonde's proposition regarding the effect of making two bases or two exponents equal. The two most important, viz. (1) and (2), he leaves without proof, and the 4th he says he would at once deduce from the 3rd,—doubtless by choosing the expansion in which the first factor of every term would be of the form

$$(aa,\ \overline{\alpha\beta})$$

and therefore equal to zero.

The proof of the 2nd theorem, viz.,

$$(abc\ldots r, \overline{\alpha\beta\gamma\ldots\rho}) \;=\; \overline{(abc\ldots r}, \alpha\beta\gamma\ldots\rho),$$

is by the method of so-called induction, and may be illustrated in a later notation by considering the case

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$$

From theorem (1) we have

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix},$$

$$= -b_1\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix} + b_2\begin{vmatrix} a_1 & a_3 \\ c_1 & c_3 \end{vmatrix} - b_3\begin{vmatrix} a_1 & a_2 \\ c_1 & c_2 \end{vmatrix},$$

$$= c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + c_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}.$$

But by hypothesis all the determinants on the right here may have their rows changed into columns; and this being done we have by addition and the use of theorem (1)—

$$3\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},$$

and thence the identity required. (IX. 4)

To this proof the following note is appended (p. 207) :—

> "Cette démonstration quoiqu'assez simple semble reposer cependant sur un artifice de calcul: mais en cherchant une démonstration *directe*, j'ai rencontré une difficulté d'un genre particulier. En effet, on trouve facilement que l^{me} terme de l'une des fonctions en question est aussi égal ou au même terme de l'autre, ou généralement au m^{me}, et que, dans le dernier cas, le m^{me} terme de la première est aussi égal au l^{me} de la seconde, abstraction faite des signes. (IX. 5) Mais l'identité de ces derniers (qui est de rigueur) exige des explications très-longues et beaucoup moins élémentaires que la démonstration que je viens de donner."

The remaining six or seven pages of the paper are more interesting, and concern the subject of vanishing aggregates of products of pairs of determinants. The theorems were suggested by taking, as we now say, a determinant of even order having its last n rows identical with its first n rows, *e.g.*, the determinant

$$(abab, \overline{1234}),$$

and using theorem (3) to expand it in terms of minors formed from the first n rows and their complementary minors. When n is even, a proof is thus obtained, as we have seen in the footnote to the account of Bézout's paper of 1779, that the first half of the expansion is equal to zero. When n is odd, the method fails, although the proposition is still true.* Reiss's enunciation is as follows (p. 209) :—

* It is worthy of note in passing, that a common method does exist for establishing the two cases,—a method quite analogous to Reiss's, but difficult of suggestion to one who used his notation, or indeed to any one who had no notation suitable for determinants whose elements had special numerical values. All the change necessary is to make the last n elements of the first column each equal to zero. This causes no difference in the result when n is even, *e.g.*, from the identity

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ . & a_2 & a_3 & a_4 \\ . & b_2 & b_3 & b_4 \end{vmatrix} = 0$$

we have, as before,

$$|a_1b_2|.|a_3b_4| - |a_1b_3|.|a_2b_4| + |a_1b_4|.|a_2b_3| = 0 ;$$

and when n is odd, the second half of the terms which previously gave trouble do not occur.

"Théorème V.—Soient les échelles

$$\begin{pmatrix} a\ b\ \ldots\ r, & a & , & b & , \ldots & r \\ 1\ 2\ \ldots\ n, & n+1, & & n+2, & \ldots & 2n \end{pmatrix} \text{ et } \begin{pmatrix} \alpha\ \beta\ \gamma\ \ldots\ \alpha^n, & \alpha^{n+1} & , \ldots & \rho \\ 1\ 2\ 3\ \ldots\ n\ , & n+1, & \ldots & 2n \end{pmatrix},$$

qu'on fasse avec les élémens $\beta, \gamma, \ldots, \rho$ toutes les combinaisons de la classe $(n-1)$, et qu'on les substitue successivement dans le premier facteur du produit

$$(ab\ldots r, \alpha\beta\gamma\ldots\alpha^n) \,.\, (ab\ldots r,\ \alpha^{n+1}\ldots\rho)$$

au lieu de $\beta\gamma\ldots\alpha^n$; qu'on remplace maintenant dans l'autre facteur les exposans $\alpha^{n+1}\ldots\rho$ par tous ceux qui ne se trouvent pas dans le premier, en ayant soin de les écrire suivant l'ordre indiqué par les échelles. Si l'on donne au premier produit le signe $(+)$, et qu'on détermine les signes de tous les autres d'après (II), la somme algébrique en sera $=0$, que le nombre n soit pair ou impair." (XXIII. 8)

An example of it is

$$\begin{aligned} &(abc, 123)(abc, 456) - (abc, 124)(abc, 356) \\ +&(abc, 125)(abc, 346) - (abc, 126)(abc, 345) \\ +&(abc, 134)(abc, 256) - (abc, 135)(abc, 246) \\ +&(abc, 136)(abc, 245) + (abc, 145)(abc, 236) \\ -&(abc, 146)(abc, 235) + (abc, 156)(abc, 234) = 0, \end{aligned}$$

the left-hand side being nothing more than the first ten terms of one of the expansions of the vanishing determinant

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{vmatrix},$$

or the other ten terms with their signs changed. Reiss's proof is lengthy and troublesome, the method being to expand each factor in terms of the a's and their complementary minors, perform the the multiplications (*e.g.*, in the special case just given the multipli-

cation of $a_1|b_2c_3| - a_2|b_1c_3| + a_3|b_1c_2|$ by $a_4|b_5c_6| - a_5|b_4c_6| + a_6|b_4c_5|$, &c.) and show that the terms of the final aggregate occur in pairs which annul themselves.

The next theorem is of still greater interest, because it is that peculiar generalisation of the preceding which in later times came to be known as the *Extensional.* The way in which it is established is also noteworthy, viz., by deducing it as a special case from the theorem of which, as we have said, it may be viewed as a generalisation. The authors words are (p. 213) :—

> "Ce théorème nous conduit à une relation qui existe dans le cas le plus général, savoir si $\nu - n$ est un nombre quelconque ou positif ou négatif. Supposons $\nu > n$, et $\nu - n = \mathrm{N}$; soient les échelles,
>
> $$\begin{pmatrix} a\,b \ldots r, & a, & b, \ldots r, & \mathrm{A}, & \mathrm{B}, \ldots \mathrm{R} \\ 1\,2 \ldots \mathrm{N}, & \mathrm{N}+1, & \mathrm{N}+2, \ldots 2\mathrm{N}, & 2\mathrm{N}+1, & 2\mathrm{N}+2, \ldots \nu \end{pmatrix}$$
>
> et
>
> $$\begin{pmatrix} \alpha\,\beta \ldots \alpha^{\mathrm{N}}, & \alpha^{\mathrm{N}+1}, \ldots \rho, & \mathrm{A}, & \mathrm{B}, \ldots \mathrm{P} \\ 1\,2 \ldots \mathrm{N}, & \mathrm{N}+1, \ldots 2\mathrm{N}, & 2\mathrm{N}+1, & 2\mathrm{N}+2, \ldots \nu \end{pmatrix}.$$
>
> Qu'on fasse avec les élémens $\beta, \gamma, \ldots \alpha^{\mathrm{N}}, \alpha^{\mathrm{N}+1} \ldots \rho$ toutes les combinaisons de la classe $\mathrm{N} - 1$; qu'on les substitue successivement au lieu de $\beta \ldots \alpha^{\mathrm{N}}$ dans le premier facteur du produit
>
> $$(ab \ldots r\mathrm{AB} \ldots \mathrm{R},\ \alpha\beta \ldots \alpha^{\mathrm{N}}\mathrm{AB} \ldots \mathrm{P})$$
> $$\times (ab \ldots r\mathrm{AB} \ldots \mathrm{R},\ \alpha^{\mathrm{N}+1} \ldots \rho\,\mathrm{AB} \ldots \mathrm{P});$$
>
> qu'on remplace dans l'autre facteur les exposans $\alpha^{\mathrm{N}+1} \ldots \rho$ par tous ceux qui ne se trouvent pas dans le premier : qu'on détermine enfin le signe de chaque produit d'après (II) : la somme algébrique en sera = 0. (XXIII. 9) (XLV. 6)
>
> "En effet, supposons les échelles
>
> $$\begin{pmatrix} a\,b \ldots r, & \mathrm{A}, & \mathrm{B}, \ldots \mathrm{R}, & a, & b, & \ldots r, & \mathrm{A}, & \mathrm{B}, \ldots & \mathrm{R} \\ 1\,2 \ldots \mathrm{N}, & \mathrm{N}+1, & \mathrm{N}+2, \ldots \nu - \mathrm{N}, & \nu - \mathrm{N}+1, & \nu - \mathrm{N}+2, & \ldots \nu, & \nu+1, & \nu+2, \ldots & 2\nu - 2\mathrm{N} \end{pmatrix}$$
>
> et
>
> $$\begin{pmatrix} \alpha\,\beta \ldots \alpha^{\mathrm{N}}, & \alpha^{\mathrm{N}+1}, \ldots \rho, & \mathrm{A}, & \mathrm{B}, \ldots \mathrm{A}^{\nu-3\mathrm{N}}, & \mathrm{A}^{\nu-3\mathrm{N}+1}, \ldots \mathrm{P}, & \mathrm{A}, \ldots & \mathrm{P} \\ 1\,2 \ldots \mathrm{N}, & \mathrm{N}+1, \ldots 2\mathrm{N}, & 2\mathrm{N}+1, & 2\mathrm{N}+2, \ldots & \nu - \mathrm{N}, & \nu - \mathrm{N}+1, \ldots \nu, & \nu+1, \ldots & 2\nu - 2\mathrm{N} \end{pmatrix}.$$

Formons avec ces élémens la fonction décrite dans le dernier théorème : la somme totale en sera donc = 0, et le premier terme aura la forme

$$(ab \ldots r\mathrm{AB} \ldots \mathrm{R},\ \alpha\beta \ldots \rho\mathrm{A} \ldots \mathrm{A}^{\nu-3N})$$
$$\times (ab \ldots r\mathrm{AB} \ldots \mathrm{R},\ \mathrm{A}^{\nu-3N+1} \ldots \mathrm{PA} \ldots \mathrm{P}).$$

Or, on voit facilement que tous les termes qui ne contiennent pas dans chaque facteur *tous* les exposans A,B, . . . P, s'évanouiront séparément, parce qu'il y aura des exposans identiques dans l'un ou l'autre des facteurs. Il ne restera donc que les termes qui, contenant α dans le premier facteur, y épuisent successivement toutes les combinaisons de la classe N–1 des élémens $\beta, \gamma, \ldots \rho$. Mais les signes de ces termes sont évidemment déterminés comme ils devaient l'être; partant la somme algébrique de tous les termes est = 0, ce qu'il fallait démontrer.

This will be best understood by considering a special example. Going back to the previous theorem, and selecting its simplest case, we have

$$|a_1 b_2| . |a_3 b_4| \ - \ |a_1 b_3| . |a_2 b_4| \ + \ |a_1 b_4| . |a_2 b_3| \ = \ 0.$$

Now what the new theorem asserts in regard to this is that we may with impunity *extend* each of the determinants occurring in it, provided the extension be the same throughout. For example, choosing the extension $\xi_5\, \zeta_6\, \eta_7$,* we can, in virtue of the new theorem, assert the truth of the identity

$$|a_1 b_2 \xi_5 \eta_6 \zeta_7| . |a_3 b_4 \xi_5 \eta_6 \zeta_7| \ - \ |a_1 b_3 \xi_5 \eta_6 \zeta_7| . |a_2 b_4 \xi_5 \eta_6 \zeta_7|$$
$$+ \ |a_1 b_4 \xi_5 \eta_6 \zeta_7| . |a_2 b_3 \xi_5 \eta_6 \zeta_7| \ = \ 0.$$

That the two may be viewed as cases of the same theorem will be apparent when it is pointed out that just as the first is derivable from

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ . & a_2 & a_3 & a_4 \\ . & b_2 & b_3 & b_4 \end{vmatrix} = 0,$$

* In Reiss's notation the extension is $\mathrm{A}_\mathrm{A}\, \mathrm{B}_\mathrm{B} \ldots \mathrm{R}_\mathrm{P}$.

so the second is derivable in exactly the same way from a perfectly similar identity,* viz.

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_5 & a_6 & a_7 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_5 & b_6 & b_7 \\ \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 & \xi_7 & \xi_5 & \xi_6 & \xi_7 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 & \eta_6 & \eta_7 & \eta_5 & \eta_6 & \eta_7 \\ \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & \zeta_5 & \zeta_6 & \zeta_7 \\ . & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_5 & a_6 & a_7 \\ . & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_5 & b_6 & b_7 \\ . & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 & \xi_7 & \xi_5 & \xi_6 & \xi_7 \\ . & \eta_2 & \eta_3 & \eta_4 & \eta_5 & \eta_6 & \eta_7 & \eta_5 & \eta_6 & \eta_7 \\ . & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & \zeta_5 & \zeta_6 & \zeta_7 \end{vmatrix} = 0.$$

Many more products than three (126 in fact) arise in the latter case; but, for the reason stated by Reiss, only three of them do not vanish.

JACOBI (1829, 1830).

[Exercitatio algebraica circa discerptionem singularem fractionum, quae plures variabiles involvunt. *Crelle's Journal*, v. pp. 344–364].

[De resolutione aequationum per series infinitas. *Crelle's Journal*, vi. pp. 257–286.]

By such memoirs as these, in which Jacobi continued to use determinants, the functions were kept before the mathematical

* It is perhaps a little more readily seen to be derivable from

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & . & . & . \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & . & . & . \\ \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 & \xi_7 & . & . & . \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 & \eta_6 & \eta_7 & . & . & . \\ \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & . & . & . \\ . & a_2 & a_3 & a_4 & . & . & . & a_5 & a_6 & a_7 \\ . & b_2 & b_3 & b_4 & . & . & . & b_5 & b_6 & b_7 \\ . & \xi_2 & \xi_3 & \xi_4 & . & . & . & \xi_5 & \xi_6 & \xi_7 \\ . & \eta_2 & \eta_3 & \eta_4 & . & . & . & \eta_5 & \eta_6 & \eta_7 \\ . & \zeta_2 & \zeta_3 & \zeta_4 & . & . & . & \zeta_5 & \zeta_6 & \zeta_7 \end{vmatrix} = 0.$$

world. For the present it will suffice to note in regard to them that although general determinants in Laplace's notation occur (p. 351, &c.), the real interest of the papers arises from the fact that use is made in them of that special form which afterwards came to be associated with Jacobi's name. His introductory words concerning it are as follows (pp. 348, 349) :—

"Vocemus porro Δ determinantem differentialium partialium sequentium :

$$\begin{array}{llll} \frac{\partial u}{\partial x}, & \frac{\partial u}{\partial x_1}, & \frac{\partial u}{\partial x_2}, \ldots, & \frac{\partial u}{\partial x_{n-1}} \\ \frac{\partial u_1}{\partial x}, & \frac{\partial u_1}{\partial x_1}, & \frac{\partial u_1}{\partial x_2}, \ldots, & \frac{\partial u_1}{\partial x_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial u_{n-1}}{\partial x}, & \frac{\partial u_{n-1}}{\partial x_1}, & \frac{\partial u_{n-1}}{\partial x_2}, \ldots, & \frac{\partial u_{n-1}}{\partial x_{n-1}}. \end{array}$$

Erit *e.g.* pro tribus functionibus u, u_1, u_2, tribusque variabilibus x, y, z :

$$\begin{aligned} \Delta = & \frac{\partial u}{\partial x}\cdot\frac{\partial u_1}{\partial y}\cdot\frac{\partial u_2}{\partial z} - \frac{\partial u}{\partial x}\cdot\frac{\partial u_1}{\partial z}\cdot\frac{\partial u_2}{\partial y} - \frac{\partial u_1}{\partial y}\cdot\frac{\partial u_2}{\partial x}\cdot\frac{\partial u}{\partial z} \\ & - \frac{\partial u_2}{\partial z}\cdot\frac{\partial u}{\partial y}\cdot\frac{\partial u_1}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{\partial u_1}{\partial z}\cdot\frac{\partial u_2}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial u_1}{\partial x}\cdot\frac{\partial u_2}{\partial y}, \end{aligned}$$

quam patet expressionem casu, quo u, u_1, u_2 sunt expressiones lineares, in expressionem ipsius Δ supra exhibitam redire."

MINDING (1829).

[Auflösung einiger Aufgaben der analytischen Geometrie vermittelst des barycentrischen Calculs. *Crelle's Journal*, v. pp. 397–401.]

Unlike Jacobi, Minding was unaware, apparently, of the existence of a theory of determinants. The functions occur at every step of his investigation, yet he makes no use of their known properties to obtain his results.

He deals with four problems in his memoir, the second two being the analogues, in space, of the first two. Nothing noteworthy

occurs in connection with the latter save that use is made of the identity,

$$\frac{\beta'\gamma'' - \beta''\gamma'}{a'} = a(b'c'' - b''c') + a'(b''c - bc'') + a''(bc' - b'c),$$

where $\beta' = ba' - b'a$, $\beta'' = b'a'' - b''a'$,

$\gamma' = ca' - c'a$, $\gamma'' = c'a'' - c''a'$.

This identity, it may be remembered, we have noted under Lagrange as an elementary case of the theorem afterwards well known regarding a minor of the adjugate determinant. Strange to say, it makes only its second appearance here fifty-six years afterwards. In the interim, too, no other special case of the theorem seems to have been established.

The third is that if P, P′, P″, P‴, be four points in space, given by the equations,

$$\begin{aligned} q\ \mathrm{P} &= a\ \mathrm{A} + b\ \mathrm{B} + c\ \mathrm{C} + d\ \mathrm{D}, \\ q'\ \mathrm{P}' &= a'\ \mathrm{A} + b'\ \mathrm{B} + c'\ \mathrm{C} + d'\ \mathrm{D}, \\ q''\ \mathrm{P}'' &= a''\ \mathrm{A} + b''\ \mathrm{B} + c''\ \mathrm{C} + d''\ \mathrm{D}, \\ q'''\ \mathrm{P}''' &= a'''\ \mathrm{A} + b'''\ \mathrm{B} + c'''\ \mathrm{C} + d'''\ \mathrm{D}; \end{aligned}$$

then for the bulk of the tetrahedron P P′ P″ P‴, we have

$$\frac{\mathrm{PP'P''P'''}}{\mathrm{ABC\ D}} = \frac{\mathrm{A} + \mathrm{A}' + \mathrm{A}''}{qq'q''q'''},$$

where

$$\mathrm{A} = \partial'(\beta''\gamma''' - \beta'''\gamma''), \quad \mathrm{A}' = \partial''(\beta'''\gamma' - \beta'\gamma'''), \quad \mathrm{A}'' = \partial'''(\beta'\gamma'' - \beta''\gamma'),$$

and

$$\begin{aligned} \beta' &= a'b - ab', & \gamma' &= a'c - ac', & \partial' &= a'd - ad', \\ \beta'' &= a''b' - a'b'', & \gamma'' &= a''c' - a'c'', & \partial'' &= a''d' - a'd'', \\ \beta''' &= a'''b'' - a''b''', & \gamma''' &= a'''c'' - a''c''', & \partial''' &= a'''d'' - a''d'''. \end{aligned}$$

The transformation of $\mathrm{A} + \mathrm{A}' + \mathrm{A}''$ into the form

$$a'a''|ab'c''d'''|$$

—a transformation all-important for Minding's purpose — is not made: but in the remark,

"Man kann den Ausdruck $A+A'+A''$ leicht entwickeln, und wird ihn dann durch $a'a''$ theilbar finden,"

there is evidently a foreshadowing of the identity

$$\begin{vmatrix} |a'\ b\ |, & |a'\ c\ |, & |a'\ d\ | \\ |a''\ b'\ |, & |a''\ c'\ |, & |a''\ d'\ | \\ |a'''b''|, & |a'''c''|, & |a'''d''| \end{vmatrix} = -a'a''|ab'c''d'''|.$$

The fourth theorem, concerning the tetrahedron enclosed by four given planes,

$$\begin{aligned} &A+xB+yC+(a\ +b\ x+c\ y)C, \\ &A+xB+yC+(a'\ +b'\ x+c'\ y)C, \\ &A+xB+yC+(a''+b''x+c''y)C, \\ &A+xB+yC+(a'''+b'''x+c'''y)C, \end{aligned}$$

is made dependent on the third. The intersections Π, Π', Π'', Π''' of the four triads of planes are found to be given by

$$\begin{aligned} q\ \Pi &= (b\ c\ )A + (c\ a\ )B + (a\ b\ )C + (a\ b\ c\ )D, \\ q'\ \Pi' &= (b'\ c'')A + (c'\ a'')B + (a'\ b'')C + (a'\ b'\ c'\ )D, \\ q''\Pi'' &= (b''\ c''')A + (c''\ a''')B + (a''\ b''')C + (a''\ b''\ c'')D, \\ q'''\Pi''' &= (b'''c\ )A + (c'''a\ )B + (a'''b\ )C + (a'''b'''c''')D, \end{aligned}$$

where

$$\begin{aligned} (bc') &= b(c'-c'') + b'(c''-c) + b''(c-c'), \\ (ca') &= c(a'-a'') + c'(a''-a) + c''(a-a'), \\ (ab') &= a(b'-b'') + a'(b''-b) + a''(b-b'), \end{aligned}$$

and

$$\begin{aligned} (abc) &= a(bc') + b(ca') + c(ab'), \\ &= a(b'c''-b''c') + a'(b''c-bc'') + a''(bc'-b'c). \end{aligned}$$

Hence, by the third theorem,

$$\frac{\Pi\Pi'\Pi''\Pi'''}{A\,B\,C\,D} = \frac{A+A'+A''}{qq'q''q'''(b'c'')(b''c''')},$$

where now

$$A=\delta'(\beta''\gamma'''-\beta'''\gamma''),\quad A'=\delta''(\beta'''\gamma'-\beta'\gamma'''),\quad A''=\delta'''(\beta'\gamma''-\beta''\gamma'),$$

and

$$\beta' = (b'c'')(ca') - (bc')(c'a''), \quad \beta'' = \ldots, \quad \beta''' = \ldots,$$
$$\gamma' = (b'c'')(ab') - (a'b'')(bc'), \quad \gamma'' = \ldots, \quad \gamma''' = \ldots,$$
$$\partial' = (b'c'')(abc) - (bc')(a'b'c'), \quad \partial'' = \ldots, \quad \partial''' = \ldots.$$

Minding then continues (pp. 399, 400) :—

"Man setze

$$a'''(bc') - a(b'c'') + a'(b''c''') - a''(b'''c) = \mathrm{M}.$$

"Nach den nöthigen Reductionen erhält man:

$$\beta' = -(c'' - c')\mathrm{M}, \quad \gamma' = -(b' - b'')\mathrm{M}, \quad \partial' = -(b'c'' - b''c')\mathrm{M},$$
$$\beta'' = +(c''' - c'')\mathrm{M}, \quad \gamma'' = +(b'' - b''')\mathrm{M}, \quad \partial'' = +(b''c''' - b'c)\mathrm{M}$$
$$\beta''' = -(c - c''')\mathrm{M}, \quad \gamma''' = -(b''' - b)\mathrm{M}, \quad \partial''' = -(b'''c - bc''')\mathrm{M}$$

"Hieraus erhält man weiter:

$$\mathrm{A} = -\mathrm{M}^3(b''c' - b'c'').(b''c'''),$$
$$\mathrm{A}' = -\mathrm{M}^3(b'''c'' - b''c'''). \{(b''c''') - (b'''c)\},$$
$$\mathrm{A}'' = -\mathrm{M}^3(bc''' - b'''c).(b'c').$$

"Eine weitere Reduction ergiebt:

$$(bc''' - b'''c)(b'c'') - (b'''c)(b'''c'' - b''c''') = (c'''b' - c'b''').$$

"Hieraus folgt $\mathrm{A} + \mathrm{A}' + \mathrm{A}'' = \mathrm{M}^3(b'c'')(b''c''')$, und als Resultat:

$$\frac{\Pi\Pi'\Pi''\Pi'''}{\mathrm{A\,B\,C\,D}} = \frac{\mathrm{M}^3}{qq'q''q'''}."$$

The first point to be noted here is, that since

$$(bc'), \quad (ca'), \quad (ab'),$$

are in modern notation

$$\begin{vmatrix} b & b' & b'' \\ c & c' & c'' \\ 1 & 1 & 1 \end{vmatrix}, \quad \begin{vmatrix} c & c' & c'' \\ a & a' & a'' \\ 1 & 1 & 1 \end{vmatrix}, \quad \begin{vmatrix} a & a' & a'' \\ b & b' & b'' \\ 1 & 1 & 1 \end{vmatrix},$$

the identity

$$a(bc') + b(ca') + c(ab') = a(b'c'' - b''c') + a'(b''c - bc'') + a''(bc - bc)$$

is the same as

$$a\begin{vmatrix} b & b' & b'' \\ c & c' & c'' \\ 1 & 1 & 1 \end{vmatrix} + b\begin{vmatrix} c & c' & c'' \\ a & a' & a'' \\ 1 & 1 & 1 \end{vmatrix} + c\begin{vmatrix} a & a' & a'' \\ b & b' & b'' \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{vmatrix},$$

—a disguised special case of Vandermonde's theorem (XII.), the four elements of one row being each unity. (XII. 11)

The next point is, that since the expression denoted by M, viz.,

$$a'''(bc') - a(b'c'') + a'(b''c''') - a''(b'''c)$$

is in modern notation

$$-\begin{vmatrix} a & a' & a'' & a''' \\ b & b' & b'' & b''' \\ c & c' & c'' & c''' \\ 1 & 1 & 1 & 1 \end{vmatrix},$$

the identity

$$\delta' = -(b'c'' - b''c')\mathrm{M}$$

is the same as

$$\begin{vmatrix} \begin{vmatrix} b' & b'' & b''' \\ c' & c'' & c''' \\ 1 & 1 & 1 \end{vmatrix} & \begin{vmatrix} b & b' & b'' \\ c & c' & c'' \\ 1 & 1 & 1 \end{vmatrix} \\ \begin{vmatrix} a' & a'' & a''' \\ b' & b'' & b''' \\ c' & c'' & c''' \end{vmatrix} & \begin{vmatrix} a & a' & a'' \\ b & b' & b'' \\ c & c' & c'' \end{vmatrix} \end{vmatrix} = \begin{vmatrix} b' & b'' \\ c' & c'' \end{vmatrix} \cdot \begin{vmatrix} a & a' & a'' & a''' \\ b & b' & b'' & b''' \\ c & c' & c'' & c''' \\ 1 & 1 & 1 & 1 \end{vmatrix},$$

and therefore is, like its eight companions, a fresh case of the theorem regarding a minor of the adjugate.* (XX. 2)

DRINKWATER, J. E. (1831).

[On Simple Elimination. *Philosophical Magazine*, x. pp. 24–28.]

Up to this date, almost 140 years after the publication of Leibnitz's letter to De L'Hôpital, no English mathematician's name

* Instead of following Minding's lengthy process, a mathematician of the present time would of course observe that the coefficients of A, B, C, D are the principal minors of M, and using Cauchy's theorem would at once reach the desired conclusion, viz., that the determinant of them = M^3.

occurs in connection with the subject of determinants,—a fact most significant of the comparative neglect of mathematical studies in Britain during the 18th century. Apart from the contents, therefore, some little interest attaches to Drinkwater's short paper, as being the first sign to us of that revival which, as is well known otherwise, had taken place some few years before.

Drinkwater knew of the investigations of Cramer, Bézout, and Laplace; and professed only to put the elements of the subject "in a more convenient form." His rule of signs is stated and illustrated as follows (p. 25) :—

> "Write down the series of natural numbers 1 2 3 4 . . . n, and underneath it all the permutations of these n numbers, prefixing to each a positive or negative sign according to the following condition :—
>
> "Any permutation may be derived from the first by considering a requisite number of figures to move from left to right by a certain number of single steps or descents of a single place. If the whole number of such single steps necessary to derive any permutation from the first be even, that permutation has a positive sign prefixed to it; the others are negative. For instance, 4 2 1 3 . . . n may be derived from 1 2 3 4 . . . . n, by first causing the 3 to descend below the 4, requiring one single step: then the 2 below the new place of the 4, another single step; lastly, the 1 below the new place of the 2, requiring two more steps, making in all 4. Therefore this permutation requires the positive sign."

In this there is essentially nothing new: it at once recalls a theorem of Rothe's (III. 8). In the following paragraph, however, we find the discussion of a point not previously dealt with. The words are (p. 25) :—

> "The same permutation may be derived in various ways, and it is necessary, therefore, to show that this rule is not inconsistent with itself: thus the same permutation 4 2 1 3. . . n might have been obtained by first marching 1 through three places, then 2 through two; and, lastly, 3 through one, making six in all, an even number as before. Without accumulating instances, it is plain, if q be the smallest number of

steps by which any number p reaches the place it is intended finally to occupy in that permutation, that if p should advance in the first instance m places beyond this, it must subsequently return through m places: or, which is the same thing, it must at a later period of the march, allow m of those which it has passed to repass it, so that it will regain its proper place after the number of steps has been increased from q to $q+2m$, which, by the rule, require the same sign as q. The same reasoning applies to every other figure; and hence the consistency of the rule is evident. (III. 25)

He then establishes four properties of the functions, viz. (1) Vandermonde's theorem regarding the effect produced on the *function* by transposition of a pair of letters; (2) Bézout's recurrent law of formation; (3) Scherk's theorem regarding the partition of one of the functions into two; and (4) Scherk's theorem regarding the removal of a constant factor from one of the functions. The two latter theorems, which, as we have seen, had been stated for the first time only six years before, are given by Drinkwater in the following form (p. 27):—

(8) If any factor in $f\{XYZT\ldots(n)\}$, as X, be divided into two parts, $X = V+W$, the function may be similarly divided, so that

$$f\{(V+W)YZT\ldots(n)\} = f\{VYZT\ldots(n)\} + f\{WYZT\ldots(n)\},$$

placing each part of X in the same relative position (which in this example is the first) which X itself occupied before the division. (XLVII. 2)

(9) If any quantity which does not vary from one equation to the other, and which, therefore, is not liable to be affected with an index, is found under the symbol, it may be considered a constant coefficient of every term of the developed function; and written as such on the outside of the symbol: of this nature are the unknown quantities themselves, so that for instance,

$$f\{XYxZT\ldots\ldots(n)\} = xf\{XYZT\ldots(n)\},$$

and so of like quantities." (XLVIII. 2)

After these preliminaries the problem of the solution of n linear

equations in n unknowns is taken up. The method followed is essentially the same as Scherk's.

MAINARDI (1832).

[Trasformazioni di alcune funzioni algebraiche, e loro uso nella geometria e nella meccanica. Memoria di Gaspare Mainardi. 44 pp. Pavia, 1832.]

In his preface Mainardi explains that the algebraical functions referred to in the title are "*funzioni risultanti* o *determinanti*." But although he thus speaks of them as if they were known to mathematicians by name, and mentions the researches of Monge, Lagrange, Cauchy, and Binet in regard to them, he does not take for granted that his reader has a knowledge of any of their properties. The one theorem on determinants,—the multiplication-theorem,—which forms the basis of the whole memoir, is consequently sought to be established without the use of any previously proved theorem. The attempt, as might be expected, is interesting.

The first two sections (pp. 9–29) of the three into which the memoir is divided may be passed over without much comment. The first deals with the multiplication-theorem for two determinants of the 2nd order, and with those applications of it to geometry which arise on making the elements of each determinant the Cartesian co-ordinates of two points in a plane. No proof is considered necessary for this simple case, the opening paragraph of the memoir being;—

"Rappresentate con x_m, x_n, x_a, x_b; y_m, y_n, y_a, y_b otto quantità qualsivogliano, ed indicati per brevità il binomio

$$x_m \cdot x_a + y_m \cdot y_a \quad \text{col simbolo} \quad (x_m x_a),$$

il binomio

$$x_n \cdot x_b + y_n \cdot y_b \qquad \text{con} \qquad (x_n x_b)$$

e simili, si proverà facilmente essere

$$\text{(a)} \qquad \begin{aligned}&(x_m y_n - x_n y_m)(x_a y_b - x_b y_a)\\ &= (x_m x_a)(x_n x_b) - (x_m x_b)(x_n x_a).\end{aligned}$$"

All the seven other paragraphs are geometrical.

The second section in like manner opens with an algebraical theorem, viz. (p. 13)—

$$\begin{aligned}
&\{x_m(y_p - y_n)\}\{x_a(y_c - y_b)\} \\
&+\{x_m(z_p - z_n)\}\{x_a(z_c - z_b)\} \\
&+\{y_m(z_p - z_n)\}\{y_a(z_c - z_b)\} \\
&= (x_m x_a)(x_p x_c) - (x_m x_c)(x_p x_a) + (x_n x_a)(x_m x_c) \\
&- (x_n x_c)(x_m x_a) + (x_p x_a)(x_n x_c) - (x_p x_c)(x_n x_a) \\
&+ (x_m x_b)(x_p x_a) - (x_m x_a)(x_p x_b) + (x_n x_b)(x_m x_a) \\
&- (x_n x_a)(x_m x_b) + (x_p x_b)(x_n x_a) - (x_p x_a)(x_n x_b) \\
&+ (x_m x_c)(x_p x_b) - (x_m x_b)(x_p x_c) + (x_n x_c)(x_m x_b) \\
&- (x_n x_b)(x_m x_c) + (x_p x_c)(x_n x_b) - (x_p x_b)(x_n x_c), \quad \text{(XXIX. 2)}
\end{aligned}$$

where $\{x_m(y_p - y_n)\}$ and $(x_m x_a)$ stand for

$$(x_m y_p - x_p y_m) + (x_n y_m - x_m y_n) + (x_p y_n - x_n y_p)$$

and

$$x_m x_a + y_m y_a + z_m z_a$$

respectively; and the remainder is occupied with the applications of the theorem to geometry and dynamics. Each factor of the left-hand side of the identity is evidently a determinant of the third order, and the three pairs of lines on the right-hand side are each the expansion of a determinant of the same order: so that in the notation of the present day the identity may be written

$$\begin{vmatrix} x_m & y_m & 1 \\ x_n & y_n & 1 \\ x_p & y_p & 1 \end{vmatrix} \cdot \begin{vmatrix} x_a & y_a & 1 \\ x_b & y_b & 1 \\ x_c & y_c & 1 \end{vmatrix} + \begin{vmatrix} x_m & z_m & 1 \\ x_n & z_n & 1 \\ x_p & z_p & 1 \end{vmatrix} \cdot \begin{vmatrix} x_a & z_a & 1 \\ x_b & z_b & 1 \\ x_c & z_c & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} y_m & z_m & 1 \\ y_n & z_n & 1 \\ y_p & z_p & 1 \end{vmatrix} \cdot \begin{vmatrix} y_a & z_a & 1 \\ y_b & z_b & 1 \\ y_c & z_c & 1 \end{vmatrix} = \begin{vmatrix} (x_m x_c) & (x_m x_a) & 1 \\ (x_n x_c) & (x_n x_a) & 1 \\ (x_p x_c) & (x_p x_a) & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} (x_m x_a) & (x_m x_b) & 1 \\ (x_n x_a) & (x_n x_b) & 1 \\ (x_p x_a) & (x_p x_b) & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} (x_m x_b) & (x_m x_c) & 1 \\ (x_n x_b) & (x_n x_c) & 1 \\ (x_p x_b) & (x_p x_c) & 1 \end{vmatrix}.$$

There has been no previous instance of an identity perfectly similar to this; the nearest approach to such being, as the numbering shows, a result obtained by Binet in 1811. The exact character of the affinity between the two, and the general theorem which both foreshadow, will be most readily brought into evidence by a little additional transformation. Taking first the right-hand side of the identity, we observe that the three determinants have only twelve elements among them, being obtainable in fact from a single array of four rows and three columns. Their sum may consequently be put in the form

$$\begin{vmatrix} 1 & (x_m x_a) & (x_m x_b) & (x_m x_c) \\ 1 & (x_n x_a) & (x_n x_b) & (x_n x_c) \\ 1 & (x_p x_a) & (x_p x_b) & (x_p x_c) \\ 0 & 1 & 1 & 1 \end{vmatrix}.$$

Secondly, we observe that the first factors on the left-hand side are similarly obtainable from

$$\begin{matrix} x_m & y_m & z_m & 1 \\ x_n & y_n & z_n & 1 \\ x_p & y_p & z_p & 1 ; \end{matrix}$$

and the second factors from

$$\begin{matrix} x_a & y_a & z_a & 1 \\ x_b & y_b & z_b & 1 \\ x_c & y_c & z_c & 1 ; \end{matrix}$$

and as the so-called product of these arrays is equal to the said left-hand member diminished by

$$\begin{vmatrix} x_m & y_m & z_m \\ x_n & y_n & z_n \\ x_p & y_p & z_p \end{vmatrix} . \begin{vmatrix} x_a & y_a & z_a \\ x_b & y_b & z_b \\ x_c & y_c & z_c \end{vmatrix},$$

Mainardi's theorem may be put in the much altered form—

$$\begin{vmatrix} 1 & (x_m x_a) & (x_n x_a) & (x_p x_a) \\ 1 & (x_m x_b) & (x_n x_b) & (x_p x_b) \\ 1 & (x_m x_c) & (x_n x_c) & (x_p x_c) \\ 0 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} x_m & y_m & z_m & 1 \\ x_n & y_n & z_n & 1 \\ x_p & y_p & z_p & 1 \end{vmatrix} . \begin{vmatrix} x_a & y_a & z_a & 1 \\ x_b & y_b & z_b & 1 \\ x_c & y_c & z_c & 1 \end{vmatrix}$$

$$- \begin{vmatrix} x_m & y_m & z_m \\ x_n & y_n & z_n \\ x_p & y_p & z_p \end{vmatrix} . \begin{vmatrix} x_a & y_a & z_a \\ x_b & y_b & z_b \\ x_c & y_c & z_c \end{vmatrix}.$$

The constitution of the 3rd section is quite like that of the others, the first paragraph dealing with the multiplication-theorem for the case of determinants of the 3rd order, the second paragraph with the same theorem for determinants of the 4th order, and the remaining eight paragraphs with geometrical applications. The mode of proof of the multiplication-theorem is partly indicated by saying that any particular case is made dependent on the case immediately preceding it; but its exact character can only be understood by a somewhat minute examination. The investigation for the case of determinants of the 3rd order stands as follows (p. 29):—

"Si considerino i due polinomj

$$(l)\qquad \begin{aligned} x_m(y_nz_p - y_pz_n) + x_n(z_my_p - y_mz_p) + x_p(y_mz_n - y_nz_m) &= \{x_m, y_n, z_p\}, \\ x_a(y_bz_c - y_cz_b) + x_b(z_ay_c - z_cy_a) + x_c(y_az_b - y_bz_a) &= \{x_a, y_b, z_c\}. \end{aligned}$$

Se ne effettui il prodotto, il quale, mediante l'equazione (a) del primo articolo, si potrà disporre sotto la forma seguente

$$(h)\qquad \begin{aligned} & x_mx_a(y_ny_b)(y_py_c) - x_mx_a(y_ny_c)(y_py_b) \\ + & x_nx_a(y_my_c)(y_py_b) - x_nx_a(y_my_b)(y_py_c) \\ + & x_px_a(y_my_b)(y_ny_c) - x_px_a(y_my_c)(y_ny_b) \\ + & x_mx_b(y_ny_c)(y_py_a) - x_mx_b(y_ny_a)(y_py_c) \\ + & x_nx_b(y_my_a)(y_py_c) - x_nx_b(y_my_c)(y_py_a) \\ + & x_px_b(y_my_c)(y_ny_a) - x_px_b(y_my_a)(y_ny_c) \\ + & x_mx_c(y_ny_a)(y_py_b) - x_mx_c(y_ny_b)(y_py_a) \\ + & x_nx_c(y_my_b)(y_py_a) - x_nx_c(y_my_a)(y_py_b) \\ + & x_px_c(y_my_a)(y_ny_b) - x_px_c(y_my_b)(y_ny_a). \end{aligned}$$

Esaminando ora la quantità

$$\begin{aligned} & x_mx_a\{x_nx_b(y_py_c) + x_px_c(y_ny_b) + x_nx_bx_px_c \\ & \qquad - x_nx_c(y_py_b) - x_px_b(y_ny_c) - x_nx_px_bx_c\} \\ + & x_nx_a\{x_mx_c(y_py_b) + x_px_b(y_my_c) + x_mx_cx_px_b \\ & \qquad - x_mx_b(y_py_c) - x_px_c(y_my_b) - x_mx_bx_px_c\} \\ + & x_px_a\{x_mx_b(y_ny_c) + x_nx_c(y_my_b) + x_mx_bx_nx_c \\ & \qquad - x_mx_c(y_ny_b) - x_nx_b(y_my_c) - x_mx_cx_nx_b\}, \end{aligned}$$

e le due espressioni che si traggono da questa, cambiando, prima a in b, b in c, c in a; poscia a in c, c in b, b in a; con facilità si scorge che la somma di questi polinomj è nulla identicamente, per cui si potrà aggiungere al prodotto (h) senza punto alterarlo. Fatta quest' addizione, l'aggregato altro non sarà che lo stesso polinomio (h), ove si supponga che i simboli $(y_n y_b)$, $(y_p y_c)$, ecc. rappresentino rispettivamente i trinomj seguenti

$$x_n x_b + y_n y_b + z_n z_b, \quad x_p x_c + y_p y_c + z_p z_c, \quad \text{ecc.}$$

Se ora si ordineranno le espressioni (l) portando fuori dalle parentesi y ovvero z in luogo di x, formeremo il prodotto delle medesime così scritte, ed opereremo come sopra, il risultato sarà il polinomio che si desume da (h) cambiando le x che sono fuori dalle parentesi in y ovvero in z egualmente accentate. Se faremo per ultimo la somma di queste tre espressioni, tal somma si caverà dal polinomio (h) scrivendo $(x_m x_a)$ ovvero $(y_m y_a)$ invece di $x_m x_a$; $(x_p x_a)$ in luogo di $x_p x_a$ ec. ec. e sarà eguale al triplo prodotto delle expressioni (l).

Essendo poi quella somma divisibile per tre, effettuata la divisione per questo numero, avremo

$$\begin{aligned}(1)\quad \{x_m, y_n, z_p\}.\{x_a, y_b, z_c\} = {} & (x_m x_a)(x_n x_b)(x_p x_c) + (x_n x_a)(x_p x_b)(x_m x_c) \\ & + (x_p x_a)(x_m x_b)(x_n x_c) \\ & - (x_m x_a)(x_p x_b)(x_n x_c) - (x_n x_a)(x_m x_b)(x_p x_c) \\ & - (x_p x_a)(x_n x_b)(x_m x_c).\text{"}\end{aligned}$$

(XVII. 6)

That the essential points of this method of demonstration may be seen, let us apply it as it would be applied if adopted at the present day.

The given determinants being

$$|a_1 b_2 c_3| \quad \text{and} \quad |\alpha_1 \beta_2 \gamma_3|,$$

we should say

$$|a_1 b_2 c_3| = a_1 |b_2 c_3| - a_2 |b_1 c_3| + a_3 |b_1 c_2|,$$

and

$$|\alpha_1 \beta_2 \gamma_3| = \alpha_1 |\beta_2 \gamma_3| - \alpha_2 |\beta_1 \gamma_3| + \alpha_3 |\beta_1 \gamma_2|;$$

hence, using the multiplication-theorem as established for determin-

ants of the 2nd order, and (to save on the breadth of the page) denoting

$$a\alpha + b\beta + c\gamma + \ldots \quad \text{by} \quad \frac{a, b, c, \ldots}{\alpha, \beta, \gamma, \ldots}$$

we should have

$$|a_1 b_2 c_3| \cdot |\alpha_1 \beta_2 \gamma_3|$$

$$= a_1\alpha_1 \begin{vmatrix} \frac{b_2, c_2}{\beta_2, \gamma_2} & \frac{b_2, c_2}{\beta_3, \gamma_3} \\ \frac{b_3, c_3}{\beta_2, \gamma_2} & \frac{b_3, c_3}{\beta_3, \gamma_3} \end{vmatrix} - a_2\alpha_1 \begin{vmatrix} \frac{b_1, c_1}{\beta_2, \gamma_2} & \frac{b_1, c_1}{\beta_3, \gamma_3} \\ \frac{b_3, c_3}{\beta_2, \gamma_2} & \frac{b_3, c_3}{\beta_3, \gamma_3} \end{vmatrix} + a_3\alpha_1 \begin{vmatrix} \frac{b_1, c_1}{\beta_2, \gamma_2} & \frac{b_1, c_1}{\beta_3, \gamma_3} \\ \frac{b_2, c_2}{\beta_2, \gamma_2} & \frac{b_2, c_2}{\beta_3, \gamma_3} \end{vmatrix}$$

$$- a_1\alpha_2 \begin{vmatrix} \frac{b_2, c_2}{\beta_1, \gamma_1} & \frac{b_2, c_2}{\beta_3, \gamma_3} \\ \frac{b_3, c_3}{\beta_1, \gamma_1} & \frac{b_3, c_3}{\beta_3, \gamma_3} \end{vmatrix} + a_2\alpha_2 \begin{vmatrix} \frac{b_1, c_1}{\beta_1, \gamma_1} & \frac{b_1, c_1}{\beta_3, \gamma_3} \\ \frac{b_3, c_3}{\beta_1, \gamma_1} & \frac{b_3, c_3}{\beta_3, \gamma_3} \end{vmatrix} - a_3\alpha_2 \begin{vmatrix} \frac{b_1, c_1}{\beta_1, \gamma_1} & \frac{b_1, c_1}{\beta_3, \gamma_3} \\ \frac{b_2, c_2}{\beta_1, \gamma_1} & \frac{b_2, c_2}{\beta_3, \gamma_3} \end{vmatrix}$$

$$+ a_1\alpha_3 \begin{vmatrix} \frac{b_2, c_2}{\beta_1, \gamma_1} & \frac{b_2, c_2}{\beta_2, \gamma_2} \\ \frac{b_3, c_3}{\beta_1, \gamma_1} & \frac{b_3, c_3}{\beta_2, \gamma_2} \end{vmatrix} - a_2\alpha_3 \begin{vmatrix} \frac{b_1, c_1}{\beta_1, \gamma_1} & \frac{b_1, c_1}{\beta_2, \gamma_2} \\ \frac{b_3, c_3}{\beta_1, \gamma_1} & \frac{b_3, c_3}{\beta_2, \gamma_2} \end{vmatrix} + a_3\alpha_3 \begin{vmatrix} \frac{b_1, c_1}{\beta_1, \gamma_1} & \frac{b_1, c_1}{\beta_2, \gamma_2} \\ \frac{b_2, c_2}{\beta_1, \gamma_1} & \frac{b_2, c_2}{\beta_2, \gamma_2} \end{vmatrix}.$$

That each line of this result is not altered in substance by writing

$$\frac{a_2, b_2, c_2}{\alpha_2, \beta_2, \gamma_2} \text{ for } \frac{b_2, c_2}{\beta_2, \gamma_2}, \quad \frac{a_2, b_2, c_2}{\alpha_3, \beta_3, \gamma_3} \text{ for } \frac{b_2, c_2}{\beta_3, \gamma_3}, \text{ &c.,}$$

would probably be shown by expressing the line in the form of a determinant of the 3rd order, *e.g.*, the first line in the form

$$a_1 \begin{vmatrix} a_1 & \frac{b_1, c_1}{\beta_2, \gamma_2} & \frac{b_1, c_1}{\beta_3, \gamma_3} \\ a_2 & \frac{b_2, c_2}{\beta_2, \gamma_2} & \frac{b_2, c_2}{\beta_3, \gamma_3} \\ a_3 & \frac{b_3, c_3}{\beta_2, \gamma_2} & \frac{b_3, c_3}{\beta_3, \gamma_3} \end{vmatrix};$$

and increasing each element of the second column by a_2 times the corresponding element of the first, and each element of the third column by a_3 times the corresponding element of the first. The whole result would in this way be transformed into

$$\begin{vmatrix} a_1\alpha_1 & \dfrac{a_1,b_1,c_1}{a_2,\beta_2,\gamma_2} & \dfrac{a_1,b_1,c_1}{a_3,\beta_3,\gamma_3} \\ a_2\alpha_1 & \dfrac{a_2,b_2,c_2}{a_2,\beta_2,\gamma_2} & \dfrac{a_2,b_2,c_2}{a_3,\beta_3,\gamma_3} \\ a_3\alpha_1 & \dfrac{a_3,b_3,c_3}{a_2,\beta_2,\gamma_2} & \dfrac{a_3,b_3,c_3}{a_3,\beta_3,\gamma_3} \end{vmatrix} - \begin{vmatrix} a_1\alpha_2 & \dfrac{a_1,b_1,c_1}{a_1,\beta_1,\gamma_1} & \dfrac{a_1,b_1,c_1}{a_3,\beta_3,\gamma_3} \\ a_2\alpha_2 & \dfrac{a_2,b_2,c_2}{a_1,\beta_1,\gamma_1} & \dfrac{a_2,b_2,c_2}{a_3,\beta_3,\gamma_3} \\ a_3\alpha_2 & \dfrac{a_3,b_3,c_3}{a_1,\beta_1,\gamma_1} & \dfrac{a_3,b_3,c_3}{a_3,\beta_3,\gamma_3} \end{vmatrix}$$

$$+ \begin{vmatrix} a_1\alpha_3 & \dfrac{a_1,b_1,c_1}{a_1,\beta_1,\gamma_1} & \dfrac{a_1,b_1,c_1}{a_2,\beta_2,\gamma_2} \\ a_2\alpha_3 & \dfrac{a_2,b_2,c_2}{a_1,\beta_1,\gamma_1} & \dfrac{a_2,b_2,c_2}{a_2,\beta_2,\gamma_2} \\ a_3\alpha_3 & \dfrac{a_3,b_3,c_3}{a_1,\beta_1,\gamma_1} & \dfrac{a_3,b_3,c_3}{a_2,\beta_2,\gamma_2} \end{vmatrix}.$$

Now by either of the interchanges

$$\begin{pmatrix} a_1, & a_2, & a_3, & \alpha_1, & \alpha_2, & \alpha_3 \\ b_1, & b_2, & b_3, & \beta_1, & \beta_2, & \beta_3 \end{pmatrix}, \quad \begin{pmatrix} a_1, & a_2, & a_3, & \alpha_1, & \alpha_2, & \alpha_3 \\ c_1, & c_2, & c_3, & \gamma_1, & \gamma_2, & \gamma_3 \end{pmatrix}$$

the first columns of this,—and the first columns only,—would be affected, the a's and α's becoming b's and β's respectively in the one case, and c's and γ's in the other; and as neither interchange could affect the left-hand side of our identity, we should consequently note that thus three different expressions would be at once obtained for $|a_1b_2c_3|.|\alpha_1\beta_2\gamma_3|$. Adding these together, and combining the nine determinants of the sum in sets of three by means of the addition-theorem (XLVII.), we should have finally

$$3|a_1b_2c_3|.|\alpha_1\beta_2\gamma_3| = 3 \begin{vmatrix} \dfrac{a_1,b_1,c_1}{\alpha_1,\beta_1,\gamma_1} & \dfrac{a_1,b_1,c_1}{\alpha_2,\beta_2,\gamma_2} & \dfrac{a_1,b_1,c_1}{\alpha_3,\beta_3,\gamma_3} \\ \dfrac{a_2,b_2,c_2}{\alpha_1,\beta_1,\gamma_1} & \dfrac{a_2,b_2,c_2}{\alpha_2,\beta_2,\gamma_2} & \dfrac{a_2,b_2,c_2}{\alpha_3,\beta_3,\gamma_3} \\ \dfrac{a_3,b_3,c_3}{\alpha_1,\beta_1,\gamma_1} & \dfrac{a_3,b_3,c_3}{\alpha_2,\beta_2,\gamma_2} & \dfrac{a_3,b_3,c_3}{\alpha_3,\beta_3,\gamma_3} \end{vmatrix},$$

from which it is only necessary to delete the common factor 3.

JACOBI (1831–33).

[De transformatione integralis duplicis indefiniti

$$\int \frac{\partial\phi\partial\psi}{A + B\cos\phi + C\sin\phi + (A' + B'\cos\phi + C'\sin\phi)\cos\psi + (A'' + B''\cos\phi + C''\sin\phi)\sin\psi}$$

in formam simpliciorem $\int \frac{\partial\eta\partial\theta}{G - G'\cos\eta\cos\theta - G''\sin\eta\sin\theta}$.

[*Crelle's Journal*, viii. pp. 253–279, 321–357.]

[De transformatione et determinatione integralium duplicium commentatio tertia. *Crelle's Journal*, x. pp. 101–128.]

[De binis quibuslibet functionibus homogeneis secundi ordinis per substitutiones lineares in alias binas transformandis, quae solis quadratis variabilium constant; una cum variis theorematis de transformatione et determinatione integralium multiplicium. *Crelle's Journal*, xii. pp. 1–69].

The first two of these memoirs may be viewed as continuations of a memoir with a similar title, which appeared in the second volume of *Crelle's Journal*, and to which we have already referred. They are noted here merely in order that the thread of investigation may be preserved unbroken, for the last memoir practically swallows up, by means of its splendid generalisations, all those that had gone before.

So long as we confine ourselves, in problems of transformation, to three independent variables, the explicit employment of the theory of determinants may be dispensed with. When, however, a sufficient number of special cases have been investigated, and an alluring glimpse has thereby been got of a generalisation involving them all, he who attempts the establishment of the generalisation must have recourse to the new weapon. In this latter position Jacobi now found himself. He wished to pass from the problem of orthogonal substitution in the case of three variables to the analogous problem in which the number of variables is n, or in his own words (p. 7) :—

"Investigare substitutiones lineares huiusmodi

$$\begin{aligned} y_1 &= a_1' x_1 + a_2' x_2 + , \ldots + a_n' x_n, \\ y_2 &= a_1'' x_1 + a_2'' x_2 + \ldots + a_n'' x_n, \\ &\cdots\cdots\cdots\cdots\cdots \\ y_n &= a_1^{(n)} x_1 + a_2^{(n)} x_2 + \ldots + a_n^{(n)} x_n, \end{aligned}$$

quibus efficiatur

$$y_1y_1 + y_2y_2 + \ldots + y_ny_n = x_1x_1 + x_2x_2 + \ldots + x_nx_n,$$

simulque data functio homogenea secundi ordinis variabilium $x_1, x_2, \ldots, x_n$ transformetur in aliam variabilium $y_1, y_2, \ldots, y_n$, de qua binarum producta evanuerunt."

This being the case he introduces determinants at the outset, fixing upon a notation which is practically Cauchy's, and immediately using properties of them without proof. Much that is contained in the memoir falls to be considered later, as it concerns special forms of determinants,—those afterwards known as Jacobians, axisymmetric determinants, and, of course, determinants of an orthogonal substitution. Indeed, the half-page of introduction is almost all that is of interest at present, but even in this a new and important theorem is enunciated. The first sentence of it stands as follows:—

"Supponamus, designantibus $a_k^{(m)}$ datas quantitates quaslibet, ex n æquationibus linearibus propositis huiusmodi

$$y_m = a_1^{(m)}x_1 + a_2^{(m)}x_2 + \ldots + a_n^{(m)}x_n,$$

per notas regulas resolutionis algebraicæ haberi æquationes formæ:

$$Ax_k = \beta_k'y_1 + \beta_k''y_2 + \ldots + \beta_k^{(n)}y_n.$$

Ipsum A supponimus denominatorem communem valorum incognitarum, qui per algorithmos notos formatur: sive fit

$$A = \Sigma \pm a_1'a_2'' \ldots a_n^{(n)},$$

signo summatorio amplectente terminos omnes, qui indicibus aut inferioribus aut superioribus omnimodis permutatis proveniunt; signis eorum alternantibus secundum notam regulam, quam ita enunciare licet, ut termino cuilibet per certam permutationem *indicum* orto idem signum tribuatur, quo afficitur productum sequens conflatum e differentiis numerorum $1, 2, \ldots, n$

$$(2-1)(3-1)\ldots.(n-1).(3-2)(4-2)\ldots.(n-2).(4-3) \text{ etc.},$$

eadem *numerorum* permutatione facta."

It will be at once observed that Cauchy's italic letters S, a, b are simply changed into Greek Σ, α, β.

The next sentence is:—

"Eadem notatione adhibita, sit

$$B = \Sigma \pm \beta_1' \beta_2'' \ldots . \beta_n^{(n)},$$

ubi ipsam B e quantitatibus $\beta_k^{(m)}$ eodem modo compositam accipimus, quo A ex ipsis $\alpha_k^{(m)}$ componitur. Quibus statutis, observo fieri:

$$B = A^{n-1},$$

ac generalius:

$$\Sigma \pm \beta_1' \beta_2'' \ldots . \beta_m^{(m)} = A^{m-1} \Sigma \pm \alpha_{m+1}^{(m+1)} \alpha_{m+2}^{(m+2)} \ldots . \alpha_n^{(n)}." \quad \text{(xx. 3)}$$

As for the first theorem thus formulated, the credit of it is, of course, due to Cauchy: the second, however, is new, being indeed the theorem referred to above under Minding as having been foreshadowed by Lagrange, and left for over fifty years undisturbed. Jacobi evidently knew it in all its generality, for he adds—

"De qua formula generali cum pro variis valoribus ipsius m, tum indicibus et superioribus et inferioribus omnimodis permutatis, permultae aliae similes formulae profluunt."

The only other point to be noted at present is contained in the casual remark that the β's may be expressed as *differential coefficients* of A. When dealing later (p. 20), with a special form of determinant, he says—

"Data occasione observo generaliter, si $\alpha_{\kappa,\lambda}$ et $\alpha_{\lambda,\kappa}$ inter se diversi sunt, propositis n aequationibus linearibus hujusmodi:

$$\begin{aligned}
\alpha_{1,1}u_1 + \alpha_{1,2}u_2 + \ldots . + \alpha_{1,n}u_n &= v_1, \\
\alpha_{2,1}u_1 + \alpha_{2,1}u_2 + \ldots . + \alpha_{2,n}u_n &= v_2, \\
\ldots \quad \ldots \quad \ldots \quad \

$$\begin{aligned}
\Gamma u_1 &= \frac{\partial\Gamma}{\partial a_{1,1}}v_1 + \frac{\partial\Gamma}{\partial a_{2,1}}v_2 + \ldots\ldots + \frac{\partial\Gamma}{\partial a_{n,1}}v_n,\\
\Gamma u_2 &= \frac{\partial\Gamma}{\partial a_{1,2}}v_1 + \frac{\partial\Gamma}{\partial a_{2,2}}v_2 + \ldots\ldots + \frac{\partial\Gamma}{\partial a_{n,2}}v_n,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
\Gamma u_n &= \frac{\partial\Gamma}{\partial a_{1,n}}v_1 + \frac{\partial\Gamma}{\partial a_{2,n}}v_2 + \ldots\ldots + \frac{\partial\Gamma}{\partial a_{n,n}}v_n. \qquad \text{(VI. 6)}
\end{aligned}$$

JACOBI (1835).

[De eliminatione variabilis e duabus aequationibus algebraicis. *Crelle's Journal*, xv. pp. 101–124.]

In a memoir having for its subject Bézout's method of eliminating x from the equations

$$\left.\begin{aligned} a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1 x + a_0 = 0,\\ b_n x^n + b_{n-1}x^{n-1} + \ldots\ldots + b_1 x + b_0 = 0, \end{aligned}\right\}$$

determinants are certain to occur explicitly or implicitly; and, the author being Jacobi, one is not surprised to find them introduced near the outset and employed thenceforward. It is of course only a special form of them which appears, viz., that afterwards distinguished by the term *persymmetric;* consequently, for the present the main contents of the memoir do not concern us. Note has to be made, however, of two points—(1) that while Jacobi does not discard his former notation $\Sigma \pm a_{r_0,s_0} a_{r_1,s_1} \ldots a_{r_m,s_m}$, he introduces and uses another, viz.,

$$a\begin{Bmatrix} r_0, r_1, r_2, \ldots, r_m \\ s_0, s_1, s_2, \ldots, s_m \end{Bmatrix}; \qquad \text{(VII. 8)}$$

(2) that a page is devoted to a fuller statement of the above-mentioned theorems regarding the adjugate determinant and a minor of the adjugate. The final sentence of this statement is all that need be reproduced. It is

> "Sint igitur $r, r', r'', \ldots, r^{(n-1)}$ atque $s, s', s'', \ldots\ldots, s^{(n-1)}$ numeri omnes 0, 1, 2, . . . , $n-1$, quocunque ordine scripti; erit

$$A\begin{Bmatrix} r^{(m)}, r^{(m+1)}, \ldots, r^{(n-1)} \\ s^{(m)}, s^{(m+1)}, \ldots, s^{(n-1)} \end{Bmatrix} = L^{n-(1+m)}.\alpha\begin{Bmatrix} r, r', \ldots, r^{(m-1)} \\ s, s', \ldots, s^{(m-1)} \end{Bmatrix}, \quad \text{(XX. 4)}$$

where L stands for $\Sigma \pm \alpha_{0,0}\alpha_{1,1} \ldots \alpha_{n-1,n-1}$ and the adjugate of L is $\Sigma \pm A_{0,0}A_{1,1} \ldots A_{n-1,n-1}$. As before, no proofs of the theorems are given.

The Electrotonic Variation with Strong Polarising Currents.

By **George N. Stewart**, D.Sc., *Owens College, Manchester.*

(Read January 21, 1889.)

Let AB (fig. 1) be a piece of nerve interposed in the galvanometer circuit, and C D in the battery circuit. Then, as has long been known, on closing the battery circuit, one obtains a current in the galvanometer circuit, the direction of which in the nerve is the same as that of the polarising current. If, now, stimulation be made, say at I, this current undergoes a negative variation. Hermann, who investigated the subject, after Bernstein, was at first inclined to explain the negative variation by his law of "polarisation increment." He assumed that the excitation in passing along a polarised nerve undergoes changes in its intensity, increasing as it passes through regions under the influence of the anode, decreasing as it passes through parts dominated by the cathode.

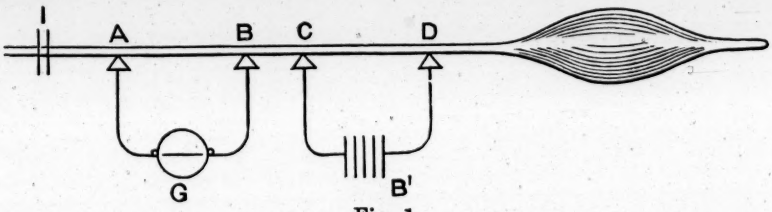


Fig. 1.

Now, if the current be ascending in the nerve (fig. 1), the electrotonic current in AB is also ascending. As B is nearer the cathode than A, the excitation will pass B in less intensity than A. Accordingly, during tetanus, B may be considered as less negative than A. In other words, B will be positive to A, and a current of

action will pass through the galvanometer from B to A.* This will have the opposite direction to the electrotonic current, and will therefore look like a diminution or negative variation of that current. Similarly, if C be the anode, and the current be descending, the excitation will pass over B in greater intensity than over A, and again there will be a negative variation of the electrotonic current. Hermann, as already stated, seemed at one time to suppose that this was a complete explanation of the phenomena. But he was afterwards led by rheotome researches to modify his view, and, while retaining the law of "polarisation increment" as an essential factor in the explanation, to postulate besides, as Bernstein had previously done in a somewhat different form, an actual diminution in the polarisation, a negative variation, so to say, in the capability of the nerve to take on polarisation between core and sheath. ("Untersuchungen über die Actionsströme der nerven," *Pflüger's Archiv*, Bd. xlii. s. 246, &c.).

According to Hermann, the electrotonic currents are branches of the polarising stream which spread beyond the electrodes, owing to the transverse resistance caused by polarisation between this hypothetical core and sheath. The greater the polarisation coefficient is, the more widely do these branches spread, the stronger are the electrotonic currents. If stimulation diminishes this polarisation coefficient it will, in general, diminish the electrotonic currents. If the excitation be confined to special parts of the nerve, then it will depend upon the ratio of the transition resistance between core and sheath to the longitudinal resistance of the nerve, and upon the magnitude and position of the unexcited or relatively unexcited parts, whether the electrotonic variation (as we may for shortness call the variation of the electrotonic currents produced by stimulation) will be negative or positive. All this he deduces from his theory, and supports by experiments with the "Kernleiter Modell." He looked, in vain, however, for a positive phase in his rheotome work on nerve, the experimental difficulties being very great.

It was not from Hermann's theoretical standpoint that I entered

* Strictly speaking, if $E_{(A)}$, $E_{(B)}$, represent the intensity of excitation at A and B, $\int E_{(A)} dt$ is $> \int E_{(B)} dt$ for corresponding limits. Considering time-integrals, B may, therefore, be looked on as positive to A during the tetanus. The galvanometer deflection produced by stimulation will be a measure of the difference of these integrals.

upon the work of which this paper is an account. But from certain experiments on the effect of stimulation on the intrapolar current during the flow, and on both extra and intrapolar currents after the opening of the polarising stream, I suspected that, if one of the galvanometer electrodes were placed very near the polarising circuit, and the strength of the current increased sufficiently, a positive electrotonic variation ought to appear on the side of the anode, but not on that of the cathode. For the explanation of those experiments it was assumed, and the assumption was supported by direct experiments on muscular contraction, that during the flow of the polarising current the conductivity of the nerve for the excitatory change is less around the cathode than around the anode, and that, with increasing strength of current, complete block occurs sooner at the former than at the latter, although eventually it prevails at both. Whether, when this last stage is reached, the whole intrapolar area has lost its conductivity, was left an open question, and need not be considered here.

Going back now to fig. 1, let us inquire what the effect would be on the side of the anode, *i.e.*, with descending current, at a time when complete block was established there, and at the same time let us suppose that the galvanometer circuit is brought quite close to the anode, so that the lower galvanometer electrode is within the non-conducting region. If stimulation be now made at I, the excitation will pass A with a certain intensity, but will altogether fail before reaching B. B will, therefore, be strongly positive to A. We leave out of account for the moment any possible effect of the excitation on the electrotonic currents as such. There will be a current of action developed in the descending direction through the nerve—that is, in the same direction as the anodic electrotonic current. If this true action current be not masked by an overwhelming negative electrotonic variation, it will appear as a positive variation of the electrotonic current.

Now let us take the case of the ascending current in fig. 1. Here the lower galvanometer electrode is in the cathodic region, and we know that even with comparatively weak currents the cathodic block appears. B will therefore, above a low limit of current density, be positive to A when the nerve is excited, and the true action stream will be descending. The cathodic electrotonic current,

however, is ascending, and the action stream will appear as a negative variation of it.

These are the considerations which led me to expect that a positive variation, if it existed, would be found with strong currents upon the anodic side, but not upon the side of the cathode. It was not overlooked that the ordinary electrotonic negative variation might be so large as to reverse the action current. Still it was hoped that, even in this case, indications might be found in the curve of the stimulation effect to show that the expected true action current was really in play.

Method of the Investigation.

The first one or two observations were made without compensating the electrotonic currents. They, indeed, give the same general results as when compensation was used. But it was obvious that it would not do to accept a positive variation on the evidence of an uncompensated anodic current. For it would be necessary to show

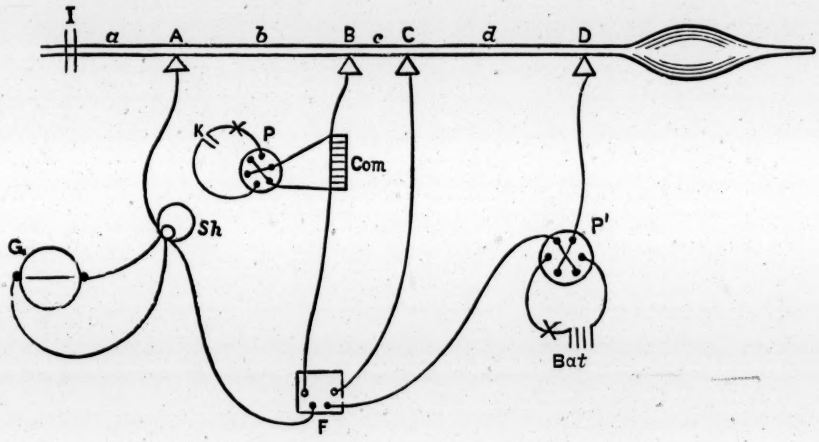


FIG. 2.—A, B, C, D are electrodes; I, stimulating electrodes; G, galvanometer; P, P', Pohl's commutators; Com., compensator; Bat., polarising battery; K, cell connected with commutator; F, is a paraffin double key by which the polarising and galvanometer circuits were closed at the same time.

that the apparent positive variation was not analogous to that which the intrapolar current undergoes when the nerve is stimulated, the so-called "charge of resistance effect." It was found that after compensation the positive variation continued in undiminished or scarcely diminished amount. Nay more, over-compensation did not abolish, nor begin to abolish it. Fig. 2 shows the arrangement

which was at first used; *a*, *b*, *c*, *d*, represent the lengths of nerve IA, AB, BC, and CD, respectively.

Experiments 1 and 2 are examples of the first method without compensation; Experiments 3 and 4, with compensation. It will be seen that on the cathodic side, *i.e.*, with ascending current, the stimulation effect has the negative sign with reference to the direction of the polarising stream. On the side of the anode the same is true up to an electromotive force of about 3 Daniells working through 9 mm. of nerve. Above this the effect becomes positive. This is so only when the distance C is small. In Experiment 4 it is seen that, with C = 6½ mm., the positive effect does not appear with 7 Daniells, nor even when C = 3½ mm. When C is reduced to 1 mm., it comes in even with 3 Daniells.

Experiment 1.

Distances—*a*, 9 mm.; *b*, 10 mm.; *c*, 2 mm.; *d*, 9 mm.

Polarising Current.	Stimulation Effect.	Polarising Current.	Stimulation Effect.
1 D Rh. 100 cm.*		1 D ↑	−12
↓	− 3	1 D ↓	−15
↑	− 3	3 D ↑	−55
1 D Rh. 2000 cm. ↓	−25	3 D ↓	+50
↑	−20	5 D ↑	−42
1 D ↓	−20	5 D ↓	+53

Experiment 2.

Polarising Current.	Stimulation Effect.	
1 D ↑	−14	Galv. shunt 10.
1 D ↓	−16	
3 D ↓	+45	
4 D ↓	+29	

No compensation in Experiments 1 and 2.

Experiment 3.—Here two sets of observations were taken on the same nerve, the distance between electrodes B and C being altered.

* The total resistance of the Rheochord was 2000 centimetre units.

1st set.—Distances—*a*, 7½ mm.; *b*, 9 mm.; *c*, 1½ mm.; *d*, 9 mm.

Polarising Current.	Stimulation Effect.	
5 D ↓	+ 45	Shunt 10.
1 D ↓	− 6	
5 D ↓	+ 27	Not compensated.
5 D ↓	+129	Compensated. No shunt.
5 D ↑	− 51	,, ,,

2nd set.—Distance—*c*, 7½ mm. Other distances the same as before.

Polarising Current.	Stimulation Effect.	
5 D ↓	−22	Compensated. No shunt.
5 D ↓	−10	,, ,,

Experiment 4.

Distances—*a*, 7½ mm.; *b*, 9 mm.; *c*, 6½ mm.; *d*, 9 mm.

Polarising Current.	Stimulation Effect.
1 D ↓	* −47, −42
2 D ↓	−41 (?)
3 D ↓	−68, −69
2 D ↓	−65, −82
1 D ↓	−28, −32
5 D ↓	−45, −50
7 D ↓	−28

Same nerve; distance *c* made 3½ mm.

Polarising Current.	Stimulation Effect.
7 D ↓	−37
1 D ↓	− 5

Same nerve; distance *c* made 1 mm.

Polarising Current.	Stimulation Effect.
1 D ↓	−22
3 D ↓	+34
5 D ↓	+26

* The bracketed numbers represent double readings.

In Experiment 4 only half of the galvanometer was in circuit. The deflections given must be doubled in order to compare with the preceding experiments.

These results suggested that it might be still better to put electrodes B and C in contact, so as practically to make them one electrode. Of course it was here necessary to attend to compensation even more strictly than before; for the danger of a direct escape of current was greater than before; but so long as the galvanometer circuit was fully compensated, even such an escape would introduce no error.

Experiment 5 gives an example of this method.

Experiment 5.

Polarising Current.	Stimulation Effect.	
1 D Rh. 1000 cm. ↑	− 80	
1 D Rh. 100 ↑	+ 6	After 30″ closure.
	− 18	,, 1′ ,,
	− 22	,, 2′ ,,
	− 28	,, 3′ ,,
	− 35	,, 4′ ,,
	− 38	,, 5′ ,,
	− 43	15″ after opening polarising current.
2 D ↑	−110	
	−133	30″ after opening.
3 D ↑	?	Owing to unsteadiness, difficult to read amount, but certainly less than − 100.
	−117	20″ after opening.
3 D ↓	+255	
	−215	Another reading. Current kept closed for 5′ before readings taken.
	+ 48	30″ after opening.

With 1 D Rh. 100 ↑ a small positive deflection was got. I have a good many times observed that when the nerve is perfectly fresh, the polarising current very weak, and the reading taken very soon after closure, a positive stimulation effect is got on the side of the cathode. This would suggest that the conductivity around the cathode is not reduced immediately on closing such a current, but may even be increased. This agrees with what I saw occasionally when stimulating in the middle of the intrapolar area, with the muscle attached. Sometimes with weak currents the descending was more favourable than the ascending for getting contraction. This never happened when the currents were fairly strong.

Werigo also, in his experiments on intrapolar stimulation, quite

different in purpose from mine and essentially different in method, found that the cathodic block took time for its establishment, and that, when it appeared, it appeared suddenly.

In the example given in Experiment 5 the initial positive effect is seen to change in 30″ into a negative effect thrice as great, and this negative effect then gradually increases with still longer time of closure.

In order to diminish, as far as possible, the irregularities in the deflection, which are always a source of trouble with strong electrotonic currents, especially on the anodic side, I thought of using the currents led off to the galvanometer from two separate nerves of the same frog to compensate one another, a method resembling somewhat in principle that which Hermann has used in some of his polarisation work. Then, on exciting one of the nerves, one ought to get the stimulation effect, weakened, of course, by the extra resistance of the second nerve. The same battery was connected with both nerves, so that irregularities in the battery itself might be eliminated. The result was very satisfactory.

Figs. 3 and 4 show the arrangement.

In the arrangement of fig. 4 two nerves were placed on two separate sets of electrodes A, B, D; A′, B′, D′, a compensator (Com.) being introduced into the galvanometer circuit.

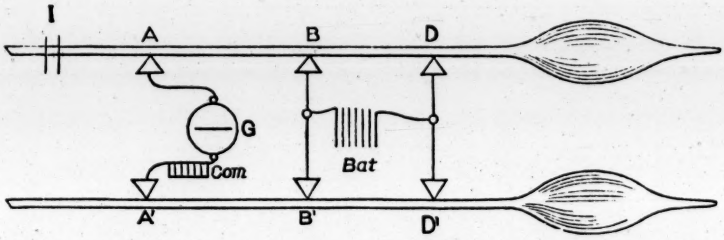


FIG. 3.—G is the galvanometer; Bat., the battery; I, the stimulating electrodes.

The pieces of nerve BD, B′D′ were made as nearly as possible equal in length, and therefore the current would have nearly the same density in each. The electrotonic currents in AB, A′B′ would be nearly equal, and they would pass through the galvanometer in opposite directions. The balance was completed by means of the compensator.

In the arrangement of fig. 4 the polarising current passed to

both nerves through the same electrodes C,D, and the density would therefore be more nearly equal in the two than with the arrangement of fig. 3. As before, a compensator was put in the galvanometer circuit. B was not an electrode, but only a movable

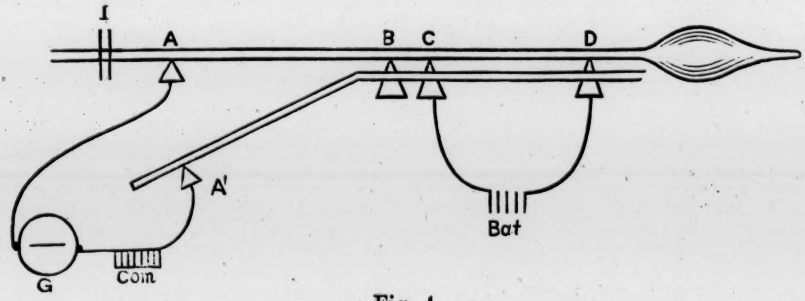


Fig. 4.

bridge of clay. If we stimulate at I, it will depend upon the distance of B from C whether the anodic effect will be positive or negative.

Experiments 6 and 7 are samples of the results got by this method.

Experiment 6.

Distances—*a*, 10 mm.; *b*, 10 mm.; *c*, 2 mm.; *d*, 13 mm.

Polarising Current.	Stimulation Effect.	
1 D ↓	−184	
3 D ↓	− 38	
5 D ↓	+ 58	
8 D ↓	+ 68	
1 D	+138	
1 D Rh. 90 cm. ↓	− 53	
5 D ↓	+ 30	
8 D ↓	+ 63	
1 D ↓	− 79	
2 D ↓	− 76	

Experiment 7 shows the change of sign on the anodic side even with 2 D. The negative effect on the side of the cathode seems here to diminish with increase of current, and this might suggest that with still stronger currents a positive phase might be found. I cannot say that I have found any trace of such an effect, and it is only in exceptional cases that the diminution in the negative effect appears.

Experiment 7.

Distances—*a*, 7½ mm.; *b*, 7½ mm.; *d*, 10 mm.

Polarising Current.	Stimulation Effect.	
2 D ↑	− 40	B and C in contact.
1 D Rh. 90 cm. ↑	− 8	
3 D ↑	− 28	
5 D ↑	− 24 − 22	
1 D ↑	− 9	
2 D ↑	− 14	
5 D ↑	− 3	
1 D ↓	−127 −114	
2 D ↓	+ 28 + 22	
5 D ↓	+126 +119	
1 D ↓	− 46 − 49	

It was now desirable to compare the amount of the intra- and extrapolar stimulation effects; and in order to avoid the uncertainty which must always exist when one tries to get quantitative comparisons from experiments made on different days with different nerves, I determined to control the other observations by means of a set in which the intrapolar and extrapolar regions of the same nerve were led off alternately to the galvanometer. It was particularly important to notice how the *ratio* between the amount of the two effects varied with varying density of polarising current when the latter was nearly strong enough to suppress the intrapolar effect altogether.

The arrangement is shown in fig. 5 for the case where the two extrapolar regions compensate each other, and the two intrapolar regions are placed one in each coil of a differential galvanometer.

A, B, C, D are, as before, the electrodes of one of the nerves; A′, B′, C′, D′ those of the other. G, G′ are the two coils of the differential galvanometer; P is a Pohl's commutator without cross wires, by means of which either AB or CD may be joined on to G; P′ is a Pohl with cross wires, to alter the direction of the polarising current; Com. is a compensator to complete the compensation when the extrapolar areas are led off; R is a rheostat to equalise the

intrapolar currents. By an arrangement not shown it could be thrown either into CD or into C′D′.

The balancing arrangement was used for the intrapolar currents in order to diminish the irregularity in the deflection, which is much more troublesome than even in extrapolar experiments. Of course, only one nerve was stimulated.

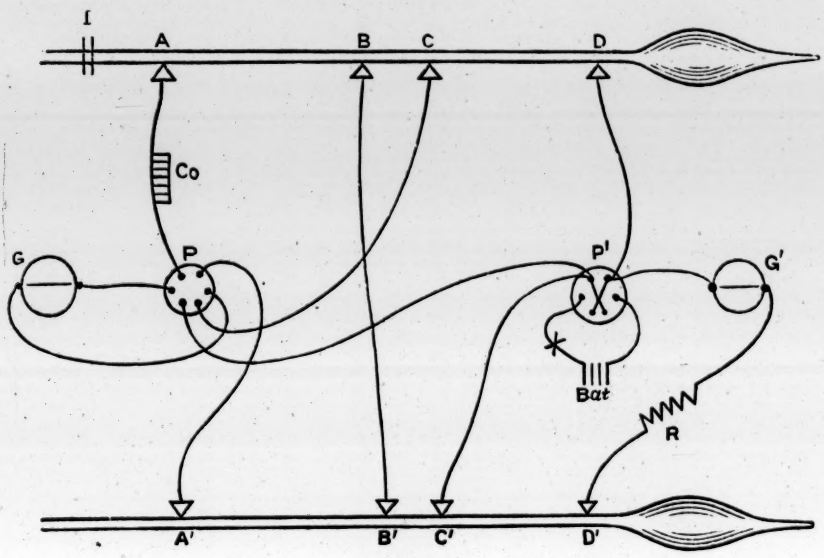


Fig. 5.

The circuit of G′ was broken by a simple key, whenever the extrapolar regions were to be connected with G. The current was then passed through CD, compensation completed in the galvanometer circuit, and the stimulation effect read off. After the nerves had recovered, the two intrapolar regions were thrown on to G and G′, the extrapolar being off. The same current was now passed again in the same direction, for the same length of time, and the stimulation effect again taken. A given number of cells would give practically the same current density in CD, whether the alternative circuit C′D′ was open or closed, since the internal resistance of the battery is very small compared with the resistance of the nerves.

Experiments 8, 9, and 10 (pp. 246, 247) are examples of this method as applied to currents near the limiting intensity. An electromotive force of about 5 Daniells working through 9 mm. of nerve gives the density corresponding to the disappearance of the intrapolar effect. This limiting electromotive force will be inversely as the length of nerve included in the circuit, if we assume that the specific resistance

of nerve in the longitudinal direction is a constant. Of course it will vary slightly even for the two nerves of the same frog, as it will depend mainly at least upon the amount and kind of the dissolved crystalloids. In my former results on the intrapolar effect I found that the limiting electromotive force varied from 8 to 9 Daniells, when the length of nerve was from 12 to 14 mm. The two sets of experiments therefore agree as well as one is entitled to expect in observations of this sort. The strength of stimulus, of course, has also to be taken into account.

Experiments 12 and 13 show how the effect in the extrapolar region reaches a maximum, while in the intrapolar it declines to a minimum. This of itself is quite enough to dispose of the possibility that the suppression of the intrapolar effect is due to the decline of excitability at the point of stimulation through the spread of anelectrotonus.

Experiment 14 is an example of stimulation on the cathodic side.

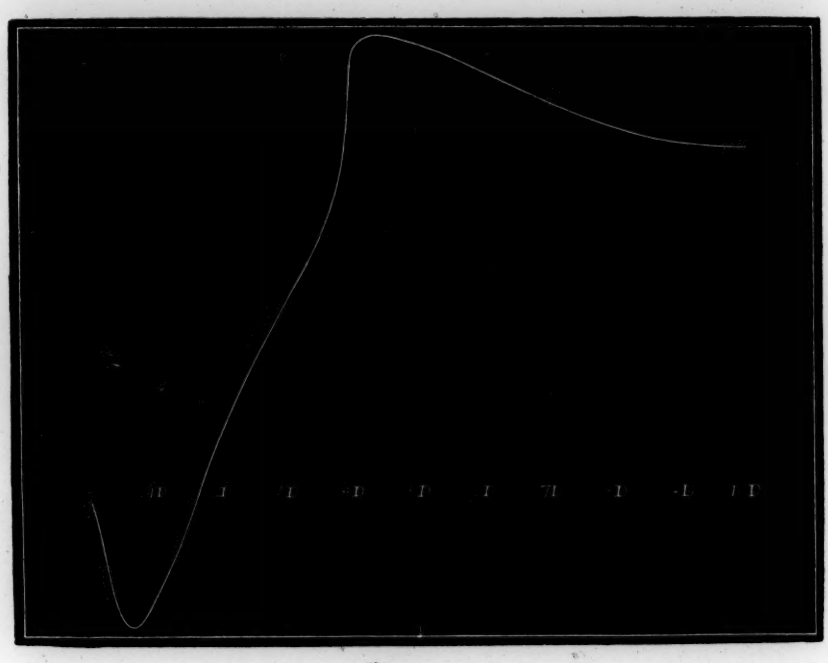


Fig. 6.

Fig. 6 shows the curve of the effect in Experiment 12 plotted to scale.

The number of points are too few to determine the details. Almost the whole of the ascent lies very nearly in a straight line.

Summary of Results.

1. With weak currents there is the ordinary negative variation both on anodic and cathodic side, however close the galvanometer and polarising circuits may be brought to each other.

2. As the strength of the polarising current is increased, the negative variation on the anodic side passes into a positive variation, which increases and apparently reaches a maximum.

3. The maximum of the positive anodic variation corresponds to a density of current which is not far from that for which the intrapolar variation is at its minimum (zero).

4. On the cathodic side the variation is always negative with currents above the very weakest. (With very weak currents, fresh nerves, and short period of flow, sometimes a small positive variation seems to be got.)

5. The greater the distance between the polarising and galvanometer circuits, the stronger must the polarising current be for which the positive anodic variation first appears.

All these results hold when the electrotonic currents are compensated.

Experiment 8.

Distances—*a*, 9 mm.; *b*, 7½ mm.; *c*, 1 mm.; *d*, 9 mm.

Polarising Current.	Extrapolar Stimulation Effect.	Intrapolar Stimulation Effect.	
4 D ↓	38	0	
4 D ↓	57	5	
4 D ↓	39		

Experiment 9.

Distances—*a*, 9 mm.; *b*, 7½ mm.; *c*, 1 mm.; *d*, 9 mm.

Polarising Current.	Extrapolar Stimulation Effect.	Intrapolar Stimulation Effect.	
4 D ↓	+214	+50	Here before passing current there was a stimulation effect of 38 in same direction as intrapolar effect.
4 D ↓	+ 81	+55	Here there was a stimulation effect of 87 in same direction before passing current.
4 D ↓	+230		
4 D ↓	+170		

Experiment 10.

Polarising Current.	Extrapolar Stimulation Effect.	Intrapolar Stimulation Effect.	
5 D ↓	+180	0	

Experiment 11.

Distance—*d*, 12 mm.

Polarising Current.		Intrapolar Stimulation Effect.	
1 D ↑ Rh. 2000	...	+66	Stimulus 85.
5 D ↑ 2000	...	0	Even with strongest stimulation.
5 D ↓ 2000	...	+55	Coils close up.
1 D ↓ 2000	...	+87	,,
5 D ↓	...	+30	,,
7 D ↓	...	+14	,,
1 D Rh. 2000	...	+85 +90	,,

Experiment 12.

Distances in Experiments 12 and 13—*a*, 9 mm.; *b*, 9 mm.; *c*, 1½ mm.; *d*, 9 mm.

Polarising Current.	Extrapolar Stimulation Effect.	Polarising Current.	Extrapolar Stimulation Effect.
1 D ↓	−26	7 D ↓	+126
2 D ↓	+25	10 D ↓	+106
3 D ↓	+62	4 D ↓	+115
4 D ↓	+135	4 D ↓	+75
5 D ↓	+138		

Experiment 13.

Polarising Current.	Intrapolar Stimulation Effect.	
1 D Rh. 50 ↓	+22	Same strength of stimulus.
1 D Rh. 2000 ↓	+41	
2 D ↓	+28	
3 D ↓	+17	
4 D ↓	+24	
5 D ↓	0	
1 D Rh. 2000	+36	

Experiment 14.

Polarising Current.	Cathodic Extrapolar Effect.
1 D ↑	−27
2 D ↑	−29
3 D ↑	−31
5 D ↑	−43 ?
10 D ↑	−44

I do not propose to discuss here, further than I have done, the real significance of the results stated, as I hope soon to have an opportunity of doing so in connection with a more extended research, embracing the effect of stimulation on the whole of the polarisation phenomena of nerve and muscle. I should just like to say, that it is by no means impossible that a real positive electrotonic variation may be mixed up with a true action current in the positive direction.

The work was done partly in the Owens College, and partly at Edinburgh in the Laboratory of Professor Rutherford, whose great kindness I take this opportunity of acknowledging.

Notice of Fundamental Tables in Trigonometry and Astronomy, arranged according to the Decimal Division of the Quadrant. By E. Sang, LL.D.

(Read June 3, 1889.)

Canon of Sines.

In January of 1878, there was laid on the Society's table the Canon of Sines to each fifth minute of the decimal division of the quadrant, computed to thirty-three for thirty places; along with a detailed record of every step in the process. During the years 1880-81, this work was continued for each single minute, but only to eighteen for fifteen places, and the record thereof to fifteen places is now submitted. When the rejected figures were from 497 to 503 a mark of interrogation is recorded, and it is believed that not a single error exists in the work. The arrangement of the sines with their first and second differences in position enables us instantly to detect an error.

That fifteen places suffice for all possible practical purposes, is made clear by this consideration, that the Earth's distance from the Sun, measured in inches, is represented by the number 6, twelve removes from the unit's place, that is 6 000 000 000 000; and that, if we take this as the radius of our circle, the figure 1 in the fifteenth decimal place will represent ·006 or the 170th part of an inch. Thus the present canon gives, on this circle, the co-ordinates of the ten thousand points in the quadrant, each true to within the three-hundredth part of an inch.

The process followed in this work differs, in one very important respect, from that used by previous computers. The sine of the smallest tabular arc has hitherto been found indirectly by help of repeated bisections; in the present work the quinquisection of the arcs has been accomplished directly by the solution of the appropriate equations of the fifth degree, according to the method described in my treatise "On the Solution of Equations of all Orders." The ease and rapidity of this method are well shown by the recorded details of the work for the various equations, to thirty decimal places.

A table of one thousand multiples of 2 ver. 1′ having been pre-

pared, the rest of the work was carried on in the usual manner with verifications at frequent intervals.

Logarithmic Sines.

The logarithmic sines were deduced from the sines themselves by help of my fifteen-place table of logarithms of numbers from 100 000 to 370 000, using the auxiliary table. Beginning at 100° 00′, the computations were made on scroll paper for each single minute down to 50° 00′, each step being verified by first, second, and third differences. The third differences were then copied into their places on the actual manuscript, and the others were thence reconstructed. In this way all errors of transcription were avoided, and any mistake in the previous work detected.

From this half of the canon, the other half, namely from 50° 00′ to 00° 00′, was deduced according to the formula $\sin a = \frac{1}{2} \sin 2a \,.\, \sec a$; the method of proceeding being to compute each tenth log sine directly, and to fill in the single terms by differences easily got from the differences already written in the first part. This operation supplied a check on all the previous work.

Logarithmic Tangents.

The logarithmic tangents were computed in the same way, that is to say, each tenth term was found directly, and the single terms by means of the preceding differences, thus furnishing another verification of the whole. But, seeing that the log tangents of the one half are the arithmetical complements of those of the other half, it was enough to write out the log tangents of arcs from 50° 00′ to 00° 00′.

With the exception of the arrangement for computing by differences, and for assuring exactitude, this is the very process used by Nepair in the construction of his Canon Mirificus; and, indeed, this volume of Logarithmic Sines and Tangents might, with all propriety, have been entitled:—

"John Nepair's wonder-working Canon, changed by his express desire, to suit the Denary System of Arithmetical Notation."

After a labour which must have occupied his leisure time for more than the quarter of a century, Nepair had published his

Canon, had experienced its utility, had received the approval of the scientific world; and yet, foreseeing the advantage of accommodating his plan to the notation in common use, he at once recommended the putting of it aside for a far better plan. No stronger evidence can be adduced in favour of discarding the time-honoured division by 90 and by 60, and substituting the decimal division throughout.

Kepler's Problem.

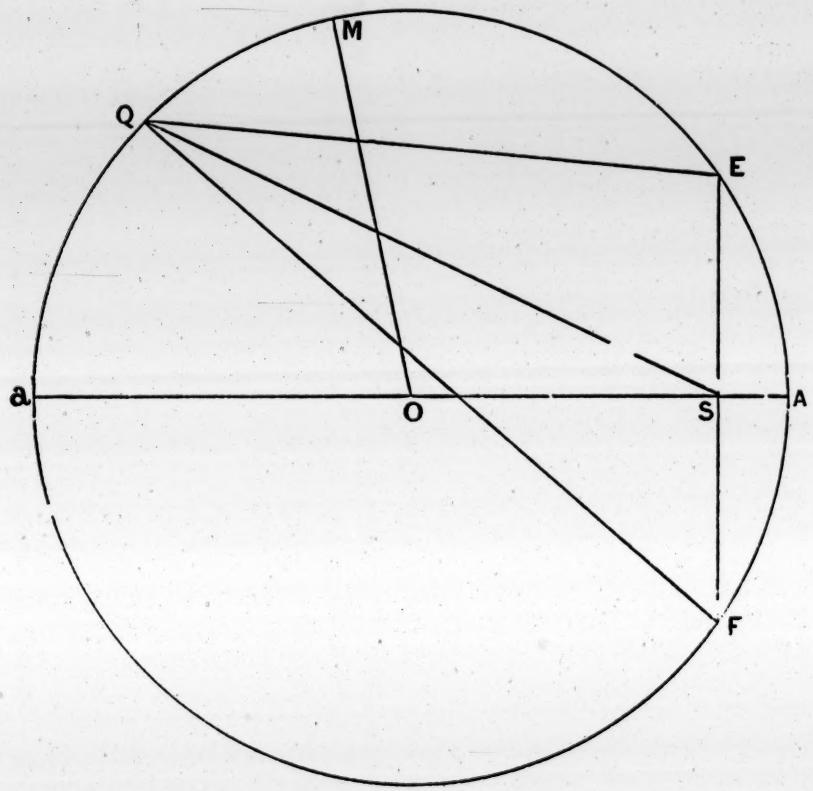
While the Canon of Sines was still in progress, circumstances led to a repetition of the often fruitlessly made attack on Kepler's famous problem, and this time an unexpectedly simple solution presented itself. The Royal Academy of Sciences of Turin did me the very great honour of giving that solution a place among their memoirs. The subject, however, may be treated more generally and even more simply, thus:—

Let us suppose ourselves to be studying the apparent relative motion of a binary system of stars; each one seems to describe round the other an ellipse, and the areas passed over by the radius vector are proportional to the elapsed times. But, since the actual orbit may be inclined anyhow to the plane on which it appears to be projected, the one star does not appear to be in the focus of the orbit of the other; nor is the diameter drawn through its apparent place, necessarily be the major axis. If we divide the periodic time into equal portions, the corresponding vectors will similarly divide the area of the ellipse, and hence the problem virtually comes to be this,—"to subdivide the area of an ellipse by lines diverging from some point within it."

The line from the eye to the revolving star defines the surface of a cone, in our imaginary case sensibly of a cylinder, and the planes passing through the eye, and along the vectors, subdivide this cylinder into wedges. If now this system be cut by any plane, the section so made will have its area also subdivided; now we can always cut a cylinder so that its section may be a circle, and thus, ultimately, the problem becomes this, "to subdivide the area of a circle by lines diverging from a point within it."

If S represent the point given within the circle described round the centre O, the diameter ASOa will represent the line of apsides, A being the perihelion, a the aphelion. Let now Q correspond to

some position of the planet, then the surface comprised between AS, SQ, and the arc AQ, is proportional to the time elapsed from the planet, being in the position A, till its reaching the position corresponding to Q, so that this surface is the planet's mean anomaly.



Draw now ESF perpendicular to AQ, then the arc AE, which has the excentricity OS for its cosine, may be called the arc of excentricity; we shall denote it by e; while AQ, the arc defining the planet's position, may be denoted by p. Having joined EQ, FQ, it is seen, by mere inspection, that the mean anomaly AEQS is half the sum of the two circular segments cut off by the chords FQ, EQ, or that

$$\text{mean anomaly} = \tfrac{1}{2}\{\text{segm}(p+e) + \text{segm}(p-e)\},$$

and so a table of circular segments would enable us to determine the mean anomaly when the position is given, and conversely the position when the mean anomaly is given.

In order to avoid the frequent multiplication and division by the

number π, we measure the areas of the segments not in parts of the square of the radius, but in parts of the surface of the circle; a superficial degree being the sector standing on a degree of arc.

For the construction of such a table, it was necessary to compute the canon of sines measured in parts of the quadrant. The sines for the single degrees were therefore computed by simple multiplication of the ordinary sines, to serve as verifications of the subsequent work. Afterwards those for each quarter degree were obtained by using the previous multiples of 2 ver. 25′ for second differences; these two operations completely checked each other. Again the sines for each fifth minute were got by help of the multiples of 2 ver. 5′. But, as the computations were carried only to the tenth decimal of the quadrant, the products by 2 ver. 1′ were not needed.

Sines Measured in Degrees.

In this way the "Canon of Sines measured in degrees" now presented was completed, the actual volume contains the whole details of the work, and it is hoped without any error exceeding two units in the tenth place.

Canon of Circular Segments.

Since the number which expresses the area of a segment in degrees of surface is the difference between those which express the arc and its sine, it follows that the second differences in the table of circular segments are identic with those of the sines; and therefore the canon of segments was constructed directly from those second differences. In the present volume it is extended to the entire circumference, that is to forty thousand minutes, and shows the value of each segment true to within two or three units in the ten-thousandth parts of the centesimal second. Its accuracy, thus, is very far beyond any requirement in actual astronomy. This work furnished another check on that for the canon of sines measured in degrees.

This table of circular segments enables us very easily to discover the mean anomaly when the angle of position is known. The converse problem, "to find the angle of position from the mean anomaly," has to be solved by approximation; which is sufficiently rapid if the first assumption be not very wide of the mark. When, for the

orbit of some particular planet, we are computing the positions at equal intervals of time, attention to the differences reduces the labour to little more than that of writing out the results. It is only when a sporadic case is presented that the approximation is attended with any difficulty.

Mean Anomalies.

In order to obviate even this difficulty, a table has been constructed of the mean anomalies for orbits of each degree of excentricity, and for every degree of the angle of position, up to 200° in each of these orbits. This table enables us, in every possible case, to get at once a first assumption so near as to make the subsequent approximation quite easily.

This table is presented in two forms. In the volume marked mean anomalies A, the values are given to four decimal places of a second. In the corresponding volume marked B, they are written only to the nearest second; but the differences and the variation from one orbit to another are inserted. Hence, by the ordinary method of interpolation for two variables, we can solve both the direct and the inverse problem with precision sufficient for all the purposes of practical astronomy.

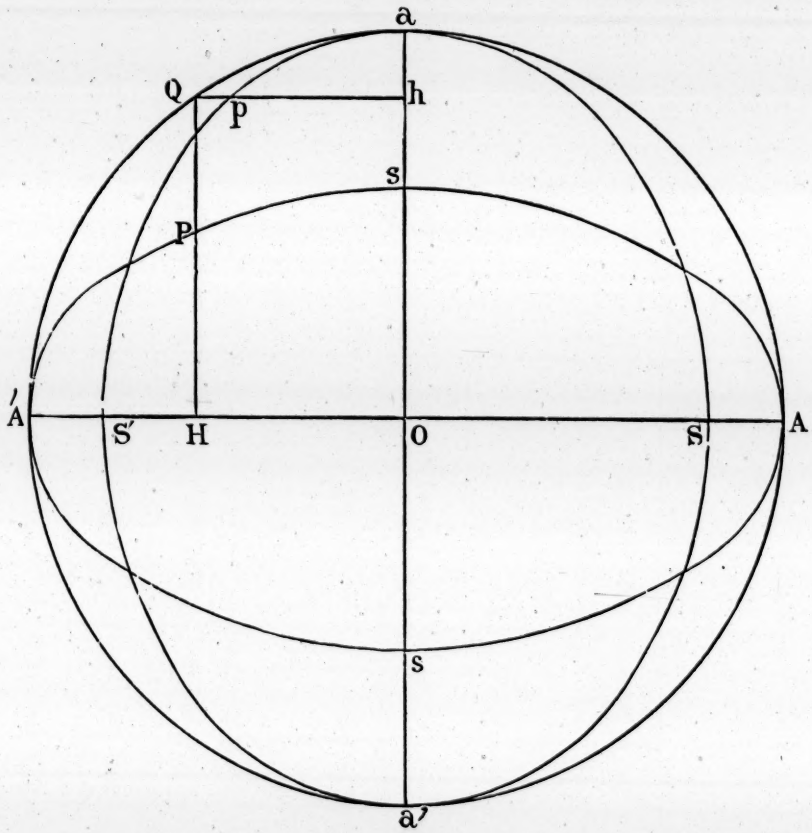
My intention was to have computed also the radii vectors and the true anomalies. For this, however, the only available trigonometrical tables were those to seven places printed in a most inconvenient form, by Callet, in his Tables Portatives. The work was scarcely begun when it became apparent that the precision attainable was not commensurate with the labour. Therefore, putting that work aside, I preferred to undertake the hopeless-looking task of computing the logarithmic sines and tangents to a greater number of places. This work is fortunately accomplished, although there still remain the transcription in the order usually adopted for convenient reference.

The application of these tables to the computation of the true anomalies, is a task far too great to be undertaken at the close of a long life, and, not without reluctance, it is left to the zeal of other computers. Enough, that I have been enabled to place within the reach of mathematicians some contributions to the progress of exact science.

The convenience of having the true anomaly, and the planet's distance alongside of the time-argument, would be so great as to dwarf altogether that of having merely the angle of position; this last mentioned forms, indeed, only a step toward the obtaining of the others. The remaining operation, implying only the solution of a right-angled triangle, is easy though laborious. It may, therefore, be not inopportune to indicate here the course most convenient to be followed in the subsequent work; particularly because that which may appear to be the most rapid in an isolated case, may not be the best for systematic work.

Distances.

If P be the planet's place in the ellipse A*s*A′*s*′, having S and S′ for its foci, and SS′ for its minor axis, and if the ordinate HP be



continued to meet the circle described on AA′ as a diameter in Q, the arc AQ is p, the arc of position, and we have

$$\mathrm{SH} = \cos p - \cos e\,;\ \mathrm{PH} = \sin p.\ \sin e.$$

From those we easily get the tangent of the true anomaly ASP, and thence the distance SP. Here the great part of the labour is in finding the logarithm of SH, the angle HSP from its log tangent, the log secant from the angle, and SP from its logarithm; that is to say, in using the tables of the logarithms of numbers, and of circular functions to a considerable number of decimal places. This labour, repeated for each of the twenty thousand cases to be tabulated, rises to a formidable total.

But if, on the perpendicular diameter AOA′, we describe another ellipse having SS′ for its minor axis, and consequently s and s' for its foci, and if from Q we draw the ordinate Qph, we have, according to the properties of the ellipse, $SP = OA \mp ph$, and conversely $SP = OA \pm PH$. Thus the computation of the ordinates in the one of the two orbits gives us, with only the labour of writing the numbers in their places, the vectors of the other orbit, and we are now enabled to compute the true anomaly from its log sine. When following this course, it will be convenient to begin with the orbit $e = 50°$, and to take the others in couples, $e = 49°$, $e = 51°$, and so on.

Our working formula then stand thus:—

$$\text{log ordinates} \begin{Bmatrix} \log \sin p + \log \sin e \\ \log \cos p + \log \cos e \end{Bmatrix}, \text{ whence ordinates}$$

$$\text{distances} \quad \begin{Bmatrix} 1 \mp \cos p \,.\, \cos e \\ 1 \mp \sin p \,.\, \sin e \end{Bmatrix} \text{whence, log distances,}$$

log sin anomaly = log ord. − log dist., whence anomaly.

If it were proposed to make these computations with all the precision obtainable from our fifteen-place tables, it might be economical, even for this single piece of work, to interpolate the logarithmic sines for each hundred-thousandth part of the quadrant.

On the Relation among the Line, Surface, and Volume Integrals. By Professor Tait.

(Read April 1, 1889.)

The fundamental form of the Volume and Surface Integral is

$$\iiint \nabla u d\varsigma = \iint \mathrm{U}\nu u ds .$$

Apply it to a space consisting of a very thin transverse slice of a cylinder. Let t be the thickness of the slice, A the area of one end, and α a unit-vector perpendicular to the plane of the end. The above equation gives at once

$$\mathrm{V}(\alpha\nabla)u . t\mathrm{A} = t\int \mathrm{V}.\alpha\mathrm{U}\nu u dl ,$$

where dl is the length of an element of the bounding curve of the section, and the only values of $\mathrm{U}\nu$ left are parallel to the plane of the section and normal to the bounding curve. If we now put ρ as the vector of a point in that curve, it is plain that

$$\mathrm{V}.\alpha\mathrm{U}\nu = \mathrm{U}d\rho , \quad dl = \mathrm{T}d\rho ,$$

and the expression becomes (after division by t)

$$\mathrm{V}(\alpha\nabla)u\mathrm{A} = \int u d\rho .$$

By juxtaposition of an infinite number of these infinitely small directed elements, α (now to be called $\mathrm{U}\nu$) being the normal vector of the area A (now to be called ds), we have at once

$$\iint \mathrm{V}(\mathrm{U}\nu\nabla)uds = \int u d\rho ,$$

which is the fundamental form of the Surface and Line Integral.

In fact, as the first of these expressions can be derived at once from the ordinary equation of "continuity," so the second is merely the particular case corresponding to displacements confined to a given surface.

The Development of Diarthrodial Joints in Birds and Mammals.

By **David Hepburn**, M.B., M.R.C.S. (Eng.), *Senior Demonstrator of Anatomy, University of Edinburgh.* *Communicated by* Professor Sir W. TURNER.

(From the Embryological Laboratory, University of Edinburgh.)

(Read May 20, 1889.)

After giving a summary of recent literature on the subject, the author then proceeded to state the nature of the material which he had employed in the present investigation.

The bird selected was the common fowl (*Gallus domest.*), and he had examined a series of microscopic sections through the limbs from the fourth day of incubation to the day of hatching.

The mammalian embryos examined were mice and rabbits, and the fingers of the human fœtus from an embryo approaching the full period of uterogestation.

Method of Preparation.—The embryo chicks were prepared, partly by hardening in picrosulphuric acid and partly in dilute solutions of nitric acid. The human embryos were also hardened in nitric acid. The embryos were then dehydrated with alcohol, stained in borax-carmine, and cut with the Cambridge rocking microtome, the average thickness of the sections being ·006 mm.

The author expressed his indebtedness to Mr George Brook, Lecturer on Embryology in the University of Edinburgh, under whose guidance and in whose laboratory the investigation had been conducted, and then proceeded to give a summary of the results which he had attained.

At the end of the fourth day of incubation the wing of the chick is in the form of a bud, ·8 mm. long, and consists of a mass of mesoblast cells enveloped in a covering of epiblast. At this stage the cells of the mesoblast present no differentiation into separate structures, but at the end of the fifth day the free extremity of the limb has assumed a bulbous form, and horizontal sections show that along certain lines a process of condensation has occurred, apparently presaging the positions of future bone matrices. The individual cells in these portions present no great change from the rest of the

surrounding mesoblast, except such variations in outline as can be accounted for by compression.

As growth proceeds, digits emerge from the bulbous extremity, and a section at the end of the ninth day shows that a process of differentiation has taken place in the condensed portions, the result of which is the formation of cartilaginous rods separated from each other by masses of undifferentiated cells termed the articular disc or inter-tissue.

It is in connection with this articular disc that the future joint cavity and its various appendages are developed. Here the joint cavity makes its appearance, and may be seen in the wing of the chick at the end of the ninth day. The cleft commences within the circumference of the articular disc and extends towards the axis of the disc, so as to divide it into two segments, each of which is applied to the end of a cartilaginous rod, and the segments are held together by its undivided periphery. The sides of the cleft are bounded by a layer of flattened cells.

When this cleft does not extend across the axis of the disc, material is left for the formation of an interarticular ligament, as may be seen in sections taken from the leg of a chick about the middle of the second week.

In the case of some joints two cavities appear, having between them a portion of the disc, which ultimately develops into a meniscus. Again, when the two cavities fuse in the axis of the disc, we have an incomplete meniscus.

Even at this early stage there is a certain amount of moulding of the ends of the cartilaginous rods which foreshadows their future shape, and as this occurs at a time when the muscular system is in abeyance, it cannot be the result of movement, and neither can we ascribe the formation of the cavity to this cause.

Tracing the changes which take place in connection with the various parts of the now partially divided articular disc, we find that the segments applied to the ends of the cartilaginous rods gradually become differentiated into hyaline cartilage, until this process has affected the whole thickness of the segment, with the exception of the row of flattened cells next the cavity of the joint. In the chick these are found still persisting at the period of hatching.

The undivided circumference of the disc has meanwhile under-

gone differentiation into fibrous tissue, and in it may be found the rows of cells and wavy fibres characteristic of that structure. This fibrous capsule also continues to be lined by a row of flattened cells continuous with those on the surface of the articular cartilages. A similar series of changes may be traced in connection with inter-articular fibro-cartilages and ligaments.

In mammals the formation of the articular disc and the appearance of the joint cavity, as seen in embryo mice and rabbits, are practically identical with those just described. There is therefore no reason to doubt that the subsequent changes in the component parts of the disc are also similar.

There is, however, one striking difference in connection with the layer of flattened cells which line the cavity, and this was observed on the articular cartilages taken from the phalangeal joints of a human fœtus approaching the full period of uterogestation. Here the usual flat cells were found lining the interior of the capsular ligament, but on tracing them towards the articular cartilages they were seen to be replaced by a narrow band, staining somewhat more freely than the hyaline cartilage, but presenting no cell structure. The free surface of this band was slightly ragged, and it appeared to be undergoing degeneration. Thus it would appear that the flat cells lining the primitive joint cavity have a double fate. Those in relation to the ligamentous structures, and thus within reach of a direct blood supply, become specialised into a synovial membrane; whereas those in relation to the articular cartilages, although present in the chick at the period of hatching, probably disappear as the result of friction; while, in the case of the mammal, they undergo degenerative changes, which lead to their early disappearance from the same cause.

Summary of Conclusions.

1. The bone matrices and the articular disc possess a tissue continuity, and are derivative of a common blastema of which the articular disc is at first the undifferentiated form.

2. The articular disc may conduct itself as follows :—

(*a*) It may develop into a plate of cartilage and form a synchondrosis, *e.g.*, the articulation between basi-occipital and basi-sphenoid bones.

(*b*) It may differentiate into fibrous tissue and form a syndesmosis or synarthrosis.

(*c*) It may partially cleave and form a joint cavity.

3. The joint cavity appears within the articular disc at a period when the process of chondrification is at some distance from the cavity.

4. If the cavity remain of small size, and the surrounding articular disc develop into fibrous tissue, an amphiarthrosis is formed, *e.g.*, the joint between vertebral bodies. (This is specially well seen in some Cetacea, and probably the epiphyseal plates on the bodies of vertebrae are also derived from the articular disc.)

5. The cavity may enlarge to form a diarthrosis.

6. When the joint cavity is single we have a simple diarthrosis; when there are two cavities we have a diarthrosis with an interposed meniscus; when the two clefts unite in the centre we have an incomplete meniscus; when the cleft is single, but does not extend across the axis of the disc, an interarticular ligament is formed.

7. The proximal and distal segments of the articular disc develop into the articular cartilages of the joint, and probably form part, if not all, of the epiphyseal ends of the bones.

8. The circumference of the articular disc develops into the capsule of the joint.

9. Interarticular fibrocartilages and ligaments are derived from the articular disc as the result of modifications of the joint cavity.

10. The cells lining the joint cavity have a double fate :—

(*a*) Those applied to the ligamentous structures are specialised as synovial membrane.

(*b*) Those upon the articular cartilage persist until the period of hatching in the chick, but undergo degeneration in the mammal; in both cases disappearing by friction as the result of movement.

Electrification of Air by Flame. By Sir William Thomson.

(Read July 15, 1889.)

In continuation of experiments on the electricity of air within doors, which I made twenty-seven years ago, and which are described in §§ 296–300 of my Electrostatics and Magnetism, a series of observations was commenced under my instructions at the end of April last, within the Natural Philosophy Class Room and Laboratory, the Bute Hall, the University Tower, and other places inside and outside the buildings of Glasgow University, by Mr Magnus Maclean, official assistant to the Professor of Natural Philosophy, and Mr Goto of Tokio, Japan, for the purpose of endeavouring to find a relation between the electrification of air within a building and the atmospheric potential in its neighbourhood outside; and of finding causes which produced or changed the electrification of any given mass of air.

A large number of series of observations have been made by Mr Maclean and Mr Goto on the potentials of water-dropping collectors within the building, and at different points outside the building. Hitherto no definite relation has been discovered between the external potential and the potential at different points within large enclosures, such as the Bute Hall and smaller rooms of the University Buildings. The weather has been for the most part very settled and the external potential almost always positive in all positions from a few feet above the ground to the top of the University Tower; while the potential within doors here, in the new University Buildings, on the top of Gilmour Hill, just as in the old College, down in the densest part of Glasgow, was always negative except sometimes in the Natural Philosophy Lecture Room and Apparatus Room, where there were considerable disturbances, undoubtedly due to the electric light wiring. In one ordinarily unused room (the Physical Apparatus Museum) 31½ feet long by 24 feet broad and about 20 feet high, practically quite free from any sensible disturbance by electric light wires, or by electrical operations being performed in the Laboratory, a remarkable result has been observed within the past fortnight. The electric potential of a water-dropper

having its nozzle at the centre of the room and about 7 feet above the floor, was always found about 2 volts negative at the commencement of the observations, and always increased to about 9 volts in the course of the first twenty minutes of a series of observations lasting generally forty minutes. During the last twenty minutes of the series the potential remained somewhat nearly constant at 9 volts. Within the room, two quadrant electrometers, each with an ordinary paraffin lamp and scale, were used; one of them for the outside water-dropper, and the other for the water-dropper within the room.

Towards ascertaining the cause of this change, an observation was made on the 4th July, between 10 and 11 A.M. The lamps were both extinguished, and one of them was lighted by a lucifer match every five minutes for the purpose of reading the electrometer deflection. It was found that in these circumstances there was not the increase of negative potential which had been found in every previous series of observations in the same place, and with all other circumstances the same, except the burning of the lamp. This single observation seemed to prove conclusively that the burning of the lamp produced a negative change of the air of the room. Subsequent experiments made by Mr Goto, with the electrometer and its lamp and scale outside, and with paraffin lamps burning or not burning within the room, have confirmed this result, and are being continued to discover whether corresponding effects are produced by other kinds of flame, or by the presence of eight or nine people in the room. Mr Maclean and Mr Goto will also continue their observations on natural atmospheric electricity, in various localities, indoors and in the open air, and will, I hope, give a paper to the Royal Society of Edinburgh early next session on the subject.

On the Placentation of the Halicore Dugong.

By Professor Sir William Turner.

(Read July 1, 1889.)

(*Abstract.*)

The only observations hitherto recorded on the placentation of the Dugong are by Paul Harting, of Utrecht, in 1878, who examined the fœtal membranes of a fœtus 27·8 cent. long. He stated that the placenta was diffused and non-deciduate.*

The gravid uterus described in this communication was presented to the author by C. W. de Vis, Esq., M.A., curator of the Queensland Museum, Brisbane, through Professor Anderson Stuart of the University of Sydney.

The uterus was bicornuate, and contained a single fœtus, 5 feet 4 inches long. The fœtus and its membranes occupied the left cornu, and there was no extension of the membranes into the right cornu.

The chorion was an elongated sac, upwards of 5 feet long from pole to pole.

The placenta formed a zone a little on one side of the equator of the chorion. The zone was $11\frac{1}{2}$ inches broad in its widest part and 6 inches at its narrowest. The rest of the chorion was smooth and free from villi. The villi were closely crowded together in the fœtal placenta; as a rule they were short, though longer villi were interspersed amongst them; they were cylindriform and filamentous in shape, and branched seldom except near their free ends.

The allantois was very extensive, and reached to the opposite poles of the chorion. Connected to the outer wall of its sac, formed by the endochorion, were a number of plate-like allantoic bodies.

The amnion was very capacious, and was completely surrounded by the allantois, except for a limited area in the region of the placenta. No amniotic corpuscles projected from its inner surface.

The uterine mucous membrane had a zone which formed the maternal placenta, and which corresponded in form, size, and

* See abstract of his paper in the *Journal of Anatomy*, vol. xiii. p. 116.

position to the zone on the chorion. This zone contained multitudes of short cylindriform crypts, in which the villi of the chorion were lodged. Longer and more deeply placed crypts were also present for the lodgment of the longer villi.

The non-placental area of the mucous membrane was smooth, and corresponded to the non-villous part of the chorion.

Uterine glands were seen both in the placental and non-placental areas of the mucous membrane. In the placental area they opened amidst the crypts by special orifices; in the non-placental area they opened obliquely on the smooth surface of the mucous membrane.

Owing to the shortness both of the chorionic villi and the uterine crypts and their simple form, it is believed that the placenta, when shed in normal parturition, would be generally non-deciduate, in the sense of the vascular walls of the crypts not being shed along with the villi; it is not unlikely, as the author showed some years ago to be the case in the sheep and cow,* that the epithelial lining of the crypts may separate more or less, and pass off entangled between the villi. It is also possible that the longer villi may carry away with them parts of the vascular walls of their crypts.

Should the placenta be non-deciduate in the sense that the vascular part of the maternal mucous membrane is not shed, then the placenta of the Dugong gives a new type of placenta—one which is both zonary and generally non-deciduate.

The diffused character of the placenta in the specimen described by Paul Harting was due to its comparatively early stage of development, for the villi had not as yet limited themselves to a denfinite zone.

The paper concluded by a comparison of the placentation of the Dugong with that more especially of the Cetacea, Carnivora, and Proboscidea, and by remarks on the bearings of the form and structure of the placenta on the classification of the Sirenia.

* *Proc. Roy. Soc. Edin.*, May 1875.

On the Geographical Distribution of some Tropical Diseases, and their Relation to Physical Phenomena.

By R. W. Felkin, M.D., F.R.G.S., *Lecturer on Diseases of the Tropics and Climatology, Edinburgh Medical School.*
(With 16 Plates.)

(Read July 15, 1889.)

The subject of the present paper has occupied my attention for some years, but I may state that what follows is the outcome of notes prepared for lectures to my students and is only a preliminary attempt to focus our present knowledge of the geographical distribution of some tropical diseases, and to indicate as far as possible the knowledge which we at present possess of those physical phenomena which influence the production of these diseases and the area of their distribution. Why, for instance, some diseases are confined to limited areas of distribution, whereas others are endemic in extensive districts, and others again periodically extend their ravages throughout clearly defined, though wide-spreading, regions.

A map was published by A. Keith Johnston in 1856 representing the geographical distribution of health and disease, chiefly in connection with natural phenomena; but although it gives a great deal of information, it does not indicate with sufficient clearness the definite areas of the various diseases referred to. Since its publication, too, our knowledge both as to the distribution of tropical diseases and the cause of disease has made considerable progress. More recently Mr Alfred Haviland, M.R.C.S., published three very valuable and instructive maps dealing with the geographical distribution in England and Wales of cancer in females, of phthisis in females, and of heart disease. These maps show very strikingly the influence exerted on these diseases by locality.

Two maps, illustrating the distribution of diphtheria and scarlet fever in England and Wales, were published by Dr E. G. Barnes in the *British Medical Journal*, July 28, 1888. In the same *Journal* for January 19, 1889, there is a report of the collective Investigation Committee of the British Medical Association, prepared by Dr

Isambard Owen, on the geographical distribution of ricketts, acute and sub-acute rheumatism, chorea, cancer, and urinary calculus in the British Islands; and various maps, illustrating the geographical distribution of ricketts, chorea, and cancer, have been prepared but not published.

In 1886 the report on the mortality and vital statistics of the United States, as returned at the tenth census (June 1, 1880), was published; and it contains some most instructive maps and charts showing the distribution of deaths from various diseases in the United States, which indicate to a certain extent the geographical distribution of those diseases, as also the localities in which they are most prevalent. None, however, of the publications to which I have referred, nor any with which I am acquainted, attempt to depict graphically the geographical distribution of tropical diseases in a manner which would give at a glance the areas throughout the world which are affected by them, and their possible or probable connection with physical phenomena. It must be remembered, too, that in the works above mentioned the various authors were dealing with civilised districts, where authentic statistics were obtainable, and where it was possible to localise the distribution of disease in a minute form, which, however desirable, it is at present completely out of my power to attempt to imitate.

I propose to treat my subject on a definite plan. I intend first to give the name of the disease and its various synonyms; secondly, a short definition of the disease, and a very brief description of it. I will then sketch out with some attempt at detail the geographical distribution of the disease, and finally point out as far as possible its relation to various physical phenomena as affecting both its causation, its area, and its epidemic spread. I may state at the outset, that I have taken the definition and description of each disease from the Dictionary of Medicine edited by Richard Quain, M.D., F.R.S., because the definitions and descriptions there given are the generally accepted ones, and in a paper of this character, which deals more or less with broad outlines and general facts, it would be out of place to enter into any personal views I may have as to either the cause, the definition, or the description of the diseases referred to.

With regard to the data concerning the geographical distribution of the diseases, I am indebted for my principal facts to the *Handbook of Geographical and Historical Pathology*, by Dr August Hirsch, and to the copious bibliography which accompanies his various chapters. I have not, however, hesitated in any case where my own information, or information gathered from other sources, modifies or supplements his data, to make use of the same, and I have as far as possible verified the facts I have given.

The maps which illustrate this paper have been specially prepared for it, and in order to ensure as great accuracy as is possible on such small maps, the areas of the distribution of the various diseases have been drawn upon large maps used for lecturing purposes, and subsequently reduced by photography to the scale suited for publication in the *Proceedings* of this Society.

The maps illustrating some of the physical phenomena with which my paper deals are adapted from various sources. The chart of the world depicting the mean annual temperature of the tropical and sub-tropical zones is taken from the *Challenger Reports*, and is based upon the most recent investigations on the subject. The chart representing the mean annual rainfall throughout the world is after the most recent map published, namely, that in the *Contributions to Meteorology*, by Professor Elias Loomis (1889).

Plate No. 16 gives the isoclinal lines with reference to pandemic waves of disease, and the prevailing winds upon the ocean. It has been compiled from two maps; one published by Dr Robert Lawson in 1888, and the other by Dr W. S. Wilson in 1881.

I trust that this paper may be of special interest to members of my own profession, who will be able to see at a glance the diseases infesting the various districts in the tropics, and who will therefore be the more readily able to give the necessary advice to patients. It should be useful to intending emigrants, and of special service to insurance companies, as indicating the areas of comparative safety or risk for the residence of their clients.

The diseases treated of are—1. Malaria; 2. Dengue; 3. Asiatic Cholera; 4. Yellow Fever; 5. Oriental Boil; 6. Endemic Hæmaturia; 7. Beri-beri; 8. Oriental Plague; 9. Dysentery; 10. Leprosy (Elephantiasis Grecorum); 11. Yaws; 12. Fungus Disease of India; 13. Elephantiasis Arabum (Barbadoes Leg); 14. Guinea-Worm;


160
140
120
100
80
60
40
20
0
80
60
40
20
0
20
40
60
ARCTIC OCEAN
MELVILLE I.
MELVILLE Sd.
GREENLAND
BAFFIN BAY
SPITZBERGEN
GREEN LAND
SEA
WRANGEL LAND
BOOTHIA
ALASKA
ARCTIC CIRCLE
ICELAND
HUDSON STR.
HUDSON BAY
BRITISH ISLES
ALEUTIAN Is
VANCOUVER
DOMINION OF CANADA
NEWFOUNDLAND
NOVA SCOTIA
NORTH
NORTH AMERICA
NORTH
SAN FRANCISCO
UNITED STATES
PACIFIC
ATLANTIC
OCEAN
MOROCCO
TROPIC OF CANCER
G. of MEXICO
MEXICO
CUBA
HAYTI
WEST INDIES
SAHARA OR
SANDWICH Is
OCEAN
CENTRAL AMERICA
CARIBBEAN SEA
C. VERD
EQUATOR
C. LOPEZ
POLYNESIA
PERU
BRAZIL
SOUTH AMERICA
SOUTH
TROPIC OF CAPRICORN
ARGENTINE REPUBLIC
CHILI
ATLANTIC
OCEAN
C. of GOOD
CHART OF T
ON MERCATORS
PATAGONIA
CAPE HORN
MALARIA.
Intermittent Variety most widely distributed.
Remittent and Pernicious Varieties in Tropics and Sub-Tropics.
ANTARCTIC CIRCLE
ANTARCTIC OC



ART OF THE **WORLD**

MERCATORS PROJECTION

15. Filaria Sanguinis Hominis; 16. Scurvy; 17. Tropical Abscess of the Liver.

In a future paper I hope to continue the subject with a more extended range of diseases, which, although to a certain extent met with in the tropics, are also to be found in the temperate zones.

I. Malarial Diseases.

(See Plate I.)

It has been hitherto the custom to subdivide the diseases due to malaria, or, in other words, to the malarial process, but I am so convinced that the same cause produces the various types or manifestations of malaria that I include them all under one heading.

Malaria (*Ital.*)—*Synon.*—Marsh Miasm.; *Fr.* Mauvais air; Intoxication des Marais; Intoxication telurique; *Ger.* Malaria.

Definition.—An earth-born poison, generated in soil, the energies of which are not expended in the growth and sustenance of healthy vegetation. By almost universal consent, this poison is regarded as the cause of all the types of intermittent and remittent fevers commonly called malarial, and of the degeneration of the blood and tissues from long residence in places where this poison is generated.

Malaria therefore includes—

A. *Intermittent Fever—Synon.*—Ague; *Fr.* Fièvre Intermittente; *Ger.* Kaltes Fieber.

Definition.—A fever of malarial origin, characterised by a sudden rise of temperature during the paroxysm, by an equally sudden fall at its termination, and by the regularity of the times of accession and apyrexia.

B. *Remittent Fever—Synon.*—Bilious Remittent; Marsh Remittent, The Jungle Fever of the East Indies; The African, Bengal, Mediterranean, Persian, or Walchern Fever; *Fr.* Fièvre Remittente; *Ger.* Bösartiges endemisches Fieber.

Definition.—A paroxysmal fever of malarial origin, in which the paroxysms do not intermit, but only, as the name implies, remit.

C. *Pernicious Malarial Fever* including—(*a*) Febris Algida; (*b*) Febris Comatosa.

Geographical Distribution.—The distribution of malaria is very extensive, although the intensity of the malarial process varies in different regions. Commencing with Africa, we find that on the west coast malaria is very prevalent in the basins of the Senegal and Gambia, and on the Guinea coast from Sierra Leone to Cape Lopez, especially in the basins of the Niger and Gaboon, on the Ivory and Gold coasts, at Fernando Po, and St Thomas. It diminishes rapidly in intensity until lat. 18° S., where it disappears. On the East coast, the malarial region commences in the south at Delagoa Bay, and extends northwards as far as 5° N. lat., including the islands of Zanzibar, Madagascar, Mauritius, Bourbon, and Seychelles. Areas of endemic malaria are also found in the lowlands of Abyssinia and in the Somali district, in the valley of the Nile, from Khartum to the Great Lakes, west of the Nile between Dongola and Khartum, and all over Tropical Central Africa, up to an altitude of 3000 feet. Egypt, Tripoli, Tunis, and Algiers are likewise affected. The malarial process is most intense upon the equatorial African coast-line until an altitude of 500 feet is reached, and it also extends in its gravest form for about 300 miles up the banks of the Zambesi, the Congo, and the Niger.

Malaria is met with in Arabia, on the east coast of the Red Sea, especially on the coast of Hedjaz, and in Yemen from Jisan south to Moccha. It is also found at Muscat, on the shores of the Persian Gulf and its islands, and in the valleys of the Euphrates and Tigris. It also exists in Syria, especially in the damp valleys of the Lebanon, in the valley of the Jordan and along the shores of the Levant; and this malarious area extends to Asia Minor, from Adana and Tarsus to the Dardanelles, including Smyrna. The disease extends all round the Caspian Sea, overspreads Persia, Beloochistan, and Afghanistan; and is met with all over India, with the exception of places having a high altitude. Ceylon too and Burmah, Siam, Sumatra, Borneo, New Guinea, the Phillipine Islands, Japan, the Andaman Islands and Australia as far south as lat. 17°, are all affected. The disease also prevails in a severe form in the tropical and sub-tropical parts of China.

In Europe we may commence by tracing malaria in the steppe lands of the Caspian. It follows the course of the Volga through Astrakan, includes the central Caucasian plain, and the countries

bordering the Black Sea to the north, the basins of the Dnieper and Dniester as far as Ekaterinoslav, the Crimea, Wallachia, and Bulgaria. There are also endemic areas of malaria in the marshy plains of western Russia, in many places in the Baltic provinces, and in the district of Novgorod. In Galicia malaria is only endemic in Cracow, Wadowice, Zolkiewo, and Zloczow; and in Poland only in the province of Agustusowo. One of the largest malarial areas of Europe follows the course of the Danube and its tributaries, from the plain of lower Austria, over a great part of Hungary. It is also endemic in the marshy districts of Croatia, in the damp valleys of Servia and Montenegro, and in the valley of the Save. In the Balkan peninsula there are various endemic areas in Roumania, on the shores of the Black Sea and of the Sea of Marmora, in Albania, and northwards along the coasts of Dalmatia and Istria. In Greece malaria occurs at many points in Boetia, Zeituni, Naupantos, and Vonitza; at Chalcis, Corinth, Mistra, Navariño, Modon, and many other places on the coast. In Crete, Cephalonia, St Maura, and Corfu malaria is also endemic. In Italy there are two areas of endemic malaria, occupying the plain of the Po and its tributaries, and the west coast from Pisa, as far as and including most of Calabria. In the Iberian peninsula, malaria is most severe in the southern and western coast regions, in the low-lying country of Andalusia, on the marshy banks of rivers, especially the Guadiana and Guadalquiver, on the flooded plains of the Tagus, Sado, Mondego, and other coast rivers of Portugal, on the level coasts of Granada and Murcia, and on the plains of Algara and Alemtejo. In France malaria is most prevalent in the western and southern parts of the country, especially from the basin of the Loire as far as the Pyrenees. Another area stretches along the coast of Languedoc and Provence. There are several other small malarial areas, of which the plain of Auvergne and the marshy country around Lake Indre may be noted. In Switzerland endemic malaria is only found in the southern part of the canton Ticino, and in the canton of Valais along the Rhone. In Austria the chief seats of the disease are along the Danube, and there are smaller areas in the river valleys of upper Austria, Salzburg, Styria, and Carinthia. Where the river widens at Krems we meet with a great region of malaria, which extends as far as the Black Sea. In south-west Germany

there are small and isolated areas where malaria is endemic on the marshy banks of the rivers or lakes, *e.g.*, the valleys near the Neckar in the Black Forest. The more extensive malarial regions are on the banks of the Rhine, in lower Alsace, in the Palatinate, in the Rheingau, and in the low grounds of the Danube, and its side valleys in Wurtemburg and Bavaria. In central Germany it is only endemic in a few small districts. In North Germany it is found in the basins of the Vistula, Oder, Elbe, Weser, and Rhine. Holstein and Schleswig (west coast districts), the coast belt west of the Elbe, the moorlands of Hanover and Oldenburg, the low grounds of Westphalia, and the plains of Rhenish Prussia are also infected. In the Netherlands malaria is mostly found in the provinces of Grönland, Friesland, and Zealand, on the coast belt of the provinces of north and south Holland, and in the provinces of Drenthe Overyssel. Belgium, West Flanders, East Flanders, and Antwerp are affected. The disease also occurs in Laaland and Falster, on the Hvalöen islands, and in the neighbourhood of Fredericstadt. In Sweden it is endemic at three principal points—around Lakes Maler and Werner, on the east coast of Torhamn, and at the mouth of several coast streams, such as the Angermanna-elf, the Dal-elf, and Gotta-elf. In Britain, the East Riding of Yorkshire, the Fen district, Essex and Kent, the banks of the Thames in Surrey, and the south marsh of Somersetshire, are slightly infected by malaria.

In the western hemisphere, endemic malarial fever of the severest type has its principal seats in the West Indies, on the Mexican Gulf coast, and in Brazil, but considerable regions of fever, though of a less intense kind, are met with in the northern parts of the Pacific coast of South America, and in the southern, central, and prairie States of the Union. All the West Indian Islands are affected, save Antigua, St Vincent, and Barbadoes, the Bahamas and the Bermuda group, in which islands it is rarely seen.

In South America the worst centre of malaria is the east coast, including the ports of Carthagena, Maracaybo, and Puerto Cabello, and the country of Guiana. Another extensive area covers the whole of the north of Brazil as far as Rio de Janeiro, the banks of the Amazon, Rio Madeira, Maranhao, Paranahyba, San Francisco, Parana, Rio Doce, and their tributaries; also the island of Santa

Caterina and the marshy districts of the provinces of Piauhy, Para, Mato-Grosso, Goyas, and Minos Geraes. Endemic malaria is also found in the prairie lands of Paraguay and Bolivia, particularly in the provinces of Tucumana, Salta, and Santa Cruz. Chili and Peru are also affected, and the disease extends thence along the coast to Ecuador, probably also to New Granada.

The disease is found all over Central America, but it is less severe on the Pacific coast, except at Corinto. In Mexico malaria is most prevalent on the Atlantic coast, and is less frequently seen everywhere up to 3000 feet in height, except on the tableland proper (Anahuac). On the Pacific coast of Mexico malaria is confined to Acapulco, Tepic, and the strip of coast from San Blas to Mazatlan.

In the United States from the Rio del Norte a great malarial field extends all over the Gulf coast to Cape Florida, spreading far into the interior along the Colorado, Brazos, and Mississippi and their tributaries. In Texas the malarial region stretches from the coast into the highlands as far as Fort Duncan in Eagle Pass, and Fort Makavit. In New Mexico malaria is very widely diffused, the limit of its prevalence being Santa Fé. From the western part of the Louisiana coast, between the Sabine and the Mississippi, the malarial region extends across the zone of bluffs in that State, over a great part of Arkansas, especially along the banks of the Mississippi and Arkansas rivers, over the marshy plains of the north-east of the country towards Missouri, and over the eastern part of the Indian territory, including Fort Gibson and Fort Still. In the peninsula of Florida malaria is only met with on the Gulf coast, especially in Escambia and Tambabay, and at Fort Meade. In Georgia we find it in the creeks along the coast. In the Central States of the Union malaria is met with in South Carolina, North Carolina, Virginia, and Maryland, and to a moderate extent in Tennessee and Kentucky. It is also found within the prairie States proper, *e.g.*, in Ohio, Indiana, Illinois, Missouri, Iowa, Minnesota, Wisconsin, and Michigan. In no case, however, does malaria extend farther north than 46° 10′ N. (Fort Ripley). In the southern parts of the State of Michigan malaria skirts both shores of Lake St Clair to its junction with Lake Huron, and the southern shores of Lakes Erie and Ontario as far as the St

Lawrence. Severe malarial fever is found at Fort Gratiot, Detroit and Plymouth, on the United States side of the Lake; and at Amhurstbury, Fort Maldon, and Sandwich, on the Canadian side. On the northern side of Lake Ontario malaria extends from Hamilton to Kingston, and up the ridge of hills which runs along the shore from Burlington to the mouth of the Trent, attaining in some places an altitude of 600 feet. Endemic malaria extends also to the north-western parts of the State of New York, although there are many localities now free from fever. It is most frequently met with along the banks of the Hudson and on a narrow strip of coast. During recent years the disease has increased in the mountain districts of New York and also in Pennsylvania. In the New England States the disease is endemic at only a few points, and it is not endemic in the greater part of British North America. Malaria, as an epidemic, is met with on the banks of the St Lawrence and its tributaries, and on Lake St Peter, as well as at Montreal and Quebec, and at various coast places, such as Halifax (N.S.) and Miquelon (N.F.). In Nova Scotia (except at Halifax), in New Brunswick, and in Greenland, the disease is quite unknown. In western North America the limit of malaria reaches somewhat higher latitudes. It is met with chiefly on the slopes and valleys of the Rocky Mountains and in the territories of Wyoming, Utah, and Colorado. In California there are considerable malarial regions, especially up the valleys of the Sacramento and San Joaquin, and in the inland southern part of the State of Arizona.

The incidence of malaria throughout the world has been very well summarised by Mr W. North, whose classification I will quote.*

First Category. Highest Degree of Intensity.

Class I. Senegal; Coasts of Gulf of Guinea; West Coast of Africa, as far as the 20th parallel of S. latitude; Madagascar; the Guianas.

Class II. India; Cochin-China; Ceylon; Afghanistan; Burmah; Siam; the whole of the Malay and Philippine Archipelago; New Guinea; Nubia; parts of Abyssinia and the Soudan, and Central America.

* See *Nineteenth Century*, June 1889, p. 867.

Class III. The East Coast of Africa; Egypt; the coast-line of Arabia; Mexico; China Proper; the Brazils and Peru.

Second Category.

Class I. Tripoli; Algeria; Morocco; the Cape de Verde Islands; and the Oases of the Sahara.

Class II. Turkey, in Europe; Greece; the Islands of the Archipelago; Sardinia; Malta; Sicily, and parts of Italy.

Class III. Roumania; Hungary; Italy; Corsica; Spain; Portugal; Southern Russia, and a large part of the United States.

Third Category.

Southern Sweden; Denmark; Belgium and Holland; Germany; France; La Plata; Chili, and the Islands of Madeira; Bourbon and at Helena.

Fourth Category. No Malaria or Insignificant.

The British Islands; Norway; the southern parts of Sweden; Finland and Russia; all North America above the 50th parallel of N. latitude; Uruguay; the Argentine Republic and Patagonia; Northern China; almost all Siberia and the greater part of Japan; New Zealand and the southern part of Australia.

Remarks.—Of the malarial fevers, the intermittent is the most widely distributed type, and it will be noticed that the remittent and pernicious fevers are only met with in comparatively small areas, and that they are, as a rule, confined to tropical or sub-tropical countries. A glance at the map will show, without further specification, their distribution. It may be noted, however, that the quotidian and quartan types of intermittent fever are those which are most frequently met with in the tropics, and that the Tertian type is that form which is most widely distributed in the more temperate zones. In fact, the type of fever stands in a definite relation to the intensity of the malarial process; thus we find that the Tertian type prevails in those regions within the tropics where the milder malarial fevers are indigenous. Again, the frequency of the occurrence of the quotidian type of fever in endemics or in epidemics is in direct proportion to the severity of the disease. When an epidemic

wave of malarial fever passes over a district, the Tertian type is seen at its outbreak, whereas at the height of an epidemic, or whenever it assumes a severe character, the quotidian type obtains, and as the outbreak of sickness abates one meets with a return to the types of fever having a longer interval between the paroxsyms, so that in tropical and sub-tropical countries the fever takes on the Tertian type, and in the higher latitudes the quartan type makes its appearance. These remarks apply also to outbursts of the disease where it is endemic. In temperate zones remittent malarial fevers are exceedingly uncommon, in fact, so uncommon as to be regarded as a departure from the ordinary type and as due to exceptional causes.

All races may suffer from malaria, although the Negroes are less prone to it, always provided that they do not migrate. Indeed, it is very generally acknowledged that in all parts of the world strangers suffer more severely from malaria than does the indigenous population. The incidence of malaria is to a certain extent governed by the seasons. In those places where it is endemic it occurs all the year round; but where it is only slightly developed there are two maxima—one in spring and one in autumn, and a considerable decrease of the disease in the months between them. In regions with strongly developed malaria, there is a maximum beginning in summer, which reaches its height at the end of summer or the beginning of autumn, lasting not rarely into winter, and which so far exceeds the spring maximum, that the latter not unfrequently disappears altogether, so that there is only one minimum, winter and spring, and one maximum, summer and autumn. In tropical countries, in the worst malarious districts, the disease is most rife during the rains. The relation which malaria bears to heat is as follows—the greater the mean summer temperature, other things being of course taken into account, the more malaria, and the amount of malaria decreases with the mean annual temperature of the place, ceasing altogether with the summer isobar of 60° F. But, as Hirsch says, "in higher latitudes, the malarial fevers which have prevailed endemically or epidemically in spring undergo for the most part a considerable remission on the setting in of summer heat, and they do not revive until the cooler weather of autumn," and again, "in the regions of severe malaria the disease shows itself, and attains wide diffusion,

not at the height of summer, but only when the high temperature is declining in late summer and in autumn, and, for the tropics in particular, at the end of the hot season. As many observers state, this is directly due to the great diurnal range of temperature that occurs at that season."

The influence of rain or moisture has undoubtedly much to do with both the production and spread of malaria. With regard to the rains, we may say that the malarial poison is most virulent either when they set in after a long period of heat, or when the rains cease and give place to warm, dry weather. An endemic outbreak of malaria, and its epidemic spread are both notably diminished at the height of the rains, if they are very abundant, and it has been proved over and over again that the malarial process is developed more abundantly in wet than in dry years. These remarks hold good both as to tropical and temperate climates. But it is not with rainfall alone that we have to do, for moisture must be present in the soil in order for the production of malaria, and this saturation of the ground may be produced in various ways apart from atmospheric precipitations. Drainage from rivers, lakes, and pools, may constantly saturate the soil, and so may inundations either periodic or irregular. The irrigation of the soil exerts an undoubted influence on the production of the poison; its effect is very marked in Egypt, and in the irrigation districts of India. Lastly, the soil may be saturated by sub-soil water. This point is of importance, because it explains the occurrence of malaria in localities remote from river basins, and where the soil cannot become saturated in other ways. The occurrence of malaria in the Sahara, in Spain, Greece, Algiers, Tripolis, and Darfour, is in all probability due to this sub-soil water, arising either from springs or in other ways and resting upon impermeable strata of rock or marl.

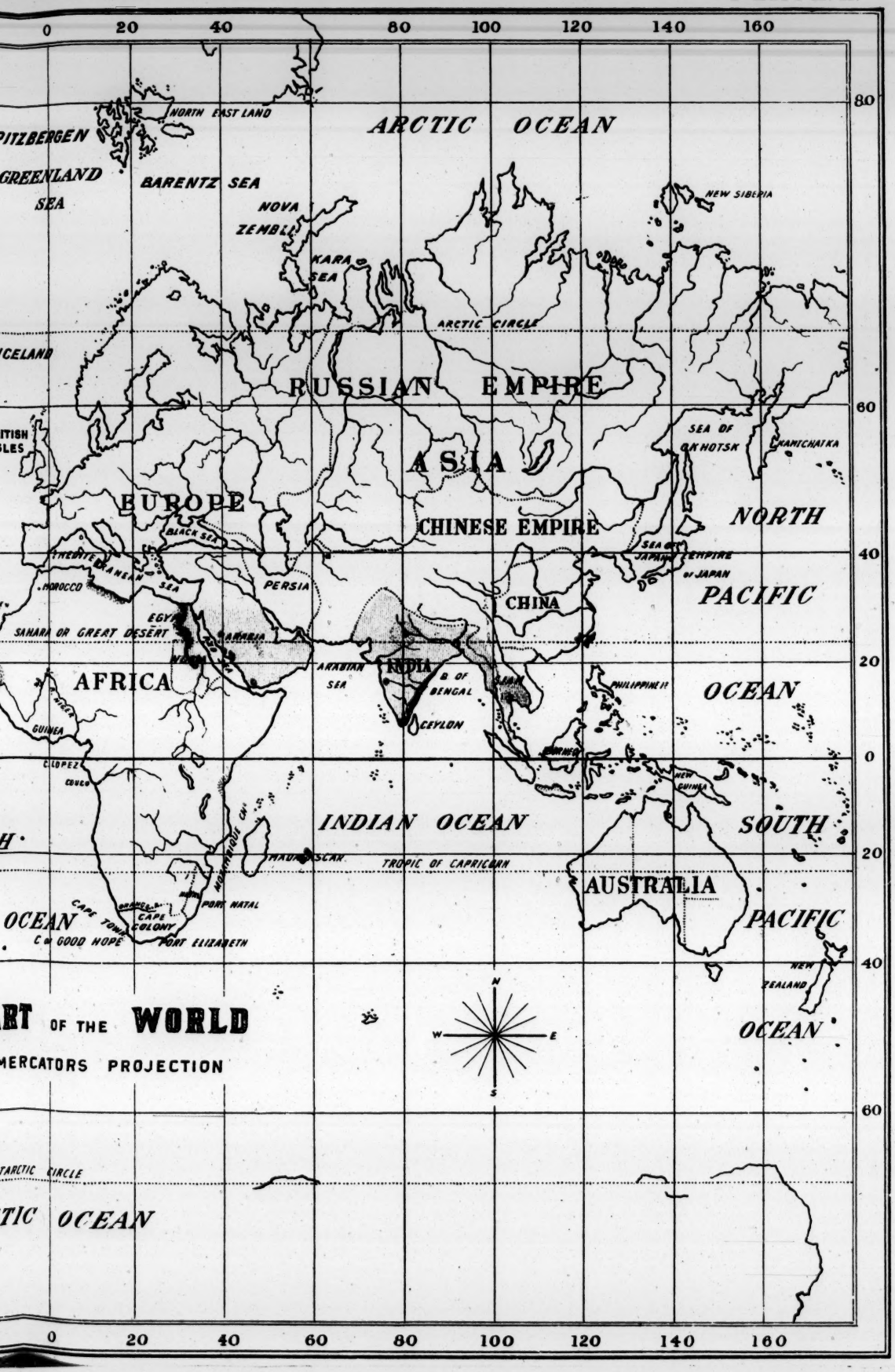
Apart from the moisture in the soil, it must possess other physical characteristics, although the geological characters of the country would appear to exert little or no influence on the production or non-production of the disease, except in so far as they affect the physical nature of the soil. Clay, loam, clayey marl, and marshy soil are most favourable to the production of malaria; a porous chalky soil is less favourable and sandy soil least so, provided always that the chalk or sand does not rest either upon clay or firm

rock. The exemption from malaria of some of the islands in the West Indies with a chalky soil is remarkable when contrasted with its special prevalence in islands of volcanic formation. The amount of organic material too, contained by the soil is bound up with the production of malaria. All other circumstances being equal, the greatest amount of malaria will be found where the amount of organic matter in the soil is greatest, the prevalence of the disease diminishing as the organic matter is found in less abundance. Changes in the soil produced by cultivation, the neglect of cultivation, and by excavations, also affect the presence or absence of malaria, but space forbids us entering into these details. In reference to marshes, however, it may be noted in passing that malaria will disappear from a marshy district if it is completely drained and dried, or if a marsh is converted into a pool or lake.

The amount of malaria, as also its severity, is affected by altitude and the configuration of the ground. The altitude at which malaria can be produced varies in different regions, being higher in the tropics than in the temperate zones. Thus in Central Africa we find that a height of some 3000 feet must be attained before one reaches a district free from malaria, whereas in the Apennines a height of 1500 feet only is required, and farther north only 500 feet. This, however, must be explained more by variation in temperature than by mere altitude. On the other hand, however, examining the configuration of the ground in plains, it is found that the disease is distinctly more virulent the lower the level of the country. This fact is so marked that often even 50 or 100 feet less makes a considerable difference to the salubrity or otherwise of a given spot.

Although winds do not exert any direct influence upon the production of the malarial poison, they act indirectly, as, for instance, by moderating temperature, &c. They act, however, directly in the diffusion of the poison or in preventing it exercising its potent effects. Wind may carry the malarious poison from a marsh to a healthy district, but it is probable that it can only thus convey it for a distance of some two or three miles. Malaria may rise to a height of some 700 or 800 feet in a calm atmosphere; wind will prevent this vertical diffusion. Probably on some islands, where from analogy we should expect to find malaria present, the constant


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ARCTIC OCEAN
MELVILLE I.
MELVILLE SD.
BOOTHIA
BAFFIN BAY
GREENLAND
SPITZBERGEN
GREENLAND SEA
WRANGEL LAND
ALASKA
ARCTIC CIRCLE
ICELAND
HUDSON STR.
HUDSON BAY
BRITISH ISLES
ALEUTIAN Is
VANCOUVER
DOMINION OF CANADA
NEWFOUNDLAND
NOVA SCOTIA
NORTH
NORTH AMERICA
NORTH
SAN FRANCISCO
UNITED STATES
PACIFIC
ATLANTIC
OCEAN
TROPIC OF CANCER
G. of MEXICO
MEXICO
CUBA
HAYTI
WEST INDIES
SANDWICH Is
OCEAN
CENTRAL AMERICA
CARIBBEAN SEA
C. VERD
GUINEA
EQUATOR
POLYNESIA
PERU
BRAZIL
SOUTH AMERICA
SOUTH
TROPIC OF CAPRICORN
ARGENTINE REPUBLIC
CHILI
ATLANTIC
OCEAN
PATAGONIA
CAPE HORN
CHART
ON MERCATORS
DENGUE.—First known 1779.
Area of diffusion limited by { 32° 47′ N. lat. / 23° 33′ S. lat.
Most prevalent in summer and early autumn.
Incubation five to six days.
ANTARCTIC CIRCLE
ANTARCTIC


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NORTH EAST LAND
ARCTIC OCEAN
SPITZBERGEN
GREENLAND SEA
BARENTZ SEA
NOVA ZEMBLA
KARA SEA
NEW SIBERIA
ARCTIC CIRCLE
ICELAND
RUSSIAN EMPIRE
SEA OF OKHOTSK
KAMTCHATKA
ASIA
EUROPE
CHINESE EMPIRE
NORTH PACIFIC OCEAN
BLACK SEA
MEDITERRANEAN SEA
MOROCCO
PERSIA
SEA OF JAPAN
EMPIRE of JAPAN
CHINA
SAHARA OR GREAT DESERT
EGYPT
ARABIA
AFRICA
ARABIAN SEA
INDIA
B. OF BENGAL
CEYLON
PHILIPPINE I.
GUINEA
CLOPEZ
CONGO
BORNEO
NEW GUINEA
INDIAN OCEAN
SOUTH PACIFIC OCEAN
MADAGASCAR
TROPIC OF CAPRICORN
AUSTRALIA
CAPE TOWN
CAPE COLONY
PORT NATAL
C. of GOOD HOPE
PORT ELIZABETH
NEW ZEALAND
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MERCATORS PROJECTION
ANTARCTIC CIRCLE
ANTIC OCEAN

winds rapidly changing the atmosphere carry away the morbific elements before they have time to do harm.

Water can convey the malarial poison, but it is unknown at present how far it can carry it or how long the poison can remain unimpaired when carried either by a stream or a current.

From what has been said, some idea may be obtained of those factors requisite for the production of the malarial poison. It is in all probability due to a micro-organism which may find entrance into the body by means of the air, by drinking water and possibly also by the consumption of food contaminated by it. It is certainly ponderable, as is proved by the effect of altitude, of barometrical pressure, and by the action which winds have in its dissemination. It is also miscible with water. We may sum up our definite knowledge of the disease by saying that it requires for its production a specific germ, suitable soil, a certain amount of moisture, a sufficiently high temperature, and a certain time for development.

An examination of the map, bearing what has been said in view, will I think show how the presence of malaria in the various quarters in which it exists is to be accounted for by physical phenomena.

II. Dengue.

(See Plate II.)

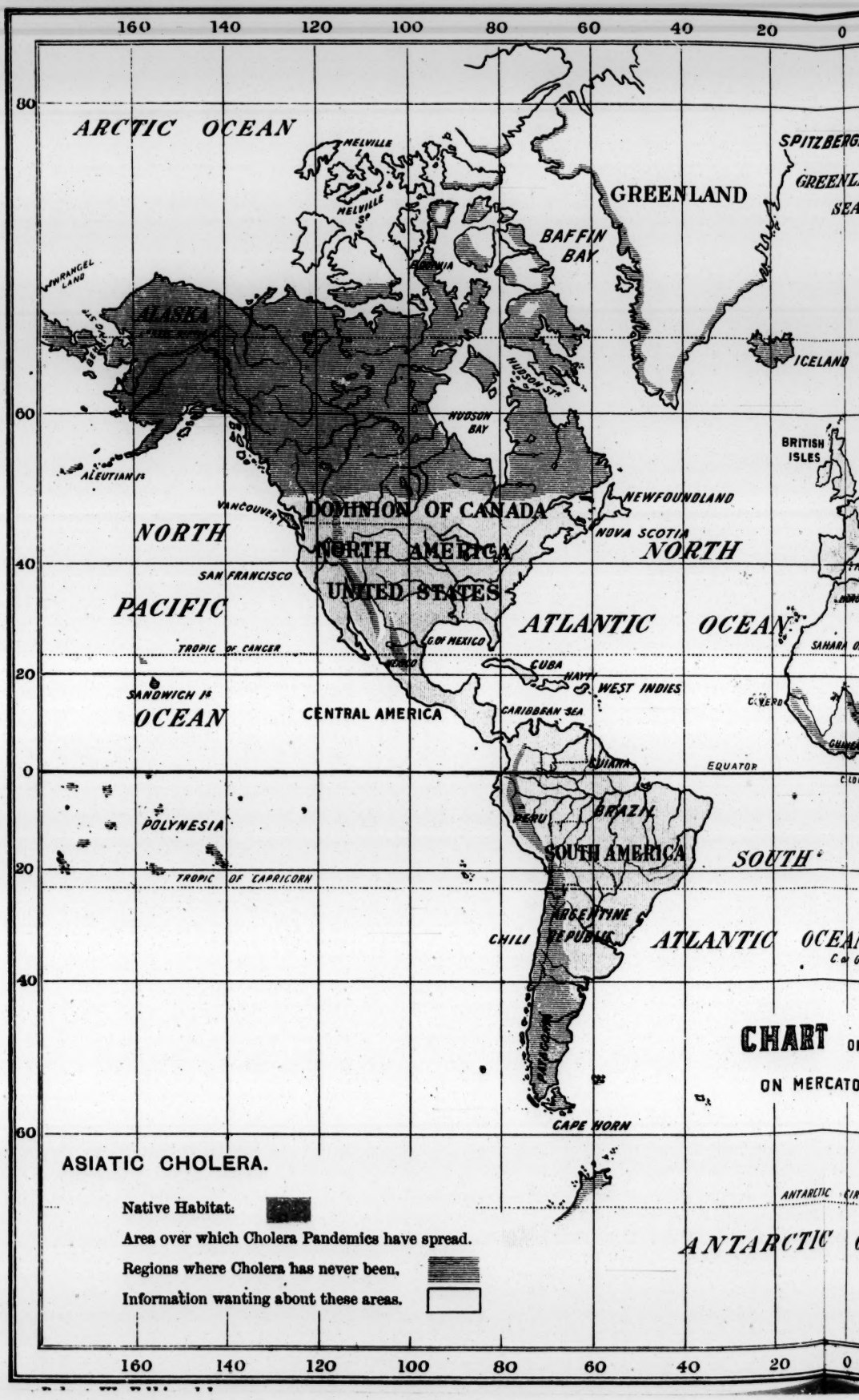
Synon.—Dandy Fever (West Indies); Third Day Fever; Red Fever; Leg Fever; Breakbone Fever; Scarlatina Rheumatica (Aitken); Aburukah, or Aburuka-bar, or Father of the Knee (Arabia); Nadak-Mariata, or the Deity (Southern India); Tootiah (Bengal); Kidniga pepo, *i.e.*, Spasmodic pains (Zanzibar); *Fr.* and *Ger.* Dengue.

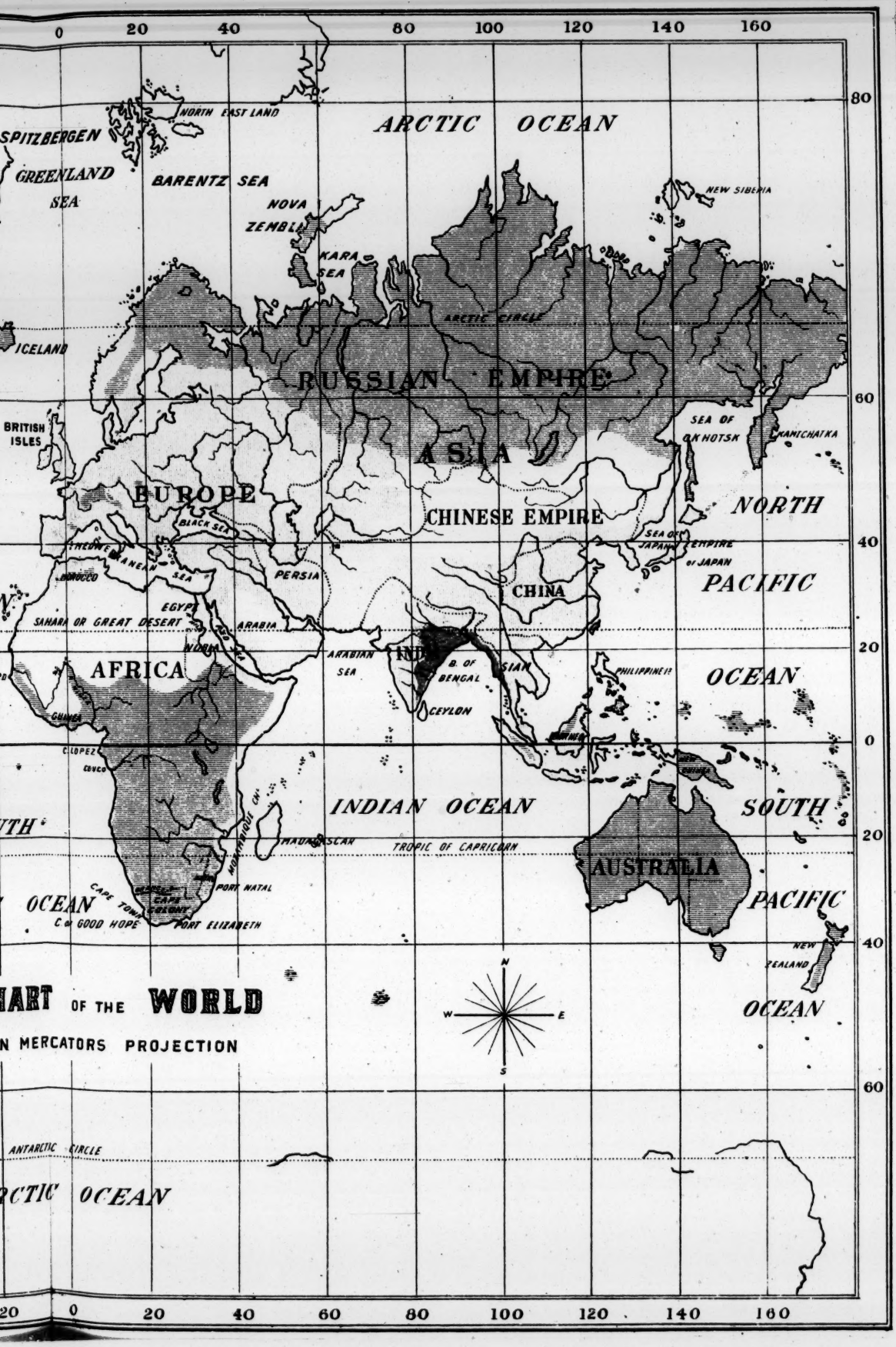
Definition.—An infectious, eruptive fever, commencing suddenly, and characterised by severe pain in the head and eye-balls, swelling and pain in the muscles and joints, prone to shift suddenly from joint to joint, catarrhal symptoms, sore throat, congested conjunctivæ, and affection of the sub-maxillary glands. The disease may remit, and is liable to relapse.

Geographical Distribution.—Dengue has been known since 1780, in which year it attained a considerable diffusion in the tropical and sub-tropical parts of both eastern and western hemispheres. It

is now known to have visited Egypt, Senegambia, and Tripoli, and the valley of the Nile as far as Khartum; also Arabia along the coasts, and Mecca. It is epidemic all over India and Further India, and in Batavia; and has appeared in Shanghai, Amoy, and the island of Formosa. It has also visited Réunion, Tahiti, Zanzibar, and the Canary Islands; and epidemics have overspread the West Indies and a great part of the Southern States of the Union, as well as the northern shores of South America. Space will not permit of indicating the various areas of diffusion during the several great epidemics of Dengue, but it must suffice to state that its greatest area of diffusion lies between 33° N. lat. and 23° 30′ S. lat.

Remarks.—The period of incubation of Dengue is probably about six days. It is a highly infectious disorder, spreading with extreme rapidity. Summer and early autumn are undoubtedly the Dengue season, and the disease appears to depend on a high temperature for its production. In the tropics as well as in the more tropical zones, nearly all the epidemics of Dengue have been in the hot weather, and as soon as a great fall of temperature takes place the disease declines rapidly. It is probable that the moisture of the atmosphere has little or nothing to do with the production of Dengue; as a rule it is chiefly confined to coast districts, to the courses of great rivers, and to places having a low altitude. It has been noticed in various epidemics of Dengue that it spreads in a curious way amongst various classes of the community. Every race, nationality, age, and sex may be attacked by the disease, although in separate epidemics a remarkable immunity has been noticed on the part of certain classes. Sometimes Europeans will be attacked, and natives enjoy comparative freedom from the disease; again, in other cases, natives will be almost solely attacked; sometimes children suffer more than adults, or the reverse may obtain. For instance, Pasque says, speaking of the epidemic at Benghazi, that it was noticeable for the decided immunity experienced by the blacks, but they are attacked as much as anyone else in Egypt and Senegal. Christie remarks that in one or two of the epidemics which he witnessed in Zanzibar, the natives suffered less than the Europeans. In the epidemic in Mauritius in 1873, hardly any children were attacked by the disease. In the Deccan epidemic of 1872 there was noticed a peculiar predisposition to


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ARCTIC OCEAN
MELVILLE I.
MELVILLE Str.
GREENLAND
BAFFIN BAY
SPITZBERG
GREENL
SEA
WRANGEL LAND
ALASKA
BEHRING Str.
ICELAND
HUDSON Str.
HUDSON BAY
BRITISH ISLES
ALEUTIAN Is.
VANCOUVER
DOMINION OF CANADA
NEWFOUNDLAND
NOVA SCOTIA
NORTH AMERICA
NORTH
SAN FRANCISCO
UNITED STATES
PACIFIC
ATLANTIC
OCEAN
TROPIC OF CANCER
GOLF MEXICO
CUBA
HAYTI
WEST INDIES
SANDWICH Is
OCEAN
CENTRAL AMERICA
CARIBBEAN SEA
GUIANA
EQUATOR
POLYNESIA
PERU
BRAZIL
SOUTH AMERICA
SOUTH
TROPIC OF CAPRICORN
ARGENTINE REPUBLIC
CHILI
ATLANTIC
CHART
ON MERCATO
CAPE HORN
ASIATIC CHOLERA.
Native Habitat.
Area over which Cholera Pandemics have spread.
Regions where Cholera has never been.
Information wanting about these areas.
ANTARCTIC
ANTARCTIC



HART OF THE WORLD

ON MERCATORS PROJECTION

Dengue in persons suffering from any surgical complaint. In Goojerat in 1824, and in Amoy in 1872, it was found that all the severe cases were limited to the native population, that the Europeans suffered to a far less extent, and that those who were attacked by the disease had it in a remarkably mild form.

As to the specific nature of Dengue, there can be no doubt, but whether due to a parasite or not is at present unknown, nor is it yet ascertained whether the poison springs up *de novo* at all points where its potency is manifested, or whether it is only epidemic at a few places, and spreads from these under favouring circumstances. At first Dengue was not thought to be contagious, but it is both infectious and highly contagious, and the diffusion of the disease could be traced in the epidemic of 1871–73, in the East from port to port, and from country to country along the highways of land and water traffic. One attack of Dengue does not confer absolute immunity from subsequent attacks, although it does so to a certain extent.

III. Asiatic Cholera.

(See Plate III.)

Synon.—Serous Cholera; Spasmodic Cholera; Malignant Cholera; *Fr.* Cholèra asiatique; *Ger.* Asiatische Cholera.

Definition.—Asiatic cholera is a specific disease, characterised by violent vomiting, rice-water evacuations, cramps, prostration, collapse, and other striking symptoms; tending to run a rapidly fatal course; and capable of being communicated to persons otherwise in sound health, through the dejecta of patients suffering from the disease.

Geographical Distribution.—With regard to the distribution of cholera, it is not advisable to proceed on exactly the same lines as with other diseases, for, although the epidemic area of cholera is very vast, its endemic area is very limited. It will be well therefore, in the first place, to specify those regions which have hitherto escaped the ravages of the disease, next to define its endemic area, and finally to make some general remarks as to its pandemic diffusion. A glance at the map will render this division clear.

Up to the present time, Australia and the islands of the Pacific

Ocean have remained unaffected by the disease. So too has the whole of Africa, with the exception of a strip of coast-land commencing at Delagoa Bay and running all round the eastern and northern coasts of the Continent as far as lat. 14° N. The islands of St Helena and Ascension have also escaped. In South America, Terra del Fuego, Patagonia, Chili, the higher slopes of the Andes and the Falkland Islands have been hitherto exempt. In North America, the whole of the country north of 50° N. lat. has remained free from cholera, as well as the highest slopes of the Rocky Mountains and the Bermudas. Greenland and Iceland have not been visited by the disease.

In Europe, we find that the Farœ Islands, the Hebrides, the Shetlands and Orkneys, Lapland, the Russian territory north of the 64th parallel, Switzerland, and the northern part of Scotland have escaped.

In Asia, the northern districts of Siberia and Kamtschatka have remained free, as also probably Mongolia and Manchuria. We see from these exceptions, what an immense area of the earth's surface has been visited from time to time by the terrible scourge of cholera. It must be remembered, however, that, although the area of distribution of cholera has been so vast, yet certain isolated districts in the various countries visited by it have remained unaffected. For instance, some mountainous districts in the south-west of France, the south-west of Germany, notably Baden and Würtemburg, and the greater part of Greece.

Even in India itself there are places where its ravages are unknown; for instance, it has not attacked the hilly regions of Bengal, but the reason for this is apparently to be found in the fact that the hill men have little or no communication with individuals from the affected area. Should they descend from their mountains, or have any communication with the inhabitants of the plain, they suffer from the disease very severely.

With regard to the home of cholera, we must define it as situated in the delta and valley of the Ganges, from which point it receives an impulse which enables it at times to become pandemic in character. There can be no doubt that cholera existed long before 1817; of its previous epidemic spread no very certain information can be given, but that it overspread India and the adjacent

countries is certain. It appears to be equally certain that, apart from the original impetus given to cholera in 1817, the commerce of the world has been the means of spreading it over such a world-wide area as indicated on the map.

The first pandemic, of which we have authentic data, occurred during the years 1817–1823, in which period the disease devastated an area from Nagasaki, 147° E., to the coast of Syria, 52° E., and from Bourbon, 21° S., to Astrakan, 46° 21′ N.

The second pandemic, which took place during the years 1826–1837, overspread nearly the whole of the countries marked on the map as being affected by cholera. From 1837–1846 Europe, Africa, and America were free from further outbreaks of the disease.

The third great pandemic occurred from 1848–1863, with a remission, as far as the eastern hemisphere is concerned, between 1850 and 1852; and during this pandemic it visited the whole of the northern hemisphere, and reached lat. 25° S. in the Old world and 30° S. in the New.

The fourth pandemic occurred during the years 1865–75, and is noticeable for the rapidity with which cholera was introduced into Europe by sea from the coast of Arabia. In the former pandemics it had always come from the east by way of Afghanistan, Persia, and Asiatic Russia. From 1875 until the limited outbreak in 1883, when cholera appeared in Italy, Spain, and the south of France from Egypt, the disease has been confined to Asiatic soil, and in the year mentioned the spread of cholera was very strictly limited.

Remarks.—Before referring briefly to some points in the origin and spread of cholera, it may be well to remark that, however the disease is produced, the researches of Koch, which have been very recently confirmed by Drs Milles and Macleod, who investigated the subject in 1875 at Shanghai, and an abstract of whose researches was read before the Royal Society of Edinburgh on Dec. 17, 1888, have almost definitely proved that the comma bacillus has a causal relation to cholera. This bacillus is constantly present in Asiatic cholera, and it has never been found anywhere but in Asiatic cholera. Granting, as we think we are right in doing, that cholera is produced by a morbific microbe, we must next refer to various facts with regard to the conditions necessary for an epidemic of

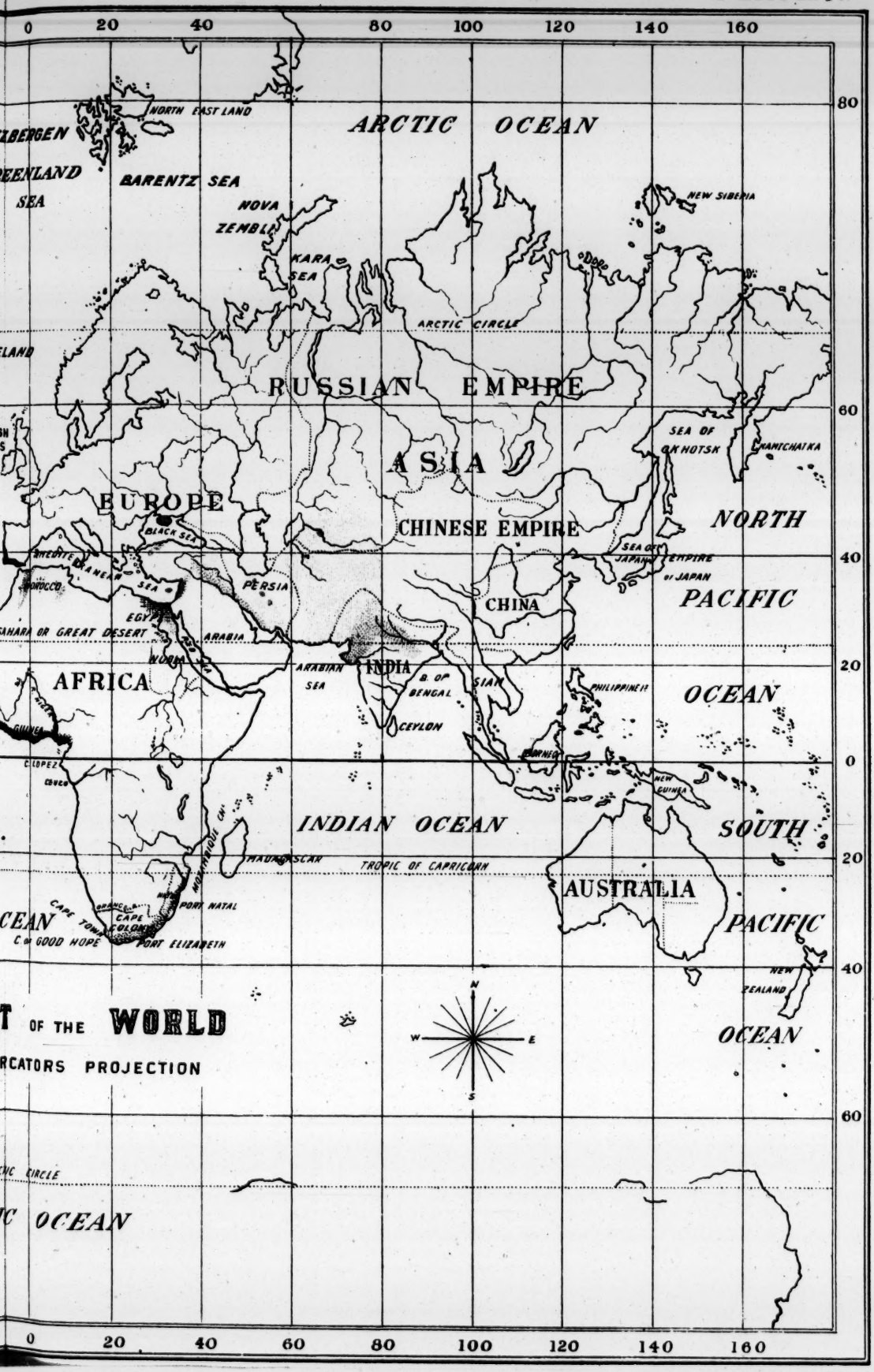
cholera to arise. Cholera may be introduced into a given area, and yet it does not necessarily follow that it will thrive in that area, and some definite local condition must exist for its propagation. With regard to the influence of altitude, it may be said that, although cholera can penetrate to a considerable height, yet, unless in very severe epidemics, the number of cases are fewer the higher it ascends; and Farr adduced the law, that "the proportion of deaths among the inhabitants from cholera is inversely as the elevation of the ground." This law holds good, not only in high altitudes, but in slight elevations in very limited areas. Ackland, writing on the mortality from cholera, mentioned the fact that in three epidemics of cholera in Oxford, in 1832, 1849, and 1854, the mortality per thousand people, living at a mean height of 30 feet above water-level, was respectively 3·3 and 1·8, as compared with 9·8 and 5·3 deaths per thousand which occurred amongst the people living in the low-lying part of the town. There are, however, exceptions to this rule, as in some epidemics hilly districts have suffered more than the lower-lying surrounding country.

Again, cholera follows the course of rivers, this probably being due to the fact that the riparian areas possess soil saturated with water and decaying organic matter. There can be no doubt, too, that cholera spreads most rapidly in countries having an alluvial or tertiary soil.

With reference to the influence of weather on cholera, we know that the disease is met with under every known variety of climate; yet doubtless a high temperature aids its spread, and most epidemics are brought to an end, or at least are most markedly reduced in extent, when the temperature falls greatly. The Vienna Conference of 1884, in which were assembled representatives of all countries, stated that there were no facts to show that atmospheric causes alone could bring on cholera, and also, that all facts go to show that in free air the generative principle of cholera rapidly loses its morbific character.

With regard to the influence of the moisture of the atmosphere upon the production or spread of cholera, it would appear that a certain amount of moisture in the atmosphere, or perhaps more correctly speaking, in the soil, is needed for its development and spread; it is probable that spring showers or sudden summer


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DOMINION OF CANADA
NEWFOUNDLAND
NOVA SCOTIA
NORTH AMERICA
NORTH
SAN FRANCISCO
UNITED STATES
PACIFIC
ATLANTIC
OCEAN
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MEXICO
CUBA
HAYTI
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EQUATOR
C. LOPEZ
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ARGENTINE REPUBLIC
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CAPE HORN
ANTARCTIC CIRCLE
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CHART OF TH
ON MERCATORS
ORIENTAL BOIL. B
ENDEMIC HŒMATURIA. C
YELLOW FEVER. A
New arrivals most liable to Yellow Fever.
Specially strangers from northern latitudes.
Negroes almost entirely exempt.
Endemic at all seasons, in Eastern Hemisphere, between 5° 7′ N. Lat. and 14° 53′ N.
In Western Hemisphere between 82° 46′ N. and 22° 54′ S.


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NOVA ZEMBLA
KARA SEA
NEW SIBERIA
ARCTIC CIRCLE
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ASIA
EUROPE
SEA OF OKHOTSK
KAMTCHATKA
CHINESE EMPIRE
NORTH PACIFIC OCEAN
BLACK SEA
MEDITERRANEAN SEA
MOROCCO
PERSIA
EGYPT
SAHARA OR GREAT DESERT
ARABIA
CHINA
EMPIRE of JAPAN
AFRICA
ARABIAN SEA
INDIA
B. OF BENGAL
SIAM
PHILIPPINE I.
CEYLON
BORNEO
NEW GUINEA
INDIAN OCEAN
SOUTH PACIFIC OCEAN
MADAGASCAN
TROPIC OF CAPRICORN
AUSTRALIA
PORT NATAL
CAPE COLONY
CAPE TOWN
C. of GOOD HOPE
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rains will give to cholera an impetus, whereas continuous rains, by completely saturating the soil, prevent its occurrence. No one knows the cause which gives it epidemic impulse, but we know that, generally speaking, cholera becomes epidemic during the intensely hot weather following heavy rains.

The wind may have a certain action on the spread of cholera, always provided that it be a moist wind, but the old idea that winds could convey cholera poison for long distances seems now to be obsolete. But indirectly the south-west monsoon certainly aids in spreading cholera, because it brings with it moisture which is necessary for its propagation, and because numbers of vessels take advantage of this monsoon to sail from the endemic area of cholera to other places. As Macnamara justly remarks, "cholera thus progresses with man along the great high-roads upon which he travels, spreading no faster than he moves, and being generated by wet, hot weather." Although the disease is thus indirectly spread by the south-west monsoon, it must not be concluded that it cannot travel against wind, for it travels just as fast against the wind as with it. Water is capable of disseminating cholera germs after they have been produced in the soil, or after it has been contaminated by the discharges of cholera patients.

The great factor in the distribution of cholera is certainly that of human intercourse. Persons suffering from the disease, though it may be only in a latent form, undoubtedly convey the poison for long distances, and it is a well known fact that troops, pilgrims, and emigrants have spread it far and wide. It is necessary, however, for an epidemic of cholera to arise, that the poison conveyed into a district should find there a fitting soil for its growth. What that fitting soil is, it is impossible yet to say.

IV. Yellow Fever.

(See Plate IV. A.)

Synon.—Yellow Jack; Bronze John; Vomito Prieto; *Fr.* Fièvre jaune; *Ger.* Gelbes Fieber.

Definition.—A pestilential contagious fever of a continuous and special type. It presents two well-defined stages. The first extends from 36 to 150 hours, and is marked by rapid circulation and

elevated temperature. The second is characterised by depression of the nervous and muscular powers, and of the circulation, with slow and often intermittent pulse; jaundice; suppression of urine, albuminuria, and desquamation of the renal epithelium; diminution of the fibrin of the blood, capillary congestion, passive hæmorrhages from the mucous surfaces and black vomit; fatty degeneration of the heart and liver; and convulsions, delirium and coma. As a general rule, it occurs but once during life.

Geographical Distribution.—Although yellow fever extends over the areas mentioned below, it must be noticed that there are only three districts where it is really endemic, *i.e.*, (*a*) in the West Indies, especially in the Greater Antilles; (*b*) on the Mexican part of the Gulf Coast; and (*c*) on the Guinea coast at Sierra Leone.

The area of distribution of yellow fever is at present limited in Africa to the west coast from 19° N. to a point on the mainland opposite Fernando Po. In the Western Hemisphere it occurs along the eastern shores of the United States from lat. 38° N., and, skirting the coast round the Gulf of Mexico and Central America, it passes along the northern coast of South America and the eastern coast as far as 32° S. On the western shores of South America yellow fever has appeared in epidemics from 5° S. to 42° S. It is also prevalent throughout the whole of the West Indies. Although in the United States and South America yellow fever chiefly infests the coast regions, exception must be made to the great rivers, such as the Mississippi, the Amazon, and the Rio de la Plata, for it extends up these rivers to varying distances. In the western hemisphere the yellow fever area is bounded on the north by 44° 39′ N. (Halifax), and on the south by 34° 54′ (Monte Video); in the eastern hemisphere by 43° 34′ N. (Leghorn), and 8° 48′ S. (Ascension); these are its extreme limits.

Remarks.—There are some curious facts with regard to yellow fever and its relation to climate which it is necessary to remember. Firstly, Negroes are congenitally exempt from it, unless they leave the tropics for any length of time and then return; if they do this their immunity seems to be lost. It appears, too, that Mongolians escape yellow fever. All other races, however, suffer from it, and it is noteworthy that the further north from its area a person was born the more likely is he to suffer from it,

should he come within reach of an epidemic. New arrivals are most exposed to this disease; should they escape it at first they are the less liable to suffer from it, the longer they reside in one place; but if they travel about, this acclimatisation appears to be lost. Whatever may be the real cause of yellow fever, its origin seems to be connected with heat, for although it occurs in sporadic and epidemic forms at all seasons in the tropical part of the yellow fever zone, the disease is greatest, and takes an epidemic spread at the hottest period of the year, a temperature of at least 70° F. being required for its production. Frost puts an end to an epidemic at once, and storms, heavy rains, or cold weather check its progress. Although heavy rainfall will stay an epidemic, yet moisture in the atmosphere would seem to be necessary for the production of the poison, for in dry years or during seasons of long-continued drought the number of yellow fever cases are always remarkably few. Winds influence yellow fever by their effect upon the atmosphere, but it does not appear that they are capable of conveying the poison, at least to any distance. On looking at the chart, it will at once be seen that yellow fever is most prevalent on the sea coast and along the courses of the great rivers; it always spreads from centres of dense population, and the greatest number of cases occur in the dirtiest and most overcrowded parts of large towns. Altitude, certainly, has an important influence on the spread of yellow fever, and it is only in very severe epidemics that it leaves the plains. The protection which altitude confers against the disease is almost certainly due to the lower temperature of elevated spots, because where yellow fever has made its appearance in highly situated regions, it has always been in localities noted for the exceptional heat of the days.

A certain saturation of the atmosphere is an essential condition for an epidemic of yellow fever. It is probable that it does not occur until a high dew-point, the minimum being upwards of 74, exists, and it is certain that epidemics cease before the dew-point descends to 58. The geological characters of the soil have apparently nothing to do with the production of yellow fever, and all those conditions of soil which we found to be necessary for the production of the malarial poison, exert no influence in producing yellow fever. Electricity has a curious influence upon

persons suffering from the disease, even should they be almost convalescent. It is said that should a thunderstorm occur, severe symptoms are immediately manifested in persons suffering from the disease, or a relapse occurs in the apparently convalescent.

Yellow fever poison clings to the ground, and its diffusion may be barred by streams, walls, and, some say, by much travelled thoroughfares, and it does not appear that the water-supply of cities aids its spread. Its period of incubation is variable, and may be said to be between twenty hours and several weeks; this varies in different epidemics, and the most common period of incubation is from twenty to one hundred hours.

Preventive inoculations are now being largely practised against yellow fever with very great success, and there seems to be every reason to hope that at last a method of preventing the disease will be firmly established, although up to the present its true origin has escaped scientific inquiry.

The disease may be spread by fomites, and its toxic power retained for very long periods.

V. Oriental Sore or Boil.

(See Plate IV. B.)

Synon.—Aleppo Evil; Mycosis Cutis Chronica (Carter); Lupus Endemicus (Lewis and Cunningham); Oriental Sore (Fox); Mooltan and Biscara Boil; Date Disease; Caneotica; Liblib; Yemen and Cochin-China Sores; Scinde Boil; Parangi; Mal d'Alep; *Fr.* Bouton d'Alep; *Ger.* Veule von Alep.

Definition.—An indurated, indolent, and very intractable sore; papular in the early, encrusted or fungating in the advanced stages; spreading by ulceration of the skin, single or multiple; and often occupying extensive surfaces of the exposed parts of the body, such as the face, neck, and extremities. It is capable, if innoculated, of reproducing the disease, and it also affects dogs and horses.

Geographical Distribution.—In Europe the boil is met with in Crete, in Cyprus, and in the Crimea. In Africa there is a considerable area in Morocco, on the banks of the Muluia, where the disease abounds, as it does also in numerous oases of the Algerian desert and in the Tunisian Sahara. It is occasionally met with in

Egypt between Suez and Cairo. In Asia it is more widely distributed; it is seen at Broussa, and in Syria it is endemic, chiefly between Killis and Aleppo. In Mesopotamia it is endemic over the whole plain between the Euphrates and the Tigris, extending from Diarbekir to Bagdad and Bassara. In Persia it is endemic at Teheran, Kashan, and Ispahan; whereas in Hamadan it is not so frequently seen. There is also a small endemic focus in the district of Elizabethpol.

The sore is met with in Tashkend and skirts the river Tchirtchik, and in all probability it exists in Turkestan, Afghanistan, and Beloochistan. Another very important endemic area extends along the Indus from the Punjaub southwards through Scinde as far as Goojerat and the Gulf of Cambay, and to the east through Rajpootana and the North-West Provinces as far as Delhi, Meerut, Lucknow, and Gwalior.

Remarks.—Objection may be taken by some to classing under one heading a sore having so many names, but it seems to me that, taking all things into consideration, the various designations all refer to one and the same disease, and that their different features are simply modifications produced by varieties of climate; their various manifestations are also most probably, to some extent at least, influenced by racial characteristics and by the habits of the patients attacked. In each locality where these sores obtain, they vary considerably in their appearance with the character of the season. Two theories have been, and still are, advanced as to their cause, some authorities considering that they are a local manifestation of a cachectic condition due to a residence in unhealthy localities or badly drained towns in certain parts of the tropics. Others again, and notably Carter, consider that the disease is distinctly due to a parasite, and this view is supported by cogent facts. Carter has found in the sores spheroids and mycelium; the disease is localised, and this fact is against it being the outward manifestation of a constitutional state. The disease can be innoculated, and it is contagious. Again, many facts lead one to suppose that the parasite is introduced into the body, either during ablution or by the bite of some insect.

The area of the distribution of this disease prevents us entertaining the idea that its production is influenced by the physical

characters of the soil, nor has altitude anything to do with its production. With regard to water, however, it may be that we should consider it as a cause, for there are instances on record where a change of the source of the water supply has exerted a marked influence upon the number of cases seen. When, however, we come to inquire as to what substance in the water produces or might produce the disease, we are met by various statements which are contradictory, or at any rate give no certain clue on which to base a definite opinion. For instance, the gypsum found in the water at Aleppo is given as a cause: the abundance of nitrates in the water of the Punjaub, or the presence of sulphurated hydrogen due to putrefying matters, is blamed. Hard water is suspected by others, and in Algiers the excessive amount of choride of sodium is considered suspicious; lastly, the amount of earthy salts contained in some waters is said to cause the sore. But there are many other observations which cause grave doubts as to the correctness of any of these views.

Meteorological conditions must, we believe, exert a not inconsiderable power in the production of Oriental sores. It will be noticed that the distribution of the disease is over arid regions, but in these regions it is strictly localised to various foci. In the subtropical regions the disease makes its appearance in the late autumn, and it is met with in the winter in the tropical zone. Where it is found at all seasons of the year, the climate is characterised by hot, dry air during the day, sometimes heavy dew at night, and rapid fluctuations of the thermometer. It must, however, be remembered, that in some places the Oriental sore attacks its victims in the season of the year when vital powers are lowest. It is important not to mistake phagedænic tropical ulcers for the Oriental sore, as the former are due to constitutional debility, induced either by anæmia, malaria, scurvy, fatigue or want, in persons residing or travelling in swampy regions, where the atmosphere is hot and moist.

VI. Endemic Hæmaturia.

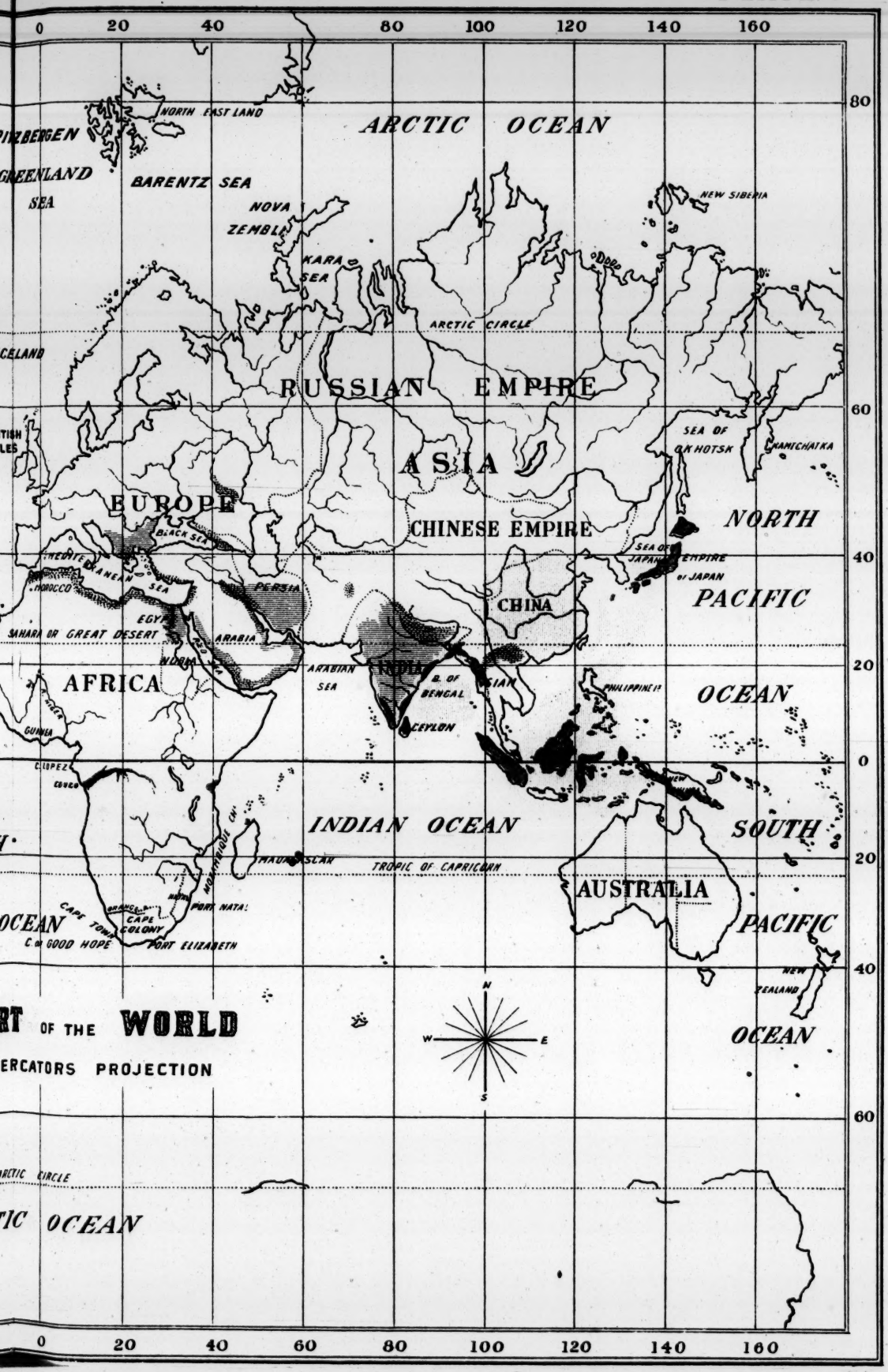
(See Plate IV. C.)

Synon.—Distoma Hæmatobium; Bilharzia Hæmatobia.

Definition.—Endemic hæmaturia is caused by the entrance into


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BERI-BERI.
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ORIENTAL PLAGUE.
B
ANTARCTIC CIRCLE
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PLATE V



RT OF THE **WORLD**

ERCATORS PROJECTION

the body of a trœmatoid hœmatozoon, which is found in the portal system, the mesentery, bladder, &c. It produces hœmaturia and anæmia more or less profound.

Geographical Distribution.—The distoma has, so far as we know, a peculiar and very limited area of distribution. It is found in Mauritius, where the disease it causes was first described in 1812 by Chanotin. It is strictly limited to the delta of the Nile and to various points of the White Nile between 6° N. and the Albert Nyanza. It is also indigenous at the Cape, where it is strictly confined to the coast territory and to the banks of streams for a distance of some 10 or 20 miles from the sea. Its chief seat is in the south-eastern districts of Cape Colony near Algoa Bay, especially at Uitenhage and Port Elizabeth, the neighbourhood of King William's town and East London in Kaffraria, as well as at several places in Natal; for example, on the banks of the Umlasi, the Ungeni between port Natal and Pietermaritzburg, and the Umhloti. It is probably also found in various other places in Central Africa.

Remarks.—A knowledge of the geographical distribution of this parasite is of great importance, because, by taking proper precautions when residing in its limited area of production, it is possible to escape its ravages. There can be no doubt that it exists in stagnant pools, in the shallow water of declining rivers, near estuaries, and at the sea coast. It is affected by season, being found in the water in the summer, and it is during the summer too that most people are affected by the disease. It is curious to notice its preference for the male sex, and it is most commonly seen in them between the ages of five and thirty-five years. At Pietermaritzburg the majority of youths are affected by the parasite. As it can only obtain entrance into the body by means of drinking water or in bathing, the necessary precautions should be taken, and Europeans who may contract the disease should immediately remove from the infected area.

VII. Beri-Beri.

(See Plate V. A.)

Synon.—Barbiers; Loempoe (Java); Kak-ké (Japan); Maladie des Sucreries (French Antilles); Sleeping Sickness (west coast of Africa); the Bad Sickness of Ceylon.

Definition.—A disease characterised by anæmia, anasarca, degeneration of muscular tissue, effusion into the serous cavities, debility, numbness, pain and paralysis of the extremities, especially the lower; precordial anxiety, pain, and dyspnœa, and in some cases drowsiness or sleepiness. Beri-beri occurs in a chronic and an acute form.

Geographical Distribution.—Beri-beri, like other similar diseases, has a wide distribution in many countries situated in the tropical and sub-tropical zones, but, although its area of distribution is so extensive, it is strictly limited in its endemicity. It is to be found both in the eastern and western hemispheres. Looking at the eastern hemisphere first, we find that one of its chief habitats is in Japan. There it was limited, until fifty years ago, to the coast towns, but since that time it has been met with practically all over the islands. In China it is not so frequently seen now as it used to be, but it is endemic in Burmah, in Singapore, and in the Calabash islands. In the Malay Archipelago, we find that most of the islands are endemically affected, especially Sumatra, Banka, Borneo, Labuan, Celebes, and some of the Molucca group of islands. We meet with it, too, on the west coast of New Guinea, and on the extreme east of Java, as well as in the prisons of Batavia in Passuruan and Samarang. With regard to India, Beri-beri infests a strip of country, 100 miles broad, on the coast from Grandjam to Masulipatam, while it is more rare on the Coromandel and Malabar coasts, and in the plain of the Carnatic. It is also met with at various isolated spots in the provinces of Decca and Assam, and it is seen occasionally in Calcutta. It exists in Ceylon, especially at Trincomalee and Candy. As an epidemic, it is met with in Mauritius, Réunion, Nossi Bé, Zanzibar, and probably on the Congo. Passing to the western hemisphere, we find that Beri-beri has been undoubtedly imported from the east. Thus, at various times it has been epidemic in Guadaloupe, Cuba, Cayenne, Paraguay, and San Francisco. Beri-beri may be said to be endemic over the greater part of Brazil, where it commenced in the Bahia Province, and it is also met with in Guiana. It must be mentioned, too, that Beri-beri often breaks out on board ship, especially in transports, coolie ships, and vessels trading in the Malay Archipelago, Bay of Bengal, and with Japan.

Remarks.—Many have been the theories started to explain the cause of Beri-beri, but, owing to the limited well-defined areas in which the disease is endemic, most of them are unsatisfactory. Its epidemic spread, however, is probably influenced by climate, and seems to coincide with conditions of high atmospheric moisture and extreme thermometric variations. Some parasite will, doubtless, ere long, be proved to be the real cause of the disease, and it is probable that its production will be traced to the soil, for in those places where Beri-beri is endemic the soil abounds in saline materials, such as magnesia, lime, chlorides, alumina, and iron. Although all races and persons of all ages are attacked by Beri-beri, the dark races suffer most, and adult men far more than women and children. Indeed, so great is the disproportion between male and female sufferers, that the cases seen in women are about one to thirty-one in men. Although it is, as a rule, rare for children under fifteen to be attacked by Beri-beri, yet there are epidemics on record in which children seem to have suffered most. Whatever be the cause of Beri-beri, a residence of eight or ten months in an endemic area appears to be necessary before a person can be attacked by it, and it is also a remarkable fact that it attacks by preference persons in an apparently robust condition according to some authorities, others think debility a predisposing cause.

VIII. Oriental Plague.

(See Plate V. B.)

Synon.—The Pest; Inguinal, Bubolic, Glandular, Oriental, Indian, Pali, and Levantine Plague; Oriental Typhus; Septic Pestilence; *Fr.* La Peste; *Ger.* Die Pest.

Definition.—A specific fever, attended by bubo of the inguinal or other glands, and occasionally by carbuncles.

Geographical Distribution.—In olden times Oriental Plague had a very wide distribution; now it is met with in much narrower limits, and it is to its present distribution that we refer.

In Africa its area is distinctly limited to the northern coast belt, including Morocco, Algiers, Tunis, Tripoli, and Lower Egypt. In Egypt it has never gone beyond the first cataract of the Nile. In Russia it is met with in Astrakan along the Volga, and outbreaks

sometimes occur in Turkey. In Asia, epidemics of plague arise in Syria, Caucasia, Mesopotamia, and Persia; also in Arabia, on the coasts, and inland as far as Mecca. Epidemics have visited Hindostan, and there are endemic centres of plague on the southern slopes of the Himalaya, in the provinces of Kumaon and Gharwal and in Peshawur. It is probably endemic in the mountain valleys of Yunnan in China, and in Burmah.

Description.—In recent outbreaks of plague, three varieties have been described—(*a*) Abortive or larval plague; (*b*) Plague proper; (*c*) Fulminant plague. The usual characters of plague may be rapidly summed up as follows:—After a day or two's lassitude, shivering, and vomiting of a black material, high fever is experienced, with great pain in the axillary and inguinal regions, where buboes soon form. Often, too, the body assumes a livid hue, which gave plague the name of "black death." The aspect of the plague patient is peculiar; the face is haggard, the eyes retracted, and the conjunctivæ red.

Remarks.—In considering the causes of plague, there can be no doubt that the influence of the seasons is very marked. The disease commences to make its appearance in the winter, and cases become more numerous as spring sets in; but extreme heat or extreme cold usually puts an end to an epidemic. It is probable that plague has no relation to soil, and with regard to altitude, moderately high situations are more prone to be affected by the disease than are low-lying places, although from this it must not be understood that plains escape. There can be no doubt, however, that want, filth, and overcrowding are necessary to the production of plague, and it is well known that hygienic measures both prevent its appearance and stop its epidemic progress. In conclusion, it may be noted that the closer the association of healthy people with the sick the more liable are they to contract the disease; hence, persons residing in the same house with plague-stricken patients are much more likely to be attacked than others, and clothing and bedding may carry the infection.

IX. Dysentery.

(See Plate VI.)

Synon.—Fr. Dysenterie; *Ger.* Dysenterie.

Definition.—A specific febrile disease, characterised by considerable nervous prostration and inflammation of the solitary and tubular glands of the large intestine, sometimes ending in resolution, but frequently terminating in ulceration, occasionally in more or less sloughing or gangrene; always accompanied by tormina and tenesmus.

Geographical Distribution.—The distribution of tropical dysentery is very wide, and to a great extent coincides with the area in which malaria is endemic. Commencing with its existence on the west coast of Africa, we find that it is extremely prevalent in Senegambia, on the Sierra Leone coast, in Upper Guinea, and on the Gold and Slave coasts, as well as throughout the area watered by the Niger. In all these regions it affects natives as well as Europeans. It is not so frequently met with in the Cameroons, and from thence southward to Cape Lopez it is also less frequent. From Cape Lopez along the Congo coast endemic areas of dysentery are only to be found in isolated spots. Fernando Po is severely affected by this disease, as are also the islands of St Iago and Nicolao. In Madeira it is only epidemic. Passing on to the Cape of Good Hope, we again meet with a wide area of its distribution, the natives being especially affected by it; but it is to be noted that the disease is more severe in the interior of the country than at the coast. On the east coast of Africa dysentery is endemic at Mozambique, Madagascar, Réunion, and Mauritius; also at Zanzibar and along the adjacent coast, but it is much less severe in Mayotte, Nossi Bé, and St Marie. It is very prevalent all over Abyssinia, except in the dry open tablelands, and it is met with throughout the whole of the southern Soudan and Nubia; it also passes down the valley of the Nile to the Delta. It is endemic in Algiers and along the coast regions of Morocco, Tunis, and Tripoli.

In Asia dysentery is met with in the valleys of Syria, in the plain of Mesopotamia, and in many parts of Persia, but it is most severe on the western and southern coasts of Arabia. It is found in the deep mountain valleys of Beloochistan and Afghanistan, and throughout

all India, being least severe in the Presidency of Bombay. It occurs in Ceylon, in Further India, in most of the islands of the Malay Archipelago, and on the southern and eastern coast zones of China. In the islands of the Japanese Empire the disease is only epidemic.

In Australia dysentery is endemic only on the west coast, although it occurs in slight epidemics in Melbourne, Sydney, Tasmania, and New Zealand. Endemic dysentery is also met with in New Caledonia, the Fiji Islands, Tahiti, the Mangareva group, and in the Hawaiian Islands.

Passing to the western hemisphere, we find that in South America dysentery is endemic in French and Dutch Guiana, in Brazil, especially on the coast of the provinces of Maranhâo, Piauhy, and Parahiba, and in the northern and central parts of the country. It is seen in Paraguay and on the coast of the Argentine Republic, as well as in the Provinces of Tucuman and Salta. The coast of Chili is also infested by dysentery, and the disease extends northwards along the coast of Peru; it is, however, most severe in the forest region on the eastern slopes of the Cordillera, on the Peruvian Pampas, and in the marshy country bordering the Amazon. This area ends at the coast-line of Ecuador, Granada, and Venezuela.

Dysentery is also endemic in Central America; that is to say, in Panama, Costa Rica, Nicaragua, the Mosquito shore, San Salvador, Guatemala, and Mexico. It is found all over Mexico until the Anahuac Plateau is reached. In the West Indian islands, Cuba and Hayti are its principal seats, Jamaica being less subject to it, and it is probably met with in the other islands.

With regard to the United States of America, it is difficult to say where the disease is endemic, but it occurs more or less all over the States, including California, and is most frequent along the Atlantic and the Gulf coasts. In British North America, Prince Edward Island and Vancouver Island are affected.

In Europe, endemic dysentery is confined to a few spots chiefly in the southern peninsulas and islands. It is common in Andalusia, on the tableland of Estremadura and New Castile, in Aragon, and in the southern parts of Galicia and Catalonia. It is prevalent throughout Italy, especially in the southern provinces and in Sicily; also in Malta. It is found in Greece in the Peloponnese, and in Constantinople, Roumelia, and Asia Minor. In France it is

endemic in some parts of Guyenne and Provence, in Lyonaise and Auvergne, in a few valleys of the Vosges, in the marshy districts of the Brenne, and in Sologne and Guer. In Sweden it is probably endemic at Jönköping, Skaraborg, Elfsborg, Wermland, Göteborg, and Bohus, and in the island of Gottland. In Russia it casually occurs in the Baltic Provinces, but more especially in Trans-Caucasia.

Remarks.—That endemic dysentery is due to some microbe, there can be little or no doubt; at least the researches of Prior and Cartullis almost definitely prove it. It is, however, difficult to say with exactitude what gives rise to the morbific microbe. Although, as above stated, endemic dysentery occupies very nearly the same geographical distribution as malaria, yet it has some points of difference. For instance, its endemic spread can proceed to far higher latitudes than does malaria.

Various facts prove that the disease is influenced by climate, for it is endemic at all seasons in hot tropical regions, and it is also found that in more temperate latitudes outbursts of the disease occur chiefly, one might almost say solely, during the late summer and early autumn season. Extreme cold puts an end to epidemic dysentery just as it does to yellow fever. All this shows that heat is necessary for its production. But, again, we find that fluctuations in temperature exert a marked influence in its production and spread. Where hot days and cold nights obtain in tropical regions dysentery is most prevalent. This fact has been observed again and again in various wars, as, for instance, in Ashantee, Abyssinia, China, and more recently still in the Soudan. In Sweden the disease rarely attains any considerable epidemic diffusion, unless the summer has been remarkable for great heat; and even the severe epidemics confined to limited portions of the country mostly coincide with extremely hot summer weather. It is difficult to decide what influence the moisture in the atmosphere has upon the production of dysentery, as authorities differ very much on this point, but we cannot help believing that moisture does play a part in either the production of the disease or in predisposing persons to suffer from it, and that marshy districts and the neighbourhood of large rivers where morning fogs are of frequent occurrence, are certainly injurious, even if the moisture itself does

not aid in producing the dysenteric poison. It may, of course, be that swampy districts exert their influence more indirectly than directly, for, doubtless, malaria does predispose individuals to attacks of dysentery, and the marshes may give rise to malaria which is only too often followed by intestinal disease. It has been stated by Annersley that dysentery rages in Bengal in the rainy season, and it is well known that the disease is most prevalent in lower Egypt at the time of the overflow of the Nile. It does not seem that elevation or configuration of the ground, nor the geological formation or the physical characters of the soil, have any connection with the production of dysentery.

Although, as we have just remarked, malaria may predispose to dysentery, we do not find, in considering the geographical distribution of the two diseases, that the points in which they are severally most virulent coincide, which one would naturally expect to be the case were the diseases produced by the same factors, or if they had a very intimate relation the one to the other.

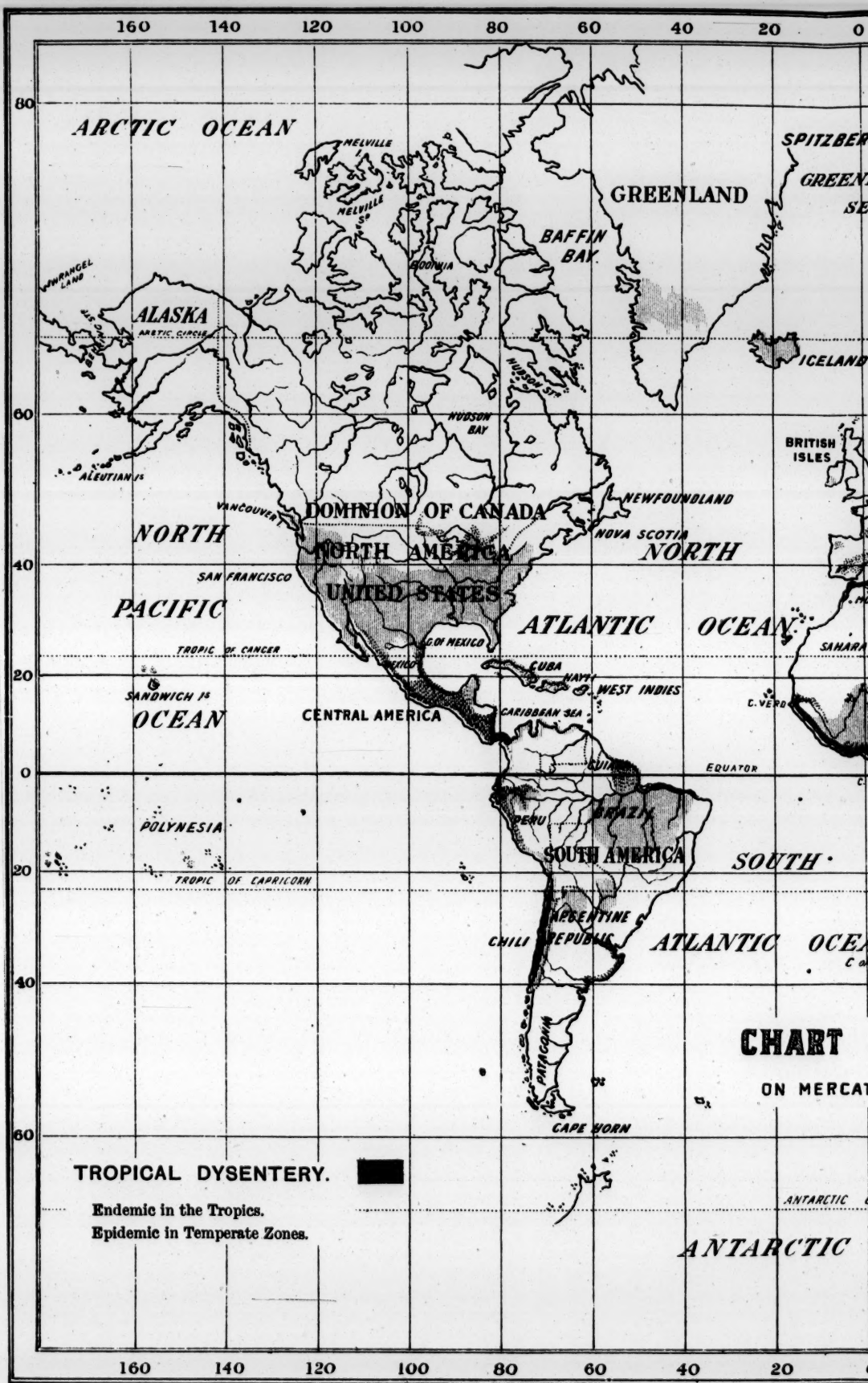
It can only be said, in conclusion, that contaminated drinking water may frequently serve to introduce the virus of dysentery into the system, and that it appears to be proved that a person suffering from the disease may introduce it into a previously healthy community.

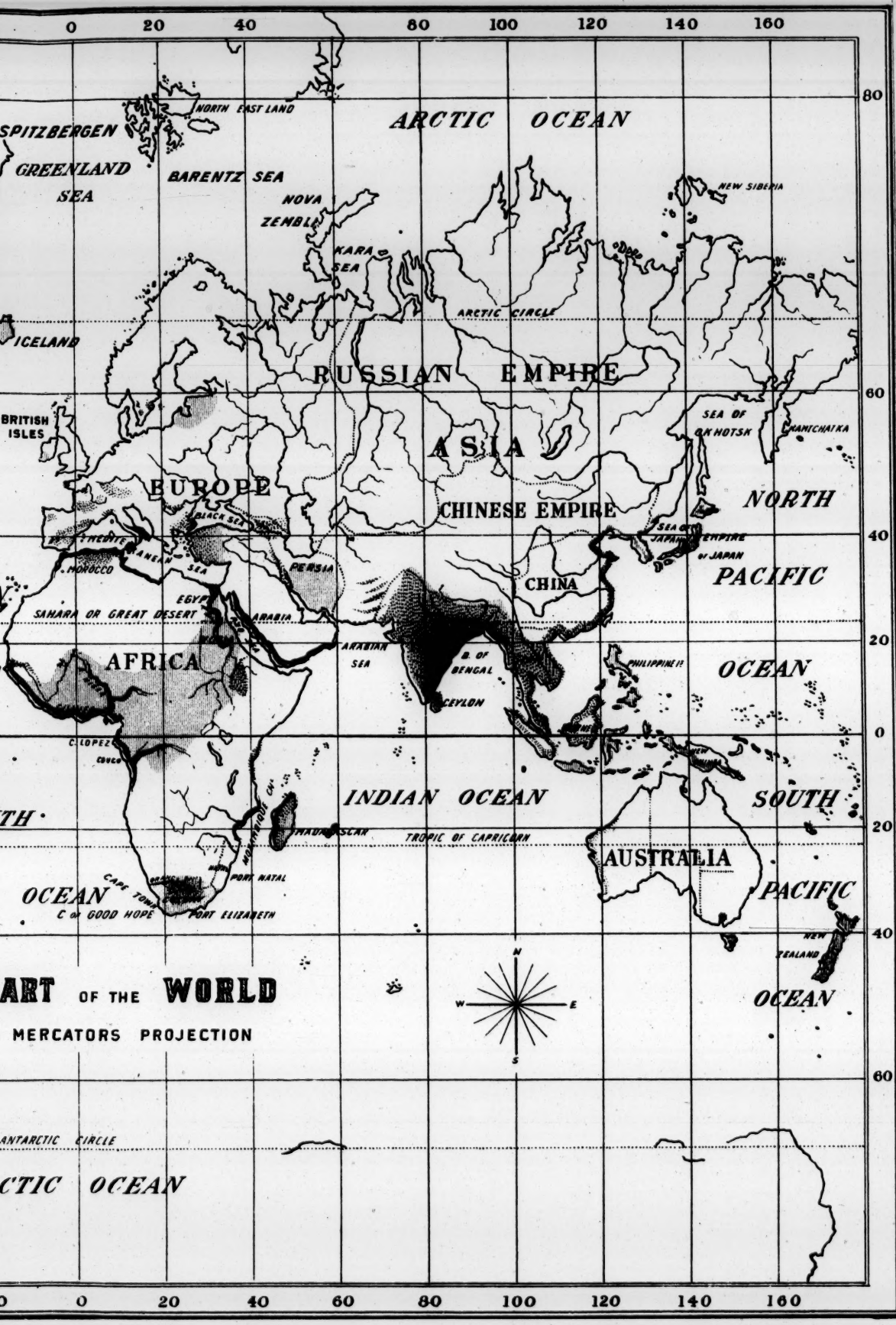
X. Leprosy.

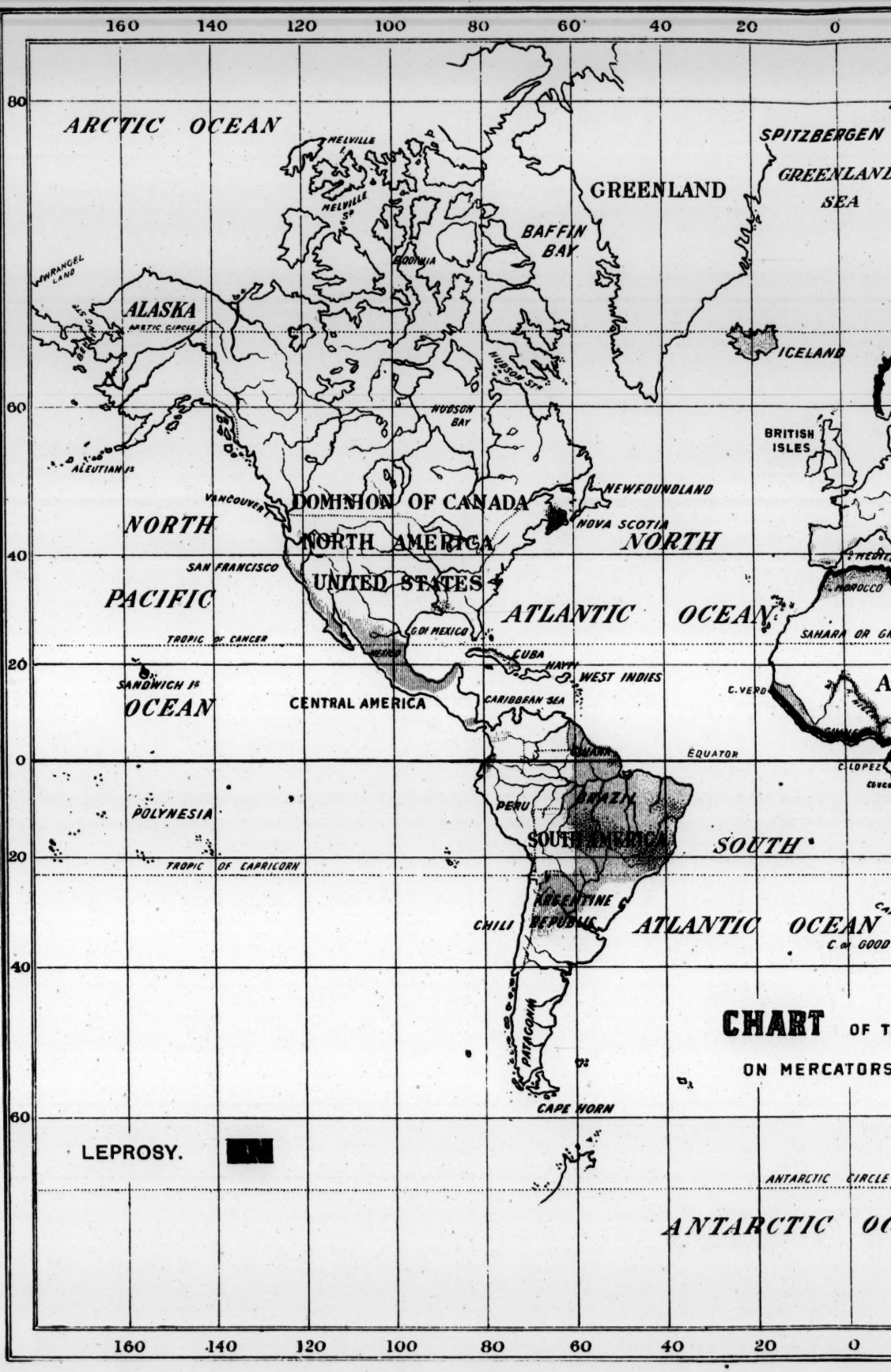
(See Plate VII.)

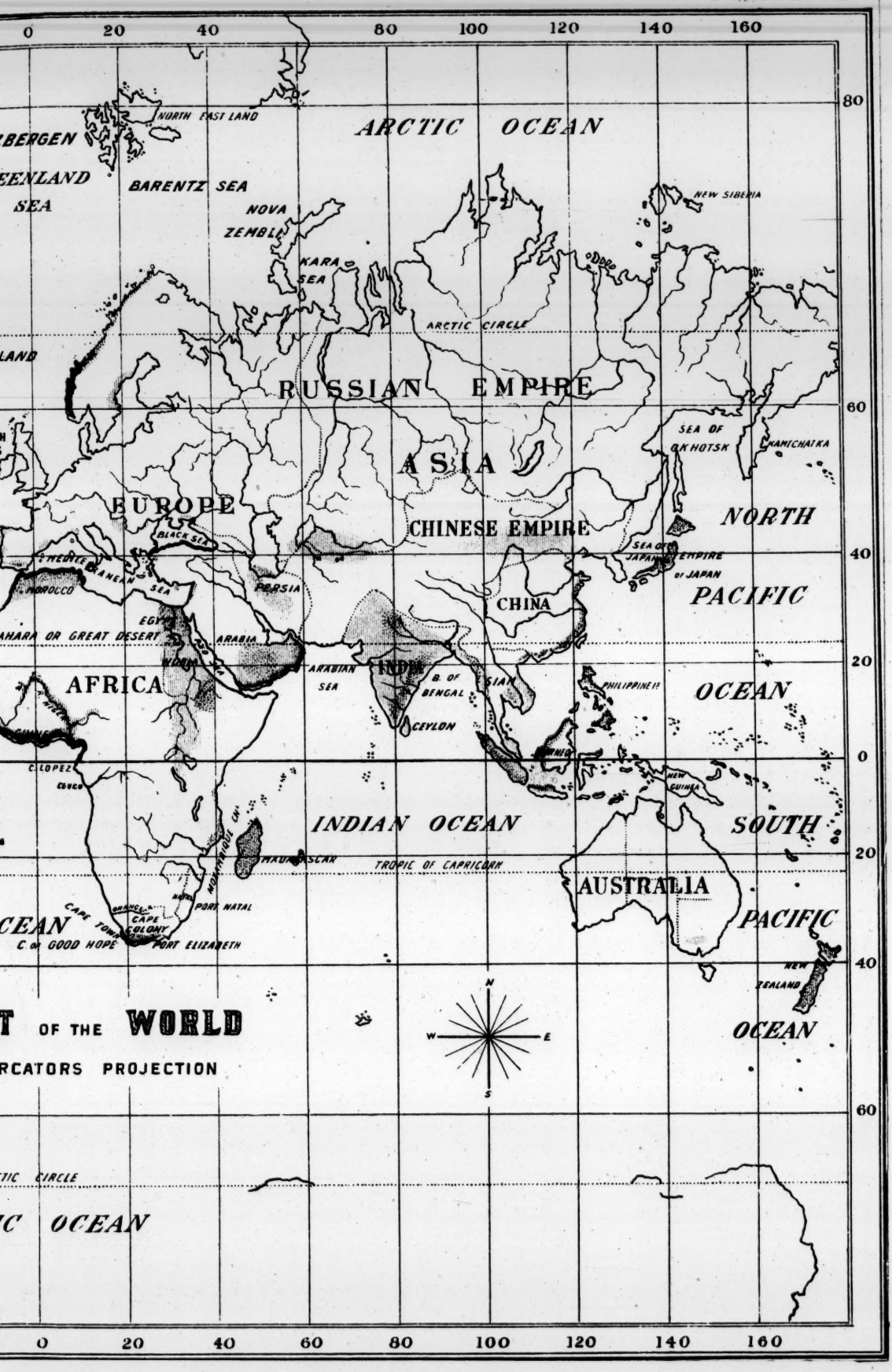
Synon.—Elephantiasis Grecorum; Lepra; Lepra elephantia; Black Leprosy; Red Leprosy; Elephantiasis tuberosa, anæsthetica, nodosa, mutilans, leontina, satyria; Joint Evil; the Myckle Ail or Great Disease; *Fr.* La Lèpre; *Ger.* der Aussatz; *Scand.* Spedaklalskshde; *Norway*, Likpra.

Definition.—A specific disease, endemic in many parts of the world, characterised by the slow development of nodular growths in connection with the skin, mucous membranes and nerves, and (in the last case) by the supervention of anæsthesia, paralysis, and a tendency to ulcerative destruction and gangrene (Bristowe). Two types of leprosy are described—the tubercular and anæsthetic varieties; the first variety is more frequently seen in temperate climates, the latter in the tropics.


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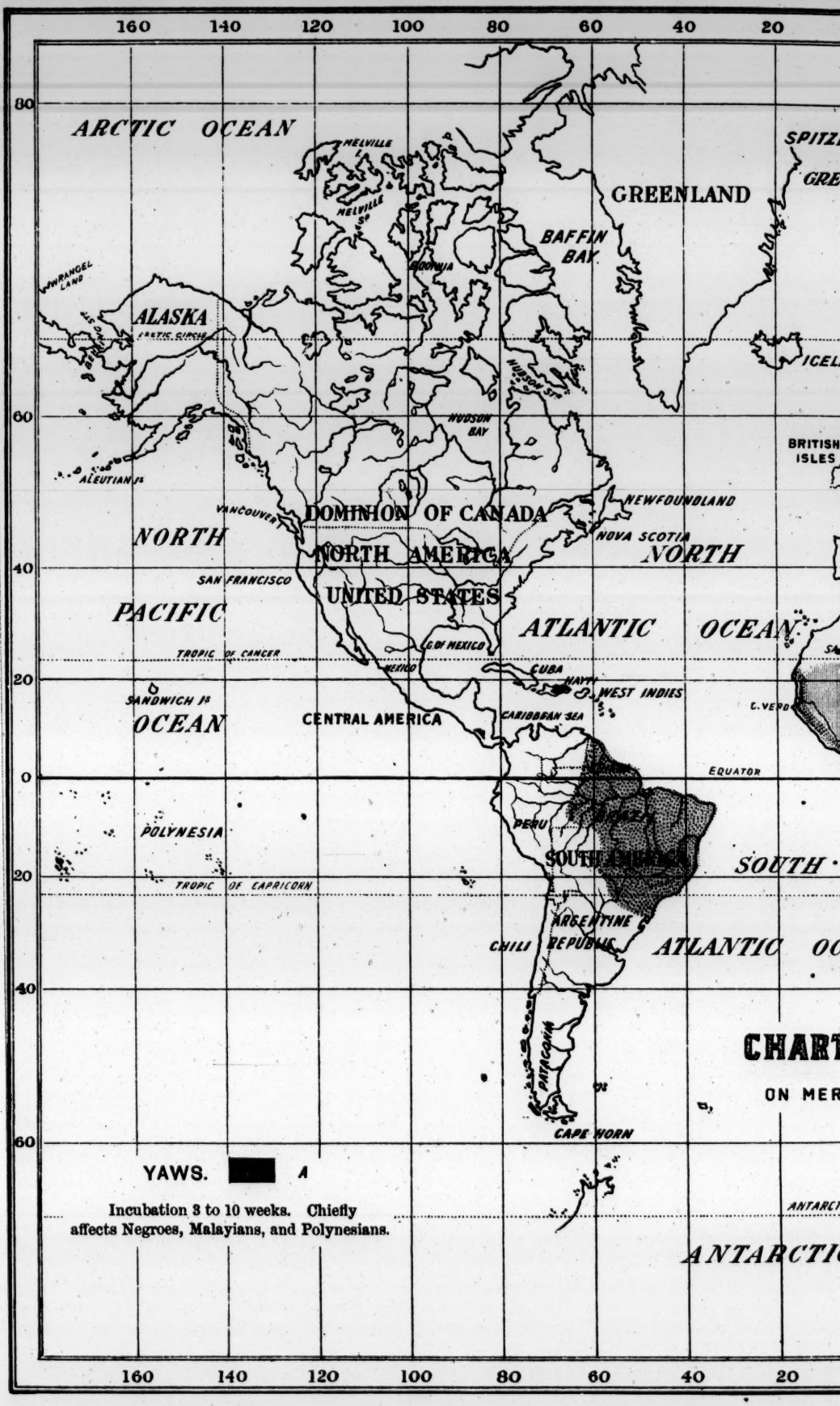

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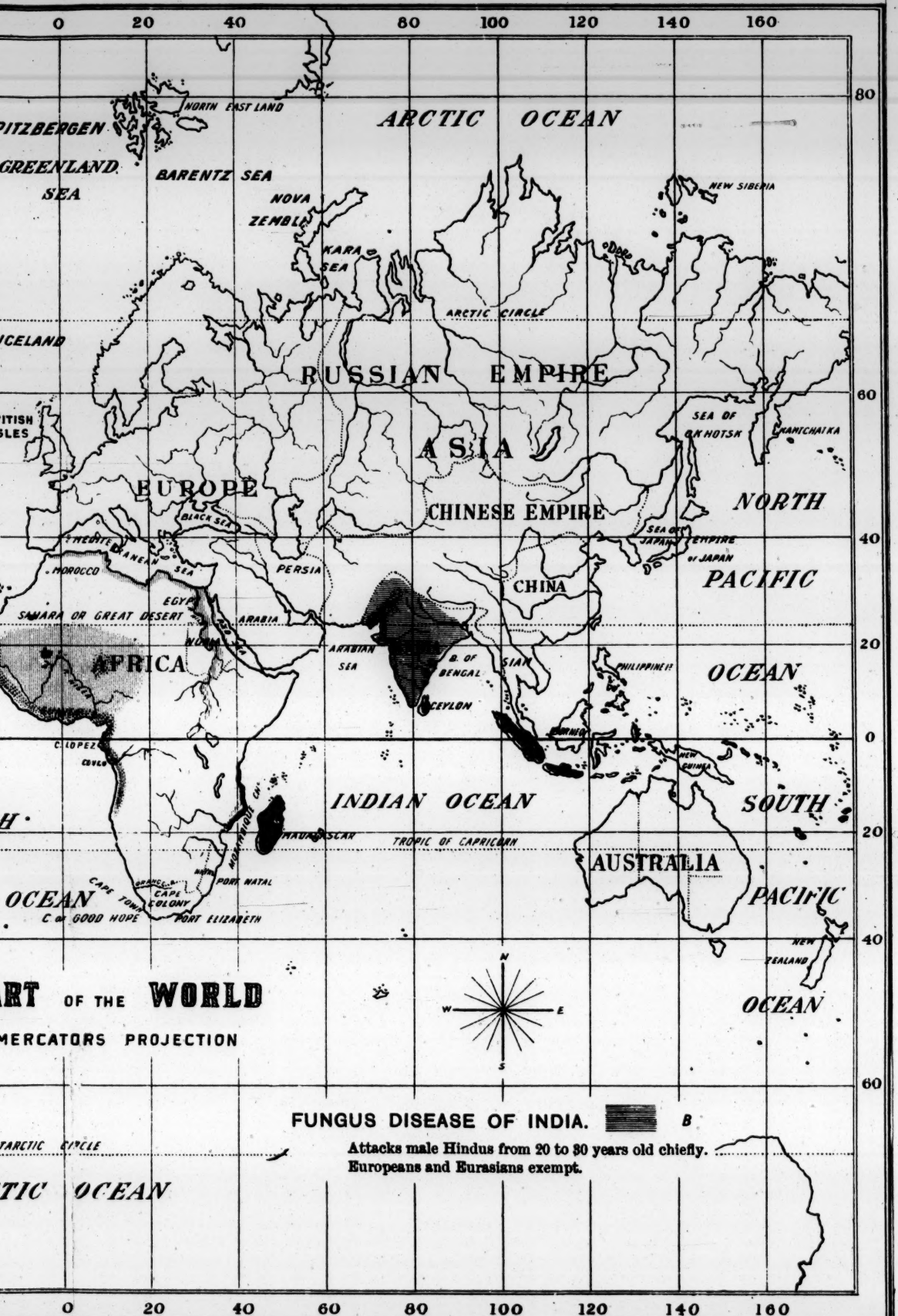
Geographical Distribution.—Leprosy is endemic at the present time in Egypt, and throughout the whole of the basin of the Nile, as well as on the shores of the Mediterranean and Red Seas. It is very prevalent in Abyssinia, on the coast, on the plains, and in the hill districts. It is also endemic at Zanzibar, Mozambique, Madagascar, St Marie, Mauritius, Réunion, St Helena, in the Canary Islands, and on the west side of the island of Madeira. It is met with in Algiers and Morocco, but is rarely seen in the Azores, and Tripoli and Tunis are said to be free from it. On the west coast of Africa leprosy is endemic in a very extensive area, extending from Senegambia to Cape Lopez. In this region it exists all over Senegambia and Sierra Leone; it overspreads the districts watered by the Niger and the Binué, as well as the whole of the Cameroons district. Leprosy is not met with on the Loango coast, and Angola as well as the Congo are free from it, as also is the province of Natal, but it is endemic to a considerable extent at the Cape and in Zululand. Passing on to the endemic area of leprosy in Asia and the Archipelagos adjoining it, it is to be noticed that India and the eastern parts of Asia are the most affected. In Nearer Asia the disease is endemic in a few limited areas; *e.g.*, on the southern coast of Arabia, especially at Muscat, as well as in the centre of the country. The mountainous districts of Persia, Syria, and Cyprus are affected, as are also some parts of Turkestan, especially Samaracand, Miankal, and Hissar. In Asia Minor it is now only endemic in isolated spots —at Smyrna, in the neighbourhood of Sinope, and on the shores of the Black Sea. It is met with all over India, but is least prevalent in the Madras Presidency. The disease is fairly common in Ceylon, but chiefly on the south and south-western coasts, and endemic areas are found in British Burmah, in the peninsula of Malacca, in Siam, and Cochin-China. In the East Indian Archipelago the most important areas of endemic leprosy are on the west coast of Java and in its mountainous regions, the disease being more rarely met with on the southern and eastern coasts. Leprosy is also endemic in the Andamans and Nicobars, in the elevated and inland regions of Sumatra, on the west coast of Borneo, in Celebes (province of Menehasse), in Flores, in the interior of Timor, in Banda, and in the Philippines. In the Chinese Empire the areas of endemic leprosy are chiefly confined to the southern and eastern coasts; it is rarely met with in the

interior, except towards the northern part of the empire. In Japan leprosy is very prevalent, only the Loo-Choo Islands being free from it. It exists in New Zealand, in the Hawaiian group, and sometimes in Tahiti.

In Europe leprosy only occurs endemically, in small and, for the most part, clearly defined areas; in Spain it is confined to the provinces of Catalonia, Andalusia, Galicia, Asturia, and Granada. In Portugal it is met with in the provinces of Beira, Estremadura, and Algarve. In Italy it is endemic in two places—viz., at the Riviera-di-Ponete and at Comacchio. It is also found in Sicily. In the Balkan Peninsula leprosy still exists in small centres on the coast of the Ejalet of Salonica (Thessaly and Macedonia), and a few cases occur in Constantinople, which are probably imported. In Greece it is sometimes seen in the neighbourhood of Parnassus, and on the islands of Samos, Rhodes, Chios, and Mitylene, but the chief seat of leprosy in this region is Crete. In Roumania and Hungary occasional cases of leprosy occur, but it has ceased to be endemic there. In Sweden leprosy is met with in the districts of Angermanland, Mendelpad, Helsingland, Upland, and Bohus, but it has diminished very greatly in recent years. The most considerable area of endemic leprosy in Europe is the west coast of Norway, from Stavanger up to Tromsöe, most of the cases belonging to the departments of Söndre and Nordre Berghus. The disease is also met with in Iceland. Turning to the western hemisphere, there are three endemic areas of leprosy in North America—Mexico, Louisiana, and New Brunswick and Nova Scotia, and it is found amongst the Chinese immigrants in California. It also occurs in Costa Rica and in the West Indies, particularly in Cuba, Jamaica, St Bartholomew, St Kitts, Nevis, Antigua, Guadaloupe, St Vincent, Barbadoes, Trinidad, and the Bahamas. Leprosy is endemic in the elevated regions of Ecuador, and in various parts of Guiana, but its headquarters in South America are undoubtedly in Brazil, the provinces of Maranhao and Rio Grande being almost exempt. From the southern provinces of Brazil the area extends over Paraguay and the northern parts of the Argentine Republic, especially throughout the provinces of Entre Rios and Salta, and it stretches across the continent as far as the eastern frontier of Bolivia.

Remarks.—There seems to be no doubt now that leprosy is


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Incubation 3 to 10 weeks. Chiefly affects Negroes, Malayians, and Polynesians.
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ART OF THE **WORLD**

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FUNGUS DISEASE OF INDIA. B

Attacks male Hindus from 20 to 30 years old chiefly.
Europeans and Eurasians exempt.

both contagious and hereditary, and that it is caused by the bacillus leprae, but for the present we must confess ignorance as to its origin. It should be stated, however, that the production of leprosy has been ascribed to extremes and frequent and rapid transitions of temperature, associated with high degrees of atmospheric moisture, but a glance at the chart will disprove this idea. Again, it has been said that leprosy must bear a special relation to the sea coast; but although in isolated cases it does occur chiefly near the sea, its area of distribution completely disproves this theory. Various articles of diet have been blamed as its cause—fish diet, salt or rotten fish, immoderate use of pork, and the use of decomposed rice or maize. It is, however, impossible, after studying the subject, to arrive at the conclusion that any of these causes is the true one. The mere fact of the very definite isolation of the areas of endemic leprosy goes against these theories.

XI. Yaws.

(See Plate VIII. A.)

Synon.—Framboesia; Button Scurvy; Verruga-Peruviana; Peruviana Wart; Buba or Boba, and Patta (West Indies); Framosi (Calabar); Tetia (Congo); Tonga or Coco (Fiji); Lupani and Tono (Samoa); *Fr.* and *Ger.* Pian.

Definition.—Yaws consists of an eruption of yellowish or reddish-yellow tubercles, which gradually develop into a moist exuding fungus without constitutional symptoms, or with such only as result from ulceration and prolonged discharge, namely, debility and prostration. It is epidemic, and contagious by actual contact. The period of incubation of the poison varies from three to ten weeks, and as a rule it only occurs once in a life-time.

Geographical Distribution.—In Africa, yaws is to be met with on the west coast, from Senegambia in the north, as far south as Angola, together with the westerly Soudan, where it is especially frequent in Timbuctoo and Bornu. It is occasionally seen in the Nile Valley, as well as on the northern and north-eastern African coast-line. It is very frequently met with in Madagascar, Mozambique, and the Comoros. In the East Indies it is chiefly seen in the Moluccas, Java, Sumatra, and Macassar; it is also

endemic in Ceylon, New Caledonia, Fiji, and Samoa; and it is met with amongst the Hindoo population of Pondicherry. In the West Indies it is endemic in San Domingo, Jamaica, Barbadoes, Martinique, Guadaloupe, Sta Lucia, and Dominica. It is found all over Brazil and in Guiana, and is said to be rather frequent at Punta Arenas in Central America.

Remarks.—Yaws is distinctly a tropical disease, and the poison, whatever it may be, depends for its production on extreme heat and moisture; but although these factors would appear necessary for its production, there must be other causes, for in some countries, as in India for example, where the same temperature and moisture exist, yaws is unknown. Negroes chiefly suffer from its ravages, but no race is exempt, although it must be admitted that it attacks Europeans with comparative rarity.

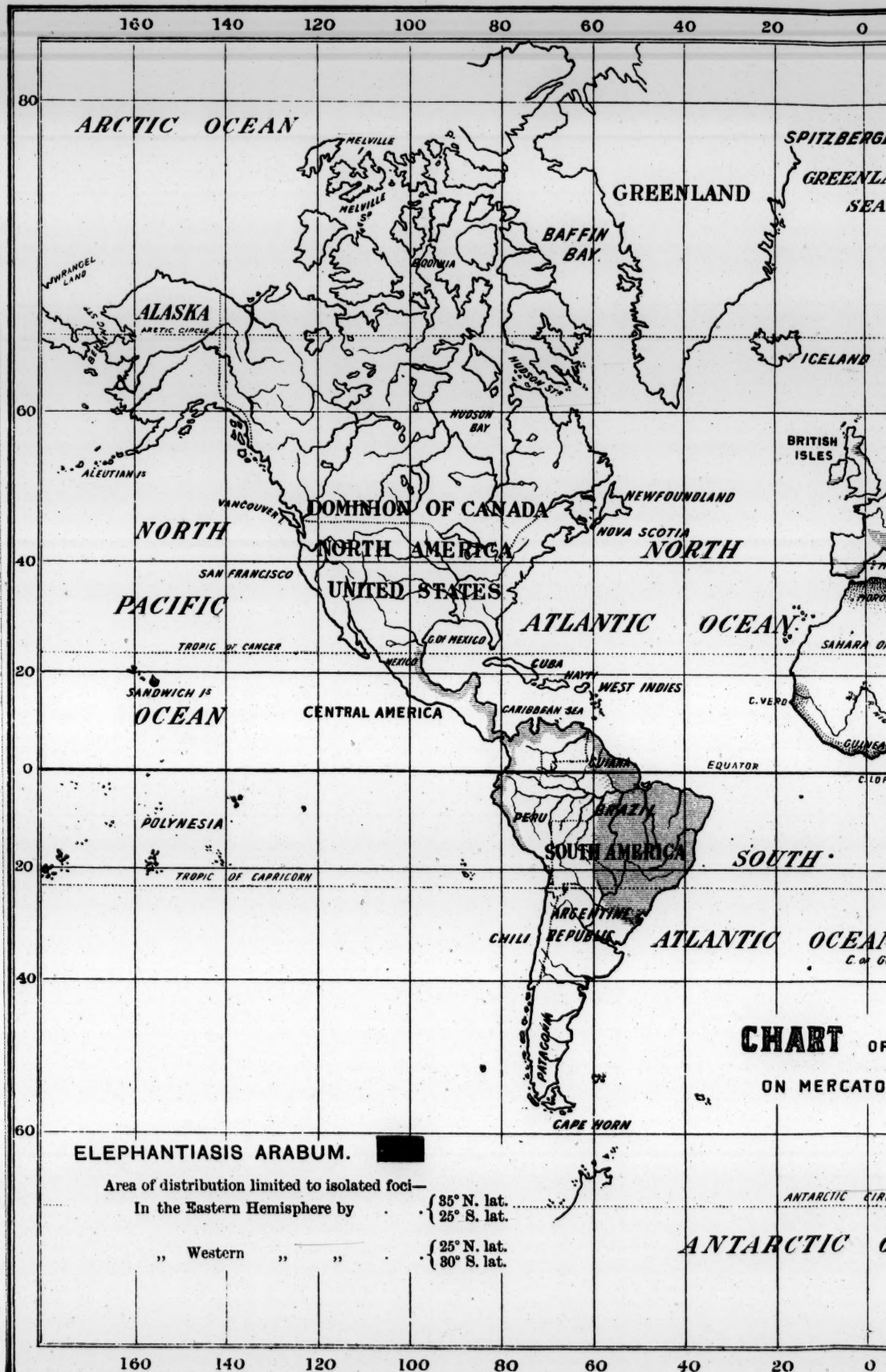
XII. Fungus Disease of India.

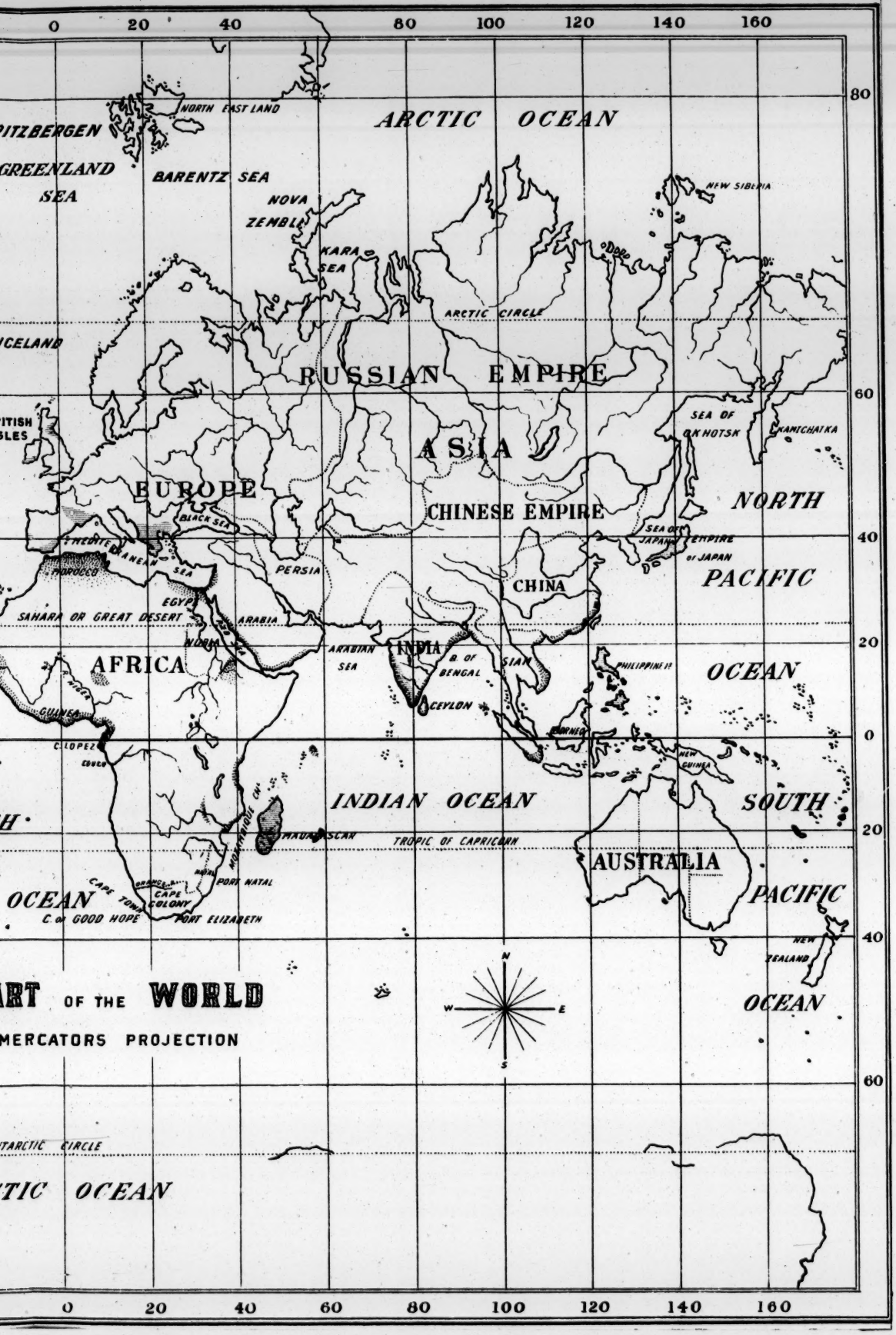
(See Plate VIII. B.)

Synon.—Madura Foot; Mycetoma; Morbus tuberculosos pedis; Ulcusgrave; Podelkoma; *Fr.* Dégénérescence endémique des os du pied; Pérical; Keerenugra.

Definition.—A diseased condition of the hands and feet, occurring in India, characterised by enlargement and distortion of the affected extremity, due to thickening of the cutaneous tissues, with degeneration and subsequent fracture of the osseous structures. There are two varieties of this malady—one, the pale or ochroid form; the other, the melanoid or dark form.

This disease has been recognised since 1712, when Kampfer first called attention to it, but little definite was known about it until Goodfrey in 1846, and Vandyke Carter in 1860, investigated it thoroughly. In all probability, as Carter assumes, the disease is due to a parasite, but authorities still differ as to the nature of the parasite, and also as to the nature of the method by which it finds entrance into the hand or foot affected. It seems to be clear that it is not a constitutional disease, but the various theories which have been put forward as to its precise cause cannot be reconciled. Hindoos of all classes are affected by the disease. Mahommedans


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ELEPHANTIASIS ARABUM.
Area of distribution limited to isolated foci—
In the Eastern Hemisphere by 35° N. lat. 25° S. lat.
„ Western „ „ 25° N. lat. 30° S. lat.


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NOVA ZEMBLA
NEW SIBERIA
KARA SEA
ARCTIC CIRCLE
ICELAND
RUSSIAN EMPIRE
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KAMTCHATKA
ASIA
EUROPE
CHINESE EMPIRE
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EMPIRE of JAPAN
MEDITERRANEAN SEA
PERSIA
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PHILIPPINE I.
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GUINEA
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are rarely attacked by it, and as yet there is no case on record of a European or half-breed suffering from it.

Geographical Distribution.—Broadly speaking, this disease is met with in its dark variety in Madras, Bombay, the west and north-west of India, whilst cases of the pale variety occur all over India. The Malabar coast and inland places near it are chiefly affected, and the disease is reported as being present at Pondicherry, Bellary, Tanjore, Guntoor, Madura (whence one of its names), Cuddapah, Trichinopoly, and Combacumum. It is also met with on the slopes of the western Ghauts, in Rutnagherry, Poonah, and other parts of the Bombay Presidency, as well as in Kattivar, Goojerât, and Cutch; in Kurrachee and other places in Scinde; in Bawalpur, Bicanir, and other parts of Rajpootana; and in the Punjaub at Jhelum, and the North-West Provinces at Sarsa and Hissar. It is very rare in Bengal, and the cases met with in Calcutta are all imported.

Remarks.—It is most difficult to refer the cause of this disease to definite physical phenomena. It appears, however, that it is associated with certain definite local conditions, although what these conditions are it is hard to say. At the places where it occurs there is a heavy rainfall, the altitude of the district is not high, and as a rule the soil is moist, dolometic, and rich in vegetable matters. At the same time, it must be noticed that at Cuddapah the soil is clayey limestone, that at Pondicherry it is clay, and at Tanjore and in the places where the disease is known on the Malabar coast, the soil is alluvial. It is highly probable that the disease has an intimate relation to the soil; those most affected by it are persons employed in agriculture, and who go barefooted, exposing themselves thereby to wounds on the feet, which would readily permit the tissues to be invaded by a parasite, if a parasite, as we believe, causes the disease.

XIII. Elephantiasis Arabum.

(See Plate IX.)

Synon.—Barbadoes Leg; Cochin Leg; Bucnemia indica; Pachydermia; *Fr.* Éléphantiasis; *Ger.* Elephantiasis.

Definition.—A non-contagious disease, characterised by recurrence of febrile paroxysms, attended by inflammation and progressive

hypertrophy of the integument and areolar tissue, chiefly of the extremities and genital organs; occasionally by swelling of the lymphatic glands, enlargement and dilatation of the lymphatics, and in some cases by the co-existence of chyluria, and the presence in the blood of certain nematoid hæmatozoa; together with various symptoms indicative of a morbid or depraved state of nutrition.

Geographical Distribution.—Although elephantiasis may be occasionally seen in all parts of the world, it is endemic in circumscribed areas, in tropical and sub-tropical countries; in these areas of distribution it is not uniformly present, but is almost always limited to well-defined foci. In India, the disease is frequently met with along the littoral of Lower Bengal, in Pondicherry, and at a few other points on the Coromandel coast. It is especially frequent in the district of Tanjore, but most of all on the Malabar coast (principally in Travancore and Cochin). It occurs at Ramghar, Chota-Nagpore, Sirgooja, and in the district of Tirhoot. In Ceylon the headquarters of the disease are on the coast, especially between Colombo and Matura. In the East Indies the places most severely affected are Sumatra, Banka, the Nicobars, and Philippines. In Further India it is met with in Penang and Cochin-China. In China elephantiasis is principally seen on the southern and south-eastern coast districts, especially at Canton, Amoy, Shushan, and Shanghai. Some of the worst regions of endemic elephantiasis are to be found in the Polynesian Archipelago, *e.g.*, the northern part of New Caledonia, the Tonga and Fiji groups, the Samoa group, Wallace Island, the Society Islands, especially Tahiti, and Raiatea, and the Gambier group. Elephantiasis is less frequently seen in the Marquesas and Hawaiian islands. In equatorial and sub-tropical Africa and the adjoining islands, elephantiasis is very common, especially in Réunion, Mauritius, Seychelles, Madagascar, Nossi-Bé, the Mozambique and Zanzibar coasts, the coasts of Senegambia and Liberia, and the Guinea coast as far as the equator. Further inland elephantiasis is met with in the Cameroons, in Bornu, and Sego; and a few cases occur in Tunis, Algiers, and Egypt, not far from the sea coast, and in the swampy valleys of the interior of Abyssinia. Throughout the whole of the upper Nile valley and the adjacent districts isolated cases only are met with, save in the Bari and Madi districts, where

it is more frequently seen. It is said to be often found to the west of Lake Nyassa. In the western hemisphere elephantiasis is met with in New Granada, Venezuela, and Peru, in those parts of Brazil which are mostly tropical in character, on the coasts and marshy levels of Guiana, on the Gulf coast of Central America and of Mexico. Elephantiasis is also seen in the following islands of the West Indies:—Barbadoes, Martinique, Guadaloupe, Trinidad, St Vincent, and St Bartholomew.

Remarks.—Although sporadic cases of elephantiasis are met with occasionally in Turkey, the south of France, Lisbon, and the south of Spain, as well as on the east coast of Scotland and in some parts of the south of Ireland, the endemic area of elephantiasis is from 35° N. to 25° S. in the eastern hemisphere, and 25° N. to 30° S. in the western. We must, therefore, consider that the disease for the most part depends upon high temperature and much atmospheric moisture for its production. Where cases occur outside the limits indicated above, it is in connection with moist soil and humid atmosphere, such as is met with on sea coasts and the banks of rivers. Climate not only appears to influence its production, but variations in temperature undoubtedly bear some relation to its growth, and some observers have maintained that it has a lunar periodicity. At any rate, as Hirsch says, "the more flat and damp the ground is in a tropical or sub-tropical piece of country, the more suited does it seem to be for the endemic existence of elephantiasis." Various theories have been advanced to explain the production of this disease. It has been said to be due to fish forming the staple of diet, to the drinking of water rich in saline constituents or tainted by organic matters; and others have thought that it is a form of malarial poisoning, but there are numerous facts which prevent these views from obtaining general assent. It is, however, agreed by nearly all observers that the disease attacks principally the male sex, of dark races, over twenty years of age. Before concluding this note, it may be remarked that there is an increasing number of observers who believe that the cause of elephantiasis is the *filaria sanguinis hominis*; the maps show how far the distribution of this parasite is identical with that of elephantiasis.

XIV. Guinea-Worm.

(See Plate X.).

Synon.—Dracunculus; Filaria medinensis.

Definition.—The Guinea-worm is a nematoid parasite, usually measuring from 1 to 3 feet in length, and having a breadth of $\frac{1}{10}$ of an inch. It infests the feet and legs, as well as other parts of the body that are much exposed.

Geographical Distribution.—As a general rule, the Guinea-worm is only found in the tropical parts of the eastern hemisphere, and even there the area of its endemic distribution is limited. It may, however, be conveyed from place to place, and, although rarely, become propagated in a fresh locality. In Africa, the principal area in which the Guinea-worm is found extends from Senegal as far as Cape Lopez. In Senegambia it is met with, not only on the coast, but in that more elevated region which extends from Bakel to Galam, but the parasite does not infest the banks of the Casamance. The Sierra Leone coast is less extensively infested by Guinea-worm than the Grain coast, Ivory coast, Gold and Slave coasts; it is met with on the shores of the Niger and Gaboon. It is to be noticed that on these coasts various places, such as Cape Coast Castle, Elmina, Cormantia, and Accra, are especially affected, whereas the surrounding country is very often free from the parasite. The Guinea-worm is found throughout Sennaar, the southern district of Kordofan, and the whole of the Bahr-el-Ghazel, the district between Dem Suliman and the Sobat being very extensively affected. It is doubtful if it exists south of latitude 6° N., and in Abyssinia the parasite is limited to the sea coast.

In Asia the endemic seats of the Guinea-worm are Arabia-Petræa, a few points on the coast of Hedjaz and Yemen, and the south coast of Persia. It is said to have been met with in the Bay of Skanderoom, and it is known in some parts of Turkestan, in Khiva, Bokhara, and Kokaun, on the shores of the Sir-Daria and on the northern shore of the Caspian (lat. 47° N). In India the Guinea-worm is most widely diffused in the northern division of the west coast, the Rajpootana States, and the western parts of the Deccan. It rarely occurs in the North-West Provinces, where it is only known at Dehra Dhun, Sirsa, and Hansi. The parasite is also rarely met

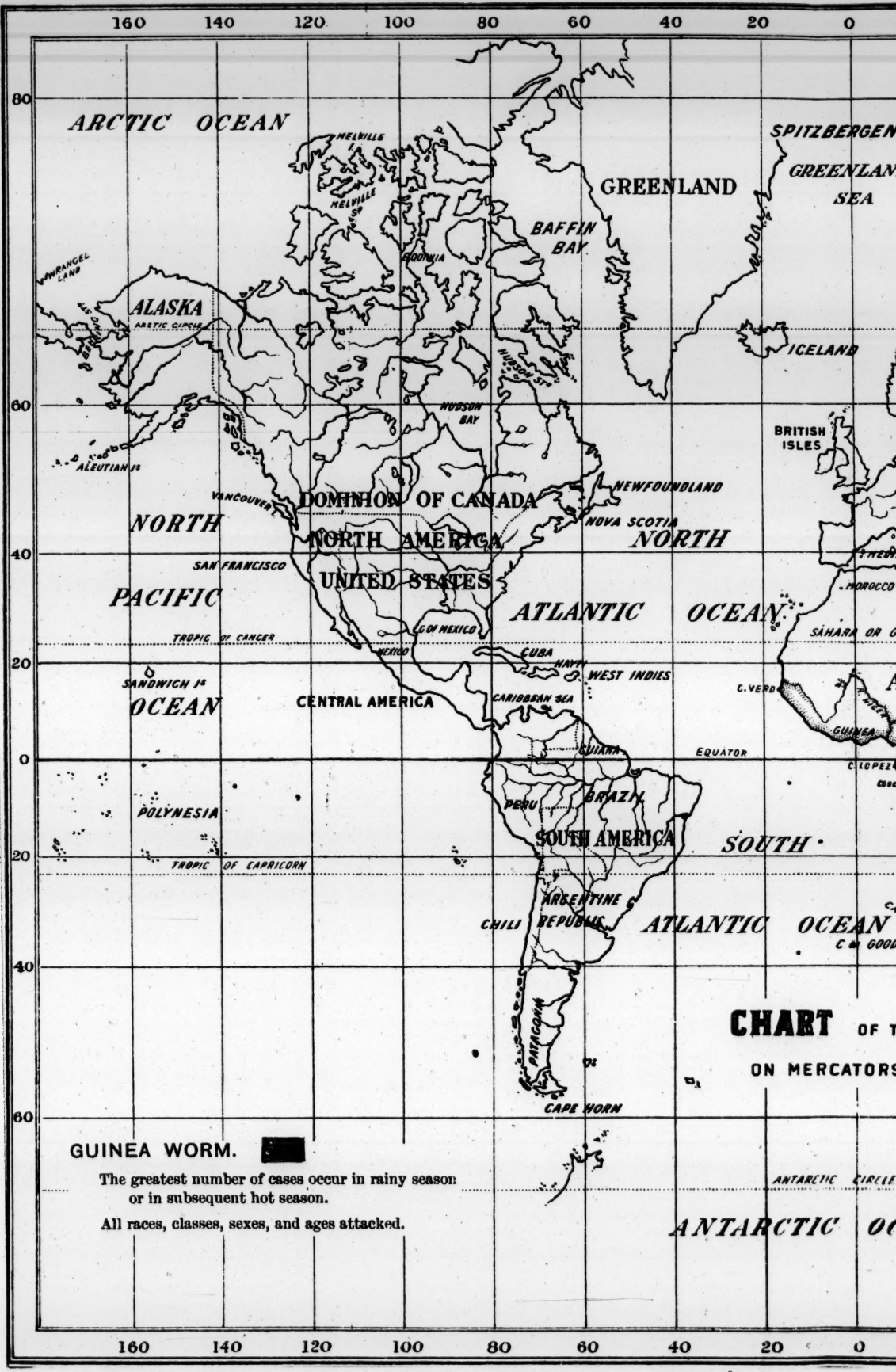


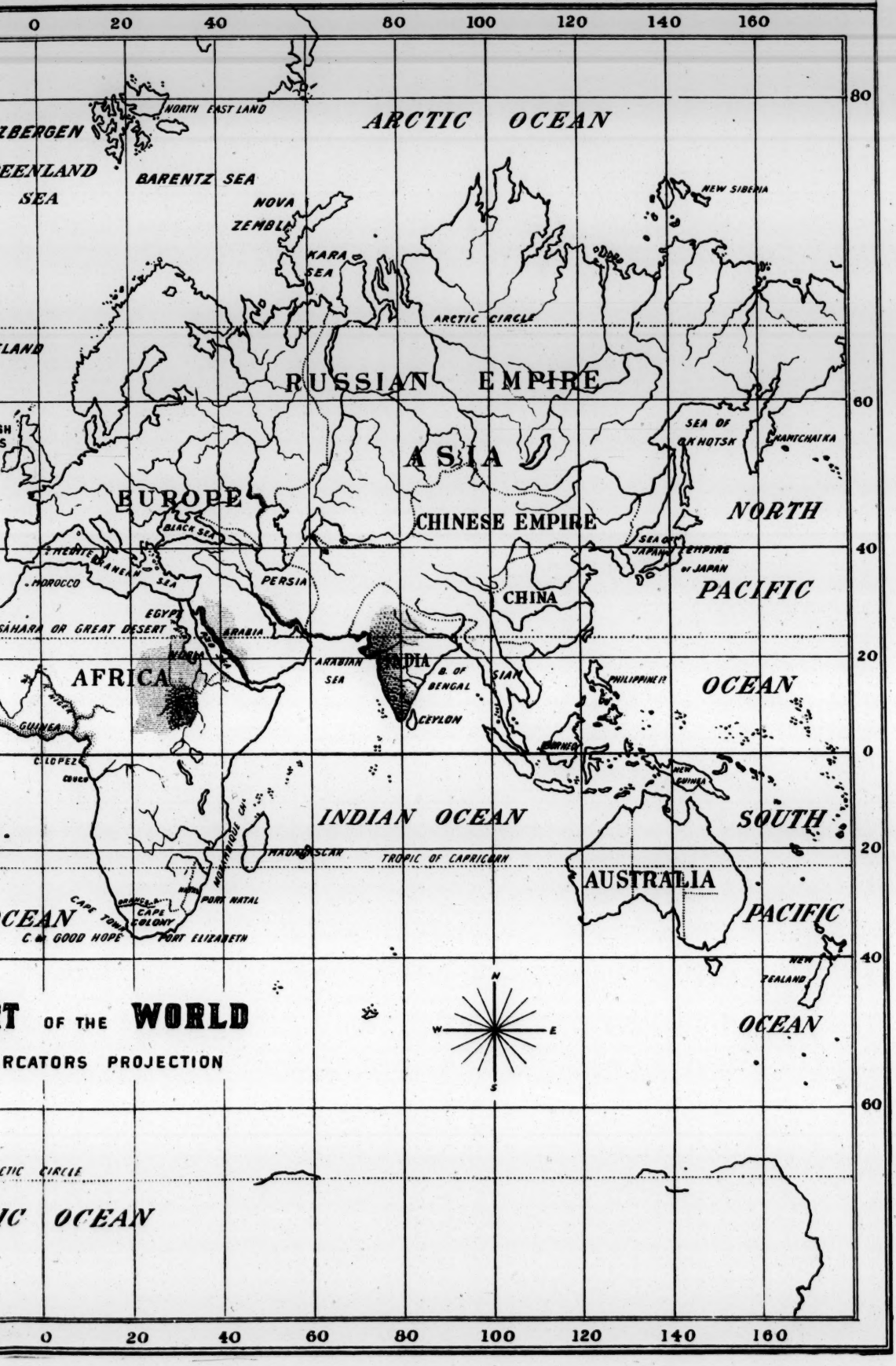
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GUINEA WORM.

The greatest number of cases occur in rainy season or in subsequent hot season.

All races, classes, sexes, and ages attacked.



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with on the coast of the Madras Presidency, being chiefly seen near Madras and Pondicherry. It is known too in the plain of the Carnatic, from Mysore between the eastern and western Ghauts towards Cape Comorin. On the western sea-board the worm is found in the Bombay Presidency from lat. 18° N. up to Goojerat, at Rutnagherry, Matunga, Bombay, and Daman; also at Baroda, Caira, and Jambosir, and at Bhooj in Cutch. Great centres of the disease are met with in the Mewar and Marwar (Rajpootana States), in the district of Chanda, at Dhoolia in Khandeish, at Nagpore, in Behar, at Aurungabad, Jalnah, Hyderabad, and Secunderabad, on the east side of the western Ghauts and in the adjoining districts of the Deccan, where it is especially prevalent at Ahmednuggar, Jedjhuri, Baramati, Poona, Satara, Aculcota, Tasgoon, Miraj, and Beejapore, the district of Savant-Warri, Balgâm, Darwar, and Bellary.

In the New World the disease was imported from the west coast of Africa into Guiana, Brazil, and the West Indies, but it has almost disappeared except at the island of Curaçoa, and in the small town of Feira da Santa Anna in the province of Bahia.

Remarks.—It is useful to remember certain facts with regard to Guinea-worm. In the first place, all races of both sexes and at all ages may suffer from it, but it certainly attacks by preference natives who go about barefooted, and it is most usual in middle life. In all probability it gains entrance into the body by direct contact of an exposed part of the body with water or mud, the parasite penetrating the pores of the skin. There have been many attempts made to connect the occurrence of the Guinea-worm with the geological features of the soil where it is found, but, although on the whole it is most prevalent in localities composed of secondary trap rock, it is also met with where the geological structure is sand on sandstone. There is a certain relation between heat and moisture and the production of the parasite. It is found in places where there is a mean temperature in summer of about 88° F. On the whole, the Guinea-worm is most frequently met with at the beginning of the rains, and in the hot weather succeeding them, although it appears to be the case that in India it is most usually found during the rains. It is impossible to say how long is the period of the incubation of the Guinea-worm, but it probably varies from one month to a year, and with regard to the part of the

body in which it makes its appearance, in at least 95 per cent. of the cases, it is met with in the lower extremities. In a person suffering from Guinea-worm the first symptom usually noticed is a cord-like substance felt beneath the skin. There is usually some pain, and more or less fever, the temperature of the patient being sometimes exceedingly high. As the worm commences to make its exit from the body, a small blister forms at the point it selects for its escape; this blister is often surrounded by a distinct rash.

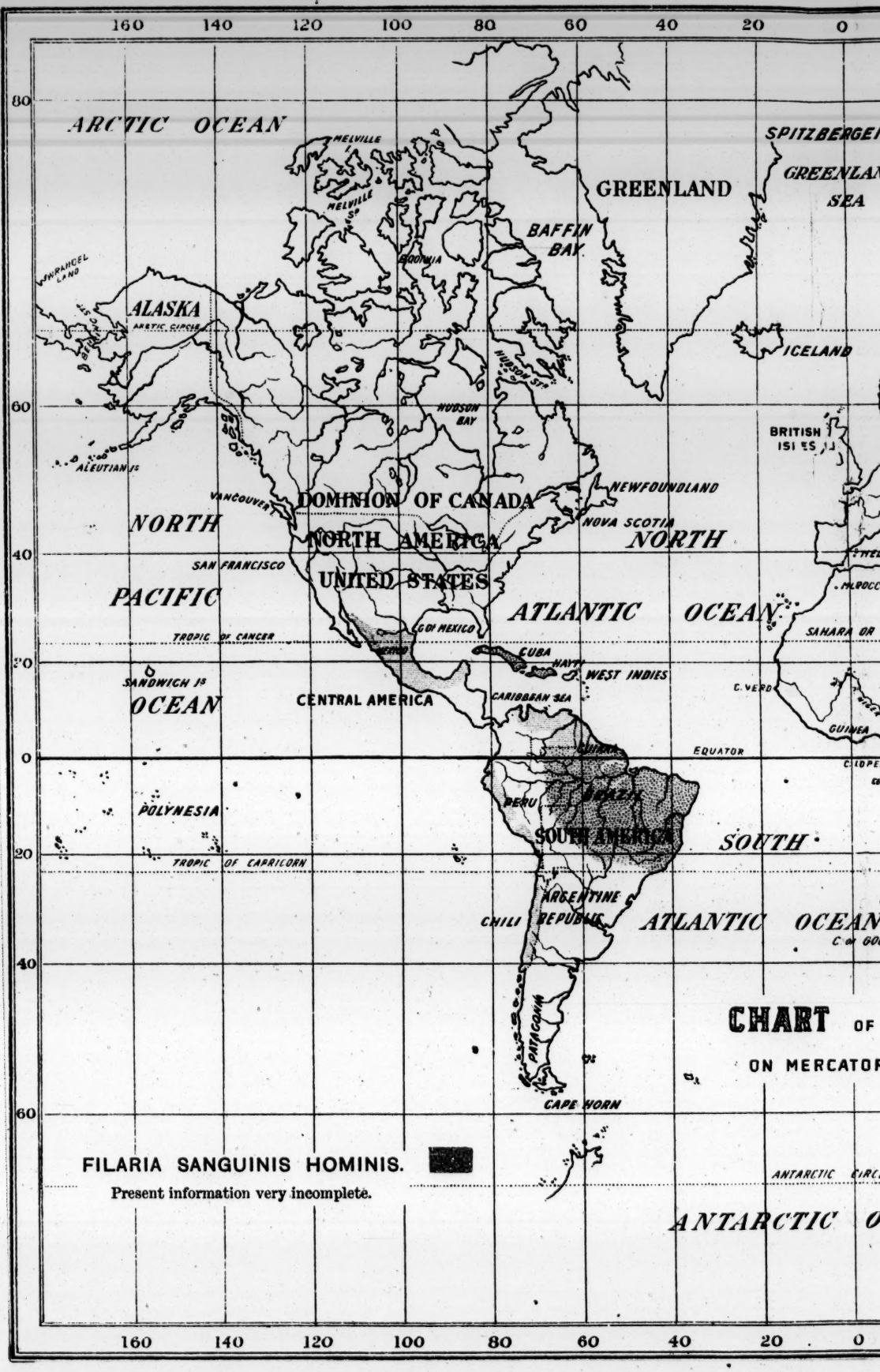
XV. Filaria Sanguinis Hominis.

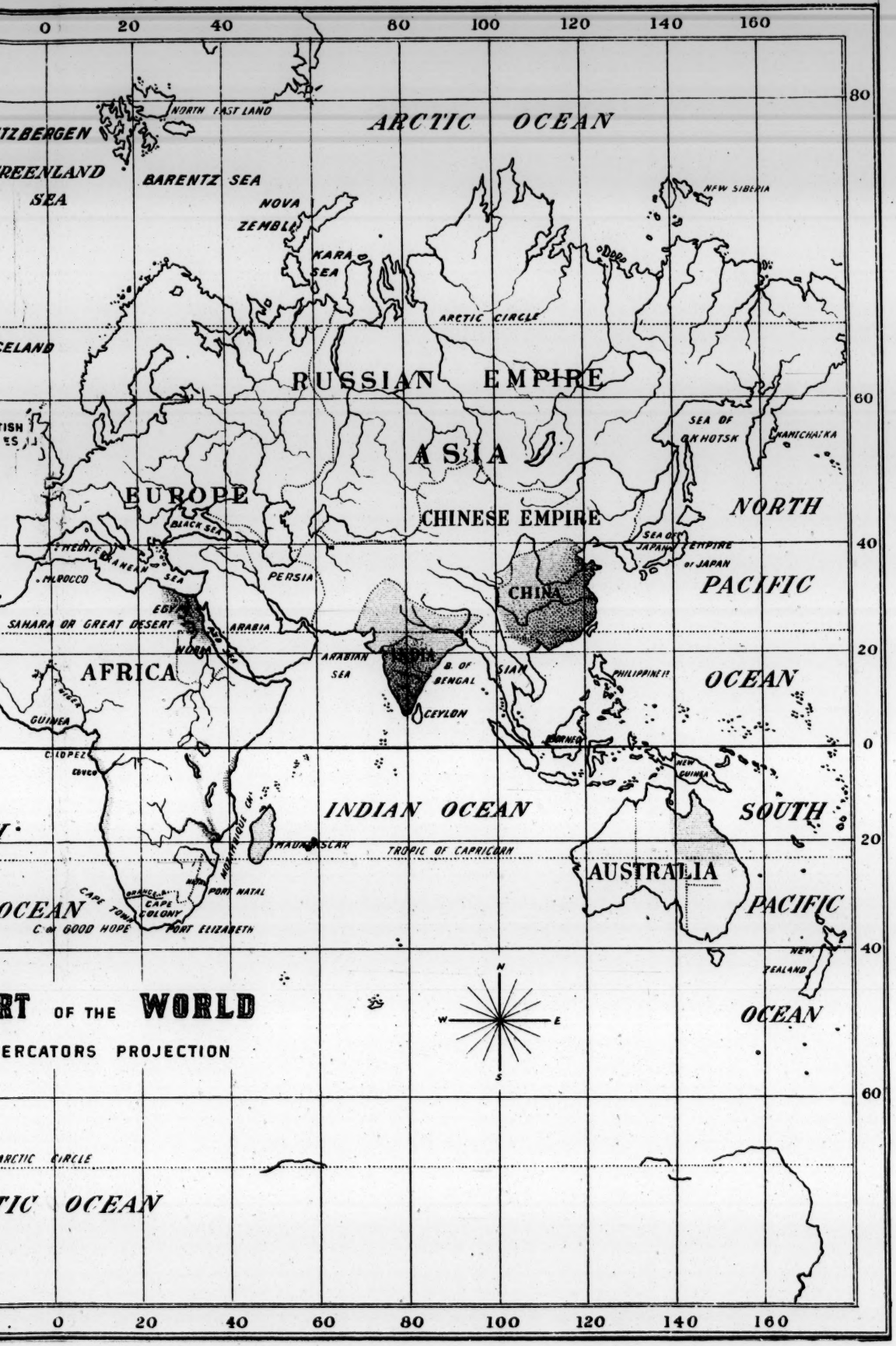
(See Plate XI.)

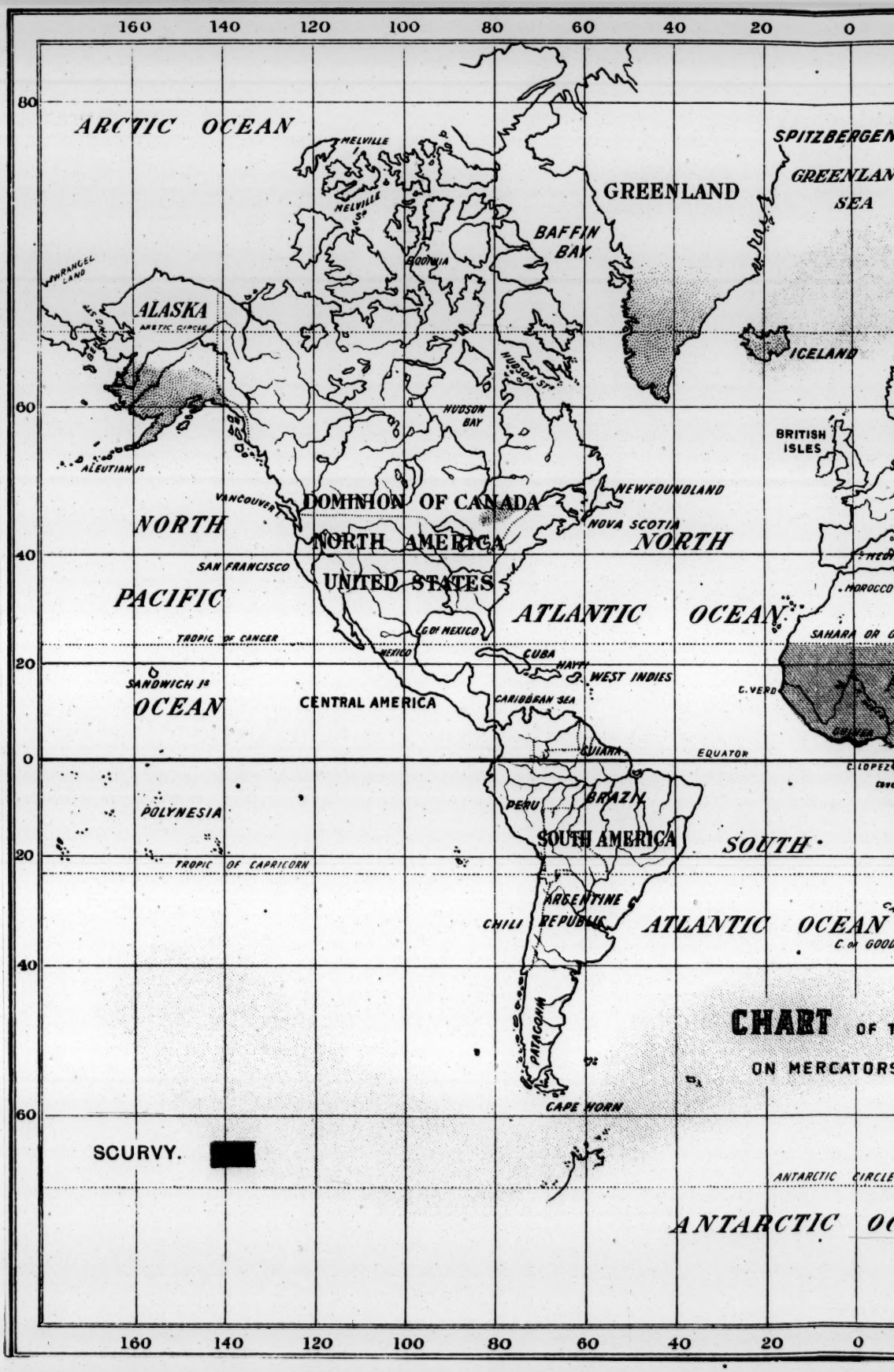
Definition.—The *Filaria sanguinis hominis* is a nematoid hæmatozoon, about the thickness of a hair, and from 3 to 4 inches in length, which is found in the blood of some animals and men.

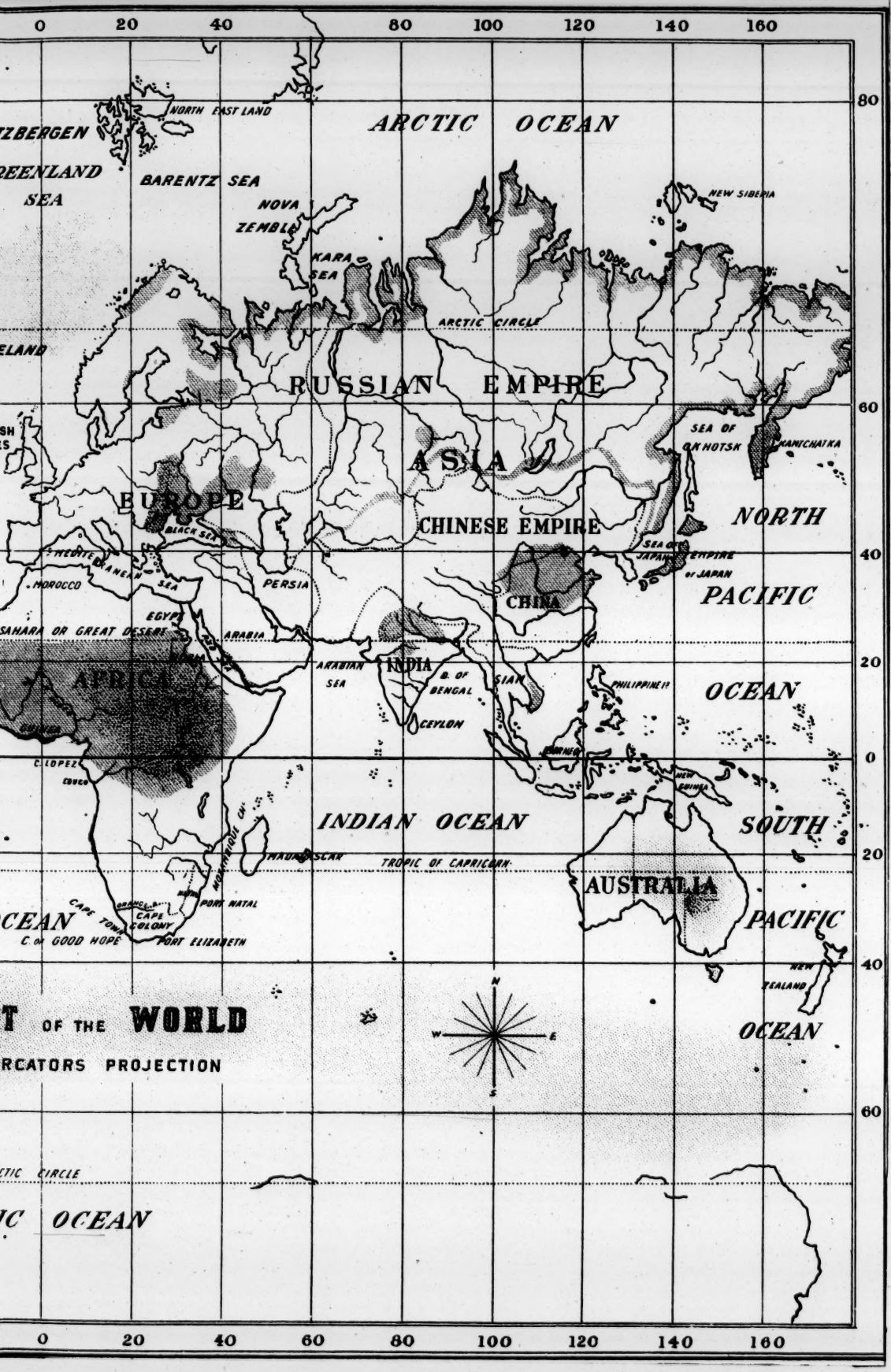
Its geographical distribution is of importance, because when the parasites gain entrance into a human being, in the tropics at any rate, they frequently induce chyluria, lymphorrhagia, elephantiasis, lymphangiechodes, chylous hydrocele and varicocele, and possibly also true elephantiasis arabum. It may be incidentally remarked, that the filaria are almost invariably totally absent from the blood during the day—*i.e.*, from 9 A.M. to 6 P.M. They commence to appear between 6 or 7 in the evening, rapidly increase in number until their maximum is reached about 2 or 3 A.M., and then gradually disappear.

Geographical Distribution.—As far as we are yet aware, the *Filaria sanguinis hominis* occurs principally, if not entirely, in tropical regions. It is known to be indigenous in the following regions, but it is not improbable that future observations will show its distribution to be rather greater than that which we can now give to it. Commencing with Brazil, it is most extensively distributed throughout the tropical parts of that country, excepting however the southern districts and the province of Sta-Caterina. In all probability, too, it occurs in Chili, Peru, Venezuela, and Mexico, as also in Guiana. It certainly occurs in Barbadoes, Cuba, Mauritius, St Domingo, and St Thomas. In Africa it is found in Egypt, on the shores of the Zambezi and Lake Nyassa, the Zanzibar coast, Mauritius, and Réunion; it is probably indigenous on the west coast, and it has been met with in various districts of the Negro part


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of the Egyptian Soudan (Bari and Madi districts, Bahr-el-Ghazal, and Uganda). In Mayotte and Madagascar it almost certainly occurs, and it probably exists in Queensland. The *Filaria sanguinis hominis* is extensively found throughout China and India, but curiously enough, the Dutch East Indies and Formosa are free from it.

Remarks.—From what has been said, it will be seen that the habitat of the *Filaria sanguinis hominis* is confined to the tropical or sub-tropical regions, and it is also of great interest to notice that the mosquito forms its intermediate host. As was mentioned above, the *Filaria* are found during the night in the blood, and consequently in the subcutaneous capillaries, where the mosquitos are able to reach them. In the insect's stomach they undergo developmental changes, and they are probably discharged with the larvæ of the mosquito into drinking water, and by this means again conveyed into the human subject.

XVI. Scurvy.

(See Plate XII.)

Synon.—Scorbutus; *Fr.* Scorbut; *Ger.* Scharbock.

Definition.—Scurvy is characterised clinically by intense general debility; sponginess and swelling of the gums; ecchymoses closely resembling bruises, about the thighs and legs; a brawny hardness about, and sometimes a contraction of, the muscles of the calf; pearly conjunctivæ; and a sallow aspect, somewhat akin to mild jaundice.

Geographical Distribution.—Apart from the occurrence of scurvy at sea and amongst troops on campaign, and outbreaks of it, which occasionally occur all over the world in prisons, and among emigrants who have rapidly settled in given districts, scurvy is still endemic in some parts of the world. In Russia we find one of the chief seats of the disease at the present time. It is endemic in the Baltic provinces and at St Petersburg, in the governments of Olonetz and Novgorod, along the shores of the Arctic Ocean and other parts of the Siberian littoral, such as the Amoor region and at Kamtchatka. It is endemic in Asiatic Russia, along the Chinese frontier, and also at Tomsk. It is met with in the government of Kasan, but more especially in the southern provinces of the empire—

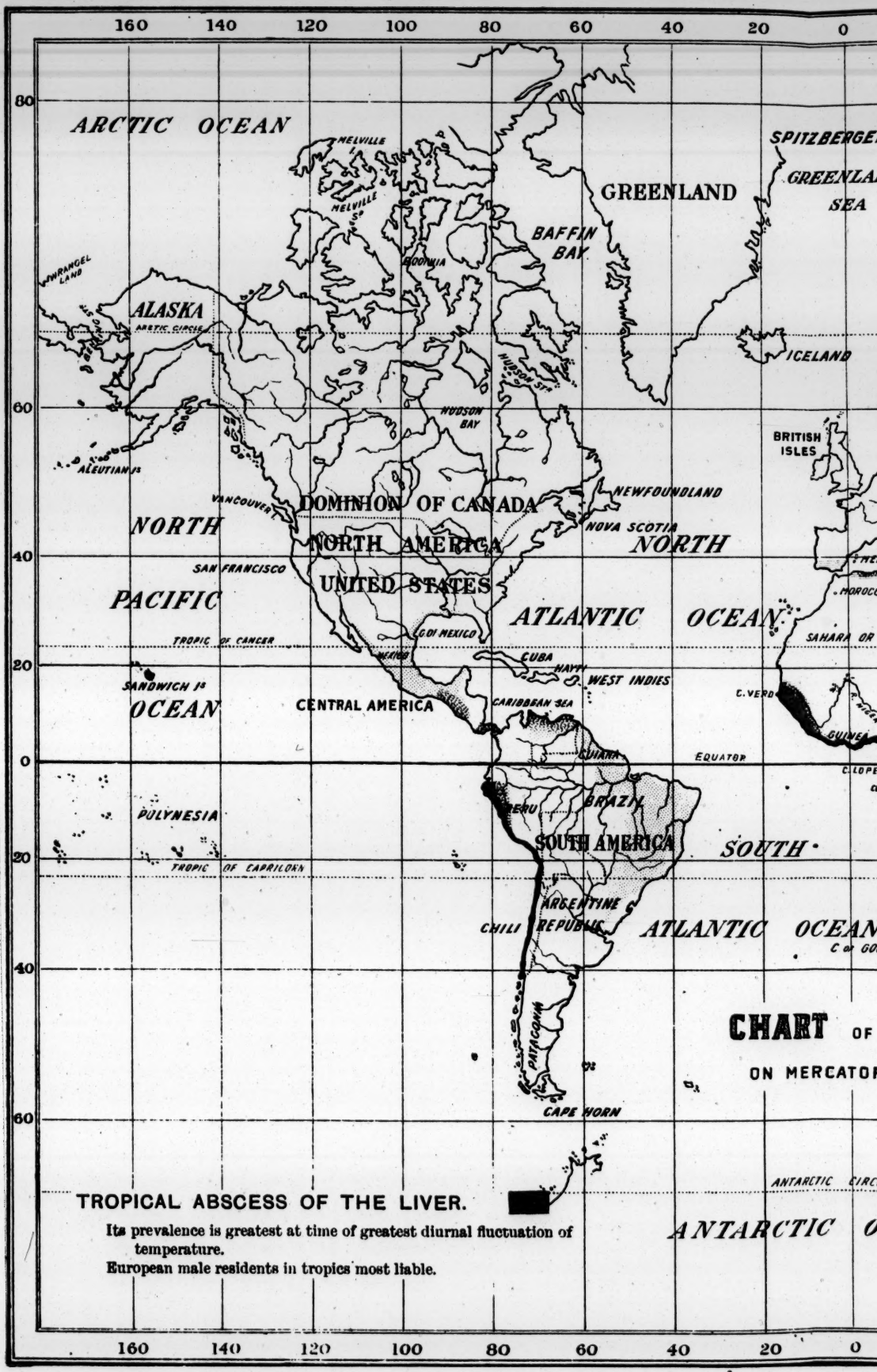
e.g., Jekaterinoslav, the Steppes of Saratov, the Ukraine, the adjoining districts of western and little Russia, and in the Crimea; also in Kutais (Trans-Caucasia). The area of endemic scurvy in southern Russia joins an endemic district in Roumania. In Sweden scurvy may be said to be endemic only in the neighbourhood of Umea, in the districts of Udewalla and Jemtlandslän; and in Norway it is endemic in Finnmarken.

Scurvy is endemic in various countries in Asia. We meet with it on the Yemen coast of Arabia, especially at Aden; in some parts of India, *e.g.*, North-West Provinces, Rajpootana, and Malwa; in Cochin-China, the northern part of China, particularly at Pekin, and in Japan. In Australia it is endemic in the north-west of New South Wales; and it occurs endemically on the Darling Downs.

In Africa scurvy is met with in the eastern Soudan, and throughout nearly the whole of the rainy zone of Africa, especially during the overflow of the Nile. On the west coast of Africa it is also endemic in Benguela, and on the Gold and Ivory coasts.

In the western hemisphere, endemic centres of scurvy are found in the most northern latitudes, in Greenland, Alaska, and the Ottawa district of Canada.

Remarks.—There are several points in regard to the geographical distribution of scurvy which are well worthy of notice. Apart from its production in jails and camps, during sieges, and in armies on the march, it is also met with amongst explorers and at sea. It is seen too very frequently in damp low-lying localities, where the soil is highly impregnated with saline matter, chiefly nitrate of potash, and where consequently the production of vegetables is scanty; or else in arid regions where cultivation is not practised. A number of general causes, apart from mere locality, predispose to this disease. Malaria, fatigue, a humid atmosphere, great variations in temperature, bad water, insufficient clothing, and residence in crowded or ill-ventilated rooms or cabins, all seem to invite the onset of the malady, although, as a matter of fact, its absolute occurrence is due to the want of fresh anti-scorbutic vegetables or of lime-juice which, when not too old, takes their place.



TROPICAL ABSCESS OF THE LIVER.

Its prevalence is greatest at time of greatest diurnal fluctuation of temperature.

European male residents in tropics most liable.


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XVII. Tropical Abscess of the Liver.

(See Plate XIII.)

Synon.—Hepatic Abscess; *Fr.* Abscés du Foie; *Ger.* Leberabscess.

Definition.—An abscess of the liver, due to hepatitis, sometimes, but not always, associated with malaria or dysentery.

Geographical Distribution.—Tropical abscess of the liver is met with all over the Indian Peninsula, except in the very highest altitudes, but it is most frequent in the Madras Presidency. It also occurs in Ceylon, in Burmah and in the Peninsula of Malacca. In Cochin-China, the Chinese ports, and in the southern and subtropical parts of Japan it is not so frequently met with, and it is exceedingly rare in the northern parts of Japan. The chief seats of the disease in the East Indies are the coast of Java, Sumatra, and Borneo, and the island of Luchon (Philippines). It is much less frequently found in Banka, Celebes, Moluccas, the Riouw-Lingga Archipelago, and the Andamans. It occurs in Polynesia, especially in New Caledonia, and more rarely in the Hawaiian Islands and Tahiti. Tropical abscess of the liver is very prevalent in the tropical parts of Persia and Arabia, especially in the neighbourhood of the Persian Gulf and Red Sea.

In African regions the disease is most often found in Réunion, Mauritius, Madagascar, Mozambique, Nubia, and Egypt; less frequently on the Zanzibar coast; it is very prevalent in Senegambia, on the coast of the Bight of Benin and the Bight of Biafra, on the Slave and Gold Coasts, and on the island of Fernando Po. But along the coast further south it becomes less prevalent, and is extremely rare south of the Congo.

In South America it is found in Chili, especially in the north, in the coast and forest regions of Peru, in Venezuela, and rarely in Brazil and Guiana. It occurs in Panama, and is said to be known in Costa Rica, Guatemala, and Salvador. The disease is sometimes to be seen in Mexico, more especially on the west coast.

In Europe abscess of the liver occurs in Turkey and Greece, southern Italy, and southern Spain.

Remarks.—Although abscess of the liver is occasionally met with in temperate zones, it is of endemic occurrence only in the tropics.

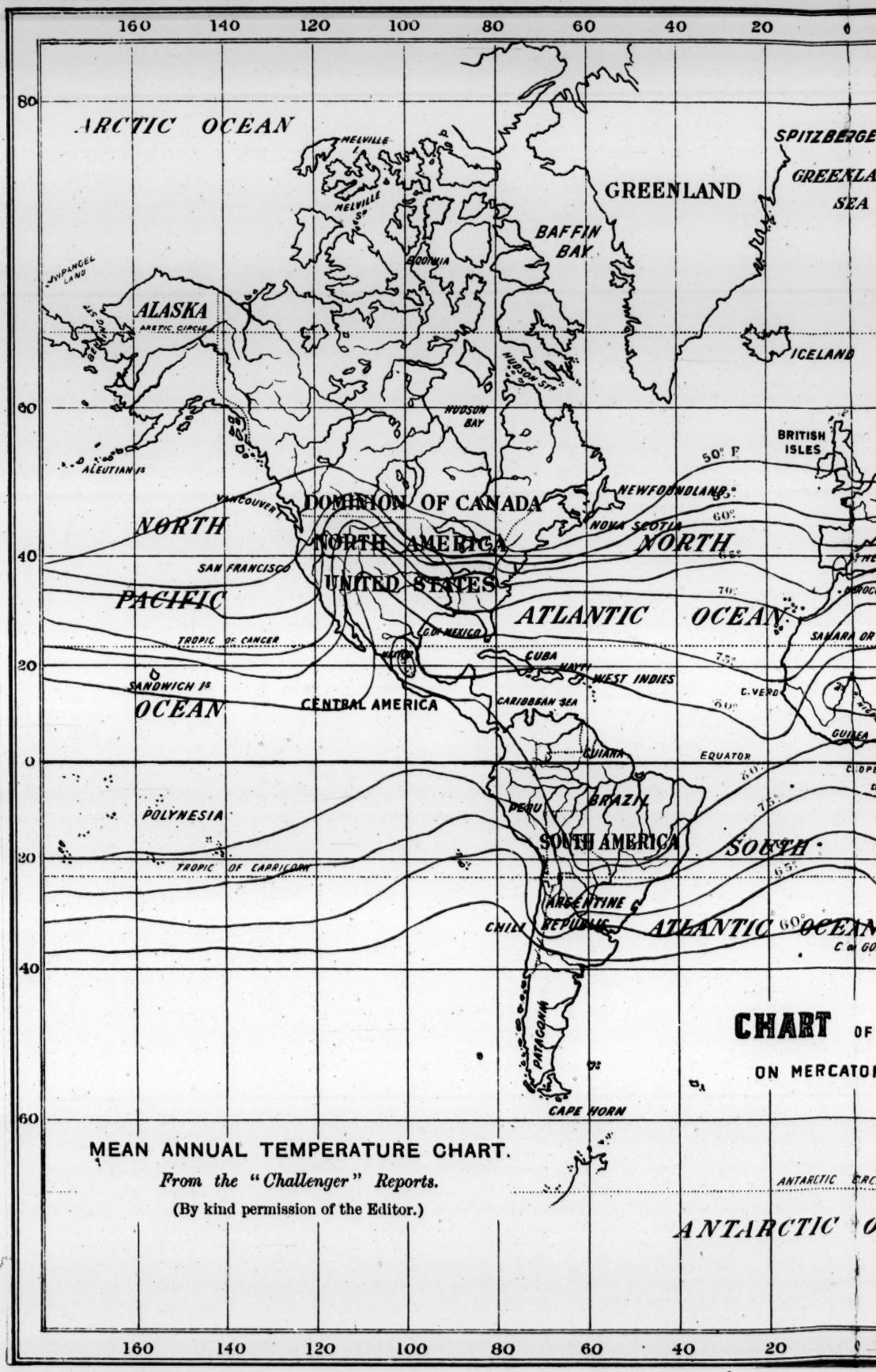
The statistics showing the frequency of the disease are chiefly based upon the records which refer to Europeans in the tropics, but it also affects the natives of tropical places.

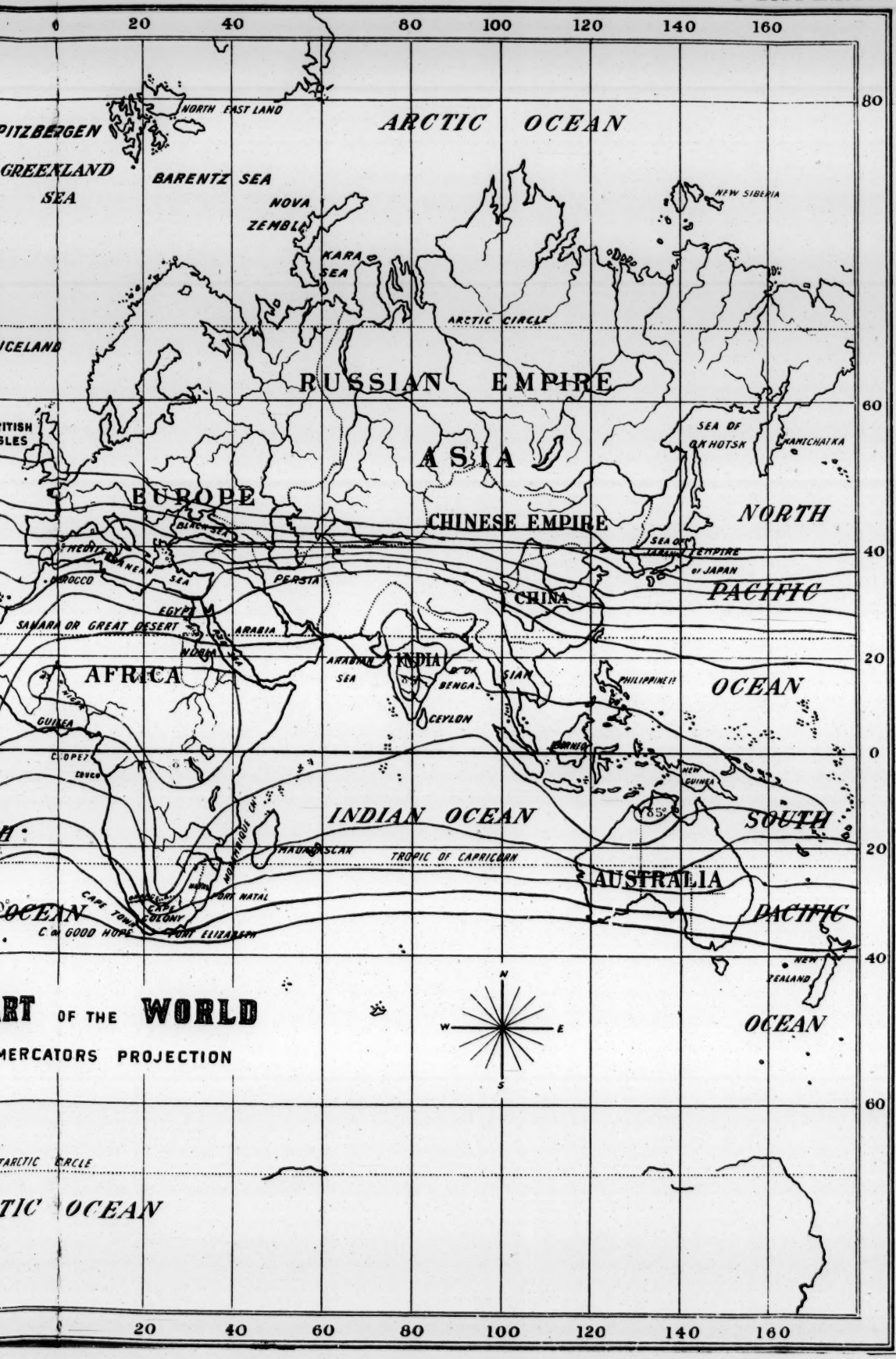
There is no doubt that congestion of the liver in the tropics is brought about by heat and malaria, and that it most frequently occurs in the cold or rainy season. It is to a great extent caused by the great diurnal fluctuations of temperature, the days being very hot and the nights cold, but this is not the only cause. Malaria may, and probably does, predispose to tropical abscess of the liver, on account of the congestion of the liver caused by it, and yet the areas of distribution of the two diseases do not coincide. Alcohol is another cause of the disease in question; this is shown by the fact that it occurs chiefly in Europeans, and in Europeans of the male sex; also that natives who ape European customs of drinking suffer severely, and that Mohammedans are very rarely affected.

It was at one time very generally thought that dysentery was the great cause of tropical abscess, but this is not the case. The two diseases do occur together sometimes, and abscess of the liver may cause dysentery, but many things go to prove that dysentery is not the cause of abscess of the liver. In very many instances *post-mortem* examinations have revealed no intestinal lesion at all in the case of deaths from abscess of the liver. Dysentery occurs in temperate climates, but is not there followed by abscess of the liver, as would be the case were it the cause of that disease. Again, in the tropics women suffer very rarely from abscess of the liver, but are as liable to dysentery as men are. Dysentery is very common in Egypt, whereas abscess of the liver is of very rare occurrence there. Lastly, children are very frequently the victims of dysentery, but not of abscess of the liver.

To what, then, is abscess of the liver due? To the action of excessive heat (and possibly too to the amount of moisture in the air), to chills caused by great diurnal fluctuations of temperature, and to the consumption of a too stimulating diet with regard to both food and drink.

A few general remarks must bring this paper to a close. I have endeavoured as briefly as possible to call attention to the various physical phenomena which may influence the production or spread


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MEAN ANNUAL TEMPERATURE CHART.
From the "Challenger" Reports.
(By kind permission of the Editor.)
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of the various diseases dealt with. Although it must be allowed that such factors as heat, moisture, the character of the soil, and so forth, do exert a great effect on the production of disease, it is most difficult to estimate justly or with accuracy the parts they severally play in the etiology of disease. A wide field for further research presents itself in this direction, and one of my objects in compiling this paper has been to draw attention to a subject which deserves close and extended investigation.

In looking at the maps which I have drawn, it will be seen, on comparing them with Plate XIV., that all the diseases depicted, with the exception of scurvy, are endemic in various areas where the highest mean annual temperature of the globe is to be found, and it will also be noticed that they are most prevalent in regions having a rainfall of over 50 inches in the year.

In investigating the etiology of tropical diseases, certain other factors have to be taken into account, factors which I have been obliged almost entirely to exclude from this paper, as, had I dealt with them, it would have involved considerable space and a much larger number of maps. The factors I refer to are the races inhabiting tropical regions, and their habits and customs, the geology and physical geography of the countries, and the character of the vegetation met with. The regions where the diseases I have included in my list are endemic are characterised by distinctive features with regard to food supply. They are chiefly, although not entirely, within the zone where a vegetable diet obtains, and where tropical grains and fruits are indigenous.

In conclusion, I should like to mention Dr Lawson's theory of pandemic waves of disease coinciding with the isoclinal lines which are found depicted upon Plate XIV. For instance, in regard to fever, he believes there is a factor which determines its appearance at points more and more northward in successive years, and he apparently proves that this factor, whatever it may be, revives periodically every second year, or at some multiple of two years, and then passes over a more or less extensive portion of the earth's surface, giving epidemic impulse on its way to various diseases, and finally disappearing in the north. He found that the position of these pandemic waves, as he calls them, could be defined from year to year, approximately at least by referring them to lines of equal

magnetic dip (isoclinal lines).* I think that this theory is well worthy of study.

I must reserve for the present any further elaboration of the subject of this paper, hoping to be able to resume it at a future time.

My thanks are due to Dr John Murray for permission to use Plate No. XIV., and to Dr Buchan, Dr Woodhead, and Mr J. G. Bartholomew for some assistance and advice. My indebtedness to authors are acknowledged in the text.

EXPLANATION OF PLATES.

PLATE I.

Chart of the world showing the area of endemic and epidemic malaria. The colouring depicts, as nearly as possible on such a small scale, the severity of the disease in various regions.

PLATE II.

Chart showing the endemic habitat of Dengue and also the area of its epidemic spread. The darker colour shows where the disease is endemic.

PLATE III.

Chart showing—I. The native habitat of Asiatic cholera ; II. The area over which pandemic waves of cholera have spread ; III. Regions to which cholera has never penetrated ; IV. Districts from which no information is obtainable.

PLATE IV.

Chart showing—A, the area over which yellow fever is endemic and epidemic ; B, the districts in which Oriental boils or sores are endemic ; C, the limited areas in Africa and in the Mauritius in which, so far as yet known, endemic hæmaturia occurs.

PLATE V.

Chart showing—A, the areas in which Beri-Beri is met with; B, the districts which have been visited by Oriental plague during recent years.

PLATE VI.

Chart showing the geographical distribution of tropical dysentery, its prevalence being indicated, as far as possible, in the colouring. Its epidemic occurrence in non-tropical countries is also shown.

PLATE VII.

Chart showing the distribution of leprosy at the present time, its greatest prevalence being distinguished by the colour.

PLATE VIII.

Chart showing—A, the geographical distribution of Yaws ; B, the region where the Fungus disease of India is met with.

* The *Milroy Lectures.* Lawson, 1888.

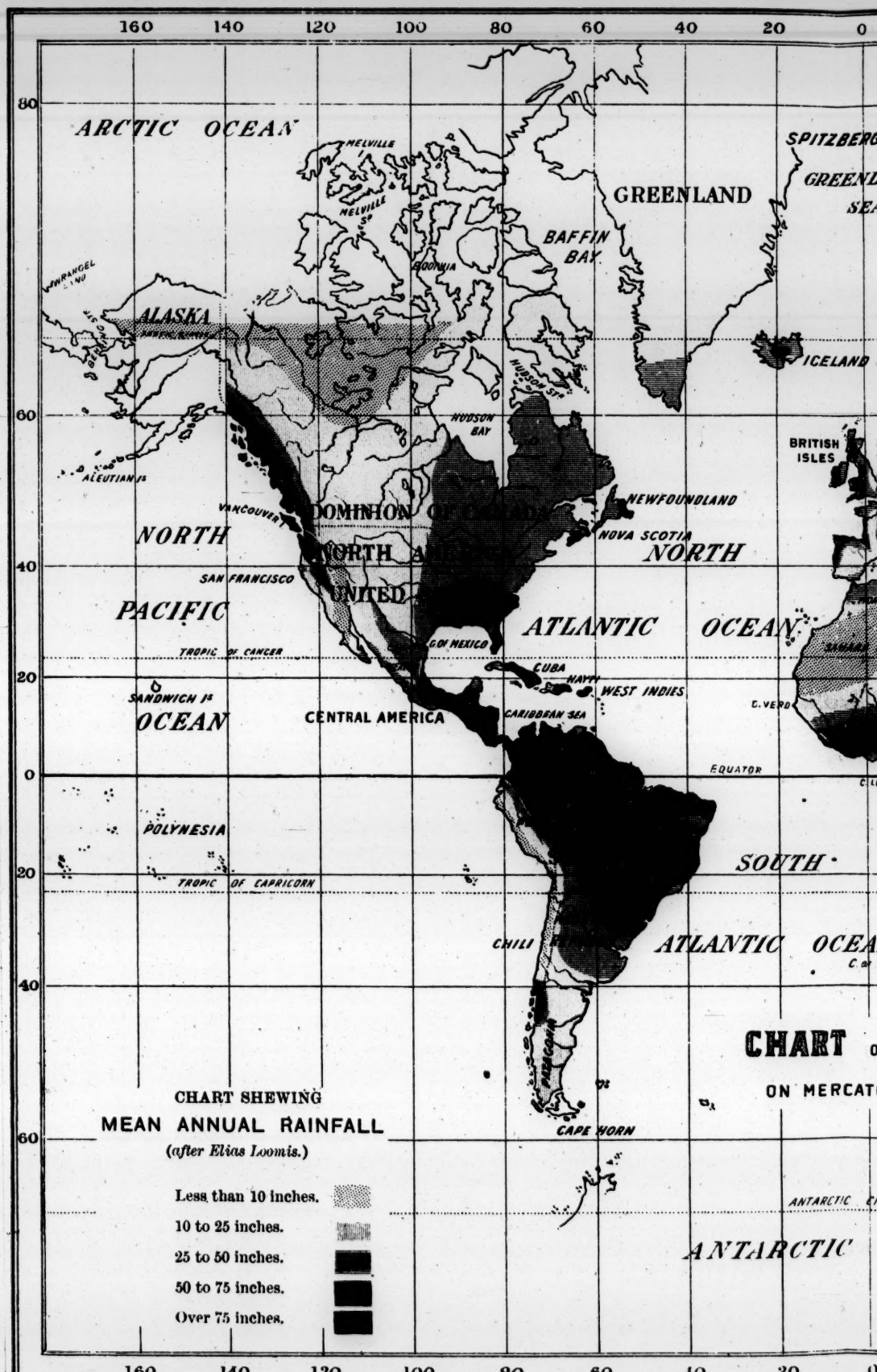
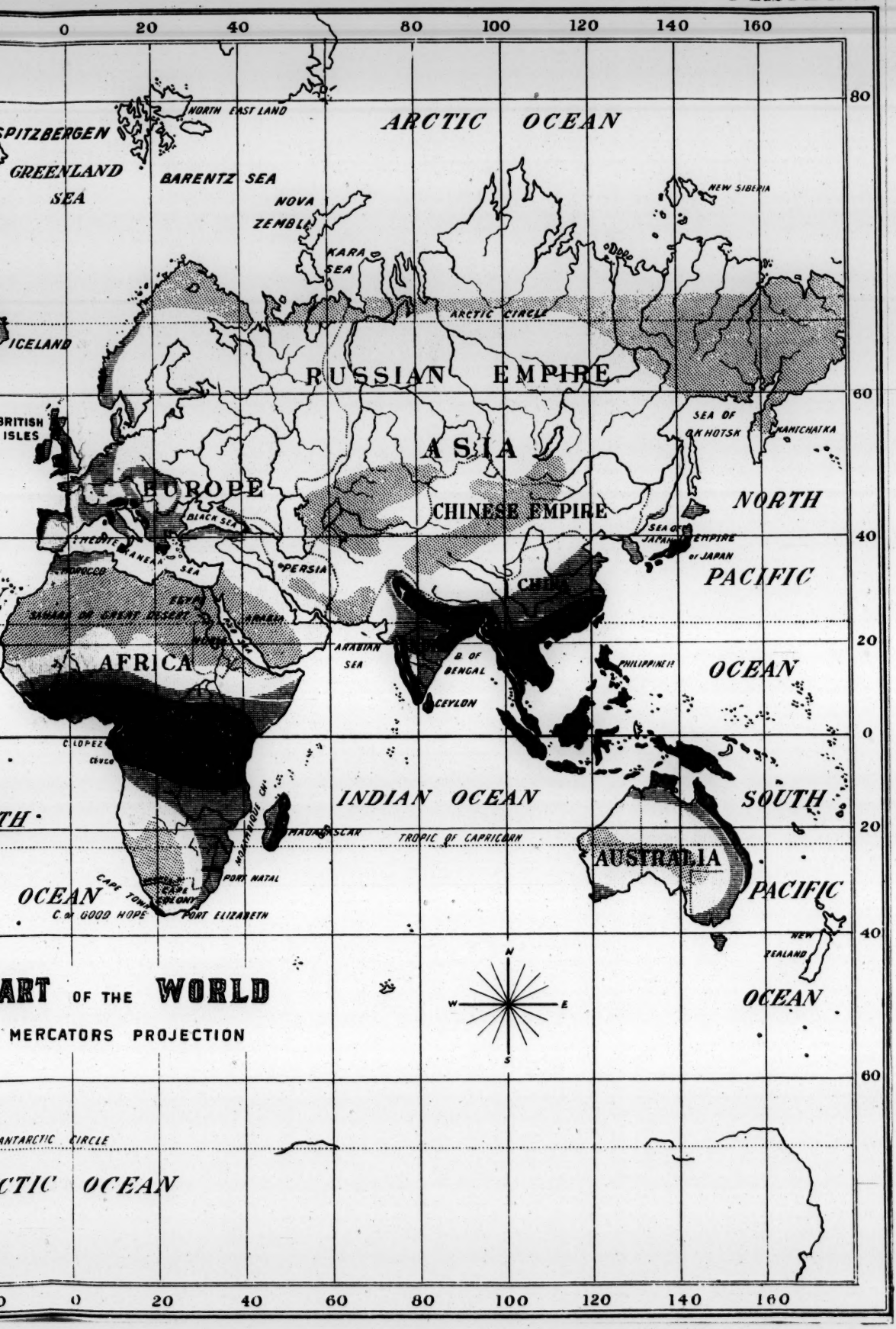

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NEWFOUNDLAND
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TROPIC OF CANCER
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WEST INDIES
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C. VERD
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POLYNESIA
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ATLANTIC
OCEA
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ON MERCAT
CAPE HORN
ANTARCTIC
ANTARCTIC
CHART SHEWING
MEAN ANNUAL RAINFALL
(after Elias Loomis.)
Less than 10 inches.
10 to 25 inches.
25 to 50 inches.
50 to 75 inches.
Over 75 inches.

PLATE XV.


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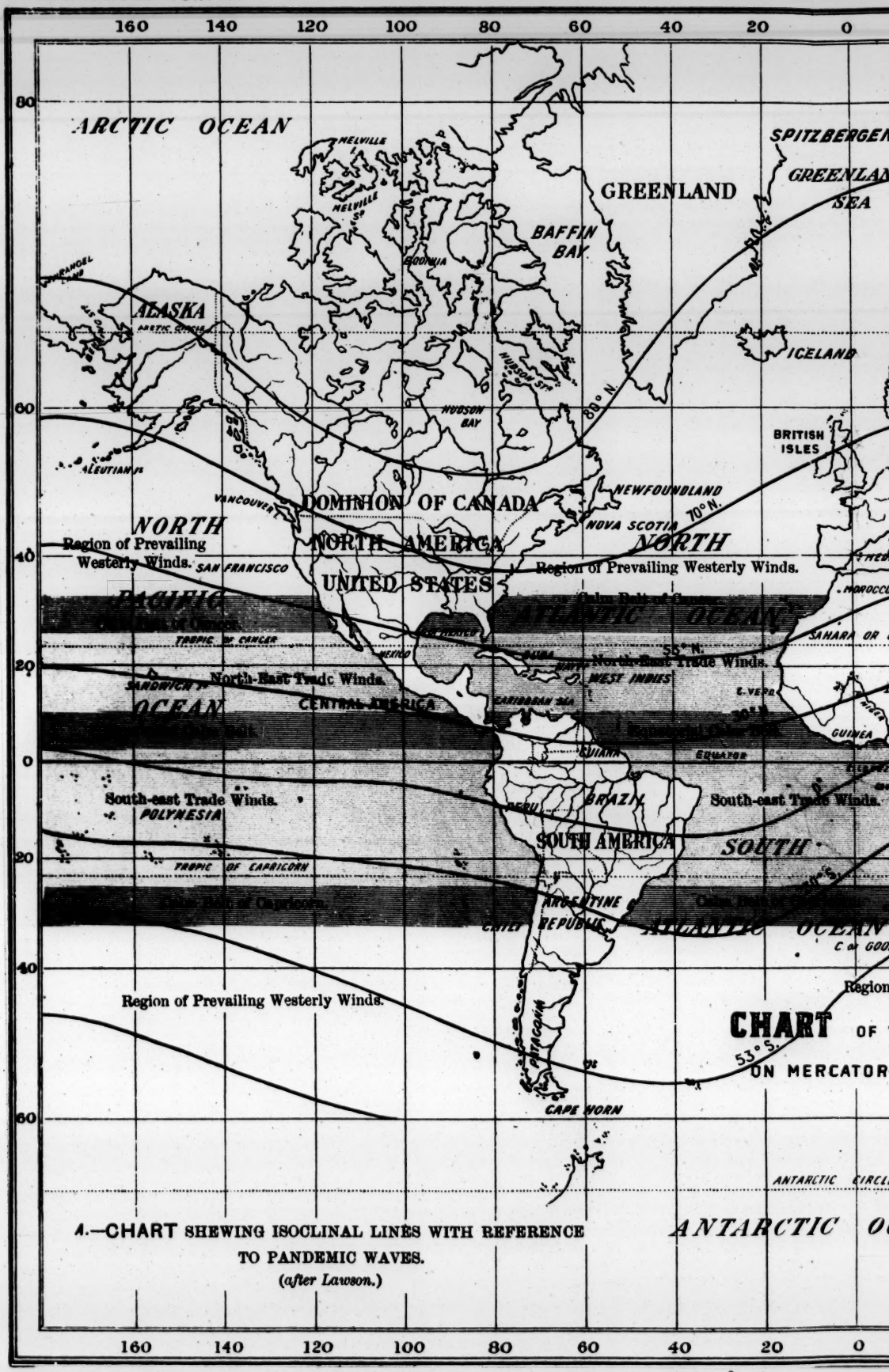
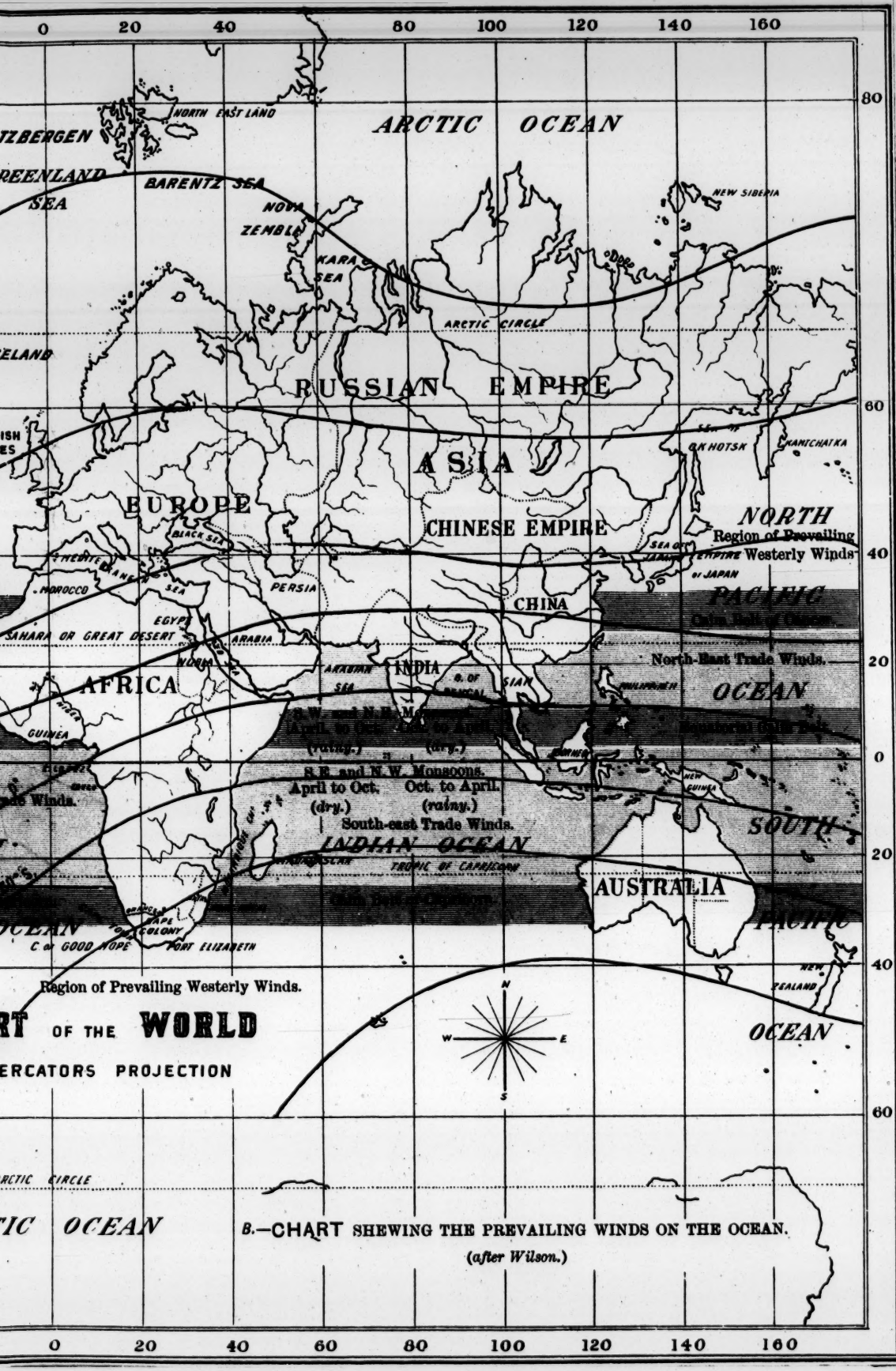


CHART OF
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A.—CHART SHEWING ISOCLINAL LINES WITH REFERENCE TO PANDEMIC WAVES.
(*after Lawson.*)



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ERCATORS PROJECTION

B.—CHART SHEWING THE PREVAILING WINDS ON THE OCEAN.
(*after Wilson.*)

PLATE IX.

Chart showing the geographical distribution of *Elephantiasis arabum* (Barbadoes leg).

PLATE X.

Chart showing the distribution of Guinea-worm.

PLATE XI.

Chart showing the geographical distribution of the *Filaria sanguinis hominis*, so far as at present known.

PLATE XII.

Chart showing the occurrence of scurvy on land, as also the seas in which it is still sometimes met with in badly-found sailing ships.

PLATE XIII.

Chart showing the distribution of tropical abscess of the liver. The regions in which it is most prevalent are indicated by a darker shade.

PLATE XIV.

Chart of the world illustrating the mean annual temperature of the tropical and sub-tropical zones.

PLATE XV.

Chart showing the mean annual rainfall throughout the world.

PLATE XVI.

Chart showing—A, the isoclinal lines with reference to pandemic waves of disease ; B, the prevailing winds on the ocean.

Quaternion Note on a Geometrical Problem.

By Professor Tait.

(Read June 4, 1889.)

The problem referred to is that of inscribing in a sphere a closed n-sided polygon, whose sides shall pass respectively through n given points which are not on the surface. Hamilton evidently regarded his solution of this question as a very tough piece of mathematics (see his *Life*, vol. iii. pp. 88, 426). In preparing a new edition of my *Quaternions*, I was led to a mode of treating this question which enables us to dispense with the brilliant feats of analysis which seem to be required in Hamilton's method.

The quaternions which Hamilton employed were such as change the radius to one corner of the polygon into that to the next by a

conical rotation. In the present Note I employ the quaternions which *directly* turn one side of the polygon to lie along the next. The successive sides are expressed as ratios of one of these quaternions to the next.

Let ρ_1, ρ_2, &c., ρ_n be (unit) vectors drawn from the centre of the sphere to the corners of the polygon; α_1, α_2, ... α_n, the points through which the successive sides are to pass. Then (by Euclid) we have

$$(\rho_2 - \alpha_1)(\rho_1 - \alpha_1) = 1 + \alpha_1^2 = A_1, \text{ suppose.}$$
$$(\rho_3 - \alpha_2)(\rho_2 - \alpha_2) = 1 + \alpha_2^2 = A_2,$$
$$\&c. \quad = \quad \&c.$$
$$(\rho_{n+1} - \alpha_n)(\rho_n - \alpha_n) = 1 + \alpha_n^2 = A_n.$$

These equations ensure that if the tensor of any one of the ρs be unit, those of all the others shall also be units. Thus we have merely to eliminate $\rho_2, \ldots, \rho_n$; and then remark that (for the closure of the polygon) we must have

$$\rho_{n+1} = \rho_1.$$

That this elimination is possible we see from the fact already mentioned, which shows that the unknowns are virtually mere unit-vectors; while each separate equation contains *coplanar* vectors only. In other words, when ρ_1 and α_1 are given, ρ_2 is determinate without ambiguity.

We may now write the first of the equations thus:—

$$(\rho_2 - \alpha_2)(\rho_1 - \alpha_1) = A_1 + (\alpha_1 - \alpha_2)(\rho_1 - \alpha_1) = q_1, \text{ suppose.}$$

Thus the angle of q_1 is the angle of the polygon itself, and in the same plane. By the help of the second of the above equations this becomes

$$A_2(\rho_1 - \alpha_1) = (\rho_3 - \alpha_2)q_1;$$

whence

$$q_2 = A_2(\rho_1 - \alpha_1) + (_2 - \alpha_3)q_1 = (\rho_3 - \alpha_3)q_1.$$

By the third, this becomes

$$(\rho_4 - \alpha_3)q_2 = A_3 q_1;$$

whence

$$(\rho_4 - \alpha_4)q_2 = A_3 q_1 + (\alpha_3 - \alpha_4)q_2 = q_3.$$

The law of formation is now obvious; and, if we write

$$q_0 = \rho_1 - \alpha_1, \quad \beta_1 = \alpha_1 - \alpha_2, \quad \beta_2 = \alpha_2 - \alpha_3, \quad \&c.,$$

we have

$$\left.\begin{aligned} q_1 &= A_1 + \beta_1 q_0, \\ q_2 &= A_2 q_0 + \beta_2 q_1, \\ q_3 &= A_3 q_1 + \beta_3 q_2. \end{aligned}\right\} \quad \&c. \qquad \ldots\ldots (1)$$

We have also, generally,

$$\rho_m - a_m = \frac{q_{m-1}}{q_{m-2}},$$

or

$$\rho_m = \frac{q_{m-1} + a_m q_{m-2}}{q_{m-2}} = \frac{A_{m-1} q_{m-3} + a_{m-1} q_{m-2}}{q_{m-2}} = \frac{p_{m-2}}{q_{m-2}}, \text{ suppose.} \quad (2).$$

From (1), and the value of q_0, we see that all the values of q are *linear* functions of ρ_1 of the form

$$q_m = r_m + s_m \rho_1 \qquad \ldots\ldots (3).$$

By (2) $p_{m-1} = A_m q_{m-2} + a_m q_{m-1}$

$$= (1 + a_m^2) q_{m-2} + a_m \{A_{m-1} q_{m-3} + (a_{m-1} - a_m) q_{m-2}\}$$

$$= q_{m-2} + a_m (A_{m-1} q_{m-3} + a_{m-1} q_{m-2})$$

$$= q_{m-2} + a_m p_{m-2}$$

Similarly $q_{m-1} = p_{m-2} - a_m q_{m-2}$ $\qquad \ldots\ldots (4)$

But the first equations in (1) give at once

$$\left.\begin{aligned} p_0 &= 1 + a_1 \rho_1 \\ q_0 &= -a_1 + \rho_1 \end{aligned}\right\} \text{ whence } \left.\begin{aligned} q_0 &= +p_0 \rho_1 \\ p_0 &= -q_0 \rho_1 \end{aligned}\right\}.$$

$$\left.\begin{aligned} p_1 &= a_2 - a_1 + (1 + a_2 a_1) \rho_1 \\ q_1 &= 1 + a_2 a_1 - (a_2 - a_1) \rho_1 \end{aligned}\right\} \text{ or } \left.\begin{aligned} q_1 &= -p_1 \rho_1 \\ p_1 &= q_1 \rho_1 \end{aligned}\right\}.$$

This suggests that

$$\left.\begin{aligned} q_m &= (-)^m p_m \rho_1 \\ p_m &= (-)^{m+1} q_m \rho_1 \end{aligned}\right\} \qquad \ldots\ldots (5).$$

By (4) we have

$$p_{m-1} = q_{m-2} + a_m p_{m-2},$$

$$q_{m-1} = p_{m-2} - a_m q_{m-2}.$$

Let m be odd, then we should have by (5)

$$p_{m-2} = A + B\rho_1,$$

$$q_{m-2} = B - A\rho_1;$$

whence

$$p_{m-1} = B - A\rho_1 + a_m (A + B\rho_1),$$

$$q_{m-1} = A + B\rho_1 - a_m (B - A\rho_1);$$

or

$$p_{m-1} = \mathrm{B} + a_m \mathrm{A} - (\mathrm{A} - a_m \mathrm{B})\rho_1,$$
$$q_{m-1} = \mathrm{A} - a_m \mathrm{B} + (\mathrm{B} + a_m \mathrm{A})\rho_1.$$

These agree with (5), because $m-1$ is even. And similarly we may prove the proposition when m is even.

If now, in (2), we put $n+1$ for m, we have

$$\rho_{n+1} = \rho_1 = \frac{\mathrm{C} + \mathrm{D}\rho_1}{\mathrm{D} - \mathrm{C}\rho_1} \text{ if } n \text{ be even,}$$

$$= \frac{\mathrm{C} - \mathrm{D}\rho_1}{\mathrm{D} + \mathrm{C}\rho_1} \text{ if } n \text{ be odd,}$$

C and D being quaternions to be calculated (as above) from the data. The two cases require to be developed separately.

Take first, the odd polygon:—

then $$\rho_1 \mathrm{D} + \rho_1 \mathrm{C} \rho_1 = \mathrm{C} - \mathrm{D}\rho_1,$$

or $$\rho_1(d+\delta) + \rho_1(c+\gamma)\rho_1 = c + \gamma - (d+\delta)\rho_1,$$

if we exhibit the scalar and vector parts of the quaternions C and D. Cutting out the parts which cancel one another, and dividing by 2, this becomes

$$d\rho_1 + \mathrm{S}\delta\rho_1 + \rho_1 \mathrm{S}\gamma\rho_1 - c = 0,$$

which, as ρ, is finite, divides itself at once into the two equations

$$\mathrm{S}\gamma\rho_1 + d = 0,$$
$$\mathrm{S}\delta\rho_1 - c = 0.$$

These planes intersect in a line which, by its intersections (if real) with the sphere, gives two possible positions of the first corner of the polygon.

For the even polygon we have

$$\rho_1 \mathrm{D} - \rho_1 \mathrm{C} \rho_1 = \mathrm{C} + \mathrm{D}\rho_1,$$

or $$\mathrm{V}\rho_1\delta - \rho_1 \mathrm{S}\gamma\rho_1 - \gamma = 0;$$

which may be written

$$\mathrm{V}.\rho_1(\delta - \mathrm{V}\gamma\rho_1) = 0.$$

This equation gives

$$\rho_1 = (x+\gamma)^{-1}\left(\delta + \frac{\mathrm{S}\gamma\delta}{x}\right),$$

where x is to be found from

$$x^2 - \gamma^2 = \frac{S^2\gamma\delta}{x^2} - \delta^2.$$

The two values of x^2 have opposite signs. Hence there are two real values of x, equal and with opposite signs, giving two real points on the sphere. Thus *this* case of the problem is always possible.

The Solubility of Carbonate of Lime in Fresh and Sea Water.

By **W. S. Anderson**, *Chemist at Marine Station, Granton.*

(Read May 20, 1889.)

At Dr Murray's request, I have during the past winter continued the investigation of Messrs Irvine and Young on the solubility of carbonate of lime in its different forms in sea water (the results of which they submitted to this Society in May 1888); and the following notes of the work done and the results obtained by me, under Mr Irvine's guidance, in the laboratory of the Marine Station, Granton, may be of interest.

At his request, I have satisfactorily checked the results already laid before you. This has also, I understand, been done by Professor Thoulet of Nancy.

In that paper special attention was given to the solubility of amorphous and artificially crystallized carbonate of lime, and the various forms of coral in sea water.

The later experiments with Iceland spar show it to be much less soluble than the above-named forms of carbonate of lime in sea water.

As shown in the Table, calcite is less soluble in sea water than in pure water, the former dissolving of the impalpable powder only 0·0082 grammes per litre; while distilled water, free from carbonic acid, dissolved during the same time 0·0251 grammes per litre (more than three times as much). There is hardly any notable difference in the solubility of calcite, whether in the form of impalpable powder, or in the condition of coarse powder, or large crystals, in sea water; the solubility being only a trifle less with massive than with the more finely divided variety (see Table). With distilled water there is a very marked difference, the powdered

spar dissolving to about double the extent. A very important factor to be taken into consideration in conducting these experiments is the time of exposure. When the same sea water stands over carbonate of lime for a lengthened period a curious and interesting reaction sets in, and the carbonate of lime it has dissolved appears gradually to diminish in quantity and be thrown out of solution again. This was observed by Professor Dittmar, who, in his paper on the composition of sea water ("Challenger" Report), says —"It seems that under certain abnormal conditions sea water dissolves lime largely in addition to what it contains normally, and subsequently will redeposit even more than the surplus lime in crystals of carbonate." This result was also found by my predecessor, Mr A. Drysdale.

Of course such a condition as this last may not occur in nature, as Dr Murray states, where the sea water is in continual circulation by tides, currents, &c., but it will help to explain the gradual petrifaction of the porous masses of dead coral reef, which being constantly supplied with salt water saturated with amorphous carbonate of lime on standing, depositing, will gradually fill up the interstices, and produce the massive condition all old coral formations exhibit. This would take place in comparatively shallow waters and while in contact with carbonate of lime, but in deeper waters, and under greater pressure, any carbonic acid present might be called into play, as shown by Mr Reid in his paper to the Society in February 1888.

As is well known, carbonic acid water has a powerful solvent action on calcspar; the more finely divided it is the greater the solubility. One litre of water, saturated at ordinary temperature and pressure, dissolved in twenty-four hours, of the massive 0·0815 grammes, and of the powder 0·472 grammes per litre, or nearly six times as much (see Table).

In order, if possible, to throw some light upon the condition in which the carbonate is present in sea water, a series of experiments was undertaken on the solubility of carbonate of lime in solutions of the different salts said to enter into the constitution of sea water.

A hard variety of coral skeleton (*Oculina coronalis*) was finely powdered, and the solutions allowed to act upon portions of it separately for four days.

As in the case of Iceland spar, the solubility was greater in distilled water than in sea water (some experiments conducted by Professor Thoulet, Nancy, confirming this result in the case of crystalline varieties of carbonate of lime).

As shown in the Table, the magnesium salts dissolved the largest quantity of carbonate of lime, the solution of sodium chloride coming next, the calcium sulphate solution dissolving the least. Calcium sulphate appears rather to retard the solubility. It was to be expected that the magnesium salts would dissolve a comparatively large amount of the carbonate and the sulphate of lime very little, for such is their action towards the amorphous, or non-crystalline, form of carbonate of lime. A solution of calcium sulphate dissolves very little amorphous carbonate, but a solution of magnesium chloride, holding the same amount as is present in normal sea water, takes up a large quantity of it, forming a clear solution, which on standing throws out the greater part of the carbonate of lime in a crystalline form. If a stronger solution of magnesium chloride be used, rhombohedral crystals of carbonate of lime are obtained large enough for their form to be seen distinctly with the eye.

This experiment is indirectly important, as if there is an interchange between the sodium chloride and the carbonate of lime entering the ocean, as is held by Tornœ, we might reason by analogy that since magnesium chloride dissolves more carbonate of lime than sodium chloride (although there is seven times the amount of the second than of the first in sea water), an interaction between these salts would also take place, as—

$$MgCl_2 + CaCO_3 = MgCO_3 + CaCl_2.$$

But such is not the case. Magnesium carbonate is not thrown down along with the calcium carbonate, as would be expected if such a reaction took place. It seems to be nothing more than a question of solubility.

Sea water acts in very much the same manner as a solution of magnesium chloride, as amorphous carbonate of lime is soluble to the extent of 0·6 grammes per litre, which may be taken as the greatest amount of carbonate of lime in its most soluble form that sea water can dissolve without the help of free carbonic acid, in this case acting

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as a vehicle for the carbonate of lime, as shown by the gradual crystallising out of the greater part that at first dissolved. After standing twenty hours, this solution only held 0·186 grammes $CaCO_3$, and after four days 0·162 in place of 0·6 grammes above referred to.

This soluble action of sea water on amorphous carbonate of lime has nothing to do with carbonic acid. An artificial sea water, free from carbonic acid and carbonates of any kind, will dissolve up quite as much. It is distinctly confined to the soluble action of the salts present.

In regard to the solubility of the coral skeleton in the various salts of sea water, it will be seen from the Table that if all these salts be acting together on the substance in *one* solution, the soluble action of the mixture is about the same as that of sea water. That is to say, normal sea water, and that artificially prepared by adding in proper proportions the salts present in sea water to distilled water free from carbonic acid, will dissolve practically the same amount of carbonate of lime. But with solutions of the individual salts, the results are in some cases higher, as in sodium chloride and magnesium chloride, and lower, as in sulphate of potassium and sulphate of lime. This difference is most probably owing to the sulphate of lime, which, as before said, seems to have a deterring action on the solubility, although when the carbonate of lime is once dissolved, the subsequent addition of the sulphate has no precipitating effect.

Curiously enough, if these results of the solubilities of the coral be added up as before mentioned, the resulting figure comes to very nearly the amount of carbonate, taken as lime, given by Professor Dittmar as being present in sea water:—

$NaCl$ . .	= 0·0525		
$MgCl_2$ . .	= 0·0746		
$MgSO_4$ . .	= 0·0712		
K_2SO_4 . .	= 0·0296		
$CaSO_4$ . .	= 0·0209		
	0·2488		
Subtract amount dissolved by 4 litres extra water used, = 0·0285 × 4 =	0·1140	Sea water contains—	0·1210
	0·1348	And will dissolve of same $CaCO_3$.	0·0237
			0·1447
			0·1348
			0·0099

Leaving only a difference of 1 centigramme per litre in favour of sea water.

From this it appears to be a reasonable conclusion that the carbonate of lime present in sea water as such, is there simply owing to its solubility in the river water flowing into it, the salts present helping or retarding the solution as the case may be.

It would seem, according to this view, that (except in special cases) the whole of the soluble carbonate of lime in sea water can be accounted for without the help of carbonic acid as a solvent, although, doubtless, its local action at great depths and in presence of decaying organic matter is notable. (See Reid's paper, and Irvine and Young's paper, Table II., *Proceedings of Royal Society.*)

It will be seen by referring to the Table accompanying this paper that the amount of carbonate of lime dissolved by the various salts present in sea water amounts to 0·1348; whilst the total amount of carbonate of lime figured as present in sea water, added to what it can yet dissolve, is 0·1447, making the difference of 1 centigramme per litre in favour of sea water. The carbonic acid is therefore free to perform its true function, which is to support the enormous flora present in the ocean. Messrs Irvine and Woodhead, in their paper read before this Society, May 1888, indicate this as follows:—"The relation between plant and animal life in the ocean is much the same as that between plant and animal life on land, so far as interchange of carbon is concerned, considering the requirements of marine plant life in the form of carbon, which it can only obtain from the sea in the condition of carbonic acid."

The behaviour of a solution of carbonate of lime in sea water on standing in a closed vessel, where it is impossible carbonic acid could escape, seems to prove beyond question that its solubility has nothing to do with the existence of free carbonic acid or bicarbonates, as the major portion is thrown out of solution.

Again, all the solutions of the various salts present in sea water, referred to in the Table, were made up according to the proportions in which they exist in sea water, with distilled water absolutely free from carbonic acid.

[TABLE

TABLE.—*Solubility of Carbonate of Lime in Distilled Water, free from Carbonic Acid, in Sea Water, and in Solutions of Salts entering into the Composition of Sea Water; 1 litre Water or Solution being used at Temperature 10°–15° C. Results in Grammes per Litre.*

	Distilled Water.			Sea Water.			Carbonic Acid Water, Atmospheric Pressure.		
	Amount Dissolved.	Exposure in Hours.	Number of Determinations.	Amount Dissolved.	Exposure in Hours.	Number of Determinations.	Amount Dissolved.	Exposure in Hours.	Number of Determinations.
Calcspar, massive,	0·0146	120	2	0·0046	120	4			
Do. do.	.	.	.	0·0075	47	3			
Do. do.	.	.	.	0·0000	396	6	0·0815	24	2
Do. granular,	.	.	.	.	.	.	0·1285	24	2
Do. fine-grained,	.	.	.	.	.	.	0·2036	24	2
Do. impalpable powder,	0·0251	46	2	0·0052	120	4	0·4720	24	2
Do. do.	.	.	.	0·0082	47	2			
Do. do.	.	.	.	0·0000	396	6			
Coral, impalpable powder,	0·0285	96	2	0·0237	96	3			
Amorphous calcium carbonate,	0·2480	.	2	0·6100	.	2			

	Solutions of—					Exposure in Hours.	Number of Determinations.	Solution of—	Exposure in Hours.	Number of Determinations.
	NaCl	$MgCl_2$	$MgSO_4$	K_2SO_4	$CaSO_4$			NaCl $MgCl_2$ $MgSO_4$ K_2SO_4 $CaSO_4$		
Coral, impalpable powder,	0·0525	0·0746	0·0712	0·0296	0·0209	96	2	0·0342	96	2
Amorphous calcium carbonate,	.	.	.	.	.	.	.	0·6100	.	2

Secretion of Carbonate of Lime by Animals. Part II. By Robert Irvine, F.C.S., and G. Sims Woodhead, M.D.

(Read May 6, 1889.)

In a paper read before this Society, on May 7, 1888, on the "Secretion of Carbonate of Lime by Animals," we gave the result of experiments made upon domestic fowls, which established the fact that they could elaborate carbonate from sulphate of lime in the formation of the calcareous covering of their eggs.

§ 1. The observation of this example of adaptability was followed up by experiments continued during the spring and summer of 1888,

various salts of lime being added to the food of the hens, and we found that the shells of the eggs laid by them, thus fed, invariably consisted of carbonate of lime, always in the same proportion, practically, as found in normal egg shells, mere traces of phosphate and sulphate of lime being present.

We found that (unless when carbonate of lime itself was given) we obtained the most favourable results with phosphate of lime, which was added to the food with the same precautions as already described, and seemingly the fowls had no difficulty in assimilating this salt, and producing from it eggs with normal shells, thus bearing out the view we hazarded in our former paper that the lime is carried to the secreting surfaces of the duct as phosphate of lime and soda. It would thus appear that lime salts, in whatever condition they are absorbed, are during the digestive processes converted into phosphate, and carried as that salt to the secreting surfaces in the oviduct, at which point it probably meets with, and is decomposed by, nascent or combined carbonic acid; at the same time, structureless or amorphous carbonate of lime is elaborated, possibly in combination with protoplasmic matter.*

Before concluding this portion of the paper it will be interesting to note the observations made with a view to determine whether birds can produce shells from salts of the metals having analogous properties to those of calcium. Compounds of strontium and magnesium were, under the conditions already fixed, administered to the laying hens. Salts of barium could not be employed on account of their poisonous nature. The result invariably was that

* A most interesting observation on the changes through which the lime salts may go, is that afforded by the transference of lime from the shell of the chick's egg (Lehmann, *Physiological Chemistry*, vol. i. p. 417), where it is found as the carbonate, to the yolk and developing chick embryo, where it appears in the form of the phosphate. Prout, *Phil. Trans.*, 1822, p. 365, pointed out further, that the amount of phosphorus in the yolk remains constant throughout the whole course of development of the chick, but that there is a steady and continuous increase in the amount of lime; and Lehmann argues that, as the egg shell becomes both lighter and more brittle, the lime is derived not from without, as Prout suggested, but directly from the shell. He says, "The phosphorus exists chiefly in the yolk, where it occurs as glycero-phosphoric acid, which during incubation is gradually decomposed, so that the liberated phosphoric acid unites with the lime which passes over by endosmosis from the shell into the egg to form this salt." It is evident from this that dialysis plays an important part in the process of lime distribution from the egg shell and its membrane to the growing embryo.

the animals could not produce eggs with shells, the only protection to the egg proper being the strong membrane on which the calcareous covering or outer shell is, in ordinary circumstances, deposited.

In each case these experiments were continued for about fourteen days, and between each change of such chemical feeding a course of carbonate of lime was administered, so as to bring the eggs again into a normal condition (*i.e.*, eggs having true shells). We found it took about forty-eight hours for the hen to absorb and elaborate sufficient calcareous matter to reproduce the shell (after absorption of lime salts), and a similar period of lime starvation again to deprive the egg of its shell covering.

The results of these experiments prove, beyond question, that egg-producing animals can form perfect eggs only when they have calcareous matter in the form of salts of lime in sufficient quantity present in their food, and that the lime salt found surrounding and protecting such eggs is generally carbonate, although this may, in some cases, be partly replaced by phosphate of lime.

A series of experiments of a similar nature was made by Papillon (see *Comptes rendus*, vol. lxxi. p. 372), who "fed a pigeon and two white rats for two months on food containing phosphate of strontium, aluminium, and magnesium." The ash from the bones of the pigeon was found to contain 8·45 per cent. of strontia, ·66 magnesia, and 6·95 alumina. H. Weiske-Proskau (*Zeitschrift f. Biologie*, vol. viii. p. 229) repeated these experiments, but failed to find the slightest trace of strontia or any noteworthy increase of magnesia.

It is of interest to notice that as soon as these birds began to lay shell-less eggs, they contracted the habit of eating them. They were, at the conclusion of the experiments, perfectly healthy, the only noticeable point being that they were in excellent condition and very fat.

If birds, then, can assimilate and secrete carbonate from other salts of lime, we consider we had strong ground for the statement we advanced in the first portion of our paper that coral animals could do the same thing, and we should have allowed this investigation to rest at this point. Mr George Brook proposed that we should continue these experiments with marine animals

and sea water, by which means we might obtain results more in keeping with the statement we had originally made.

Dr John Murray placed all the resources of the Marine Station at Granton at our disposal, at the same time defraying the cost of the analyst, whose duty it was to watch the progress of the experiments and to perform the analyses which appear in the Appendix to this paper.

§ 2. Our first object was to obtain artificial sea water free from carbonate of lime, and this we obtained by adding to fresh water the salts present in sea water (in proportion shown in Dittmar's analyses in the "Challenger" chemical report), carefully excluding all trace of carbonate of lime. The composition of this water appears in Appendix under Table II. ("Artificial Sea Water, No. 1").

On analysis we found that the fresh water employed contained 2·68 grains of lime salts (principally as carbonate) per imperial gallon. Hydrochloric acid was added (previous to the addition of the salts) in quantity more than sufficient to decompose any carbonate of lime originally present in the water. The resulting artificial sea water No. 1 (see Appendix), which had a specific gravity of 1·026, was allowed to settle in tanks, and was then aerated by means of spray jets of a mixture of air and this water, sent into the tanks at a high pressure by the pumping engine attached to the Marine Station. This aerated water was neutral.

We collected a number of male and female common shore and edible crabs, choosing these animals as being handy and most likely to meet the general requirements of our experiments, and placed them in this water No. 1, giving them raw mussel flesh for food. Whether it was in consequence of the change from their ordinary food, or whether it was owing to the absence of vegetable and animal organisms in the water, or whatever may have been the reason, they did not thrive (except upon one another).

During the early months of summer many beautiful examples of exfoliation occurred, but in this water we never had a case where the animal could replace the calcareous exo-skeleton it had thrown off; and ultimately the undefended animal was sure to become a prey to one or other of his fellow captives.

With a view to watch their movements more carefully, they

were separated and placed in large clear glass jars filled with the same No. 1 water, in which a continuous current of aerated No. 1 water was kept circulating, and provided with flint (acid-washed) gravel, stones free from lime, and growing sea-weed. They were fed regularly with raw mussel flesh.

Some of the jars were placed in sunlight so that the water might be slightly warmed (and oxygenated), and others were kept in the dark and as cool as possible; notwithstanding the greatest care, however, the results were of a negative character so far as successful new shell formation was concerned. Indeed, the experiments up to this point failed entirely.

On reference to the first portion of our paper on this subject, it will be found that we hazarded the opinion that there might be inter-action between the chloride of sodium and lime salts present in sea water, and possibly "production of chloride of calcium, which might then be almost directly assimilated by animals, and secreted as carbonate of lime. Following out this idea we added to a portion of the No. 1 water chloride of calcium, equivalent in amount to the lime already present as sulphate in the No. 1 water. This water is referred to in the Appendix as 'No. 2, Artificial Sea Water.'" Three shore crabs, weighing from 300 to 400 grains each, were introduced into this water in glass jars provided with growing sea-weed and acid-washed gravel, through which a constant current of No. 2 water was also circulated. On the 29th of July one of these animals cast, leaving a perfect envelope (or shell) consisting of carbonate of lime and chitinous matter which had formed the exo-skeleton (see Appendix); its body then presented the soft pulpy appearance common to this condition of the animal. It continued in this state for two or three days, during which period it was very shy and was unable to feed, but gradually the animal, which had, meanwhile, increased in size very considerably, began to lose its flaccid appearance and to gain substance, so much so, that in four or five days the sac-like bag or covering which now occupied the position of the shed carapace, and which was a soft pliable membrane entirely free from calcareous matter, was hardening, and within ten days there was sufficient calcareous matter deposited on the animal to allow of its resuming its regular functions. The deposition began first and continued with greatest energy upon

those parts which are essential for the procuring of food—the chelæ or claws.

It was interesting to watch the gradual deposition of carbonate of lime upon the membranous substance. At first widely separated points of calcareous matter (apparently spherical in form) were deposited; these, widening from the nucleus, gradually formed into patches, which coalesced and ultimately became an unbroken surface of hard shell, consisting chiefly of carbonate of lime, which had been secreted from No. 2 water, although this water was absolutely devoid of that substance. The exo-skeleton thus formed was found on analysis to consist of carbonate and phosphate of lime and chitinous matter in the proportions present in normal shells.

The washed mussel flesh upon which the animals were fed was found to contain traces of lime as phosphate, but the total amount being less than 0·08 per cent. (and although no doubt, like the hens the crabs could have elaborated carbonate from this phosphate), was too minute in amount to form a factor in the case, in the limited period required by it to form the new shell.

All the three crabs confined in this No. 2 water cast successfully and formed calcareous envelopes, but ecdysis occurred so late in the season that in two of the three cases the animals had not vigour to complete the process, the cold retarding their powers to such an extent that before it was completed they sickened and died. This is in accordance with the fact that heat is a necessary element in the assimilation of lime salts from sea water by these animals, ecdysis only occurring during the summer months or in warm weather.

Bromide of magnesium was added to a portion of the No. 2 water (see Appendix, No. 3 water), and a number of crabs and fish were placed in it. These all appeared to live in comfort, taking their food readily, and thriving, and we have no doubt that the same results would have been here obtained as with those already referred to, had the experiments been begun at an earlier period of the summer. Since this paper was read many other excellent examples of ecdysis, followed by complete shell formation within a few days, took place with crabs kept in the No. 3 water.

At the suggestion of Dr John Gibson a fourth water was

made up, chloride of calcium being substituted for sulphate of lime (carbonate of lime of course being excluded). This is referred to in Appendix as "Artificial Sea Water, No. 4." Crabs placed in this water have lived during the past winter. We therefore presume that sulphate of lime is in no way necessary to maintain the lives of these animals. We also went a step towards determining the point as to what salts were absolutely necessary for the maintenance of marine life. A solution containing $2\frac{3}{4}$ per cent. of chloride of sodium in pure water (the amount present in normal sea water) was first prepared, and into this solution shore crabs were introduced, but after a few days, during which they refused to feed, they all died. Chloride of magnesium was then added to this solution in the proportion in which it occurs in sea water. In this mixture of the two salts, crabs and fish seem to live in comfort, feeding greedily, but of course ecdysis in such water was impossible. Chloride of calcium was added in amount equivalent to the lime in sea water, with similar results as regards the health of the animals; but this was done too late in the season to allow of ecdysis taking place.

§ 3. In accounting for the phenomena of ecdysis, some observers have asserted that crustaceans have reserves of calcareous matter in themselves, in the shape of abdominal and stomachical teeth "plates and stones," from which they have the power of drawing supplies as they form their new calcareous structure.

These gastroliths are undoubtedly the products of a regular epithelium just as much as is the carapace, and on careful examination they are found to present the same microscopic structure.*

These, along with the gastric armature, are thrown off into the cavity of the stomach, where they are dissolved, and whence they may be utilised for the very rapid deposition of a small quantity of the lime salts in the new chitinous covering. There is described a similar small reserve store in the lobster, in the form of small masses of rods composed of lime salts, which at the time of the moult are thrown into the gastric cavity, where they lie until they are dissolved by the gastric juice. From analysis made it is evident that these reserves are, if they play any part at all, first converted

* Max Braun, *Arbeiten aus d. Zool. Zoot. Institut. in Würzburg*, 1875, vol. xi. pp. 144–489; Vitzou, *Arch. de Biologie*, vol. x., 1882, p. 659.

into the phosphate, and are so carried in the lymph (see "Analysis of Lymph," in Appendix). It is to be noted, however, that these gastroliths are not found in the other decapods (*Brachyura*), so that any reserve store can not be in the stomach, but if present at all, and we see to what a very slight extent this can take place from the analysis, it must be in the lymph, where it is chiefly in the form of phosphate of lime.

We think that this theory may be dismissed as of comparatively little importance, as, even if the teeth and whole inner calcareous structure could be absorbed by the animals, the amount of carbonate of lime at their disposal from this source is so small (a very small fraction of the outer covering) that it could not account for any material part of the new structure. Consequently such an explanation must be abandoned. As a matter of fact this internal structure, consisting of teeth, plates, &c., is not materially affected during the process of ecdysis where the other changes are so well marked.

The radical change caused by ecdysis is very striking. The animal in the operation shedding or throwing off the whole of its outer calcareous shell or structure, along with the branchiæ and calcareous supporting plates, at the same time the bulk of the fleshy muscular matter is much increased, so much so that we ask with astonishment could the animal have ever existed in the discarded structure? We failed to find, immediately after ecdysis, carbonate of lime on any of the outer surfaces of the body, but, as already noticed, the abdominal calcareous structure is not materially affected.

The method we used to determine the amount of carbonate of lime and chitinous matter was as follows:—The flesh was separated as carefully as possible from the exo-skeleton and the latter was dried and treated with weak nitric acid, by which the inorganic matter was dissolved out. The chitinous matter left was well washed and dried. The lime and phosphoric acid were determined in the solution. In Appendix will be found a number of Tables (III., IV., and V.) in which are given the proportions of carbonate of lime and chitinous matter present in the different portions of the structures of these animals, and also the composition of the stomachical teeth.

In the crab it is found that the carapace is formed by a regular secreting membrane, which, according to Vitzou,* has the following structure. [This we have verified for ourselves, and give briefly.]

There is, most externally, a thin delicate chitinous layer in which little or no structure can be made out. Beneath this is a thick chitinous layer, infiltrated with lime salts, which must be removed by maceration in weak acid before the structure can be distinguished; when so softened and examined in thin sections, it is found to be composed of a series of layers of chitin lying very regularly, one above another, this arrangement giving rise to a series of markings more or less parallel to the surface. Running at right angles to these are other regular markings, the meaning of which we shall see immediately.

These chitinous layers rest immediately on a layer of tall columnar cells, each with a nucleus and a distinct nucleolus. If now we again examine the lines in the chitinous layer running at right angles to the surface, we find that the distance between them corresponds exactly with the breadth of one of these cells, so that each of these columnar cells may be said to secrete, time after time, its little area of chitin. In some cases the markings in the chitin running at right angles to the surface are more numerous than the intercellular spaces, in which case there is evidently a splitting, similar to that which takes place in the formation of the striated margin of a ciliated epithelial cell.

These long columnar epithelial cells send down processes which rest on a very distinct basement membrane, and then beneath this basement membrane there is either a mass of muscular tissue or a layer of connective tissue, in which are large vacuolated-looking cells. The vacuoles contain a material which gives all the reactions of glycogen.

It is evident from the above description that these columnar cells are the active agents in forming the chitinous covering of the crab, which really corresponds to the horny layer of the skin of an animal, or to the thick horny lining of the gizzard of a fowl. The only difference between these cells and those of the *retæ Malpighii* being that one part of the cell is constantly growing outwards and

* *Loc. cit.*, p. 501 *et seq.*

being converted into chitin, and then being shed as it were, whilst in the human or other animal skin the cells, as they are gradually pushed away from the parent cells, become horny, are separated, and so form the cuticle. This is a point of some importance, for the actual and direct connection of the chitinous disc is not severed for some time, and in that time the chitin becomes impregnated to a lesser or greater extent with calcareous particles, the spaces in which the particles are found being somewhat irregular. This irregularity is continued as the chitin with its lime is continued outwards. It will be remembered that in the oviduct of the hen the lining epithelium of the secreting follicles and the superficial epithelium contained lime in the form of minute granules, that these epithelial cells were extremely active, and that lime and organic matter were being thrown out on to the surface simultaneously. Exactly the same thing takes place in this secreting layer covering the crab; the epithelial cells perform a double function—the free end of the cell, instead of forming a separate secretion which could be removed at once, is converted into a chitinous material. The lower and more active protoplasmic portion of the cell separates the lime from the salts brought to it by the blood and lymph, and deposits it in the chitinous end of the cell, principally in the form of a carbonate but partially as a phosphate.

We have already indicated the process by which, we believe, this occurs (*Proc. Roy. Soc. Edin.*, vol. xv. p. 314). The chitinous shell, as age advances, becomes thicker and thicker, and as each layer is formed it becomes impregnated with carbonate of lime, and is then pushed outwards (see also Schmidt, *Scientific Memoirs*, vol. v. p. 10).

It is an exceedingly interesting fact in connection with this method of formation of chitin, that in most cases where there is a deposit of a large amount of matrix in more or less direct continuity with the secreting cell, and in close contact with the fluids of the body (blood and lymph), there is always a larger amount of phosphate of lime in the calcareous infiltration than when the matrix in which the lime is deposited is actually separated from the secreting cells. We shall have again to refer to this.

§ 4. The subject of the absorption of lime salts, and their elaboration as carbonate by marine animals, resolves itself into the question

of the change in constitution (if any) which lime salts undergo when they are carried to the ocean by rivers. Tornöe and others assert that the carbonic acid in sea water is in combination with soda; if this is so, it follows that the soluble carbonate of lime on mixing with the sea water is decomposed by the alkaline chlorides present, the result being the formation of chloride of calcium and alkaline carbonate. The results we have detailed seem to give some support to this view. If this be the case, it points to the necessity for a reclassification of the salts as generally expressed in sea-water analyses.

Such a view implies the absence of carbonate of lime from sea water, chloride of calcium taking its place, from which salt structures of carbonate of lime in a more or less pure state may be built up by marine animals and plants. Our results seem to prove beyond doubt that all lime compounds, not excepting the carbonate, are decomposed during digestion, and appear in the blood in combination with phosphoric acid as phosphate. Consequently it does not seem essential that such animals and plants should obtain the lime in the shape of carbonate.

In the case of the higher forms of crustaceans with their aggressive habits, it may be assumed that they obtain a portion of carbonate of lime in a concrete form from the animals upon which they prey, and this may help in the new formations of this body by them after ecdysis, but even then the carbonate of lime must pass through the digestive changes we have explained as preceding its reabsorption. Of course, it cannot be assumed that coral polyps have such sources of calcium carbonate at their command, but, even if their food consisted of minute organisms containing carbonate of lime, this would only remove the question as to assimilation of one lime salt, and its elaboration as another lime salt, from the one animal to the other.

In the blood of these animals we have, in solution, phosphates of lime and soda, along with alkaline chlorides, carbonates and sulphates associated with albuminous matter, carbonic acid and oxygen being also present in varying quantities (in a loosely associated or combined condition). This blood when freshly drawn is alkaline, no doubt owing to the presence of alkaline phosphates and carbonates.

The existence of alkaline phosphates would appear to preclude the presence of soluble carbonate of lime in any quantity in the blood, as when brought together in solution these salts react upon one another with the formation of insoluble phosphate of lime and carbonate of soda. Therefore, even if this change is not effected in the primary processes to which blood formation is due, it must take place at that point when the incepted lime compound (be it what it may) becomes part and portion of the blood.

That phosphoric acid combined with alkalies and alkaline earths plays an important part in the functions of the blood, and in tissue, bone, and shell formation, there can be no doubt. We find, both in the case of birds and crabs experimented upon by us, comparatively large and constant quantities present, which seem to have no particular ratio to what they absorb from their food or sea water, but rather depend upon the part this acid plays acting as a carrier of the lime as phosphate or phosphatic albuminate to the required point where it is to be transformed in the presence of nascent carbonic acid (or carbonates) into carbonate of lime, the phosphoric acid, especially during the period of active growth of new shell formation following ecdyses, meantime not passing out of the body to any extent but re-entering the circulation to continue to assist in the process of elaboration of lime salts.

If a solution of carbonate of lime and phosphate of soda in carbonic acid be heated to 90° F., we find that there is a deposition of phosphate of lime with a comparatively small proportion of the carbonate, the proportion of this depending upon the relative amount of alkaline phosphate present. If these salts could be removed as they are formed, and in the proportions in which they are thrown down, it would appear that we should have just those proportions that we get deposited in the matrix of bone. We shall have occasion to show that we consider that there is actually a dialysis of these lime salts into the formed material in which they are found deposited.

Let us take, for example, developing bone in any animal. There are several factors to be taken into account in connection with the deposition of lime salts to bring about the calcification of bone. We have, in the first place, the presence of alkalies and alkaline earths in the blood of animals, combined with phosphoric acid and

carbonic acid. How marked are the differences in the proportions in Herbivora and Carnivora may be gathered from the analyses collected by Liebig (*Letters on Chemistry*, p. 406), from which we gather that the ash of human blood contains 32 per cent., that of the pig and the dog 36 per cent., that of the fowl 40 per cent., that of oxen and sheep not more than from 14 to 16 per cent. of phosphoric acid, though calves' blood contains about 20 per cent.

The carbonic acid varies inversely. In the ash of human blood we have only 3·78 per cent., in that of ox blood 18·85 per cent., in that of sheeps' blood 19·47 per cent. In these cases, where the proportion is large, however, the carbonate of lime is secreted by the kidney cells, and is carried off in solution in the urine instead of being secreted as a shell. In whatever proportions the acids are present they are always neutralised by a slight excess of the alkalies and alkaline earths.

Schmidt* found that the blood of the pond mussel (*Anodonta cygnea*) was slightly alkaline. He describes as present on evaporation beautiful crystals of carbonate of lime resembling gaylussite. These could not have been present originally in the alkaline fluid, and it is probable that they were produced by the formation of carbonate of ammonia from the decomposition of urea and nitrogenous organic matter.

This must occur in the case of the effete matter discharged by animals. The carbonate of ammonia produced by the decomposition of urea, &c., decomposing a portion of the sulphate of lime in the sea water with the formation of carbonate of lime equivalent in amount to the carbonate of ammonia thus formed.

According to Lehmann,† carbonate of lime in considerable quantity is found in the urine of graminivorous animals, in the saliva of the horse, and in many animal concretions. The urine of graminivorous animals often contains so large a quantity of carbonate of lime as to cause a deposit very soon after its emission. "My investigations tend to show that in the urine of the horse carbonate of potash and carbonate of lime very frequently replace one another; I have usually found the urine rendered turbid by the presence of much carbonate of lime, which contains a very small quantity of alkaline

* See Taylor's *Scientific Memoirs*, vol. v. p. 26.

† *Loc. cit.*, vol. i. p. 419 (see also references on p 240).

carbonates, often has only a very slight reaction on turmeric paper, while clear urine is usually rich in alkaline carbonates." Hence it is easy to see why urinary calculi consisting of carbonate of lime are of very common occurrence in herbivorous animals.

Carbonate of lime also sometimes occurs in human urine with an alkaline reaction, and indeed sometimes, though very rarely, we meet with urinary calculi in the human subject consisting for the most part of carbonate of lime. Prout was the first who made this observation, but similar calculi have been since found by Cooper, Prout, Smith, Gobet, and Fromhery. Lehmann also points out that in animal concretions considerable quantities of carbonate of lime may be deposited with the phosphate. Thus he mentions nasal concretions containing 21·7 per cent. carbonate and 46·7 phosphate of lime; phlebolith 8·3 per cent. to 24·3 per cent. carbonate, and 50·14 per cent. to 69·7 per cent. of phosphate of lime. In the concretion from the heart of a man with hydrothorax 23 per cent. of carbonate and 50 per cent. of phosphate of lime. In a concretion taken from a stag's heart 66·7 per cent. of carbonate and 25 per cent. of phosphate of lime. Some stony concretions from the peritoneum of a man were found to contain 34 per cent. of carbonate and only 19·32 per cent. of phosphate of lime. Lassaigne found 83·36 per cent. of carbonate of lime in a salivary concretion from a horse.

§ 5. The next factor to be considered is the mass of active cellular elements with which the blood and lymph are constantly coming in contact. These fluids must be looked upon as the carrier from which the cells obtain their nutrient material and also the vehicle for carrying off effete matter. In the immediate neighbourhood of these cells most marked changes take place in the composition of the fluids in which they are bathed. One of these changes is, that there is always an increase in the amount of carbonic acid in this position, and there is, as we have indicated, frequently, but by no means invariably, a deposition of phosphate of lime with a small quantity of carbonate. If the salts are removed at once they may remain as such, but if they are again acted upon by the phosphoric acid and the alkaline phosphates the lime salts are redissolved and we have a return to the normal condition. Let us see how this bears on the process of calcification of bone. In the first place,

take an adult healthy bone and then a growing bone. The osteoblasts lay down a matrix of formed material. The more active the cells, within certain limits, the greater the relative amount of matrix. This matrix may now be looked upon as inert or dead organic matter, and we hold that it corresponds to a membrane through which dialysis may take place, or rather the layers near the two surfaces may be so considered, and as the molecular combinations of the phosphoric acid and lime and the carbonic acid and lime take place around the osteoblasts (which, as above stated, during their active formative changes give off the carbonic acid to render the lime for the time insoluble) there is a continuous process by dialysis of separation of these lime salts, which first make their appearance in the centre of the matrix trabeculæ where the two currents meet, as it were; from this point the calcification extending towards the surface. We look upon the formed matrix then as playing the part of a dialysing membrane, that serves to separate the lime salts prepared in its immediate neighbourhood by the carbonic acid forming cells, this carbonic acid causing a throwing down of phosphates of lime with a small proportion of lime in which the phosphoric acid is usually replaced by carbonic acid. It should be observed in this connection that the carbonic acid is, when acting on the lime solution, in a nascent condition, and therefore in a much better position to combine with any lime already held by the phosphoric acid. That the phosphate of lime and the alkaline earths generally are deposited merely mechanically in the bones, is very evident, as Lehmann points out in his *Physiological Chemistry*, vol. i. p. 414, because we can so thoroughly deprive them of all mineral constituents by dilute hydrochloric acid, and as we have before pointed out, because the matrix in which they are deposited is vitally and chemically inert, though like many animal membranes it may still retain the power of allowing of dialysis. In the case of newly formed bones or parts of bones there is, according to Valentin (quoted by Lehmann, *loc. cit.*, vol. i. p. 417), "always a greater quantity of carbonate of lime before they are provided with their proper quantity of phosphate." This bears out our theory, because it is whilst the cells are most active in building up the matrix that the largest proportion of nascent carbonic acid will be present, and therefore the period at which a

rather larger proportion of the phosphoric acid will be replaced by the carbonic acid in the lime salts.

Additional evidence may be drawn from irregular formation and diseased conditions of bone. In exostosis, and in the new bone of a callus, there is always a larger proportion of the carbonate. One point, however, should be borne in mind; after the process of bone formation is completed, or when it is interfered with in any way, there will always be a slightly greater proportion of carbonate, because in the process of absorption the phosphate once formed is slightly more soluble, and is therefore more readily removed. Phosphoric acid is present in larger proportion in the tissues and fluids in old people, where the tissue changes and the formation of carbonic acid are not going on so rapidly. Once laid down in bone the carbonates are more permanent than the phosphates, *i.e.*, it is more difficult to remove them. This is most noticeable in the bones of old people, where the proportion of carbonate to phosphate of lime is always slightly higher than in adults.

Liebig laid great emphasis * on the fact that the carbonates of the alkaline earths are soluble in water containing free carbonic acid, whilst the earthy phosphates are also soluble in water containing free phosphoric acid. As a matter of fact, both sets of salts are soluble in solution of either acid. As we have already shown, if carbonic acid be passed into a solution in which phosphate of lime and an alkaline carbonate are present, as in the blood, there is immediately a deposition of that salt, partly as a pure phosphate of lime but partly also as a salt of lime in which carbonic acid has taken the place of one, or perhaps even more, of the phosphoric acid molecules. It does not matter then, whether the lime in the blood is combined in solution with phosphoric acid as in the Carnivora, or partly with carbonic acid as in the Herbivora, the addition of carbonic acid from the tissue and in the nascent condition will cause a deposition of the phosphate of lime and of double phosphate and carbonate, as the carbonic acid gradually and partially takes the place of the phosphoric acid. It might be urged, that because there is more carbonic acid in the blood of herbivorous animals, we should therefore find less or greater deposition of the carbonate or phosphate of lime on the evolution and

* *Letters on Chemistry*, p. 414.

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* *Loc. cit.*, p. 28.

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As regards the proportion of the lime salts and chitin, Schmidt * found that the amount of earthy phosphate increases in proportion to the quantity of chitinous tissue present in the basement structure :—

	Crawfish.	Squilla.	Lobster.
Chitin, . .	46·73	62·84	22·94
Lime Salts, . .	53·27	37·17	77·06
	100·00	100·00	100·00
Phosphate of Lime,	13·17	47·52	12·06*
Carbonate, . .	86·83	52·48	87·94
	100·00	100·00	100·00

He argued from this that the phosphate of lime is in intimate relation with cell-formation. We should be inclined to say rather, that as the chitin becomes older and thicker the cellular layer becomes less active, less carbonate is formed, and that there is thus a more direct passage outwards of the phosphate.

From a careful examination of the conditions under which calcification occurs in pathological processes in the human and other subjects, we have come to the conclusion that this process of dialysis plays a most important part in the separation of the lime salts from the lymph and their deposition in degenerated tissues.

First, there is in every case most serious interference with the vitality of the tissues in which calcification takes place.

We have already seen that lime salts are deposited in the formed and inanimate matrix. In cartilage the same thing occurs, and in some cases the calcification is found even in the hard cell-membrane of the cartilage cells. Lime is never found as a deposit, visible under the microscope, in living protoplasm, except near the surface of epithelial secreting cells, but it is frequently found in the formed material of cells both when it remains part of the cell and when actually separated, as where a matrix is formed.

As further examples of this deposition in matter in a state of degeneration, we may take such a condition as atheroma of an artery, in which, during the earlier stages of the disease, we have a low inflammatory condition due to altered nutrition of the tunica intima followed by fatty degeneration, and lastly, by extreme calci-

* *Loc. cit.*, p. 22.

fication of the fattily degenerated material. On careful examination of such a patch it will be found that, during the earlier stage of calcification, there is always a quantity of fibrous-looking tissue around the calcifying area. Eventually even this becomes calcified, often without the intervention of any regular fatty degeneration. Here all the conditions for dialysis are extremely favourable; there is first the dead tissue in close proximity to a blood-stream, but cut off from it, and from the lymph both from within and from the small vasa vasorum by a layer of fibrous or formed tissue, which, in the first instance, acts as a dialysing membrane between the lymph and the dead matter, and then as a medium in which more lime may be deposited. All pathologists are acquainted with the similar changes that take place in the middle coats of the medium-sized vessels of old people, in old unabsorbed infarctions, especially of the spleen, in the walls of old encysted trichina spiralis, in extra-uterine fœtus formation, especially in the superficial parts, in caseous infiltrations of the peri- or epicardium, on the pleura in old people, and in caseous tubercular masses in the lung, and in softening glands in the human subject, and especially in the Herbivora.*

If we take the last as an example, we find that we have another factor present, of which mention has already been made in connection with bone, *i.e.*, rapid new cell-formation in the immediate neighbourhood of the dialysing membrane, which in this instance consists of the fibrous layer immediately surrounding the caseous tissue. Here then are the three factors necessary:—tissue thoroughly devitalised, where we have dead albumenoid matter; secondly, there is around the mass of dead tissue, a layer of formed material as fibrous tissue, which, as in the case of bone or cartilage matrix, may form a dialysing membrane; and, lastly, there is the layer of proliferating cells always found in the region of any foreign or dead mass, which may, in such a case, be said to take the place of osteoblasts of bony tissue, as generators of, amongst other things, carbonic acid. On careful examination of a gland from a tuberculous cow, in which caseation was almost complete, but in which

* (See Ziegler's *Pathology*; translated by Macalister, vol. i. pp. 96, 97; Litten, *Der Hæmorrhagische Infarct.*, 1879; Kyber, *Virch. Archiv*, vol. lxxxi.; Payne, *Pathology*, p. 198; Coat's *Pathology*, p. 127.)

calcification had only just commenced, we found that by far the larger proportion of cretaceous material was present as minute hard points immediately beneath the fibrous capsule. In fact, when the gland was cut in two, the caseous mass could be completely "shelled out" from the dense fibrous capsule. On the surface of the central mass were small concretions, each of which appeared to fit into a corresponding depression in the capsule. Examining the centre of the mass, a far less proportion of lime was found than at the periphery. Here then is a fact favouring our view that dialysis plays a most important part in the separation of lime salts from lymph and their deposition in bone. An analysis of the solid portions of this gland gave the following results. After washing away the softer central caseous matter, the remainder, 124·60 grs., was ignited, by which there was lost 111·13 grs., leaving 13·47 grs. or 10·61 per cent. of ash.

This Ash consisted of Phosphate of Lime, . .	81·26
Carbonate of Lime, . .	14·31
Salts, ...	4·61
	100·00

It will be seen at once from this analysis that the proportion of carbonate of lime is considerably greater than in the bones of cattle, in which the proportion on the above calculation would be as 81·26 phosphate to 9·85 of carbonate.

It is to be borne in mind, however, that there is usually a large area of granulation, or small cell tissue, around the fibrous layer, and that consequently the conditions approach more nearly the secreting surface of the oviduct of the hen say, where we have the thick layer of secreting protoplasm with the evolution of a considerable amount of carbonic acid. There is, however, a considerable proportion of phosphate of lime directly separated and deposited, not in the fibrous layer in the first instance, but in the dead tissue beyond it.

The process here differs somewhat from the allied process in bone, in this respect, that in bone there is the separation always taking place from two surfaces of the trabeculæ, the deposit appearing in the middle and gradually extending outwards, whilst in the case of

the degenerated gland the separation takes place on one side of the membrane, and the deposition on the other, where there is a mass of dead albumenoid material which readily takes up all the lime salts brought to it. That there is an outward current of certain material in the case of tubercular caseous masses enclosed in a fibrous capsule is extremely probable, from what we have observed in waxy disease associated with tubercle.

The tendency to calcification in degenerated areas in the herbivorous animals has long been a subject of remark amongst comparative pathologists. This tendency is markedly exaggerated where the area is in any organ in which the tissue metamorphoses are specially active. For instance, in tubercle of the lung and udder in the cow, calcification follows caseation with remarkable rapidity, especially in the latter organ. In a case of acute miliary tuberculosis in the horse, for specimens of wh'ch we are indebted to Professor M'Fadyean, this calcification has taken place at once, and the whole of the caseous material in the centres of most of the minute tubercular nodules are infiltrated with globular particles of insoluble lime salts. Around this central portion there are usually a few hyaline-looking cells, which have evidently taken the place of the fibrous tissue as a dialysing membrane, and outside this again we have the actively proliferating granulation tissue characteristic of this process. It is a well-known fact that in febrile conditions in Man and in the Carnivora, where there is an increase in tissue metabolism, indicated by the rise of temperature of the blood, there is always, or usually, an increase in the amount of triple phosphate of lime, magnesia, and ammonia deposited in the urine. Under similar circumstances, in the Herbivora, on the other hand, we have an increase in the quantity of carbonate of lime, frequently in the form of peculiar dumb-bell shaped crystals, deposited after the urine is passed. In both cases the increased activity of the tissues leads to an increased evolution of carbonic acid, and we have a beautiful experiment in nature corresponding to our experiments relating to the precipitation of phosphate and carbonate of lime from a solution of phosphate of lime and alkaline carbonates, the proportions of the two salts precipitated, varying according to the relative amount of phosphoric and carbonic acid present. The tissues here playing the part of carbonic acid formers in the process.

The importance of this difference in the constitution of the urinary salts should be appreciated by clinicians.

To return to the examples of calcification. Another, bearing upon the point in question, is the calcification going on in the superficial layers of tissue of the new growth in a case of extra-uterine fœtation. Here, again, we have the dead matter in the centre surrounded by a layer of fibrous tissue and again by a granulation tissue layer. The calcification appears first in the superficial dead tissues immediately beneath the fibrous layer. The same thing occurs in the membranous cyst of the old trichina spiralis. In the uterine fibroid the centres of calcification are numerous, at first small, but they gradually run together; their method of formation appears to differ from any that we have examined in other positions, but even in them the evidence is not at all against the theory of dialysis into the dead tissue. In connection with this process of calcification our attention was drawn by Mr H. A. Thomson, M.B., to the peculiar solid wedge-shape "infarcts" found near the ends of tuberculous bones. König describes them as probably the result of a cutting off of the blood supply from a small wedge-shaped area, as in any other example of infarction. Numerous objections have been taken to this explanation, and the masses have been said to be due to the formation of tubercular sequestra. In all probability both processes enter into the causation of these pale hard masses. Owing to the tuberculous changes in and around the vessels, in such a case there is formed an infarcted area. This is borne out by recent experiments by Klein, Watson, Cheyne, and others. But beyond this, the infarcted area, there may or may not be a primary tubercular process. Even when there is no primary tubercle the osteoblasts and other cells first proliferate and then die *in situ;* they, then, as in other positions, become swollen and hyaline, and are then rapidly infiltrated with lime salts, and the infarcted bone assumes a much denser appearance than the normal bone surrounding it. It is conceivable that the lime salts in such a dense compact tissue can only come by some more or less mechanical process such as we have described. A similar condition occurs in certain diseases of bone, especially during the later stages of osteo-sclerosis where that condition is associated with specific disease. The conditions are the same in all cases,—dead

tissue areas with increased tissue activity in the immediate neighbourhood. Numerous other examples might be cited, but these will suffice for our present purpose.

Fokker* maintained that he had been successful for the first time in preparing albuminate of lime as a translucent gelatinous mass which "is soluble in water in solution of sodium chloride and in phosphoric acid. Its aqueous solutions are not coagulated by boiling except after the addition of neutral salts. They are precipitated by acids, the precipitate being soluble in excess." He gives a series of experiments to prove that lime albuminate is present in the blood, and that it there behaves just as do artifically prepared solutions of lime albuminate, *i.e.*, "it can be preserved for some time *in vacuo* without depositing calcium phosphate, but this is gradually thrown down by caustic alkalies and precipitated by ammonium oxalate," and he concludes that it is probable that all the lime in the serum is present as lime albuminate, and that no other lime compounds exist in the blood.

It is certainly quite possible that this may be the case, but not very probable, though the albumen may be the medium in which many of the changes are brought about in the rearrangement of the salts of the alkalies and alkaline earths in the various chemico-physiological processes. On the other hand, phosphate of lime is found in all animal tissues, in the blood, in urine, serous fluids, saliva, and in gastric juice; it is soluble in chlorides of sodium and ammonium, in combination with protein compounds, lactic acid or sugar, and we know that it may be present in solutions made up experimentally. From the fact that protein compounds keep the phosphate of lime in solution, and from what has been observed in the oviduct of the hen, and in the mantle of Anodonta, &c., it is possible, as Fokker† suggests, that the lime may be held in the blood, and perhaps also in the lymph, in part as a lime phosphate albuminate, from which the carbonate is formed by the accession at any point, but especially on a free surface, of a large quantity of carbonic acid, the phosphoric acid being again absorbed and reutilised.

Without doubt the phosphoric acid and lime appear to be associated with the albumen present in the blood, and although we

* *Pflüger's Archiv*, vol. vii. p. 274; see Watts's *Dictionary of Chemistry.*
† *Loc. cit.*

cannot with certainty isolate the compound, or, as yet, do more than indicate its composition, there is sufficient evidence that the changes we have indicated may occur through the instrumentality of such a compound.

The same thing may be said in regard to the association of carbonic acid (as alkaline carbonates) with albumen, both acids being always present in the blood of animals in varying quantities, and, as asserted by Liebig, the proportion of these acids may vary to almost any limits, without altering the general character of the blood.

Thus we have, without disturbance to its general characteristics, two distinct conditions existing in the blood of animals, and these conditions, in addition to those already mentioned, may, and doubtless do, account, to some extent, for the secretion of lime salts in the different forms in which we find them to occur—in the one case the secretion taking the form of phosphates as in bone, and in the other that of carbonate of lime, as in shell and coral formation.

In other words, when alkaline phosphates associated with lime and albumen preponderate in the blood, the lime is secreted in the form of phosphate, as in bone formation, so when the alkaline phosphates are partially replaced by an excess of alkaline carbonates, as in marine animals, the lime is secreted principally as carbonate.

In neither case are the lime compounds deposited in a pure form, as when the phosphate predominates it is always accompanied by a certain amount of carbonate, so when the carbonate predominates, phosphate of lime is always present as we found them in bone, and shell, and coral formation. It may be assumed in the one case that the albumen becomes converted into gelatine, which forms the structure in which the phosphate of lime is deposited in bone, and in the other into a chitinous mass in which the carbonate is deposited as in the exo-skeleton of crustaceans or the shell of the egg.

These views are founded upon actual experimental results, for if a solution of alkaline phosphates and soluble lime salts in carbonic acid is allowed to stand for a short time, the clear solution soon becomes cloudy, and a precipitate is formed which is found to consist of phosphate of lime, and the same thing happens with a solution of alkaline carbonates and soluble lime salts in carbonic

acid, but in this case the precipitate formed is found to be carbonate of lime, the precipitation in both cases being facilitated if the clear solutions are maintained at a temperature of 90° F. When a mixture of alkaline phosphates and carbonates and soluble lime salts are so treated, the precipitate which results is found to be a mixture of phosphate and carbonate of lime—the amounts varying with the proportion of the salts present.

Thus bone contains from 50 to 60 per cent. phosphate, and from 6 to 10 per cent. carbonate of lime, and the calcareous shells of crustaceans and eggs of birds contain from 60 per cent. to 80 per cent. carbonate with 1 to 2 per cent. phosphate of lime.

We have thus given a rational explanation of the chemical changes which probably govern the secretion of different lime salts by animals, but in such investigations, dealing with matter in its relations to protoplasmic or vital force, we can only reason by the inferences we may draw from the imperfect knowledge we obtain experimentally in the laboratory.

The ordinary methods of analysis fail to give us an accurate knowledge of the relative position occupied by matter in the complex relations existing in such conditions.

Summary.

We assume that the greater part of the carbonate of lime in the ocean-beds is the result of animal life.

Soluble carbonate of lime is present in very small quantities in sea water, but other salts of lime, especially the sulphate, are present in larger quantities.

Hens supplied with sulphate of lime, but no other lime salt, produce well-formed egg-shells composed of carbonate of lime.

In the organs of the body, gizzard crop, and fæces of a hen so fed, only a small quantity, 10·08 grs. of carbonate of lime was found, but there was a considerable quantity of phosphate of lime present. (It was afterwards found that sufficient carbonate of lime, for the formation of the shells of two eggs, could be stored up in the crop, gizzard, &c.).

In the alimentary canal sulphide of calcium and then phosphate or chloride of calcium, or lime soaps may be formed. These latter,

with cloacal mucus yielding calcium carbonate, carbonic acid gas, and hydrogen or carburetted hydrogen.

The lime may then be carried to the secreting surface of the oviduct in fowls as a soluble phosphate of lime and soda, as calcium chloride, as lime soaps (in combination with fatty acids), or even as carbonate. This latter is, however, not so probable.

It is then secreted along with urea, carbonate of ammonia, carbonic acid (in a nascent condition), &c. The nascent carbonic acid combining with the lime in presence of the urea (or carbonate of ammonia), we have a quantity of carbonate of lime deposited in the organic membrane, and the shell is formed of insoluble carbonate of lime.

In certain eggs the carbonate of lime is partially replaced by phosphate.

If this takes place in hens, it is probable that similar processes may go on in connection with the formation of carbonate of lime by marine animals, which have the sulphate of lime presented to them in the presence of chloride of sodium.

In the case of the crab, sulphate of lime is apparently not assimilated, even in the presence of chloride of sodium, and crabs which throw off their shells in artificial sea water, in which both the above salts are present, but from which chloride of calcium is excluded, do not form a new exo-skeleton of carbonate of lime. As soon as chloride of calcium is added, although the sulphate be withheld, shell formation may go on.

Shell formation in the crab is somewhat different from egg-shell formation in the hen, and occupies an intermediate position between such egg-shell formation and bone formation, as the carbonate of lime is deposited in the chitinous portion of growing epithelial cells in the crab shell. In the egg-shell it is deposited in a material quite separated from the shell, whilst in bone, the matrix in which the lime is deposited, though separated, is intimately connected with small cells, not epithelial in character.

In the egg-shell the organic and inorganic material are both secreted by and then separated from the epithelial cells. In the crab shell the organic material (chitin) remains attached to the upper part of the epithelial cells, and in this the lime salts are deposited, probably by a process of dialysis, whilst in the case of bone the

cells are not epithelial in character, the matrix, though separate, is closely associated with the cells, especially during its formation, and the lime is deposited in the matrix, also apparently by a process of dialysis.

Phosphoric acid combined with alkalies and alkaline earths is found in large and constant quantities in the blood and lymph. It acts as a carrier of lime, &c., to every point of the body where carbonic acid may be given off. Carbonate of lime is formed, and the phosphoric acid re-enters the circulation to continue its work as a carrier.

The nascent carbonic acid given off by the cells in the neighbourhood of the bone matrix, brings about precipitation of bath phosphate and carbonate of lime, which salts are dialysed into the matrix. In most of the secreting layers and in the fluids supplied to them, lime is found as some form of phosphate. Where the fluids bathe the matrix, or where there is no intervention of an epithelial secreting layer, the lime is thrown down very largely as phosphate as in bone, but where an epithelial secreting layer is found, or where there is much distance or tissue between the fluids and the seat of deposition, carbonate of lime preponderates.

In the secreting layer of the mantle of certain molluscs, or in that of the oviduct in the hen, the lime in the epithelial cells is principally phosphate, whilst the fluid bathing its outer surface and the shells themselves contain the lime, principally, in the form of a carbonate. If there is a definite interval between the secreting surface and the area of deposition, or if much chitin or other tissue is developed between the actively secreting cells and the tissue in which the lime is deposited, there is always a greater tendency to the formation and deposition of carbonate of lime.

Lime salts, of whatever form, are deposited only in vitally inactive tissue. They are found in bone matrix, in chitin, in old fibrous tissue (?), or in tissues that have undergone fatty or caseous degeneration. Wherever such *dead* tissue is found, dialysis goes on, and lime in an insoluble condition is deposited.

Although the lime is deposited in dead tissue and dialysed through a vitally inactive membrane, it is apparently separated from the fluids of the body through the activity of carbonic acid secreting protoplasm of cells.

When alkaline phosphates associated with lime and albumen preponderate in the blood, the lime so separated is in the form of phosphate, as in bone formation; when these are partially replaced by an excess of alkaline carbonates as in marine animals, the lime is secreted as carbonate.

The corals have a secreting layer of cells similar in all essential features to the secreting layers above mentioned, and the products of their activity assume the same forms, chitin, chitin infiltrated with carbonate of lime, and almost pure carbonate of lime with a small quantity of organic cementing material.

Carbonate of lime may be formed as follows:—

The carbonate of ammonia produced by the decomposition of the effete products of animals, urea, &c., decomposes a portion of the sulphate of lime in the sea water with the formation of carbonate of lime equivalent in amount to the carbonate of ammonia thus formed.

We take this opportunity of thanking Messrs Drysdale and Anderson, who, as chemists, attached to the Marine Station, Granton, gave us their hearty assistance during the period over which these investigations extended.

APPENDIX.

TABLE I.—EGG SHELL (HEN'S).

Dried at 212° F. Membrane removed.

	Salts added to Food.					
	No. 1. Sulphate of Lime.	No. 2. Phosphate of Lime.	No. 3. Silicate of Lime.	No. 4. Nitrate of Lime.	No. 5. Carbonate of Magnesia.	No. 6. Carbonate of Strontia.
Hens laid Eggs.	Normal.	Normal.	Normal.	Normal.	Shell-less.	Shell-less.
Analysis of Shell.						
Organic matter, .	6·86	5·77	About same as in 1 and 2.	Same as in 1 and 2.	...	...
Carbonate of lime, .	92·03	93·07			...	...
Phosphate of lime, .	1·11	1·16			...	...
Sulphate of lime, .	trace	trace			...	...
Silica, . . .	,,	,,			...	...
Magnesia, . .	,,	,,			...	...
Iron, . . .	,,	,,			...	...

TABLE II.

	Artificial Sea Waters.				Ordinary Sea Water.
	No. 1.	No. 2.	No. 3.	No. 4.	
Chloride of sodium, . .	2·7205	2·4804	2·4804	2·6996	2·7254
Chloride of magnesium, .	·3794	·3320	·3320	·3696	·3813
Sulphate of magnesium, .	·1551	·1357	·1357	·1491	·1660
Bromide of magnesium, .	...	...	·0079	...	·0076
Sulphate of lime, . .	·1276	·1116	·1116	...	·1262
Sulphate of potash, . .	·1026	·0898	·0898	·1125	·0863
Chloride of calcium, . .	...	·0903	·0903	·0964	...
	3·4852	3·2398	3·2477	3·4272	3·4928
Water, ...	96·5148	96·7620	96·7523	96·5728	96·5072

TABLE III.

Analyses of "Ecdysed" Crab which had lived ten days after casting in No. 2 Water, and of an ordinary Shore Crab to compare with it. Size across Carapace, 1½ inch.

	Ecdysed Crab.	Shore Crab.
Chitinous matter on exo-skeleton, . .	3·77	10·76 grains.
Carbonate of lime on exo-skeleton, . .	5·50*	39·78 ,,
Carbonate of lime of interior, including stomachical teeth, &c., ...	0·25	1·53 ,,
Teeth (mandibles), ...	0·041	0·59 ,,
Phosphate of lime on exo-skeleton, .	2·396	4·48 ,,
Phosphate of lime of flesh, lymph, and total interior structure, ...	0·18	0·39 ,,
Water, flesh, &c., . . about	350·00	350·00 ,,

* On a *newly* ecdysed crab there was no trace of lime-salt deposition, but in THIS example carbonate of lime had been deposited to some extent before the death of the animal.

TABLE IV.—EDIBLE CRAB.

Water, blood, salts, &c., ...	6646 grains.
Flesh (gave 14·56 of ash, containing 4·94–$3CaO,PO_5$)	295 ,,
Outer calcareous structure, ...	2956 ,,
Inner calcareous structure, ...	103 ,,
	10,000 ,,

Calcareous Structure—consisting of—

	Total.	Chitin.	Carbonate of Lime.	Phosphate of Lime.	Percentage.
Carapace,	817	150·32	656·80	9·87	Outer.
Chelæ,	1184	236·80	933·00	14·20	Chitin, 20·00
Ambulatory limbs,	736	147·20	579·97	8·83	CaO,CO_2 78·80
Abdominal segments,	156	31·20	122·93	1·87	$3CaO,PO_5$ 1·20
	63	12·60	49·64	·76	100·00
Outer structure weight,	2956	578·12	2342·34	35·53	Inner.
Inner structure,	103	35·00	66·98	1·02	Chitin, 34·00
					CaO,CO_2 65·00
					$3CaO,PO_5$ 1·00
					100·00
Teeth (mandibles) weighed,			17 grains.		
Stomachical teeth (horny matter),			27 ,,		

TABLE V.—Lymph from Edible Crabs.

When in condition, one of these crabs, weighing about 8000 grains, gave from its lymph—

Phosphate of lime,	11·10 grains.
Phosphoric acid (PO_5),	15·78 ,,

The latter probably present in combination with albumen or as alkaline phosphates.

Another crab, about same weight, but out of condition, gave—

Phosphate of lime,	0·898 grains.
Phosphoric acid (PO_5),	4·967 ,,

Theoretical Description of a New "Azimuth Diagram."

By Capt. Patrick Weir. *Communicated by* Sir Wm. Thomson.

(Read July 15, 1889.)

About thirteen years ago, when officer of a vessel trading from London to Australia, and in the daily practice of verifying the error of the compass by azimuth's declination, the idea first occurred to me of projecting the sun's path on the plane of the horizon; and during the voyage I succeeded in solving the problem, and constructed a diagram which I used daily while at sea for several years, until I gave up long voyages, and had no further occasion to trouble about azimuths.

I will not attempt to illustrate the correctness of my theory by any purely mathematical formulæ, but will confine myself to stating as plainly as possible, and as far as I can remember, the train of reasoning by which I succeeded in constructing the diagram.

Suppose that the latitude is 90° N., that is to say that the observer is standing on the North Pole, it is quite evident that the sun's path, projected on the plane of the horizon, would be a circle, and also that, no matter what the declination was, the sun's bearing would be the same at the same hour every day, say Greenwich time, its altitude only being affected by a change in declination.

Again, suppose the observer to be on the equator and the declination 0°, it is self-evident that the sun would rise due east, ascend on an azimuth circle to the zenith, and descend due west; so that his path, projected on the plane of the horizon, would be a straight line. Suppose, again, that the observer is still on the equator, but the sun is in declination 20° N., by calculation his rising amplitude will be E. 20° N., and setting amplitude W. 20° N., while his meridian zenith-distance will be 20° N.

Suppose the length of the line which represents the sun's path to be fixed at any length, say about 8 inches as in diagram, then all that is necessary to get the bearings to fit in as by calculation is to shift the position of the observer, in an opposite direction to the declination, a distance equal to the tangent of the declination, taking half the length of the line as radius.

Now, if the sun's path may be represented on the plane of the horizon, by a straight line for lat. 0° and by a circle for lat. 90°, it is, I might say certain, that it may be represented by an ellipse for any intermediate latitude, on something the same principle that, while the crank-pin of a steam-engine describes a circle and the crosshead travels in a straight line, any intermediate point in the connecting-rod describes an ellipse. The relative lengths of the major and minor axes of the ellipse which will correctly represent the sun's path for any particular degree of latitude may be illustrated in the following simple manner:—If a disc be held so as to throw its shadow on a plane, with the disc edge on to the light, its shadow will be a straight line, corresponding to the sun's path in lat. 0°; if held flat to the light, its shadow will be a circle, the same as the sun's path in lat. 90°; and if canted so that it is 20°

from being edge on to the light, its shadow will be an ellipse which will correspond with the sun's path projected on the plane of the horizon in lat. 20°. In the same manner, whatever may be the obliquity of the disc, its shadow will be an ellipse of the same relative dimensions as that representing the sun's path, in the latitude corresponding to that obliquity. It will at once be seen that, constructed on this principle, the major axes of all the ellipses would be equal to twice radius, consequently to each other, and would run into each other at the east and west points, and the minor axes would be equal to the sine of the angle of obliquity, or sine of lat.

This arrangement would answer very well while the declination was 0°, but in allowing for declination the position of observer would have to be shifted in the opposite direction by the following quantity:—(tan of decl. × cos of lat.). This would require a separate scale of declination to be laid down for each ellipse of latitude, which, to say the least of it, would be extremely inconvenient; so, to make the scale of declination available for all latitudes, I decided to vary the size of the ellipses instead of the scale of declination. That is to say, instead of multiplying the tan of declination by the cos of latitude, I divide both major and minor axes of any particular ellipse by the same quantity. The formula would therefore be as follows:—

$$\frac{\text{Radius}}{\text{Cos lat.}} = \text{sec lat.} = \text{major axis}, \quad \frac{\text{S. lat.}}{\text{cos lat.}} = \text{tan lat.} = \text{minor axis.}$$

It will easily be seen that this preserves the relative lengths of the major and minor axes of the ellipse for any degree of latitude, as illustrated by the shadow of the disc, because as radius : sec : : sine : tan.

This arrangement also allows the declination to be measured and marked off on the same scale as the latitude; and further, it locates the foci of all the ellipses in the ends of the straight line which represents the sun's path in lat. 0°, a great consideration when the ellipses have to be drawn with pins and a thread as I have always done them.

I have taken it for granted that the sun's path, projected on the plane of the horizon, will be exactly the same at all times of year, the whole ellipse representing said path being shifted north or

south (according as the declination is north or south), a distance corresponding to tan of declination with half the distance between the foci as radius. (If half the major axis of any ellipse be taken as radius, then it would have to be shifted a distance = tan decl. × cos lat., which would be just the same.) It is, however, obviously impossible to make the ellipses shift on the paper, so the difficulty is very simply got over by supposing the position of the observer to be removed to a corresponding distance in the opposite direction, as illustrated in the case of lat. 0°, and as laid down in the directions for using the diagram.

I have only constructed my diagram up to lat. 60°, which I consider high enough for all practical purposes, but it may be observed that for lat. 90° the major and minor axes would be infinity, which it may be said reduces the ellipse for that latitude to a circle as required by my first supposition.

Having calculated the dimensions of the ellipses, the next step is to fix the position of the sun on them at any time, and it is evident that the noon line in all latitudes will correspond with the meridian or minor axis of the ellipses, as the sun is either due north or south at noon, apparent time in all latitudes.

It is also equally certain that the six-hour line will be at right angles to it, and will correspond with the major axes of all the ellipses, as the sun will just have performed one quarter of his diurnal revolution at this time.

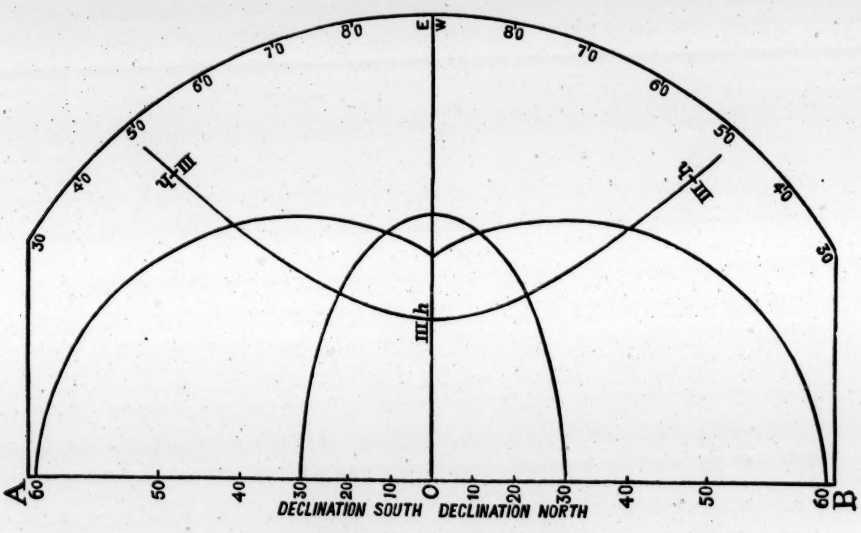
To arrive at the position of the intermediate hours, &c., I proceed as follows:—Take any ellipse of latitude and with centre 0 (see diagram), and half the major axis of the ellipse as radius, describe a circle about the ellipse; divide the circle as minutely as may be required (say, into hours and quarters), and from these divisions draw lines parallel to the meridian, cutting the ellipse, and where they cut will be the position of the sun on it at that particular time. This routine has to be gone through for, say, every fifth degree of latitude; and when the points on the ellipses for each particular period have been joined in a regular sweep, they will be found to form a curve *very much like a hyperbola.* [The hour-curves are in fact hyperbolas, confocal with the latitude ellipses. See Professor Tait's Note below.—W.T.]

For convenience in measuring off the azimuth, I have put a

graduated horizon round the diagram, but any other mechanical means might be substituted.

This completes the diagram as far as the calculating of azimuths is concerned, and, as I agree that, for practical purposes, the rising and setting circles could be very well dispensed with, I will not occupy your time with them, except to state an arbitrary rule for finding the centres of any of these circles.

To find the centre of the rising or setting circle for any degree of declination:—From 90, subtract twice the declination of circle required and remainder will give the centre of circle required on meridian or line of tangents, radius being equal to distance from point as found to focus of ellipses.



[The accompanying diagram has been prepared to illustrate Capt. Weir's paper. It shows the ellipse for latitude 30°, the hyperbola for hour-angle III. o'clock, and the rising and setting circles for declination 30°. A complete diagram, showing the ellipses for all degrees of latitude from 0° to 60°, and the hour-angle hyperbolas for every 4^m. of time, has been accurately drawn by Mr R. Wills, and is to be published by Potter, mounted on cardboard for practical use, with Capt. Weir's instructions, revised and to some degree simplified by Prof. George Darwin and Sir William Thomson.]

As compared to Saxby's spherograph and Burdwood's tables, the two methods of computing time azimuths most in use, this diagram has several advantages which I shall briefly notice.

1. *Expense.*—The spherograph is rather a costly affair (about £2, I believe), and for that reason has not come into general use. Burdwood's tables are not so expensive, but still they cost 12s. 6d. per set, whereas this diagram could be published in chart form at certainly not more than one-fifth of that price.

2. There is very little danger of making mistakes in taking off the bearing from this diagram, as it is done with parallel rulers as from a chart, the top being north and the bottom south, while the bearing is of course east or west as the time is A.M. or P.M.

3. *Accuracy.*—When the diagram is finely engraved the bearing may be taken off very accurately, as it is on a larger and more open scale than the spherograph.

4. *Convenience.*—Being in chart form, and worked with parallel rulers the same as a chart, and as the officer who uses this diagram will also probably be using a chart, it will come conveniently to his hand to use the diagram.

Note on Capt. Weir's Paper. By Prof. Tait.

(Read July 15, 1889.)

[As Sir W. Thomson was unable personally to communicate Capt. Weir's paper to the Society, he asked me to add to it a Note on the principle of the new method.]

Capt. Weir's singularly elegant construction not only puts in a new and attractive light one of the most awkward of the formulæ of Spherical Trigonometry, but it practically gives in a single-page diagram the whole contents of the two volumes of Burdwood's *Azimuth Tables.* Further, it supplies a very interesting graphical plane construction of a function of three independent variables.

In the usual notation for spherical triangles, if A be the zenith, C the pole, and B a heavenly body (whose declination is δ), C is the hour-angle (h), b the colatitude $\left(\frac{\pi}{2} - \lambda\right)$, and A the supplement of the azimuth. Hence, from the formula

$$\cot a \sin b = \cot \mathrm{A} \sin \mathrm{C} + \cos b \cos \mathrm{C},$$

we have at once

$$\tan(\text{azimuth}) = \frac{\sin h}{\sin\lambda\cos h - \tan\delta\cos\lambda}.$$

Capt. Weir, in his diagram, virtually puts

$$\left.\begin{aligned} x &= \sin h \sec\lambda \\ y &= \cos h \tan\lambda \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

so that

$$\tan(\text{azimuth}) = \frac{x}{y - \tan\delta},$$

x and y being found by the intersection of the confocal conics

$$\frac{x^2}{\sec^2\lambda} + \frac{y^2}{\tan^2\lambda} = 1, \text{ the latitude ellipse,}$$

and

$$\frac{x^2}{\sin^2 h} - \frac{y^2}{\cos^2 h} = 1, \text{ the hour-angle hyperbola.}$$

The Amplitude is the value of the azimuth at rising or setting, so that the corresponding hour-angle is to be found from

$$\cos h + \tan\lambda\tan\delta = 0.$$

With this value of h, equations (1) become

$$\left.\begin{aligned} x &= \sec\lambda\sqrt{1 - \tan^2\lambda\tan^2\delta} \\ y &= -\tan^2\lambda\tan\delta \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

Elimination of δ gives, of course, the latitude-ellipse as before. But elimination of λ gives, instead of the confocal hyperbola, the curve

$$x^2 + [y - \tfrac{1}{2}(\tan\delta - \cot\delta)]^2 = \tfrac{1}{4}(\tan\delta + \cot\delta)^2,$$

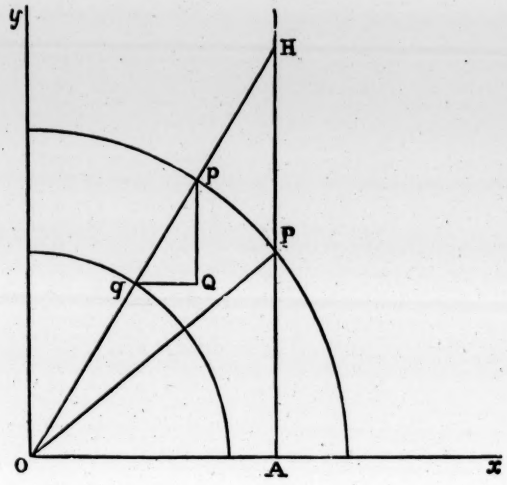
or

$$x^2 + (y + \cot 2\delta)^2 = \operatorname{cosec}^2 2\delta,$$

which is a circle passing through the common foci of the ellipses and hyperbolas.

The construction of the "Diagram" by means of (1) is, theoretically, a very simple matter. Thus, take OA as unit length on the

axis of x, and draw AP parallel to y. Make $AOP = \lambda$, and $yOH = h$. Draw the circles whose centre is O, and radii OP and AP respectively. Let OH meet them in p, q. From p and q draw lines parallel to Oy, Ox, respectively. Their point of intersection, Q,



belongs obviously to the ellipse λ, and to the hyperbola h. A somewhat similar, simple, construction can easily be given for the circle.

On the Coagulation of Egg and Serum Albumen, Vitellin, and Serum Globulin, by Heat. By John Berry Haycraft, M.D., D.Sc., and C. W. Duggan, M.B.

(*From the Physiological Laboratory of the University of Edinburgh.*)

(Read July 15, 1889.)

A large number of proteid substances, when in solution, are coagulable by heat. As the temperature of such a fluid is raised, faint opalescence at first appears, and then, at a higher temperature, masses (flocculi) of albumen separate out, in most cases, suddenly, from the fluid. It is generally held that each coagulable albumen is so affected at a definite temperature peculiar to itself; thus, egg albumen is said to become opalescent at 60° C., and to separate out in flocculi at 63° C. Unfortunately, hardly two observers agree as to the exact temperature at which opalescence and coagulation

occur; thus, keeping to the example, egg albumen, Wurtz puts the coagulation point at 73° C., and Henrijean at 60° to 61° C.

It is hardly possible to explain such differences, either on the assumption that any of the above authors had used imperfect apparatus, or, that they had been guilty of inaccurate observation. It is more probable that the conditions, under which the experiments were performed, were not always the same. What are the conditions which are capable of modifying the coagulation point of albumen? It seemed to us a not unimportant point to investigate systematically these conditions; as such investigation is calculated to throw light on the nature of coagulation itself, and may enable one to arrive at the exact specific coagulation points of the more important proteids, heated as they should always be under exactly similar conditions. The conditions modifying coagulation, which we have studied, are,—the rapidity at which the coagulation is allowed to take place, the degree of concentration of the proteid substance itself, the presence of acids and alkalies, and the presence of soluble salts.

The Rapidity at which Coagulation is allowed to take place.

This is an acknowledged factor varying the indicated temperature of coagulation, and at least one author has alluded to it in the case of the particular albumen studied by himself. If a solution of a coagulable proteid be heated quickly, the proteid will be found to coagulate at a higher temperature than if the heat be applied more slowly. Thus we found that egg albumen, diluted with one volume of water, coagulated at 64° C., when slowly heated, the temperature taking forty minutes to reach this point. Another portion of the same solution coagulated at 66° C., when heated rapidly, the experiment taking in this case only one minute. It is not difficult to explain this fact. If a drop of an albuminous fluid is mounted for microscopical examination, and, if it be heated on the stage of the microscope, the process of coagulation can be readily followed out. When opalescent, the fluid will be found to contain numbers of tiny granules. These granules increase in size, and apparently become adherent, and run together to form granular masses or flocculi. This naturally requires time, and if the fluid be heated

rapidly the temperature may materially increase above the point at which, were the fluid kept for a sufficient time, coagulation would occur. Although our experiments convince us of the general truth of this fact, it occasionally happens that an albumen slowly heated coagulates at a very high temperature, and perhaps never forms distinct flocculi, the coagulation being in the form of a thin jelly. Another portion of the same solution quickly heated coagulates in flocculi at a lower temperature. We have found this occur with some specimens of serum albumen. We are inclined to explain this occurrence on the supposition that the slow and continuous heating in these cases causes some chemico-physical change in the albumen itself, whereby its coagulation is affected.

The Influence on the Coagulation Point of the Degree of Concentration of the Albumen itself.

We find, as the result of our experiments, that in all the albuminous solutions we have investigated, the coagulation point is considerably raised by diluting the solution. A very dilute solution may not coagulate even on boiling, and egg white, diluted, but nevertheless forming a comparatively strong solution, cannot be coagulated, as Sir William Roberts long ago pointed out.

In our experiments we invariably proceeded in the same way as regards the rapidity with which the solutions were heated, so as to eliminate any fallacy which might arise on this score. The usual method for determining coagulation points was adopted. The solution was placed in a test-tube containing a thermometer which could be used as a stirrer. The test-tube was immersed in a water-bath consisting of two beakers, one within the other, and each one filled with cold tap water. The water-bath was heated by a Bunsen, the flame of which was kept always at the same height, and so arranged that it took some forty minutes for the fluid in the test-tube to reach the temperature of 80° C. All our experiments were performed in this way, so that uniformity of results was obtained. We are inclined to think, however, that the heating process was unnecessarily slow, not only on account of loss of time, but what is more important, because it permitted changes to take place in the albuminous solution, especially when acids or alkalies were present in the fluid.

The Effect of Dilution on the Coagulation Point of Egg Albumen.

Egg albumen was prepared by cutting up the glairy white of an egg and squeezing it through a linen cloth. When this was diluted with water, the dilute solutions were carefully filtered. The egg albumen was always alkaline in reaction, but we decided not to neutralise it.

In the first experiments the opalescence of the heated solution alone was observed.

(1) Undiluted egg-white became opalescent at 58° C.

(2) Egg-white, diluted with one volume of water, became opalescent at 58°·75 C.

(3) Egg-white, diluted with two volumes of water, became opalescent at 59°·75 C.

(4) Egg-white, diluted with three volumes of water, became opalescent at 60°·5 C.

(5) Egg-white, diluted with four volumes of water, became opalescent at 61°·75 C.

In the second experiment the appearance of flocculi was noted as well as the opalescence.

Opalescence appeared in the undiluted egg-white at 59° C., but did not appear so soon in the diluted portions, occurring about 1° C. higher for each dilution.

(1) The undiluted albumen coagulated with the formation of flocculi at 64° C.

(2) With one volume of water flocculi formed at 65°·5 C.

(3) With two volumes of water flocculi formed at 69° C.

(4) With three volumes of water a few flocculi formed at 80° C., the albumen never completely separating out.

(5) Greater dilutions showed opalescence, but flocculi did not appear.

The Effect of Dilution on the Coagulation Point of Serum Albumen.

Serum albumen is said by Hoppe-Seyler (iii. p. 232) to become opalescent at 60° C., and to coagulate at 72°·C. to 73° C., and Schäfer places it at 70° C. (4, p. 181).

Serum albumen was prepared in the following way:—The serum from bullock's blood was saturated by the hand with magnesium

sulphate, the precipitated globulin filtered off; by this means one obtains a solution of serum albumen in a saturated solution of magnesium sulphate. It would have been useless to dilute this solution with water, for, in that case, both the albumen and the magnesium sulphate would suffer dilution. Dilution was effected by the addition of a saturated solution of magnesium sulphate.

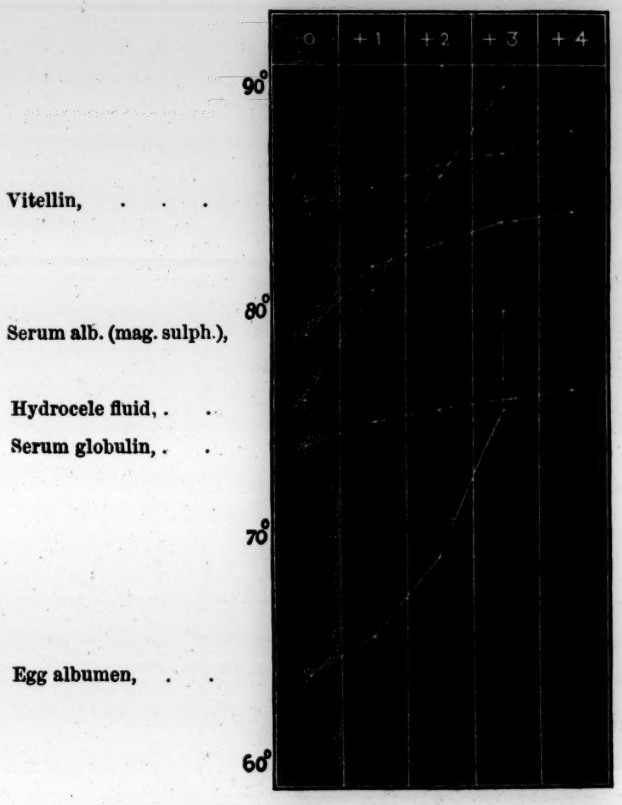


FIG. 1.—Showing the Temperature at which certain Albumens coagulate when diluted with One, Two, Three, and Four Volumes of Fluid.

(*a*) Undiluted serum albumen, saturated with magnesium sulphate, becomes opalescent at 77° C., and coagulates at 79° C.

(*b*) The same solution, diluted with one volume of a saturated watery solution of magnesium sulphate, becomes opalescent at 79° C., and coagulates at 82° C.

(*c*) When diluted with two volumes, opalescence occurs at 79° C., and coagulation at 83° C.

(*d*) When diluted with three volumes, opalescence begins at 81° C., and coagulation at 84° C.

(*e*) When diluted with four volumes, opalescence begins at 81°·5 C., and coagulation 84°·75 C.

(*f*) When diluted with five volumes, opalescence begins at 82° C., and coagulation at 85°·25 C.

The numbers quoted do not give us the correct coagulation points for diluted solutions of serum albumen; they are the coagulation points of diluted solutions plus magnesium sulphate, which raises the coagulation point considerably, as we shall subsequently see. The experiment only serves to show how coagulation varies with dilution of the albumen. In this experiment fine flocculi appeared even in the more dilute solutions, and their presence rendered the determination of the coagulation point quite easy, even in the most dilute solutions.

In order to determine the action of magnesium sulphate, serum albumen was prepared in another way.

Blood serum was diluted with two volumes of water, and a stream of carbon dioxide passed through it. The precipitate of globulin was filtered off. By this method the albumen was obtained mixed with a small quantity of globulin; its presence, however, did not prevent the recognition of the point of opalescence and the coagulation point of the albumen.

(*a*) The serum albumen became opalescent at 70° C., and coagulated in flocculi at 74°·25 C. The coagulation point being raised two or three degrees above the figure given by Hoppe-Seyler on account of its dilution.

(*b*) This solution of serum albumen, diluted with one volume of water, became opalescent at 74°, the opalescence becoming very dense at 78° C. No flocculi appeared.

On comparing these figures with those given for serum albumens in a saturated solution of magnesium sulphate, it will be seen that the former are uniformly lower, the presence of magnesium sulphate tending to elevate the coagulation point. The effect of dilution is more marked in the case of serum albumen by itself than in that of serum albumen in the saturated magnesium sulphate solution. In the first place, the coagulation becomes very imperfect in the dilute solutions; in the second place, the temperature in the dilute solution is very much raised.

The Effect of Dilution on the Coagulation Point of Vitellin.

The yolks of several eggs were dissolved in 6 per cent. solution of sodium chloride and filtered. The filtrate was poured into a large volume of distilled water, the precipitate of vitellin redissolved in saline solution, reprecipitated in distilled water, and dissolved in 5 per cent. solution of sodium chloride. In this case the vitellin, prepared from six eggs, was dissolved in 300 c.c. of the solution. In order to study the effect of dilution, a 5 per cent. solution of sodium chloride was added in all cases.

(*a*) The vitellin solution became opalescent when heated to 80° C., and coagulated at 85° C.

(*b*) When diluted with one volume of 5 per cent. solution of sodium chloride, the vitellin became opalescent at 81° C., and coagulated at 85°·5 C.

(*c*) When diluted with two volumes, it became opalescent at 82° C., and coagulated at 86°·5 C.

(*d*) When diluted with three volumes, it became opalescent at 82° C., and coagulated at 87° C.

(*e*) When diluted with four volumes, it became opalescent at 83° C., and coagulated at 88° C.

The experiment was repeated, giving a result almost precisely the same. It will be noticed that in this proteid the coagulation point does not vary to a very considerable extent with dilution.

The Effect of Dilution on the Coagulation Point of Serum Globulin.

The coagulation point of serum globulin is given by Halliburton as 75° C. (Reference 6, p. 163).

In the first experiment the globulin was precipitated from bullock's blood by magnesium sulphate. The precipitate was, after washing, dissolved in a 5 per cent. solution of magnesium sulphate. It was diluted with a 5 per cent. watery solution of magnesium sulphate. Unfortunately the opalescence was not noted down. The flocculi were well marked.

(*a*) The solution of serum globulin in a 5 per cent. solution of magnesium sulphate coagulated at 74° C.

(*b*) The solution, when diluted with an equal volume of 5

per cent. watery solution of magnesium sulphate, coagulated at 75° C.

(*c*) When diluted with two volumes, coagulated at 75°·5 C.

(*d*) When diluted with three volumes, it coagulated at 75°·5 C.

(*e*) When diluted with four volumes, it coagulated at 76°·25 C.

(*f*) When diluted with five volumes, it coagulated at 77° C.

(*g*) When diluted with six volumes, it coagulated at 77° C.

In another experiment serum globulin was prepared by passing a stream of carbon dioxide through dilute blood serum. The precipitated globulin was dissolved in 5 per cent. solution of sodium chloride. The solution of globulin not being of the same strength (a little weaker), and the salt used for its solution being a different one, the coagulation points do not correspond with those obtained in the previous experiment.

(*a*) Serum albumen, dissolved in 5 per cent. solution of sodium chloride, became opalescent at 74° C., and coagulated at 79° C.

(*b*) Serum albumen, dissolved in 5 per cent. solution of sodium chloride and diluted with one volume of a 5 per cent. watery solution of sodium chloride, became opalescent at 77°·5 C., and coagulated at 81°·5 C.

(*c*) Diluted with two volumes, opalescence commenced at 78°·5 C., and it coagulated at 82°·5 C.

(*d*) Diluted with three volumes, opalescence commenced at 79° C., and coagulated at 84° C. The albumen at this stage had begun to putrefy, and on repeating the experiments it was found that the coagulation point was raised about two degrees for (*a*), (*b*), (*c*), and that (*d*) did not coagulate even on boiling.

The Effect of Dilution on the Coagulation Point of Hydrocele Fluid.

Hydrocele fluid contains the same proteids as are found in blood plasma, namely, fibrinogen, serum globulin, and serum albumen. In a case of chronic hydrocele there may be an almost entire absence of proteid matter. The proteid substance when present varies in amount, and the coagulation point varies with it. On diluting hydrocele fluid the coagulation point is raised.

(*a*) Hydrocele fluid became opalescent at 65° C.; at 72° C. it assumed the consistence of a thin jelly which thickened, and at 76° C. flocculi separated out.

(*b*) Diluted with one volume of water, it became opalescent at 67° C., and coagulated at 81° C.

(*c*) Diluted with two volumes of water, it became opalescent at 69° C., and coagulated at 86° C.

(*d*) (*a*) Diluted with three volumes of water, it became opalescent at 73° C., and a few flocculi separated out at 90° C.

Another specimen of hydrocele fluid, apparently containing less proteid matter, became opalescent at 70° C., and coagulated with the formation of flocculi at 80°·5 C.

A third specimen became opalescent at 70° C., flocculi forming at 78° C.

General Conclusions regarding Dilution.

In the case of albumens and globulin existing in a natural condition within an animal fluid, such as white of egg, serum, or hydrocele fluid, the point of opalescence is gradually and almost uniformly raised by successive dilutions. The coagulation point, on the other hand, rises rapidly, and the more dilute fluids often refuse to form flocculi, or even to coagulate at all.

When a globulin is dissolved in an artificially prepared saline solution, both the point of opalescence and coagulating point are uniformly raised on diluting the solution. The same appears to apply to serum albumen saturated with magnesium sulphate.

The Action of Salts on the Coagulation Point of Albumen.

It is known that the addition of many neutral salts to an albuminous solution has an important action on the temperature at which it coagulates. Some salts are stated to lower and others to raise the coagulation point. It is impossible to explain at present their action, and we have accordingly commenced a somewhat systematic examination of the question. Our results are far from complete, and will subsequently, we hope, be more fully extended.

We have at present studied the action of two important salts, namely, magnesium sulphate and common salt, on the coagulation points of egg and serum albumen, vitellin, and globulin, and the action of these salts has been studied in all degrees of strength up to complete saturation.

Although we feel that it would be quite out of place to attempt general conclusions, yet we believe one or two inferences may be drawn from the facts that we have gleaned.

Some of the facts we have already obtained are sufficiently striking to justify us in thinking that a more extended investigation, made on similar lines, may throw some light on the mutual relationship existing between the albuminous and saline molecules when in solution together. We are aware of the extreme difficulty of the subject, since so little is known as yet regarding simpler problems, such as the mutual relationships that exist between simple mixtures of inorganic salts.

The Action of various Salts on the Coagulation Point of Egg Albumen.

Varenne (Reference 8) finds that many salts by their addition elevate the temperature of coagulation, such are, common salt and sulphate of magnesium; others, such as sulphate of copper and chloride of barium, lower it; while a third series, such as sulphate of sodium and chlorate of potassium, have no effect.

TABLE I. *showing the Action of various Salts on the Coagulation Point of Egg Albumen.*

Salt added.	Proportion.	Opalescence.	Coagulation.
	Per cent.	° C.	° C.
Original solution of Albumen, .	...	61	65
Lithium chloride, . . .	10	65	70
Sodium chloride, . . .	10	64	66·5
Potassium fluoride, . . .	10	66	71
Potassium chloride, . . .	10	63	68
Potassium bromide, . . .	10	67	77·75
Potassium iodide, . . .	10	67	75
Ammonium chloride, . .	10	64·5	70
Ammonium nitrate, . . .	10	71	73·5
Ammonium sulphate, . .	10	67	74
Magnesium chloride, . . .	10	69	75·5
Magnesium nitrate, . . .	10	68	70·5
Magnesium sulphate, . . .	10	65	70
Potassium nitrate, . . .	10	68	76·25
Potassium sulphate, . . .	10	65	68·5

Béchamp (Reference 5, p. 29) finds, on the other hand, that sulphate of magnesium, alum, and the salts of sodium and potassium lower the coagulation point. He came to this conclusion after working with

very dilute solutions of albumen; these did not coagulate at all, until after the addition of the salts mentioned. He added very small quantities of the salts to the albuminous solution, viz., less than one per cent. Had he worked with coagulable solutions of albumen, and had he added larger quantites of salt, his result would have been different. While, as we shall afterwards show, these salts as a rule raise the point of coagulation, it is not at all improbable that dilute uncoagulable solutions of egg albumen may be enabled to coagulate, when they otherwise would not; in fact, our results point to this conclusion. If so, it is only one of the many facts which indicate how little is at present known as to properties of the albuminous molecules and the factors which determine their solubilities.

In the preceding table we have placed some of our own results. In all cases the temperature, at which opalescence and coagulation occur, has been raised, though often, as in the case of common salt, to a very slight extent.

The Precipitation of Egg Albumen by Single and by Double Saturation with Neutral Salts.

By complete saturation of an albuminous fluid with a neutral salt the proteid may be precipitated at the temperature of the laboratory. Thus Hammarsten has shown that globulin may be precipitated from serum by the addition of magnesium sulphate. In this case the globulin is not converted into a coagulated proteid, but can again be dissolved after the magnesium sulphate has been diluted.

The Action of Magnesium Sulphate.—The egg albumen was diluted with one volume of water and freed as much as possible from membrane. A portion of this was saturated with magnesium sulphate and filtered. The saturated solution contained about 100 per cent. of magnesium sulphate. In order to obtain solutions of albumen containing a lower percentage of the salt, the saturated solution was diluted with portions of the original albumen.

The original diluted albumen became opalescent at 65° C., and coagulated, forming flocculi, at 66°·5 C.

(*a*) The saturated solution became opalescent at 78° C., and coagulated at 80° C.

(*b*) Egg albumen, containing 50 per cent. of magnesium sulphate, became opalescent at 67°·25 C., and coagulated at 68°·5 C.

(*c*) Egg albumen, containing 25 per cent. of magnesium sulphate, became opalescent at 65° C., and coagulated at 67° C.

(*d*) Egg albumen, containing 12·5 per cent. of magnesium sulphate, became opalescent at 63°·25 C., and coagulated at 65° C.

(*e*) Egg albumen, containing 6·25 per cent. of magnesium sulphate, became opalescent at 63° C., and coagulated at 65° C.

The action of this salt seems a very curious one, for while in large quantity it raises the coagulation point very considerably, small quantities seem to lower it slightly, and no doubt Béchamp

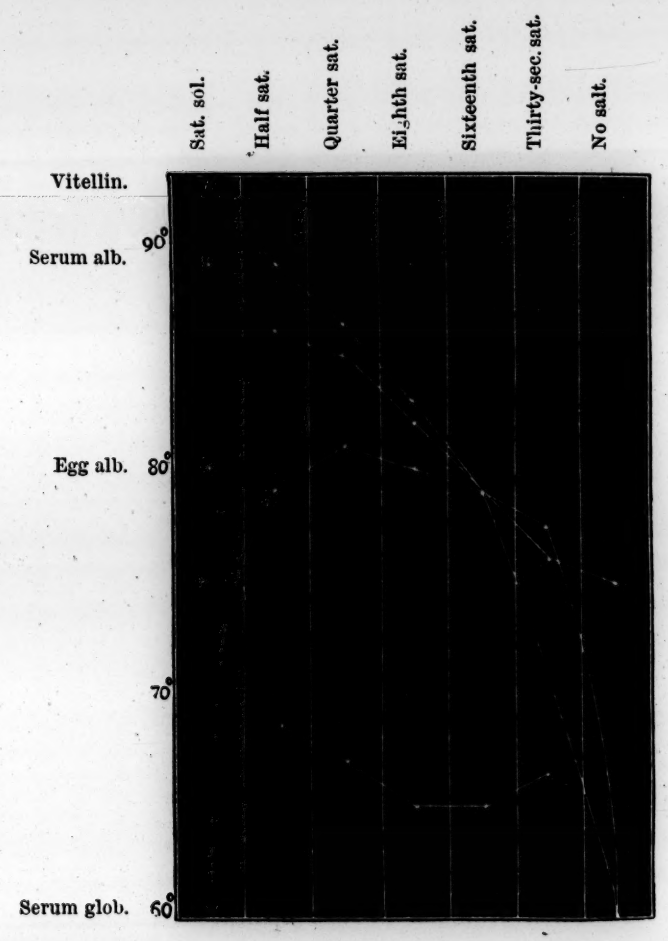


FIG. 2.—Showing the Effect of different Strengths of Magnesium Sulphate on the Coagulation Points of certain Albumens.

is correct when he states that the dilute uncoagulable albumen can readily be coagulated after the addition of the salt. He is hardly,

however, justified in speaking of magnesium sulphate as lowering the coagulation point of albumen by its presence.

It is a point of some interest to discover whether a salt, which, by its addition to an albuminous solution, raises the temperature at which coagulation occurs, will produce the same result on an albuminous solution already saturated with another salt. This we have determined to some extent.

Effect of the Addition of various Salts on the Coagulation Point of Egg Albumen already saturated with Magnesium Sulphate.

Egg albumen was diluted with two volumes of water, and saturated with magnesium sulphate. The solution was filtered, and it was found on heating to become opalescent at 79° C., coagulating at 81°·75 C. The salts added, most of which have already been studied in respect to their action on the coagulation point of egg albumen (Table I.), are seen to lower the coagulation point of egg albumen saturated with magnesium sulphate.

TABLE II. *showing the Action of various Salts upon the Coagulation Point of Egg Albumen already saturated with Magnesium Sulphate.*

Salt added.	Proportion.	Opalescence.	Flocculi.
	Per cent.	°C.	°C.
Albumen saturated with magnesium sulphate, . . .	6	79	81·75
Sodium chloride, . . .	6	72	79
Sodium iodide, ...	6	...	70
Sodium sulphate, . . .	6	79	81·5
Potassium chloride, . . .	6	72	79
Potassium bromide, . . .	6	70	74
Potassium nitrate, . . .	6	70	73·75
Potassium chlorate, . . .	6	71	74·5
Potassium sulphate, . . .	6	74	77
Ammonium chloride, . . .	6	62	73
Ammonium nitrate, . . .	6	63	65

On comparing this table with that on page 370, it will be noted, first, that those salts which on Table I. do not raise the coagulation point of egg albumen to any great extent, NaCl, KCl, K_2SO_4, and Na_2SO_4 (Varenne), do not lower the coagulation point (Table II.) to any great extent. On the other hand, salts like KBr, K_2NO_3, and NH_4NO_3, which raise the coagulation point in Table I., depress

it in Table II. It is possible still more to lower the coagulation point by the addition of larger quantities of the latter salts, until one can precipitate the albumen by double saturation at the temperature of the laboratory. On the other hand, the addition of large quantities of NaCl and Na_2SO_4 exerts very little action.

Effect of Magnesium Sulphate on the Coagulation Point of Serum Albumen.

Although Dr Halliburton has succeeded (Reference 6, p. 192) in precipitating serum albumen by double saturation by means of sulphate of magnesium in conjunction with such salts as sodium sulphate, sodium nitrate, potassium iodide, &c., magnesium sulphate in itself raises the coagulation point of serum albumen.

(*a*) Serum albumen, containing 100 per cent. magnesium sulphate, became opalescent at 84° C., and coagulated at 89° C., a slight opalescence appearing at 40° C., due to a trace of serum globulin.

(*b*) Serum albumen, containing 50 per cent. magnesium sulphate, became opalescent at 77° C., and coagulated at 86° C.

(*c*) Serum albumen, containing 25 per cent. magnesium sulphate, became opalescent at 76° C., and coagulated at 84°·75 C.

(*d*) Serum albumen, containing 12½ per cent. magnesium sulphate, became opalescent at 76° C., and coagulated at 82° C.

(*e*) Serum albumen, containing 6¼ per cent. magnesium sulphate, became opalescent at 74° C., and coagulated at 78°·25 C.

(*f*) Serum albumen, containing 3⅛ per cent. magnesium sulphate, became opalescent at 72° C., and coagulated at 76° C.

(*g*) Serum albumen, somewhat diluted in this experiment, became opalescent at 68° C., and coagulated at 75° C., without the formation of well-marked flocculi.

Sodium Chloride.—Although Hoppe-Seyler states that this salt lowers the coagulation point of serum albumen, we find that this is only the case when present in large quantity. Small quantities appear, if anything, to raise it.

A saturated solution of the same serum albumen as that used for the last experiment coagulated at 72° C., when saturated with common salt. A solution, containing 20 per cent., became opalescent at 74° C., and coagulated at 80°·5 C.

The Action of Sodium Chloride on a Solution of Serum Albumen already saturated with Magnesium Sulphate.

In this case the coagulation was lowered as sodium chloride was added in greater and greater quantity.

(*a*) Serum albumen, saturated with magnesium sulphate, became opalescent at 77° C., and coagulated at 79° C.

(*b*) The same solution, plus 10 per cent. sodium chloride, became opalescent at 72°·5 C., and coagulated at 75° C.

(*c*) The same solution, plus 20 per cent. sodium chloride, became opalescent at 70° C., and coagulated at 73° C.

A larger quantity of common salt was not added, since 20 per cent. did not dissolve readily.

The Action of Magnesium Sulphate on the Coagulation Point of Vitellin.

Some vitellin was dissolved in a dilute solution of magnesium sulphate. Some of this was saturated with the salt, the precipitate filtered off, and the filtrate tested.

(*a*) Vitellin, dissolved in a saturated solution of magnesium sulphate (100 per cent.), became opalescent at 88° C. Coagulation did not occur even on boiling, a few flocculi alone appearing.

(*b*) Vitellin, dissolved in a 50 per cent. solution of magnesium sulphate, became opalescent at 87° C., and coagulated at 89° C. with flocculi.

(*c*) Vitellin, dissolved in a 25 per cent. solution of magnesium sulphate, became opalescent at 81° C., and coagulated at 86°·5 C.

(*d*) Vitellin, dissolved in a solution containing 12·5 per cent. magnesium sulphate, became opalescent at 79° C., and coagulated at 82°·5 C.

(*e*) Vitellin, dissolved in 6·25 per cent. solution of magnesium sulphate, became opalescent at 74° C., and coagulated at 79° C.

(*f*) Vitellin did not completely dissolve in $3\frac{1}{8}$ per cent. solution of magnesium sulphate. It was not heated.

When further diluted until only about 1 per cent. magnesium sulphate was present, a distinct precipitate separated out in the cold.

This experiment was repeated with a more dilute solution of vitellin. The coagulation points at corresponding strengths of the

magnesium sulphate were all higher. The result was otherwise the same, the saturated solution requiring the greatest temperature for its coagulation.

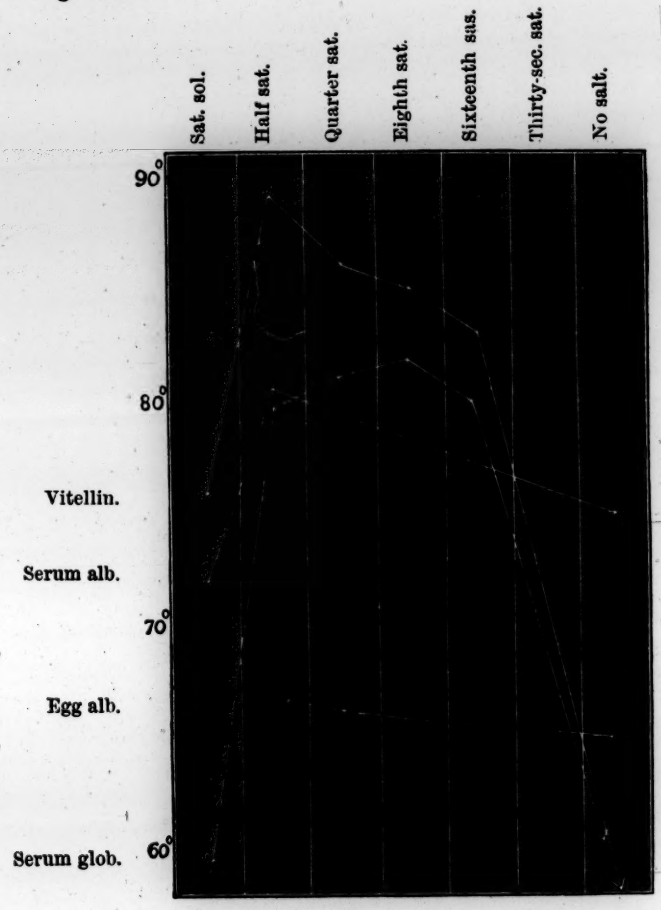


FIG. 3.—Showing the Effect of different Strengths of Sodium Chloride on the Coagulation Points of certain Albumens.

The Action of Sodium Chloride on the Coagulation Point of Vitellin.

Some vitellin was dissolved in 5 per cent. solution of common salt. It was saturated with the salt, and a precipitate of globulin filtered off.

(*a*) Vitellin, dissolved in saturated solution of common salt, became opalescent at 70° C., and coagulated at 76° C.

(*b*) Vitellin, dissolved in 20 per cent. solution of common salt, became opalescent at 83° C., and coagulated at 89° C.

(*c*) Vitellin, dissolved in 10 per cent. solution of common salt, became opalescent at 80° C., and coagulated at 86° C.

(*d*) Vitellin, dissolved in 5 per cent. solution of common salt, became opalescent at 79° C., and coagulated at 85° C.

(*e*) Vitellin, dissolved in 2·5 per cent. solution of common salt, became opalescent at 78° C., and coagulated at 83° C.

This experiment was repeated, and showed that common salt raises the coagulation point of vitellin, but that it is lowered just before the point of saturation, and that it continues to be lowered until saturation occurs.

Action on the Coagulation Point of Vitellin of both Common Salt and Magnesium Sulphate dissolved together in the Solution.

If, to vitellin in a saturated solution of common salt, some magnesium sulphate be added, the latter dissolves with difficulty, precipitating the vitellin in flocculi; on heating, other flocculi appear.

If, to vitellin in a saturated solution of magnesium sulphate, common salt be added, the coagulation point is lowered. Thus, on adding 15 per cent. of common salt, coagulation occurs at 88° C., and with a little over 20 per cent. it is lowered to 70° C.

Serum Globulin.—Serum globulin is precipitated by magnesium sulphate in excess, as Hammarsten has shown. The same observer obtained a precipitation by saturating with common salt.

The Action of Magnesium Sulphate on the Coagulation Point of Serum Globulin.

Serum globulin was precipitated from the serum of ox's blood by passing a stream of CO_2 through it. The precipitate after careful washing was dissolved in magnesium sulphate solution.

(*a*) Serum globulin is precipitated in the cold by saturating the solution with magnesium sulphate.

(*b*) Serum globulin, dissolved in a solution containing 50 per cent. magnesium sulphate, became opalescent at 74°·5 C., and coagulated at 79° C.

(*c*) Serum globulin, dissolved in a solution containing 25 per cent. magnesium sulphate, became opalescent at 78°·5 C., and coagulated at 80°·75 C.

(*d*) Serum globulin, dissolved in a solution containing 12·5 per cent. of magnesium sulphate, became opalescent at 77°·5 C., and coagulated at 80° C.

(*e*) Serum globulin, dissolved in a solution containing 6·25 per cent. of magnesium sulphate, became opalescent at 76° C., and coagulated at 78°·75 C.

(*f*) Serum globulin, dissolved in a solution containing 3·125 per cent. magnesium sulphate, became opalescent at 71°·5 C., and coagulated at 77° C.

Effect of Sodium Chloride on the Coagulation Point of Serum Globulin.

(*a*) Serum globulin, saturated with sodium chloride, is precipitated in the cold.

(*b*) Serum globulin, containing 20 per cent. sodium chloride, became opalescent at 77° C., and coagulated at 79°·5 C.

(*c*) Serum globulin, containing 10 per cent. sodium chloride, became opalescent at 79° C., and coagulated at 81° C.

(*d*) Serum globulin, containing 5 per cent. sodium chloride, became opalescent at 79° C., and coagulated at 81°·75 C.

(*e*) Serum globulin, containing 2·5 per cent. sodium chloride, became opalescent at 78° C., and coagulated at 80° C.

(*f*) Serum globulin in much smaller quantity does not dissolve to form a clear solution.

Tentative Conclusions regarding the Action of Salts.

(1) A salt may raise the temperature of coagulation if present in a certain percentage; at another percentage it may lower it. Thus common salt raises the coagulation points of both vitellin and serum globulin when present in moderately small quantity. Large quantities lower the coagulation point.

(2) If a proteid be present in a saturated solution of a salt—such as magnesium sulphate—and, if another salt be then added, which by itself would raise the coagulation point, the coagulation point may in this case be lowered. It appears, too, that salts which are most active in raising the coagulation point are most active in lowering it, when added to a solution already saturated by another salt.

Statement as to whether it is possible to speak of the Specific Coagulation Point of an Albumen.

From what has been already said, it is obvious that it is a difficult and perhaps a valueless task to attempt to determine what may be termed the "specific coagulation point" of an albumen. The coagulation point varies with the rapidity of heating, with the concentration of the fluid, with its reaction, and with the saline substances present. All that one can say is that, under such and such conditions, an albumen coagulates at such a temperature. It is probably hardly possible to obtain even two albumens under such similar conditions that their coagulation points may with advantage be compared. The nearest approach to this would perhaps be to dissolve a certain weight, say both of vitellin and another globulin such as serum globulin, in the same volume of salt solution. The coagulation points may, in this case, with advantage be compared. But what would be the value of the coagulation points so obtained for purposes of comparison with serum or egg albumen dissolved in water? The coagulation points quoted by previous writers cannot be taken in any sense as absolute values for the albumen named, modifying conditions having, as a rule, been totally disregarded. The same may be said of our own results, for the percentage strengths of the albuminous solutions used by us were in no case determined with any care. Although the forms of the curves represented in the charts are not affected by this, their altitudes in the scale of temperatures may be so to some considerable extent.

On so-called Fractional Coagulation.

So far we have been dealing with albumen in its natural condition, or mixed and possibly combined with neutral salts which we had added.

The solutions were alkaline, and, as we found, when dealing with the natural albuminous solution, difficult to coagulate, especially if in a dilute condition. Let us now consider the coagulation point of an albuminous solution to which an acid has been added. On adding an acid to an albumen solution, the coagulation is rendered, as every one knows, more easy, and it occurs at a lower temperature. The very dilute solutions, uncoagulable in the alkaline solution,

are at once coagulated after the addition of a few drops of weak acid. No one has brought this out more clearly than Dr Halliburton in a most suggestive paper (Reference 6), which will presently be quoted in relationship to fractional coagulation. He showed that the coagulation point of serum albumen varies with the amount of acid present, the greater the quantity added, the lower the coagulation point, until finally coagulation could be produced at the temperature of the laboratory. If then the coagulating point depended on the two factors, heat and the amount of acidity, it seemed to him a natural deduction, that, on keeping one of these, the acidity, a constant quantity, it might be possible to separate by fractional coagulation two or more albumens mixed together, and having different coagulation points. He investigated serum albumen, and found that if it be neutralised by the addition of some drops of a 2 per cent. solution of acetic acid, and if, further, it be rendered slightly acid by the addition of one drop of the dilute acetic acid to seventy-five drops of the albuminous solution, then it coagulates at 70° to 71° C., and if this coagulum be filtered off, and the solution again brought to the same degree of acidity, a coagulum occurs the second time at 77° to 78° C. If this coagulum be filtered off and the filtrate acidified as before, a third coagulum may be produced at 82° C. Dr Halliburton considers that the serum albumen, originally regarded as one proteid, in reality consists of three.

MM. Corin and Berard have followed this process of fractional coagulation, applying it to egg-white. This substance, commonly held to consist of albumen and globulin, they believe to consist of three albumens and two globulins.

They neutralise some egg-white, slightly acidify it, and raise its temperature, until opalescence appears; then they keep the temperature constant for a considerable time—an hour or even more. They filter off the coagula, re-acidify to the same degree, raise the temperature until opalescence occurs, and then after more prolonged heating flocculi again appear.

In this way they have succeeded, as already stated, in fractionating five proteids.

Without doubting that it may be possible to fractionate some proteids, nevertheless the results of our own work, and many of

the facts frankly stated by Dr Halliburton, seemed to throw some doubt upon the correctness of his deductions in the case of serum albumen, and this applied with equal force to the experiments conducted by MM. Corin and Berard on egg albumen.

Our previous experiments have shown that, in alkaline solution, the more dilute a solution is, the higher is its coagulation point, and we have found that we could never completely precipitate any albumen at the temperature at which flocculi first appeared. The reason of this is very simple; as soon as a solution begins to coagulate, the remnant, still soluble, is practically a more dilute solution of the same proteid, and must be heated two or three degrees more before it will begin to precipitate. In this case, also, the coagulating proteid will leave another soluble remnant, coagulable at a still higher temperature, and so on. In fact, we may venture to make this general statement—*In order to coagulate completely any proteid it must be heated to that temperature at which its most dilute solutions are coagulable.* We have not made so systematic an investigation upon the effect, on its coagulating point, of diluting acid solutions of albumen, but we have assured ourselves that the more dilute solutions coagulate at a higher temperature. One out of several experiments may be quoted the following :—

Some egg albumen was diluted with two volumes of water and carefully neutralised. It was then brought to the same degree of acidity as is recommended for fractional coagulation, 1 cubic centimetre of a 2 per cent. solution of acetic acid being added to 75 cubic centimetres of the albumen. This solution was found to coagulate at 53° C.

When diluted with one volume of water, acidulated to the same degree, it coagulated at 54° C.

With three volumes of water, it coagulated at 58° C.

With seven volumes of water, it coagulated at 62° C.

With fifteen volumes of water, it coagulated at 68° C.

It is seen, therefore, that dilution has the effect of raising the coagulation point a great many degrees, the more dilute albumen requiring a much higher temperature for its separation. This may be shown in the most striking manner by heating some of the acidulated water to between 60° and 70° C.; and dropping in some acidulated egg albumen it at first dissolves. Now divide the solu-

tion into two portions, A and B, and heat A to 75° C., and keep B at the original temperature. A will coagulate, showing that although in too dilute a solution to coagulate below 70° C., it could nevertheless coagulate, provided its temperature be raised. B will remain clear, but, if more albumen be dropped into it, a point will be reached, at which it will cease to dissolve, and then it will separate out in flocculi.

Here then, without going any further, one has come across an observation which, if it does not explain all the facts described under the head of fractional coagulation, must at any rate account for some of them.

Both Dr Halliburton and MM. Corin and Berard found that after coagulation the filtrate, which they separated from the clot, was less acid than it was before coagulation had occurred, the latter observers finding that, as a rule, it was actually alkaline. Here, again, is a factor which we cannot afford to lose sight of. If the coagulation point is lowered by acidity, as all persons are agreed, one would expect that, while coagulation is proceeding, and while *pari passu* the acidity is decreasing, that the decrease of acidity would at last bring the coagulation—at that temperature—to a standstill. In this case one would expect, that on re-acidifying to the same degree, another crop of coagula might fall at the same temperature as did the first crop.

Dr Halliburton does not mention any such coagulation, although undoubtedly it occurs, and we have found it on repeating his observations, but MM. Corin and Berard evidently find that one is produced, and in consequence they heat the albumen for an hour or more before filtering off the coagulum. After this time, they found that the albumen never gave a second coagulation at the same temperature. We cannot but conclude from this that their experiments clearly indicate that the albuminous solutions with which they worked must have been very materially changed by the temperature, nor is it at all improbable that very material changes may occur in a solution of egg albumen kept in an acid solution at a high temperature for over an hour; in one of their quoted experiments fractionation lasted over six hours.

We may, we think, make this statement, and one fully borne out by our own experiments, that during coagulation in an acid medium

the coagulation point is being continually raised, both in virtue of the albumen becoming more dilute and in virtue of its becoming less acid; these factors bring the coagulation to a standstill, but, after filtering off the coagulum, if the fluid be brought back to its original degree of acidity, and heated to the same temperature, coagula will again form, unless the albumen has undergone some physico-chemical change.

It follows, too, that it is impossible to separate two albumens from one another by heat coagulation, unless, during the process of coagulation, the degree of acidity is kept uniform by the addition of small quantities of fresh acid, and unless the coagulation point of *the most dilute solution* of one of the albumens present be below the coagulation point of the other albumen. We became more convinced of this, when repeating in detail the experiments on fractional coagulation. After keeping an albuminous solution, either egg or serum albumen, at the temperature at which flocculi appear, for five or six minutes, and then filtering off the flocculi, we found that fresh flocculi appeared, when the filtrate had been re-acidified, and again raised to the same temperature. Two or three crops might be thus removed in the case of egg-white. Keeping up the same degree of acidity, and raising the temperature, we were able to get other crops of albumen. We were struck, however, by the fact that, while dealing with the more dilute albumen, the coagulation took place with difficulty, and it was longer delayed. This was particularly the case with egg-albumen. If the fluid filtrate from the coagulated flocculi be divided into two parts, and one portion raised gradually in temperature, opalescence followed by the formation of flocculi will appear. If the other portion be raised in temperature and kept for, say, three minutes at a temperature one or two degrees below the temperature at which opalescence appeared in the first portion, it will become opalescent and perhaps form flocculi. We found, in fact, that it was impossible to get the subsequent coagulation at definite points, as indicated by the previous observers, for the coagulation point depended upon the way in which the operations had been carried out. This was especially the case, when dealing with egg albumen, and we have little doubt that MM. Corin and Berard, working with careful method, invariably raised their temperatures to points which perhaps their first experiments

had suggested. They, no doubt, produced coagula, but, had they tried the experiment, they would have obtained them equally well at a lower temperature provided they had raised the temperature more slowly. It is not difficult to fractionate egg albumen ten or twelve times.

Another point that struck us was the smaller and smaller amount of coagulation produced, as the temperature of the solution was raised and successive crops produced. This was noticed by Dr Halliburton in the case of egg albumen. It is certainly the case with egg albumen. This, of course, in itself renders it highly probable that we are dealing in both cases only with one albumen, the more dilute solutions of which are alone coagulated at the higher temperatures. Even supposing that the γ serum albumen of Dr Halliburton, of which he "in some case only found a trace," and which coagulates at 82° C., is different from α and β serum albumen, found in greater quantity, and coagulating at lower temperatures, yet fractional coagulation could not give us the means of proving this. One cannot compare the coagulating points of a dilute with a strong solution of two albumens, and presuming that γ serum albumen is a dilute solution of an albumen differing from α and β, yet its coagulation point would be lower than 82° C. in a solution of corresponding strength.

It is, of course, possible that serum albumen may consist of more than one albumen, although it is probable, from what we have brought forward, that all the albuminous matter present coagulates at the same degree of concentration—at or about the same temperature. Other methods may enable the physiologist to separate these, if they exist, from one another, and no methods have in the past yielded such valuable results as those in which separation has been obtained by the addition of neutral salts. Dr Halliburton has by this means tried to separate the α, β, and γ serum albumens from one another, and frankly states that he has failed to do so (Reference 6, p. 173).

In conclusion, we may state that the method of fractional coagulation could only be of service when the coagulation points of the albumens present are widely separated from each other. In reality, fractional coagulation has been for a long time in use, and one of the few cases in which, as far as we can see, it is at all

applicable, is the separation of serum globulin from serum albumen. Serum globulin is precipitated at the atmospheric temperature on acidifying by a stream of carbon dioxide, or by the addition of weak acetic acid. This precipitation is not a complete one, however, as Hammarsten has shown. The reason is, that, at the atmospheric temperature, part of the globulin remains in solution.

This paper contains some of the results of a research, towards the expenses of which a grant of money was voted by the Scientific Grants Committee of the British Medical Association.

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Some New Points in Connection with Muscle Contraction. By Alexander James, M.D.

(Read July 15, 1889.)

The interval which elapses between tapping a muscle or tendon and the resulting movement of the limb has been estimated by many observers—Burckhardt, Tschirjieu, Waller, Brissaud and François Franck, Eulenberg, De Watteville, &c.—but the precise signification of these so-called reflexes is not yet fully understood. What follows is intended to add to our knowledge of this subject.

The observations were made on a patient in the Royal Infirmary, aged 26, who, as the result of a blow on the left side of the neck, sustained three years previously, presented (1) greatly impaired voluntary motor power in the left arm and left leg; (2) marked jerkings on tapping the tendons of the left supinator longus, left

quadriceps femoris, and left gastrocnemius; (3) marked clonus of the left ankle and left knee-joints; and (4) to a less extent impaired voluntary motor power of the right leg, with increased ankle and knee jerks and clonus.

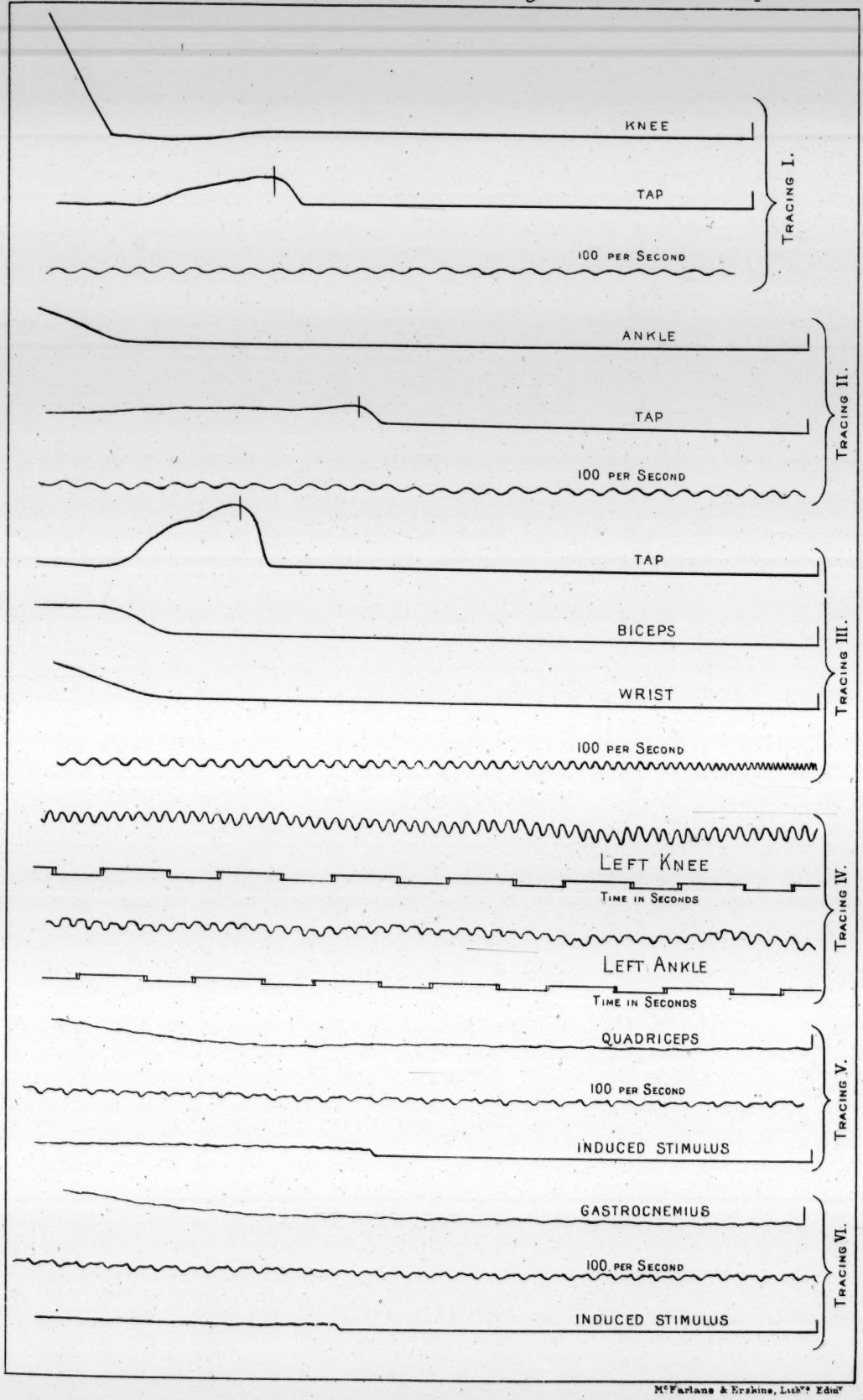
Knee Jerk.—The method of recording this, which I followed, was to connect two recording tambours, the first with a tube having a flexible extremity, which could be held over the patellar tendon, and through which the tendon could be tapped; the second with a receiving tambour, the button of which was held in contact with the leg. The first recording tambour indicated on a rapidly revolving cylinder the instant at which the tendon was tapped, and the second the moment that the limb began to move. A chronograph, vibrating 100 per second, enabled the interval between the tap and the movement to be ascertained. Care was taken that the tubes connected with the tambours were of the same length.

In both limbs the time was found to be about ·06 second—a rather longer interval than that stated by most other observers to be the case at any rate in health (see Tracing I.).

Ankle Jerk.—To ascertain this the flexible end of the tube connected with the first recording tambour was held in contact with the tendo-Achilles, whilst the button of the receiving tambour was held in contact with the ball of the great toe. In the case of both limbs the time which elapsed between the tap on the tendon and the resulting movement was about ·08 second (see Tracing II.).

Plantar Reflex.—This was ascertained in the right leg only; the cutaneous sensibility of the left being so much impaired, that its plantar reflex was practically non-existent. The method followed was to substitute for the flexible end of the tube connected with the first recording tambour an ordinary receiving tambour, to the button of which was fixed a pin point. The button of the other receiving tambour was held in contact with the lower end of the thigh. The interval between the prick thus applied to the sole of the foot and the resulting movement was ·16 second.

Wrist Jerk.—To ascertain this the flexible end of the tube connected with the first recording tambour was laid on the tendon of the supinator longus of the left arm, and the button of the receiving tambour was held over the metacarpal region of the hand. The tap applied to the tendon through the tube readily produced the desired



McFarlane & Erskine, Lith^rs Edin^r

movement, and the interval was found to be ·05 second. It was observed, further, that with each tap of the supinator longus tendon a contraction of the biceps muscle occurred. This was also timed by means of tambours employed in a similar way, the button of the receiving one being held in contact with the middle of the muscle. The interval between the tap on the supinator longus tendon and the contraction of the biceps was found to be ·045 second (see Tracing III.).

Ankle Clonus.—The method by which this was timed was the ordinary one, and is so well known that it need not be described. The rate of the clonus was found to be in both legs about 7 per second (see Tracing IV.).

Knee Clonus.—This could be easily induced in both legs by drawing down the patellæ, and was timed in the ordinary way. The rate was found to be about 8½ or 9 per second (see Tracing IV.).

From these data the following conclusions may, I think, be drawn :—

I. That (as has been pointed out by other observers) the interval between the tap on the tendon or muscle and the resulting movement is too short to enable us to regard these jerks as being ordinary reflexes in which sensory nerves, nerve centres, and motor nerves are together concerned. Thus in this patient the plantar reflex was found to be ·16 second, and the Achilles tendon jerk ·08 second. Were both of these similarly produced reflexes, the latter would have taken as long, or even a longer, interval to occur instead of a much shorter one.

II. That yet these muscle or tendon jerks cannot be regarded as direct contractions. In evidence of this is to be noticed—

(*a*) That the interval between the tap and the resulting contraction differs in different muscles, being greater in the case of the gastrocnemius than in the quadriceps femoris, and greater in the quadriceps femoris than in the supinator longus. Were they direct we should expect the interval to be the same or nearly the same in all.

(I further found in this patient that with single induction shocks, as stimuli applied directly to the muscles, the contraction of the gastrocnemius took place more rapidly than that of the quadriceps. In the tracings given the interval was in the case of the quadriceps

about ·05 second, and of the gastrocnemius about ·03 second (see Tracings V. and VI.).

(*b*) That when the tendon of the supinator longus was tapped contraction occurred in the biceps as well, and that the contraction of the biceps preceded that of the supinator longus. This could only occur as the result of some reflex in the cord.

From these special conclusions the general one which I think may be drawn is, that these muscle and tendon jerks are really reflexes, but reflexes of a nature much more simple than the ordinary ones, in which sensory nerves, nerve centres, and motor nerves are concerned. Looking upon muscle, motor nerve, and central nerve cell as being composed alike of masses of irritable protoplasm, and remembering that the masses of irritable protoplasm which compose these can conduct equally well in either direction, we can suppose that the stimulus of the tap applied to the muscle, directly, or indirectly through its tendon, produces its contraction only after the impulse so generated has traversed through muscle and motor nerve fibre to nerve cell, and down again to muscle along the same nerve fibre. In this way, *ceteris paribus*, the longer the distance between a muscle and its nerve centre the longer will be the interval between the tap and the resulting contraction. This is, of course, borne out by these observations, the ankle jerk taking the longest and the wrist jerk the shortest time to occur, the knee jerk occupying an intermediate position. But the fact that contraction of the biceps occurred when the tendon of the supinator longus was tapped, and that the contraction of the former preceded that of the latter muscle, denotes that the impulse generated in muscle, motor nerve, and nerve cell, as the result of the tap, may be reflected from the nerve cell along other nerve fibres. We must conclude, however, that for this reflex, as for a reflex along the same nerve fibre, a much shorter time is required than for one in which the afferent impulse travels along an ordinary sensory nerve.

The phenomena of clonus bear out this view. Clonus may be regarded as being a series of jerks or contractions, each jerk or contraction acting as the stimulus to the one which follows. The fact, then, that (as shown by the observations made on this patient, and as demonstrated by myself at greater length in a previous paper)* the

* "Clonus and Tendon Reflex Phenomena," *Edin. Med. Jour.*, Aug. 1880.

rapidity of clonus in a given muscle varies inversely with its distance from its centre in the cord, the ankle being slower than the knee, the knee than the elbow, &c., indicates that in the production of each jerk, nerve cell and nerve fibres are concerned equally with the muscle, and that the length of the motor nerve fibres seems to be the element of most importance in conditioning the rapidity of the clonus.

The Theory of Determinants in the Historical Order of its Development. By Thomas Muir, M.A., LL.D.

PART I. *Determinants in General* (1836–41).

(Continued from p. 234 of Vol. XVI.)

GRUNERT (1836).

[Supplemente zu Georg Simon Klügel's Wörterbuch der reinen Mathematik. Art. *Elimination* (I. Gleichungen des ersten Grades), ii. pp. 52-60.]

With Grunert it is necessary to take a long step backward. Although the memoirs of Bezout, Vandermonde, and Laplace were known to him, in addition to those of Hindenburg, Rothe, and Scherk, he advances only a short distance into the subject; his aim, indeed, is little more than the establishment of Cramer's rule for the solution of a set of simultaneous linear equations. His mode of presenting the subject, however, is fresh and interesting, the method of "undetermined multipliers" being taken to start with.

Writing his equations in the form

$$\left.\begin{aligned}
(1)_1x_1 + (2)_1x_2 + (3)_1x_3 + \ldots + (n)_1x_n &= [1]_1 \\
(1)_2x_1 + (2)_2x_2 + (3)_2x_3 + \ldots + (n)_2x_n &= [1]_2 \\
(1)_3x_1 + (2)_3x_2 + (3)_3x_3 + \ldots + (n)_3x_n &= [1]_3 \\
\ldots\ldots\ldots

and taking $p_1, p_2, p_3, \ldots, p_n$ as multipliers, he readily shows of course that if the multipliers can be got to satisfy the conditions

$$\left.\begin{aligned}
(2)_1p_1 + (2)_2p_2 + (2)_3p_3 + \ldots + (2)_np_n &= 0 \\
(3)_1p_1 + (3)_2p_2 + (3)_3p_3 + \ldots + (3)_np_n &= 0 \\
(4)_1p_1 + (4)_2p_2 + (4)_3p_3 + \ldots + (4)_np_n &= 0 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & \\
(n)_1p_1 + (n)_2p_2 + (n)_3p_3 + \ldots + (n)_np_n &= 0
\end{aligned}\right\},$$

the value of x_1 will be

$$\frac{[1]_1p_1 + [1]_2p_2 + [1]_3p_3 + \ldots + [1]_np_n}{(1)_1p_1 + (1)_2p_2 + (1)_3p_3 + \ldots + (1)_np_n};$$

in other words, that x_1 can be determined at once if a function

$$(1)_1p_1 + (1)_2p_2 + (1)_3p_3 + \ldots + (1)_np_n$$

can be formed of such a character that it will vanish when instead of the coefficients $(1)_1, (1)_2, (1)_3, \ldots, (1)_n$ we substitute the members of any one of the $n-1$ rows

$$\begin{array}{cccccc}
(2)_1 & (2)_2 & (2)_3 & \ldots & (2)_n \\
(3)_1 & (3)_2 & (3)_3 & \ldots & (3)_n \\
(4)_1 & (4)_2 & (4)_3 & \ldots & (4)_n \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
(n)_1 & (n)_2 & (n)_3 & \ldots & (n)_n;
\end{array}$$

the said function itself being the denominator of the value of x_1 and the numerator being derivable from the denominator by inserting $[1]_1, [1]_2, [1]_3, \ldots, [1]_n$ in place of $(1)_1, (1)_2, (1)_3, \ldots, (1)_n$. Further, as any one of the unknowns may be made the first, the complete solution is thus put in prospect. "Alles kommt demnach auf die Entwickelung einer Function von der angegebenen Beschaffenheit an." (XIII. 5)

Two rules, Grunert says, have been given for the construction of such a function, one by Cramer, the other by Bezout. The former he states, and illustrates by constructing the desired function for the case where $n=4$. The proof of it is then attempted, and is said at the outset to consist essentially in establishing the proposi-

tion that a permutation and any other derivable from it by the simple interchange of two indices must, according to Cramer's rule, differ in sign. This proposition is therefore attacked. The permutation

$$\ldots\ldots (k)_\alpha \ldots\ldots (1)_{\alpha+\beta} \ldots\ldots \qquad \text{A}$$

is taken in which the inferior indices are in their natural order 1, 2, 3, . . . , n, and k and 1 being interchanged, there arises the permutation

$$\ldots\ldots (1)_\alpha \ldots\ldots (k)_{\alpha+\beta} \ldots\ldots \qquad \text{B}$$

The part preceding $(k)_\alpha$ in A is called I., which thus of course also denotes the part preceding $(1)_\alpha$ in B: the part between $(k)_\alpha$ and $(1)_{\alpha+\beta}$ in A or between $(1)_\alpha$ and $(k)_{\alpha+\beta}$ in B is called II.; and the remaining part common to both A and B is called III. The number of inversions in both, when 1 and k are left out of account, is denoted by γ: the number in both due to k and the division III. is denoted by λ: the number in A due to k and the division II. by λ': and the number in both due to the division I. and k by λ''. The counting of the inversions then takes place for the two permutations. In the case of A there are the inversions due

(1)	to I. and k,	which are	λ'' in number.
(2)	to I. and II.		
(3)	to I. and 1,	. . . .	$\alpha - 1$ . . .
(4)	to I. and III.		
(5)	to k and II.,	. . . .	λ' . . .
(6)	to k and 1,	. . . .	1 . . .
(7)	to k and III.,	. . . .	λ . . .
(8)	to II. and 1,	. . . .	$\beta - 1$ . . .
(9)	to II. and III.		
(10)	to 1 and III.,	. . . .	0 . . .

and as those not counted here are γ in number, the total is seen to be

$$\alpha + \beta + \gamma + \lambda + \lambda' + \lambda'' - 1.$$

Similarly in the case of B the total is found to be

$$\alpha + \beta + \gamma + \lambda - \lambda' + \lambda'' - 2.$$

But the former total exceeds the latter by $2\lambda' + 1$, and this being an odd number, the proposition is proved. (III. 26)

Before proceeding further it is important to note that Grunert here establishes a more definite theorem than he proposed to himself, viz., the theorem of Rothe (III. 7). If he attains greater simplicity it is in part due to the fact that instead of taking *any* two indices for interchange, k and r say, he takes k and 1.

To prove now that the function constructed in accordance with Cramer's rule will satisfy the requisite conditions, it suffices to show by means of this theorem that on making any one of the $n-1$ specified sets of substitutions the function will be transformed into one consisting of pairs of terms which annul each other; in other words, to prove Vandermonde's theorem regarding the effect of making two indices alike. This is done; and then it is shown how x_κ can be got by interchanging x_κ and x_1 in all the given equations, the first step being of course to establish the fact that the denominator of x_κ and the denominator of x_1 only differ in sign.

Bezout's rule of 1764 is next taken up, and shown to be identical in effect with Cramer's. The proof, by reason of the recurring character of the former, is inductive; that is to say, it is demonstrated that, if the two rules agree in the case of n unknowns, they must also agree in the case of $n+1$. Paraphrasing the proof, but taking for shortness' sake the case where $n=4$, we say that it is agreed that both rules give in this case the signed permutations

$$1234,\ -1243,\ +1423,\ -4123,\ -1324,\ +\ldots$$

Now for the case where $n=5$ Bezout's rule directs that to the end of each of these permutations, *e.g.*, the permutation -4123, a 5 is to be put, and asserts that the result -41235 will be one of the desired permutations with its proper sign. That it is a permutation of the first five integers is manifest, and since the number of inversions in 41235 is necessarily the same as the number in 4123, its sign is correct according to Cramer's rule. In order to obtain four other permutations, Bezout's rule then proceeds to bid us shift the 5 one place and alter the sign, shift the 5 another place and alter the sign again, and so on. The result is

$$+41253,\ -41523,\ +45123,\ -54123.$$

In regard to this, it is clear as before that permutations of the

first five integers have been got, and that the altering of the sign simultaneously with the shifting of the 5 is in accordance with Cramer's rule, because every time that the 5 is moved one place to the left the number of inversions is increased by unity. The only question remaining is as to whether *all* the permutations are thus obtainable; and as it is seen that each of the 24 permutations of the first four integers gives rise to 5 permutations of the first five, we have at once grounds for a satisfactory answer. (III. 27)

LEBESGUE (1837).

[Thèses de Mécanique et d'Astronomie. Première Partie: Formules pour la transformation des fonctions homogènes du second degré à plusieurs inconnues. *Liouville's Journal de Math.*, ii. pp. 337–355.]

This simply-worded and clear exposition is a natural outcome of a study of Jacobi's memoirs on the subject. Like these it mainly concerns determinants of the special form afterwards individualised by the term axisymmetric; and, indeed, it is notable as being the first memoir in which a special name is given to a special form, the expression "déterminants symétriques" being repeatedly used for the particular determinants referred to.

His general definition is (p. 343) :—

"Si l'on considère le système d'équations

$$\begin{cases} A_{1,1}t_1 + A_{1,2}t_2 + \ldots\ldots + A_{1,n}t_n = m_1, \\ A_{2,1}t_1 + A_{2,2}t_2 + \ldots\ldots + A_{2,n}t_n = m_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ A_{n,1}t_1 + A_{n,2}t_2 + \ldots\ldots + A_{n,n}t_n = m_n, \end{cases}$$

le dénominateur commun des inconnues $t_1, t_2, \ldots, t_n$ est ce que l'on nomme le déterminant du système des nombres

$$(17) \qquad \begin{cases} A_{1,1} \quad A_{1,2} \ldots\ldots A_{1,n}, \\ A_{2,1} \quad A_{2,2} \ldots\ldots A_{2,n}, \\ \ldots\ldots\ldots\ldots \\ A_{n,1} \quad A_{n,2} \ldots\ldots A_{n,n}, \end{cases}$$

Comme ce dénominateur peut changer de signe, selon le mode de solution qu'on emploiera, on conviendra de le prendre de sorte que le terme $A_{1,1}A_{2,2}A_{3,3}\ldots A_{n,n}$ qui en fait partie, soit positif." (VIII. 3)

No use, however, is made of this for the purpose of establishing the properties of the functions, results being for the most part taken from previous investigators and merely restated. A notation for what are nowadays called the minors of a determinant is given in the following words (p. 344):— (XLI. 7)

"Ceci rappelé, si l'on représente par D le déterminant du système (17), par $[g, i]$ le déterminant du système qui se tire du système (17) par la suppression de la série horizontale de rang g et de la série verticale de rang i, et semblablement par la notation $\begin{bmatrix} g, & i \\ h, & k \end{bmatrix}$ le déterminant du système qui résulte de l'omission des séries horizontales de rangs g et i et des séries verticales de rangs i et k dans le système (17), on pourra, . . ."

REISS (1838).

[Essai analytique et géométrique. *Correspondance math. et phys.*, x. pp. 229–290.]

Reiss's memoir, the first part of which appeared in 1829, was never completed. In the course of some remarks introductory to the present essay, he says by way of excuse:—

"Je m'aperçus bientôt, et plusieurs savans me l'ont fait remarquer, que ces recherches, fussent-elles très-fécondes en résultats élégans, étaient trop abstraites pour intéresser le public qui n'apprécie les théories que selon le degré de leur utilité. J'ai donc tâché de montrer, par un exemple, de quelle manière on peut se servir de ces fonctions dans la géométrie analytique; et j'ai choisi le *tétraèdre* qui, par le concours de plusieurs circonstances qu'on aura occasion de reconnaître plus tard, permettait une application très-facile et presque immédiate des premières conséquences auxquelles j'étais parvenu."

The analytical portion of the essay is to a considerable extent identical with the original memoir. In so far as there is a difference, the change is towards greater simplicity, less seemingly aimless plunging into widely extensive theorems, and in general a better and more attractive style of exposition. Less space too is given to it,—not even half what is occupied by the portion on the tetrahedron, the main aim now being to urge on mathematicians the capabilities of the analysis in its application to geometry.

The matters falling to be noted as not having been given in the original memoir are few in number and of little importance. In restating the theorem

$$(abc \ldots r,\ \overline{\alpha\beta\gamma \ldots \rho}) = \overline{(abc \ldots r,\ \alpha\beta\gamma \ldots \rho)}$$

the remark is incidentally made that the order of the terms on the one side is never the same as that on the other except when the number of bases is 1, 2, or 3; for example, the number of bases being 4, we have

$$(abcd,\ \overline{1234}) = a_1b_2c_3d_4 - a_1b_2c_4d_3 - a_1b_3c_2d_4 + a_1b_3c_4d_2 + \ldots,$$

whereas

$$\overline{(abcd,\ 1234)} = a_1b_2c_3d_4 - a_1b_2d_3c_4 - a_1c_2b_3d_4 + \ldots + a_1c_2d_3b_4 + \ldots,$$

the difference first appearing at the fourth term. (IX. 6)

Bezout's recurrent law of formation, formerly merely enunciated, is now accompanied by a demonstration. This is not without its weak point, the cause of which, as might be expected, is the awkwardness of Reiss's rule of signs. The first paragraph, which will suffice to show its character, is as follows (p. 233):—

> "Portons notre attention d'abord, seulement sur la fonction $(abc \ldots r,\ \overline{\alpha\beta\gamma \ldots \rho})$. Si l'on se représente la manière dont on fait les permutations des n élémens $\alpha,\beta,\gamma, \ldots \rho$, on verra qu'à partir de la première, il y aura $1.2.3 \ldots (n-1)$ complexions qui commencent par α, et que, si l'on sépare cet élément par un trait vertical des autres, on aura à droite toutes les permutations des élémens $\beta,\gamma, \ldots \rho$. Les $1.2.3. \ldots (n-1)$ premiers termes de $(abc \ldots r,\ \overline{\alpha\beta\gamma \ldots \rho})$ commencent

donc tous par a^{α}, et puisque les signes de ces termes sont déterminés d'après la manière exposée plus haut, on trouvera leur somme $= a^{\alpha}(bc \ldots r, \overline{\beta\gamma \ldots \rho})$."

Vandermonde's theorem regarding the effect, on the function, of interchanging two bases is stated generally, and a demonstration is given. The mode of demonstration, which occupies one page and a half, will be readily understood by seeing it applied in later notation to the case where there are *four* bases, that is to say, where the theorem to be proved is

$$|a_{\alpha}b_{\beta}c_{\gamma}d_{\delta}| = -|b_{\alpha}a_{\beta}c_{\gamma}d_{\delta}|.$$

By repeated use of the recurrent law of formation we have

$$\begin{aligned}
|a_{\alpha}b_{\beta}c_{\gamma}d_{\delta}| &= a_{\alpha}|b_{\beta}c_{\gamma}d_{\delta}| - a_{\beta}|b_{\alpha}c_{\gamma}d_{\delta}| + a_{\gamma}|b_{\alpha}c_{\beta}d_{\delta}| - a_{\delta}|b_{\alpha}c_{\beta}d_{\gamma}| \\
&= a_{\alpha}\{\quad\quad b_{\beta}|c_{\gamma}d_{\delta}| - b_{\gamma}|c_{\beta}d_{\delta}| + b_{\delta}|c_{\beta}d_{\gamma}|\} \\
&\quad - a_{\beta}\{b_{\alpha}|c_{\gamma}d_{\delta}| \quad\quad - b_{\gamma}|c_{\alpha}d_{\delta}| + b_{\delta}|c_{\alpha}d_{\gamma}|\} \\
&\quad + a_{\gamma}\{b_{\alpha}|c_{\beta}d_{\delta}| - b_{\beta}|c_{\alpha}d_{\delta}| \quad\quad + b_{\delta}|c_{\alpha}d_{\beta}|\} \\
&\quad - a_{\delta}\{b_{\alpha}|c_{\beta}d_{\gamma}| - b_{\beta}|c_{\alpha}d_{\gamma}| + b_{\gamma}|c_{\alpha}d_{\beta}| \quad\quad\}.
\end{aligned}$$

By collecting the terms which have b_{α} for a common factor, b_{β} for a common factor, and so on, this result becomes

$$\begin{aligned}
|a_{\alpha}b_{\beta}c_{\gamma}d_{\delta}| &= -b_{\alpha}\{a_{\beta}|c_{\gamma}d_{\delta}| - a_{\gamma}|c_{\beta}d_{\delta}| + a_{\delta}|c_{\beta}d_{\gamma}|\} \\
&\quad + b_{\beta}\{a_{\alpha}|c_{\gamma}d_{\delta}| - a_{\gamma}|c_{\alpha}d_{\delta}| + a_{\delta}|c_{\alpha}d_{\gamma}|\} \\
&\quad - b_{\gamma}\{a_{\alpha}|c_{\beta}d_{\delta}| - a_{\beta}|c_{\alpha}d_{\delta}| + a_{\delta}|c_{\alpha}d_{\beta}|\} \\
&\quad + b_{\delta}\{a_{\alpha}|c_{\beta}d_{\gamma}| - a_{\beta}|c_{\alpha}d_{\gamma}| + a_{\gamma}|c_{\alpha}d_{\beta}|\}, \\
&= -b_{\alpha}|a_{\beta}c_{\gamma}d_{\delta}| + b_{\beta}|a_{\alpha}b_{\gamma}d_{\delta}| - b_{\gamma}|a_{\alpha}c_{\beta}d_{\delta}| + b_{\delta}|a_{\alpha}c_{\beta}d_{\gamma}|, \\
&= -|b_{\alpha}a_{\beta}c_{\gamma}d_{\delta}|,
\end{aligned}$$

as was to be proved. (XI. 5)

The suggestion readily arises that this process would be equally applicable in proving Vandermonde's theorem regarding the vanishing of a function in which two bases are identical, and the process, it may be remembered, was actually so employed by Desnanot.

One of the theorems given by Scherk, and later by Drinkwater, appears in the following form (p. 240), the peculiar notation adopted for a determinant with a row of unit elements being constantly employed throughout the remainder of the essay:—

"Si une des bases, par exemple a, est telle que la quantité qu'elle représente soit la même quel que soit l'exposant dont elle est affectée, c'est-a-dire, si $a^\alpha = a^\beta = a^\gamma = \ldots$, on aura

$$(abc \ldots r,\quad \alpha\beta\gamma \ldots \rho)$$

$$= a^\alpha[(bc \ldots r,\ \beta\gamma \ldots \rho) - (bc \ldots r,\ \alpha\gamma \ldots \rho) + (bc \ldots r,\ \alpha\beta\delta \ldots \rho) \mp \ldots].$$

La quantité qui se trouve sous la parenthèse, peut donc être représentée de la manière suivante :

$$(\mathrm{I}bc \ldots r,\quad \alpha\beta\gamma \ldots \rho); \qquad \text{(XLVIII. 3)}$$

en admettant une fois pour toutes que le chiffre romain I soit tel que $1 = \mathrm{I}^\alpha = \mathrm{I}^\beta = \mathrm{I}^\gamma = \ldots$ Il va sans dire que toutes les propriétés qui ont lieu pour $(abc \ldots r, \alpha\beta\gamma \ldots \rho)$ se rapportent également à

$$(\mathrm{I}bc \ldots r,\quad \alpha\beta\gamma \ldots \rho)."$$

The character of the identities used in the treatment of the tetrahedron will be learned from a glance at the following examples:—

$$a_1(\mathrm{I}bc, 123) - b_1(\mathrm{I}ac, 123) + c_1(\mathrm{I}ab, 123) = (abc, 123).$$

$$(a_1 - a_2)(\mathrm{I}bc, 123) - (b_1 - b_2)(\mathrm{I}ac, 123) + (c_1 - c_2)(\mathrm{I}ab, 123) = 0.$$

$$(ab,12)(ac, 34) - (ab, 34)(ac, 12) = -a_1(abc, 234) + a_2(abc, 134),$$
$$= +a_3(abc, 124) - a_4(abc, 123).$$

$$(\mathrm{I}ab, 123)(\mathrm{I}ac, 124) - (\mathrm{I}ab, 124)(\mathrm{I}ac, 123) = -(a_1 - a_2)(\mathrm{I}abc, 1234).$$

$$(\mathrm{I}ab, 123)(abc, 124) - (\mathrm{I}ab, 124)(abc, 123) = +(ab, 12)(\mathrm{I}abc, 1234).$$

The first of these we have already seen used by Minding; the second is nothing more than the manifest identity,

$$\begin{vmatrix} . & 1 & 1 & 1 \\ a_1 - a_2 & a_1 & a_2 & a_3 \\ b_1 - b_2 & b_1 & b_2 & b_3 \\ c_1 - c_2 & c_1 & c_2 & c_3 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} 1 & 1 & 1 & 1 \\ a_1 & a_1 & a_2 & a_3 \\ b_1 & b_1 & b_2 & b_3 \\ c_1 & c_1 & c_2 & c_3 \end{vmatrix} = 0;$$

the third is evidently the equatement of two expansions of

$$\begin{vmatrix} a_1 & a_2 & . & . \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} a_3 & . & . & a_4 \\ a_3 & a_1 & a_2 & a_4 \\ b_3 & b_1 & b_2 & b_4 \\ c_3 & c_1 & c_2 & c_4 \end{vmatrix};$$

the fourth is a case of the fifth: and the fifth is itself a case of a theorem (*C'*) of Desnanot's.

CATALAN (1839).

[Sur la transformation des variables dans les intégrales multiples. *Mémoires couronnés par l'Académie royale . . . de Bruxelles*, xiv. 2me partie, 49 pp.]

The first of the four parts into which Catalan's memoir is divided bears the title "*Valeurs générales des inconnues dans les équations du premier degré, et propriétés des dénominateurs communs*," and in the introduction it is said to contain several remarkable new properties of the functions called *resultants* by Laplace "et connues aujourd'hui sous le nom de *déterminants*."

His method of dealing with the opening problem is to derive the solution of n equations with n unknowns from the solution of $n-1$ equations with $n-1$ unknowns; or more definitely, to show that if the multipliers $\lambda_1, \lambda_2, \lambda_3$ necessary for the solution of the set of equations,

$$\left.\begin{aligned} a_1x_1 + b_1x_2 + c_1x_3 &= a_1 \\ a_2x_1 + b_2x_2 + c_2x_3 &= a_2 \\ a_3x_1 + b_3x_2 + c_3x_3 &= a_3 \end{aligned}\right\},$$

be the determinants of the systems

$$\begin{matrix} a_2 & b_2 \\ a_3 & b_3, \end{matrix} \qquad \begin{matrix} a_3 & b_3 \\ a_1 & b_1, \end{matrix} \qquad \begin{matrix} a_1 & b_1 \\ a_2 & b_2, \end{matrix}$$

then the multipliers $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ necessary for the solution of the set

$$\left.\begin{aligned} a_1x_1 + b_1x_2 + c_1x_3 + d_1x_4 &= a_1 \\ a_2x_1 + b_2x_2 + c_2x_3 + d_2x_4 &= a_2 \\ a_3x_1 + b_3x_2 + c_3x_3 + d_3x_4 &= a_3 \\ a_4x_1 + b_4x_2 + c_4x_3 + d_4x_4 &= a_4 \end{aligned}\right\}$$

are the determinants of the systems

$$\begin{matrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \end{matrix} \quad \begin{matrix} a_3 & b_3 & c_3 \\ a_4 & b_4 & c_4 \\ a_1 & b_1 & c_1 \end{matrix} \quad \begin{matrix} a_4 & b_4 & c_4 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{matrix} \quad \begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix}.$$

(XIII. 6)

This of course means that in the first case

$$a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 = 0,$$
$$b_1\lambda_1 + b_2\lambda_2 + b_3\lambda_3 = 0,$$

and $$x_3 = \frac{\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3}{\lambda_1 c_1 + \lambda_2 c_2 + \lambda_3 c_3};$$

and in the other

$$a_1\lambda_1 + a_2\lambda_2 + a_3\lambda_3 + a_4\lambda_4 = 0,*$$
$$b_1\lambda_1 + b_2\lambda_2 + b_3\lambda_3 + b_4\lambda_4 = 0,$$
$$c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4 = 0,$$

and $$x_4 = \frac{\lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + \lambda_4 a_4}{\lambda_1 d_1 + \lambda_2 d_2 + \lambda_3 d_3 + \lambda_4 d_4}.$$

The proof is disappointingly weak and unsatisfactory, and, what is still more surprising, rests at one point on a manifest inaccuracy. He says (p. 9)—

> "Par un calcul direct, on vérifie la formule (6) et les relations (5) pour le cas de trois équations. En même temps, l'on reconnait que
>
> "1° Le dénominateur de la valeur de x_3, par exemple, renferme toutes les combinaisons trois à trois des coefficients, chaque combinaison ne contenant ni deux fois la même lettre, ni deux fois le même indice.
>
> "2° Deux termes qui, dans l'expression de ce dénominateur, peuvent se déduire l'un de l'autre par une permutation tournante ont même signe.
>
> "3° Deux termes qui ne diffèrent que par le changement d'une lettre en une autre, et réciproquement, sont de signes contraires.
>
> "4° Par suite, le dénominateur est le même pour toutes les

* Note, however, the error in sign of λ_2 and λ_4.

inconnues, pourvu que l'on prenne convenablement le signe du numérateur."

He then proceeds—

"Supposons donc que pareille vérification ait été faite pour $n-1$ équations entre $n-1$ inconnues, je dis qu'elle se fera encore dans le cas de n équations."

Now although the statement in 2° is true for the case of three equations, it is not true generally, and therefore cannot be proved.*

The theorems which follow this introductory matter concern a special determinant, viz., the determinant of the system,

$$\begin{matrix} a_1 & b_1 & c_1 & \ldots\ldots & k_1 & l_1 \\ a_2 & b_2 & c_2 & \ldots\ldots & k_2 & l_2 \\ \cdot & \cdot & \cdot & \cdot\ \cdot\ \cdot\ \cdot & \cdot & \cdot \\ a_n & b_n & c_n & \ldots\ldots & k_n & l_n \end{matrix},$$

in which the elements are connected by the $\frac{1}{2}n(n-1)$ relations

$$\left.\begin{array}{l} a_1b_1 + a_2b_2 + a_3b_3 + \ldots + a_nb_n = 0 \\ a_1c_1 + a_2c_2 + a_3c_3 + \ldots + a_nc_n = 0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ a_1l_1 + a_2l_2 + a_3l_3 + \ldots + a_nl_n = 0 \\ b_1c_1 + b_2c_2 + b_3c_3 + \ldots + b_nc_n = 0 \\ b_1d_1 + b_2d_2 + b_3d_3 + \ldots + b_nd_n = 0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ b_1l_1 + b_2l_2 + b_3l_3 + \ldots + b_nl_n = 0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ k_1l_1 + k_2l_2 + k_3l_3 + \ldots + k_nl_n = 0 \end{array}\right\}.$$

Such determinants are only a little less special than determinants of an orthogonal substitution, and thus naturally fall to be considered later along with those of the latter class.

* In the proof he is fortunate (or unfortunate) enough to use another special case in which the statement is true. He says:—"Les deux termes $a_7b_6c_1d_3e_5f_2$ et $e_7f_6a_1b_3c_5d_2$ qui entrent dans D_4, et qui se déduisent l'un de l'autre par une permutation tournante entre les lettres ont même signe."

GARNIER (1814).

[Analyse Algébrique, faisant suite à la première section de l'algèbre. 2e édition, revue et considérablement augmentée. xvi. +668 pp. Paris.]

The title of Garnier's chapter xxvii. (pp. 541–555) is "*Développement de la théorie donnée par M. Laplace pour l'élimination au premier degré.*" It consists, however, of nothing but a simple exposition, confessedly borrowed from Gergonne's paper of 1813, and six pages of extracts from Laplace's original memoir of 1772. As forming part of a popular text-book, it probably did more service in bringing the theory to the notice of mathematicians than a memoir in a recondite serial publication could have done; and we certainly know that Sylvester, who afterwards did so much to advance the theory, expresses himself indebted to it.

SYLVESTER (1839).

[On Derivation of Coexistence: Part I.* Being the Theory of simultaneous simple homogeneous Equations. *Philosophical Magazine*, xvi. pp. 37–43.]

Sylvester was apparently first brought into contact with determinants while investigating the subject of the elimination of x between two equations of the mth and n^{th} degrees. At the close of a paper on this subject (*Phil. Mag.*, xv. p. 435) he says—"I trust to be able to present the readers of this magazine with a *direct* and *symmetrical* method of eliminating any number of unknown quantities between any number of equations of any degree, by a newly invented process of symbolical multiplication, and the use of *compound* symbols of notation." These last words, indicative of the method, exactly describe the matter dealt with in the paper we have now come to, and as will soon be seen, the functions which are the outcome of the said "compound symbol" of operations are determinants.

It would also appear that Sylvester was unacquainted with any

* Misprint for II., as an expression in the paper itself shows.

of the important memoirs of his predecessors regarding the functions: the twenty-seventh chapter of Garnier's *Analyse Algébrique*, to which he refers, may very probably indicate the extent of his knowledge.

Premising that he is going to use such symbols as a_1, a_2, . . . he calls the letter a the "base," and the complete symbol "an argument of the base," a_1 being the first argument, a_2 the second, and so on. Taking then a number of expressions, "each of which is made up of one or more terms, consisting solely of linear arguments of different bases, *i.e.*, characters bearing indices below but none above," *e.g.*, the expressions,

$$a_1 - b_1, \quad a_1 - c_1;$$

he alters them by writing the index-numbers *above*, *e.g.*,

$$a^1 - b^1, \quad a^1 - c^1;$$

takes the product of these resulting expressions in its expanded form

$$a^2 - a^1b^1 - a^1c^1 + b^1c^1;$$

and then reverses the operation on the index-numbers, thus finally obtaining

$$a_2 - a_1b_1 - a_1c_1 + b_1c_1.$$

The full series of these operations he indicates by the letter ζ, and denotes by the name of "*zeta-ic multiplication.*" Thus, as results in zeta-ic multiplication, we have

$$\zeta(a_1 - b_1)(a_1 - c_1) = a_2 - a_1b_1 - a_1c_1 + b_1c_1,$$

and

$$\zeta(a_1 + b_1)^2 = a_2 + 2a_1b_1 + b_2.*$$

Further ζ_{+r} is used to denote that, after the operations ζ have been performed, the indices are all to be increased by r, the result of so doing being called the zeta-ic product *in its* r^{th} *phase.*

He nexts recalls a notation previously introduced by him for the functions which came later to be known shortly as difference-products; denoting, for example,

* He would not even hesitate to extend the use of the symbol, denoting, for example,

$$1 - \frac{a_2}{1.2} + \frac{a_4}{1.2.3.4} - \ldots \text{ by } \zeta\cos(a_1).$$

$$(b-a)(c-a)(c-b) \quad \text{by} \quad \mathrm{PD}(abc),$$

$$(b-a)(c-a)(c-b)(d-a)(d-b)(d-c) \quad \text{by} \quad \mathrm{PD}(abcd),$$

and $$\therefore \quad abc(b-a)(c-a)(c-b) \quad \text{by} \quad \mathrm{PD}(0abc).$$

Lastly, he combines the two notations; and any reader who remembers Cauchy's mode of solving a set of simultaneous linear equations can with certainty predict the result of the union to be determinants. A new notation and a new name for the functions thus come into being together, the determinant of the system

$$\begin{matrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix}$$

being represented by

$$\zeta abc\mathrm{PD}(abc) \quad \text{or} \quad \zeta\mathrm{PD}(0abc), \qquad \text{(VII. 9)}$$

and being called a *zeta-ic product of differences.* (XV. 7)

These special zeta-ic products being reached, the rest of the paper is taken up with an account of some of their properties, and the application of them to the discussion of simultaneous linear equations. Some of the matter may be passed over as being already familiar to us, although its earlier appearances were certainly made in a less picturesque dress. The first really fresh theorem concerns the zeta-ic multiplication of a determinant $\zeta\mathrm{PD}\,(0abc \ldots l)$ by those symmetric functions of $a, b, c, \ldots, l$, which we would denote by

$$\Sigma a, \ \Sigma ab, \ \Sigma abc, \ \ldots .$$

but which Sylvester writes in the form

$$\mathrm{S}_1(abc \ldots l), \quad \mathrm{S}_2(abc \ldots l), \quad \mathrm{S}_3(abc \ldots l), \quad \ldots .$$

In his own words it stands as follows (p. 39):—

> "Let $a, b, c, \ldots l$, denote any number of independent bases, say $(n-1)$; but let the argument of each base be periodic, and the number of terms in each period the same for every base, namely (n), so that

$$a_r = a_{r+n} = a_{r-n} \qquad a_n = a_0 = a_{-n}$$
$$b_r = b_{r+n} = b_{r-n} \qquad b_n = b_0 = b_{-n}$$
$$c_r = c_{r+n} = c_{r-n} \qquad c_n = c_0 = c_{-n}$$
$$\cdots\cdots \qquad \cdots\cdots$$
$$l_r = l_{r+n} = l_{r-n} \qquad l_n = l_0 = l_{-n},$$

r being any number whatever. Then

$$\zeta_{-1}\mathrm{PD}(0abc\ldots l) = \zeta\big(\mathrm{S}_1(abc\ldots l).\zeta\mathrm{PD}(0abc\ldots l)\big)$$
$$\zeta_{-2}\mathrm{PD}(0abc\ldots l) = \zeta\big(\mathrm{S}_2(abc\ldots l).\zeta\mathrm{PD}(0abc\ldots l)\big)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$\zeta_{-r}\mathrm{PD}(0abc\ldots l) = \zeta\big(\mathrm{S}_r(abc\ldots l).\zeta\mathrm{PD}(0abc\ldots l)\big).\text{"}$$

The limitation made upon the arguments of the base would seem to imply that the theorem only concerned determinants of a very special kind. Such, however, is not the case. A special example in more modern notation will bring out its true character. Let the determinant chosen be

$$|\,a_1b_2c_3d_4\,|,$$

and let the symmetric function be

$$ab + ac + ad + bc + bd + cd.$$

Multiplying the two together "zeta-ically," that is to say, in accordance with the law

$$a_r \times a_s = a_{r+s},$$

we find that 120 of the total 144 terms of the product cancel each other, and that the remaining 24 terms constitute the determinant

$$|\,a_1b_2c_4d_5\,|,$$

the identity thus reached being

$$\zeta\big(|\,a_1b_2c_3d_4\,|\,.\,\Sigma ab\big) = |\,a_1b_2c_4d_5\,|.$$

Now Sylvester's ζPD notation being unequal to the representation of the determinant $|\,a_1b_2c_4d_5\,|$ in which the index-numbers do not proceed by the common difference 1, he would seem to have been compelled to give a periodic character to the arguments of the

bases in order to remove the difficulty. At any rate the difficulty *is* removed; for the number of terms in the period being 5 the index-numbers 4 and 5 become changeable into -1 and 0, and thus we can have

$$\begin{aligned} |\, a_1 b_2 c_4 d_5 \,| &= |\, a_1 b_2 c_{-1} d_0 \,|, \\ &= |\, a_{-1} b_0 c_1 d_2 \,|, \end{aligned}$$

—a determinant in which the index-numbers proceed by the common difference 1, and which is obtainable from $|\, a_1 b_2 c_3 d_4 \,|$ by diminishing each index-number by 2. Sylvester's form of the result thus is

$$\zeta \{ S_2(abcd) . \zeta \mathrm{PD}(0abcd) \} = \zeta_{-2}(0abcd) .*$$

Following this comes the application to simultaneous linear equations, or as they are called "equations of coexistence." The system is represented by the typical equation

$$a_r x + b_r y + c_r z + \ldots + l_r t = 0,$$

in which r can take up all integer values from $-\infty$ to $+\infty$, there being really, however, only n equations, because of the periodicity imposed on the arguments of the bases. One so-called "leading theorem" is given in regard to the system, its subject being the derivation of an equation linear in $x, y, z, \ldots, t$ by a combination of the equations of the system. The theorem is enunciated as follows (p. 40):—

> "Take $f, g, \ldots, k$ as the *arbitrary* bases of new and absolutely independent but periodic arguments, having the same

* It is rather curious that Sylvester overlooks the fact that the legitimate equatement of two zeta-ic products implies an identity altogether independent of the existence of zeta-ic multiplication. Thus, the identity just discussed is essentially the same as the identity

$$\begin{vmatrix} a & a^2 & a^3 & a^4 \\ b & b^2 & b^3 & b^4 \\ c & c^2 & c^3 & c^4 \\ d & d^2 & d^3 & d^4 \end{vmatrix} \times (ab + ac + ad + bc + bd + cd) = \begin{vmatrix} a & a^2 & a^4 & a^5 \\ b & b^2 & b^4 & b^5 \\ c & c^2 & c^4 & c^5 \\ d & d^2 & d^4 & d^5 \end{vmatrix},$$

where the index-number denotes a power and the multiplication is performed in accordance with the ordinary algebraic laws. From this point of view the above quoted proposition of Sylvester's involves an important theorem regarding the special determinants afterwards known by the name of *alternants*.

index of periodicity (n) as $a, b, c, \ldots, l$, and being in number $(n-1)$, *i.e.*, one fewer than there are units in that index.

"The number of *differing* arbitrary constants thus *manufactured* is $n(n-1)$.

"Let $Ax+By+Cz+\ldots+Lt=0$ be the general *prime* derivative from the given equations, then we may make

$$\begin{aligned} A &= \zeta PD(0afg \ldots k) \\ B &= \zeta PD(0bfg \ldots k) \\ C &= \zeta PD(0cfg \ldots k) \\ &\cdots\cdots\cdots \\ L &= \zeta PD(0lfg \ldots k)" \end{aligned} \qquad \text{(XIII. 7)}$$

As in the case of the other theorems, no demonstration is vouchsafed. In order, however, that the connection between it and previous work may be more readily manifest, it is desirable to indicate how it would most probably be established now. Taking the case where the number of unknowns is *three* and the number of given equations *four*, viz.—

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \\ a_4x + b_4y + c_4z &= 0 \end{aligned}\right\},$$

we should form an array of $4(4-1)$, *i.e.* 12, arbitrary quantities,

$$\begin{matrix} f_1 & g_1 & h_1 \\ f_2 & g_2 & h_2 \\ f_3 & g_3 & h_3 \\ f_4 & g_4 & h_4 \end{matrix},$$

from which we should select the multiplier $|f_2g_3h_4|$ for the first given equation, the multiplier $|f_1g_3h_4|$ for the second equation, and so on. The multiplication then being performed we should by addition obtain

$$1a_1f_2g_3h_4|\,x + |b_1f_2g_3h_4|\,y + |c_1f_2g_3h_4|\,z = 0,$$

which is what Sylvester would call "the general prime derivative of

the four given equations," the process being an instance of what he would similarly term the "derivation of coexistence."

By proper choice of the arbitrary quantities it may be readily shown, as Sylvester proceeds to do, that the theorem gives (1) the result of the elimination of n unknowns from n equations; (2) the *two* equations of condition in the case of $n+1$ equations connecting n unknowns; (3) the ratio of any two unknowns in the case of $n-1$ equations connecting n unknowns; and (4) the relation between any three unknowns in the case of $n-2$ equations connecting n unknowns. For example, the equations being

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}\right\},$$

the theorem gives the general derivative

$$\begin{vmatrix} a_1 & f_1 & g_1 \\ a_2 & f_2 & g_2 \\ a_3 & f_3 & g_3 \end{vmatrix} x + \begin{vmatrix} b_1 & f_1 & g_1 \\ b_2 & f_2 & g_2 \\ b_3 & f_3 & g_3 \end{vmatrix} y + \begin{vmatrix} c_1 & f_1 & g_1 \\ c_2 & f_2 & g_2 \\ c_3 & f_3 & g_3 \end{vmatrix} z = 0,$$

which is true whatever $f_1, f_2, f_3, g_1, g_2, g_3$ may be. By putting $f_1, f_2, f_3, g_1, g_2, g_3 = b_1, b_2, b_3, c_1, c_2, c_3$, this takes the form

$$|a_1b_2c_3|x + |b_1b_2c_3|y + |c_1b_2c_3|z = 0,$$

whence the equation of condition, or resultant of elimination,

$$|a_1b_2c_3| = 0.$$

As a corollary to one of the deductions from the leading theorem, —the deduction numbered (3) above,—the following proposition of a different character is given (p. 42):—

"If there be any number of bases $(abc \ldots l)$, and any other, two fewer in number, $(fg \ldots k)$,

$$\begin{aligned} &\zeta\mathrm{PD}(afg \ldots k) \times \zeta\mathrm{PD}(bc \ldots l) \\ +&\zeta\mathrm{PD}(bfg \ldots k) \times \zeta\mathrm{PD}(ac \ldots l) \\ +&\zeta\mathrm{PD}(afg \ldots k) \times \zeta\mathrm{PD}(bc \ldots l) \\ &\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots \\ +&\zeta\mathrm{PD}(lfg \ldots k) \times \zeta\mathrm{PD}(abc \ldots) = 0, \end{aligned}$$

a formula that from its very nature suggests and proves a wide extension of itself." (XXIII. 10)

It belongs evidently to the class of vanishing aggregates of products of pairs of determinants, of which so many instances have presented themselves. There is a manifest misprint in the third product, which should surely be

$$\zeta\mathrm{PD}(cfg \ldots k) \times \zeta\mathrm{PD}(ab \ldots l);$$

and there is an error in the signs connecting the products, which, instead of being all + , should be + and − alternately. When the determinants involved are of the third order, the theorem in the later notation is

$$|a_1 f_2 g_3| . |b_1 c_2 d_3| - |b_1 f_2 g_3| . |a_1 c_2 d_3| + |c_1 f_2 g_3| . |a_1 b_2 d_3| - |d_1 f_2 g_3| . |a_1 b_2 c_3| = 0,$$

which is readily recognised as an identity given by Bezout.

With this theorem the paper proper ends, but in a postscript an additional theorem of a curious character is given. As enunciated by the author—even his double mark of exclamation being reprinted—it is (p. 43) :—

"Let there be $(n-1)$ bases $a, b, c, \ldots, l$, and let the arguments of each be "recurrents of the nth order," that is to say, let

$$a_i = \phi\left(\cos . \frac{2\pi i}{n}\right), \quad b_i = \psi\left(\cos . \frac{2\pi i}{n}\right), \quad c_i = \chi\left(\cos . \frac{2\pi i}{n}\right),$$

$$\ldots\ldots, \; l_i = \omega\left(\cos . \frac{2\pi i}{n}\right).$$

Let R_r denote that any symmetrical function of the rth degree is to be taken of the quantities in a parenthesis which come after it, and let $\mathfrak{S}$ indicate any function whatever. Then the zeta-ic product,

$$\zeta\left(\zeta\mathrm{R}_r(abc \ldots l) \times \zeta_\varepsilon \mathfrak{S}\mathrm{PD}(0abc \ldots l)\right)$$

is equal to the product of the *number*

$$\mathrm{R}_t\left(\left(\cos.\frac{2\pi}{n}+\sqrt{-1}.\sin\frac{2\pi}{n}\right),\left(\cos.\frac{4\pi}{n}+\sqrt{-1}.\sin\frac{4\pi}{n}\right)\right.$$
$$\left.\left(\cos.\frac{6\pi}{n}+\sqrt{-1}.\sin\frac{6\pi}{n}\right).....\right.$$
$$\left.\cos.\left(\frac{(2n-1)\pi}{n}+\sqrt{-1}.\sin\frac{2(n-1)\pi}{n}\right)\right)$$

multiplied by the zeta-ic phase

$$\zeta_{\epsilon-t}\mathfrak{S}\mathrm{PD}(0abc\ldots l)\ !!"$$

Unfortunately the meaning of the proposition is seriously obscured by misprints and inaccurate use of symbols. Instead of "γ^{th}" degree we should have t^{th} degree; ζ preceding $\mathrm{R}_t\,(abc\ldots l)$ is meaningless, and should be deleted; ζ preceding $\mathfrak{S}\mathrm{PD}\,(0abc\ldots l)$ in the first member of the identity is unnecessary when a ζ has already been printed at the commencement; and the subscript ϵ, although giving an appearance of greater generality, serves no purpose whatever. Making the corrections thus suggested, and denoting

$$\cos\frac{2\pi}{n}+\sqrt{-1}\sin\frac{2\pi}{n},\quad \cos\frac{4\pi}{n}+\sqrt{-1}\sin\frac{4\pi}{n},\quad .....,$$

which are the roots of the equation

$$x^{n-1}+x^{n-2}+x^{n-3}+\ldots+x+1=0,$$

by $\alpha, \beta, \gamma, \ldots \lambda$, we are enabled to put the theorem in the more elegant form

$$\zeta\left\{\mathrm{R}_t(a,b,c\ \ldots,\ l).\mathfrak{S}.\mathrm{PD}(0,a,b,c,\ \ldots,\ l)\right\}$$
$$=\zeta_{-t}\left\{\mathrm{R}_t(\alpha,\beta,\gamma,\ \ldots,\ \lambda).\mathfrak{S}.\mathrm{PD}(0,a,b,c,\ \ldots,\ l)\right\}$$

It is readily seen to be a generalisation of the first theorem of the paper, into which it degenerates when $\mathfrak{S}$, instead of being any function of $a,b,c,\ \ldots\ l$, is a constant, and R_t, instead of being *any* symmetric function, is one of the series $\Sigma a, \Sigma ab, \Sigma abc, \ldots$. As, however, the constant $\mathrm{R}_t(\alpha,\beta,\gamma,\ \ldots\ \lambda)$ on the right-hand side will then be one of the series $\Sigma\alpha, \Sigma\alpha\beta, \Sigma\alpha\beta\gamma, \ldots$ and will not therefore be $+1$ unless when t is even, there must be an inattention to sign in one or other theorem. The matter can be more appropriately inquired into when we come to the subject of alternants, because, as has been pointed out in a recent footnote, it is to this

branch of the subject that identities between two zeta-ic multiplications of difference-products really belong.

This early paper, one cannot but observe, has all the characteristics afterwards so familiar to readers of Sylvester's writings,—fervid imagination, vigorous originality, bold exuberance of diction, hasty if not contemptuous disregard of historical research, the outstripping of demonstration by enunciation, and an infective enthusiasm as to the vistas opened up by his work.

SYLVESTER (1840).

[A method of determining by mere inspection the derivatives from two equations of any degree. *Philosophical Magazine*, xvi. pp. 132–135.]

The two equations taken are

$$\left.\begin{aligned} a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = 0 \\ b_n x^n + b_{n-1}x^{n-1} + \ldots + b_1 x + b_0 = 0 \end{aligned}\right\},$$

and rules are given for attaining three different objects, viz. (1) a rule for absolutely eliminating x; (2) a rule for finding the prime derivative of the first degree, that is to say of the form $Ax - B = 0$; (3) a rule for finding the prime derivative of any degree. The first of these concerns the process afterwards so well known by the name "dialytic." Only part of it need be given (p. 132):—

> "Form out of the a progression of coefficients m lines, and in like manner out of the b progression of coefficients form n lines in the following manner: Attach $m-1$ zeros all to the *right* of the terms in the a progression; next attach $m-2$ zeros to the right and carry 1 over to the left; next attach $m-3$ zeros to the right and carry 2 over to the left. Proceed in like manner until all the $m-1$ zeros are carried over to the left, and none remain on the right. The m lines thus formed are to be written under one another.
>
> Proceed in like manner to form n lines out of the b progression by scattering $n-1$ zeros between the right and left.
>
> If we write these n lines under the m lines last obtained, we

shall have a solid square $m+n$ terms deep and $m+n$ terms broad." (LIV. 1)

The rest of the rule deals of course with the formation of the terms from this square of elements, the old and familiar method being followed of taking all possible permutations and separating the permutations into positive and negative. As applied by Sylvester in the case of the elimination of x between the equations

$$\left.\begin{aligned} ax^2 + bx + c &= 0 \\ lx^2 + mx + n &= 0 \end{aligned}\right\},$$

that is to say, as applied to the development of the determinant of the system

$$\begin{matrix} a & b & c & 0 \\ 0 & a & b & c \\ l & m & n & 0 \\ 0 & l & m & n \end{matrix},$$

the method is lengthy.

No hint at an explanation of this or either of the two other rules is given. The principle at the basis of them all, however, is essentially that of the preceding paper. A single example will make this plain, and will at the same time serve to give a better idea of the two remaining rules than could be got by mere quotation.* Let the two given equations be

$$\left.\begin{aligned} ax^3 + bx^2 + cx + d &= 0 \\ \alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \epsilon &= 0 \end{aligned}\right\},$$

and suppose that it is desired to obtain their "prime derivative" of the 2nd (r^{th}) degree, that is to say, the derivative of the form

$$Ax^2 + Bx + C = 0.$$

Taking the first equation followed by $m-r-1$ equations derived from it by repeated multiplication by x, and then the second equation followed by $n-r-1$ equations derived from it in like manner, we have $m+n-2r$ equations,

$$\left.\begin{aligned} ax^3 + bx^2 + cx + d &= 0 \\ ax^4 + bx^3 + cx^2 + dx &= 0 \\ \alpha x^4 + \beta x^3 + \gamma x^2 + \delta x + \epsilon &= 0 \end{aligned}\right\},$$

* The third rule is incorrectly stated.

from which we have to deduce an equation involving no power of x higher than the 2nd. To do so we employ, as just stated, exactly the same method as was used in obtaining the "leading theorem" of the preceding paper. That is to say, we form multipliers

$$\begin{vmatrix} a & b \\ \alpha & \beta \end{vmatrix}, \quad -\begin{vmatrix} . & a \\ \alpha & \beta \end{vmatrix}, \quad \begin{vmatrix} . & a \\ a & b \end{vmatrix},$$

effect the multiplications, and add, the result being

$$\begin{vmatrix} . & a & b \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix} x^2 + \begin{vmatrix} . & a & c \\ a & b & d \\ \alpha & \beta & \delta \end{vmatrix} x + \begin{vmatrix} . & a & d \\ a & b & . \\ \alpha & \beta & \epsilon \end{vmatrix} = 0 . \text{ (LIV. 2)}$$

This is what Sylvester's third rule would give. His second rule is simply a case of the third, viz., where $r=1$; and his first rule is another case, viz., where $r=0$. Had he followed the order of his former paper, he would have called the third rule his "leading theorem," and given the others as corollaries from it.

RICHELOT (May 1840).

[Nota ad theoriam eliminationis pertinens. *Crelle's Journal*, xxi. pp. 226–234.]

Just as Jacobi (1835) brought determinants to bear on Bezout's abridged method of eliminating x from two equations of the nth degree, so did his fellow-professor Richelot, in treating of the other method of elimination, Euler's and Bezout's, discovered in the same year (1764). Euler's method, it will be remembered, consists in transforming the problem into the simpler one of eliminating a set of unknowns from a sufficient number of *linear* equations; and Richelot in a few lines (p. 227) points out that this may, of course, be done by equating to zero the determinant of the system of equations. An investigation connected therewith occupies the main portion of the paper.

Sylvester's method (1840) is described in passing, and the principle at the basis of it given. We have just seen that, when originally made known by the author, it was merely in the form of

a rule without any explanation. Although no doubt exists as to the mode in which it was obtained, still this first published description of the mode by Richelot deserves to be put on record. The whole passage in regard to it is as follows (p. 226):—

"Quam æquationem* inveniendi methodi diversæ a geometris adhibentur, ex quarum numero eius, quæ a clarissimo *Sylvester* in diario *The London and Edinburgh Philosophical Magazine and Journal of Science* nuper exposita est, mentionem faciendi hanc occasionem haud prætermittere velim. Ibi illius eliminationis problema reducitur ad problema eliminationis $m+n-1$ quantitatum ex systemate $m+n$ æquationum linearium. Multiplicata enim æquatione $f_1=0$ ex ordine per $y^{n-1}, y^{n-2}, \ldots, y^0$, nec non æquatione $f_2=0$ ex ordine per $y^{m-1}, y^{m-2}, \ldots, y^0$, adipiscimur systema $m+n$ æquationum linearium inter quantitates $y^{m+n-1}, y^{m+n-2}, \ldots, y^0$, quarum $m+n-1$ prioribus eliminatis, æquatio inter coefficientes † a' et a'' prodit. Quæ eliminatio facillime ita instituitur, ut determinantem harum $m+n$ æquationum linearium ponamus $=0$. Determinans vero, cum quantitates a' et a'' in æquationibus ipsæ tantum lineariter involvantur, et quantitates a' in n, nec non quantitates a'' in m ceteris æquationibus solis reperiantur, respectu illarum dimensiones $n$tæ est, respectuque harum $m$tæ. Unde concluditur, eam positam $=0$, esse quæsitam illam æquationem finalem $X=0$, quæ omni factore superflua careat. Notissima enim est proprietas ab *Eulero* inventa æquationis $X=0$, quod eius dimensio respectu quantitatum a' est $=n$, atque respectu quantitatum a'', $=m$, ita ut quæque functio integra evanescens, inter quantitates a' et a'', has dimensiones quadrans, pro genuina æquatione finali habenda sit." (LIV. 3)

Taking Sylvester's example,

$$\left.\begin{aligned} ax^2 + bx + c &= 0 \\ ax^2 + \beta x + \gamma &= 0 \end{aligned}\right\},$$

* *I.e.*, æquationem finalem.

† The equations are taken in the form

$$f_1 = a'_m y^m + a'_{m-1} y^{m-1} + \ldots + a'_0 = 0,$$
$$f_2 = a''_n y^n + a''_{n-1} y^{n-1} + \ldots + a''_0 = 0.$$

and doing as Richelot here directs, we should first multiply both members of the first equation by x^{2-1} and by x^{1-1}, then both members of the second by x^{2-1} and by x_{1-1}, thus obtaining

$$\begin{aligned} ax^3 + bx^2 + cx &= 0, \\ ax^2 + bx + c &= 0, \\ ax^3 + \beta x^2 + \gamma x &= 0, \\ ax^2 + \beta x + \gamma &= 0, \end{aligned}$$

and finally eliminate from these four equations x^3, x^2, x^1, by equating to zero the determinant of the system.

The statement "Ibi illius . . . . . linearium," which seems to contradict what we have above said in regard to the absence of explanation in Sylvester's paper, is not literally true. Richelot may have meant by it that Sylvester's result *implied* that the problem had been transformed as stated.

CAUCHY (1840).

[Mémoire sur l'élimination d'une variable entre deux équations algébriques. *Exercises d'analyse et de phys. math.*, i. pp. 385–422.]

After the appearance of the special papers on this subject by Jacobi, Sylvester, and Richelot, a review of the whole matter could not but be a desideratum. This was supplied by Cauchy in the singularly clear and able memoir which we have now reached. After an introduction of four pages there is an account (1) of Newton's method as expounded by Euler in 1748; (2) of Euler and Bezout's method of 1764; (3) of Bezout's abridged method; and (4) of a method * by means of the differences of the roots of the equations.

Euler and Bezout's method is shown to lead to the same determinant as Sylvester's, and the cause is made apparent. Cauchy's says (p. 389):—

"Supposons, pour fixer les idées, que les fonctions $f(x)$, $F(x)$

* Euler's, although not called so.

soient l'une du troisième degré, l'autre du second, en sorte qu'on ait

$$f(x) = ax^3 + bx^2 + cx + d,$$
$$\mathrm{F}(x) = \mathrm{A}x^2 + \mathrm{B}x + \mathrm{C}.$$

Alors u, v devront être de la forme

$$u = \mathrm{P}x + \mathrm{Q},$$
$$v = px^2 + qx + r;$$

et, si l'on élimine x entre les deux équations

$$f(x) = 0, \quad \mathrm{F}(x) = 0,$$

l'équation résultante sera précisément celle qu'on obtiendra, lorsqu'on choisera les coefficients

$$p,\ q,\ r,\ \mathrm{P},\ \mathrm{Q}$$

de manière à faire disparaître x de la formule

$$uf(x) + v\mathrm{F}(x) = 0, \tag{2}$$

par conséquent de la formule

$$(\mathrm{P}x + \mathrm{Q})f(x) + (px^2 + qx + r)\mathrm{F}(x) = 0,$$

que l'on peut encore écrire comme il suit :

$$\mathrm{P}xf(x) + \mathrm{Q}f(x) + px^2\mathrm{F}(x) + qx\mathrm{F}(x) + r\mathrm{F}(x) = 0. \tag{3}$$

Les valeurs de

$$p,\ q,\ r,\ \mathrm{P},\ \mathrm{Q}$$

qui remplissent cette condition sont celles qui vérifient les équations linéaires,

$$\left\{\begin{array}{llll} a\mathrm{P} & +\mathrm{A}p & & = 0, \\ b\mathrm{P} + a\mathrm{Q} & +\mathrm{B}p & +\mathrm{A}q & = 0, \\ c\mathrm{P} + b\mathrm{Q} & +\mathrm{C}p & +\mathrm{B}q + \mathrm{A}r & = 0, \\ d\mathrm{P} + c\mathrm{Q} & & +\mathrm{C}q + \mathrm{B}r & = 0, \\ \quad + d\mathrm{Q} & & & +\mathrm{C}r = 0. \end{array}\right. \tag{4}$$

Donc, pour obtenir la résultante cherchée, il suffira d'éliminer les coefficients

$$\mathrm{P},\ \mathrm{Q},\ p,\ q,\ r$$

entre les équations (4), ou, ce qui revient au même, d'égaler à zéro la fonction alternée formée avec les quantités que présente le tableau

$$(5)\qquad \begin{cases} a, & 0, & A, & 0, & 0, \\ b, & a, & B, & A, & 0, \\ c, & b, & C, & B, & A, \\ d, & c, & 0, & C, & B, \\ 0, & d, & 0, & 0, & C. \end{cases}$$

On arriverait encore aux mêmes conclusions en partant de la formule (3). En effet, choisir les coefficients P, Q, p, q, r, de manière à faire disparaître de cette formule les diverses puissances

$$x,\quad x^2,\quad x^3,\quad \ldots,\quad x^{m+n-1},$$

de la variable x, c'est éliminer ces puissances des cinq équations,

$$(6)\quad xf(x)=0,\ f(x)=0,\ x^2\mathrm{F}(x)=0,\ x\mathrm{F}(x)=0,\ \mathrm{F}(x)=0,$$

ou

$$(7)\qquad \begin{cases} ax^4+bx^3+cx^2+dx & =0, \\ \quad ax^3+bx^2+cx+d & =0, \\ \mathrm{A}x^4+\mathrm{B}x^3+\mathrm{C}x^2 & =0, \\ \quad \mathrm{A}x^3+\mathrm{B}x^2+\mathrm{C}x & =0, \\ \quad \mathrm{A}x^2+\mathrm{B}x+\mathrm{C} & =0. \end{cases}$$

C'est donc égaler à zéro la fonction alternée formée avec les quantités que présente le tableau,

$$(8)\qquad \begin{cases} a, & b, & c, & d, & 0, \\ 0, & a, & b, & c, & d, \\ \mathrm{A}, & \mathrm{B}, & \mathrm{C}, & 0, & 0, \\ 0, & \mathrm{A}, & \mathrm{B}, & \mathrm{C}, & 0, \\ 0, & 0, & \mathrm{A}, & \mathrm{B}, & \mathrm{C}. \end{cases}$$

Or cette fonction alternée ne différera pas de celle que nous avons déjà mentionnée, attendu que, pour passer du tableau (5) au tableau (8), il suffit de remplacer les lignes horizontales par les lignes verticales, et réciproquement." (LIV. 4)

Bezout's abridged method for the equations

$$\left.\begin{aligned} a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n &= 0 \\ b_0x^n + b_1x^{n-1} + \ldots + b_{n-1}x + b_n &= 0 \end{aligned}\right\}$$

is shown to lead to the final equation

$$\mathrm{S}=0,$$

where S is "une fonction alternée de l'ordre n formée avec les quantités que renferme le tableau,

$$\begin{Bmatrix} A_{0,0} & A_{0,1} & \dots & A_{0,n-2} & A_{0,n-1} \\ A_{0,1} & A_{1,1} & \dots & A_{1,n-2} & A_{1,n-1} \\ \dots & \dots & \dots & \dots & \dots \\ A_{0,n-2} & A_{1,n-2} & \dots & A_{n-2,n-2} & A_{n-2,n-1} \\ A_{0,n-1} & A_{1,n-1} & \dots & A_{n-2,n-1} & A_{n-1,n-1} \end{Bmatrix}$$"

in which

$$\begin{aligned} A_{0,l} &= a_0 b_{l+1} - b_0 a_{l+1}, \\ A_{1,l} &= a_1 b_{l+1} - b_1 a_{l+1} + A_{0,l+1}, \\ A_{2,l} &= a_2 b_{l+1} - b_2 a_{l+1} + A_{1,l+1}. \\ &\dots \dots \dots \dots \dots \end{aligned}$$

In connection with this, however, no reference is made to Jacobi's paper of 1835.

The fourth method, which occupies much the largest space (pp. 397–422), is not a determinant method.

SYLVESTER (January 1841).

[Examples of the dialytic method of elimination as applied to ternary systems of equations. *Cambridge Math. Journ.*, ii. pp. 232–236.]

In returning to extend the method, here and generally afterwards called "dialytic," Sylvester takes occasion to say that "the principle of the rule will be found correctly stated by Professor Richelot of Königsberg in a late number of *Crelle's Journal*." It may be noted, too, that he now for the first time uses the word *determinant*.

Only the first and last of the four examples need be given, as the subject strictly belongs to the application rather than the theory of determinants. Even these, however, will suffice to show the masterly grip which Sylvester had of his own method.

"To eliminate x, y, z between the three homogeneous equations

$$Ay^2 - 2C'xy + Bx^2 = 0 \quad (1),$$
$$Bz^2 - 2A'yz + Cy^2 = 0 \quad (2),$$
$$Cx^2 - 2B'zx + Az^2 = 0 \quad (3).$$

Multiply the equations in order by $-z^2, x^2, y^2$, add together, and divide out by $2xy$; we obtain

$$C'z^2 + Cxy - A'xz - B'yz = 0 \quad (4).$$

By similar processes we obtain

$$A'x^2 + Ayz - B'yx - C'zx = 0 \quad (5),$$
$$B'y^2 + Bzx - C'zy - A'xy = 0 \quad (6).$$

Between these six, treated as simple equations, the six functions of x, y, z, viz., $x^2, y^2, z^2, xy, xz, yz$, treated as *independent* of each other, may be eliminated; the result may be seen, by mere inspection, to come out

$$ABC(ABC - AB'^2 - BC'^2 - CA'^2 + 2A'B'C') = 0,$$

or rejecting the special (N.B. not *irrelevant*) factor ABC, we obtain

$$ABC - AB'^2 - BC'^2 - CA'^2 + 2A'B'C' = 0." \quad (\text{LIV. } 5)$$

The example, however satisfactory as illustrating the dialytic method, cannot be passed over without a note in regard to the unaccountable blunder made in developing the determinant involved. In later notation the determinant is

$$\begin{vmatrix} . & C & B & -2A' & . & . \\ C & . & A & . & -2B' & . \\ B & A & . & . & . & -2C' \\ A' & . & . & A & -C' & -B' \\ . & B' & . & -C' & B & -A' \\ . & . & C' & -B' & -A' & C \end{vmatrix}.$$

Now neither of the factors given by Sylvester are really factors of this, the truth being that it

$$= -2(ABC + 2A'B'C' - BB'^2 - CC'^2 - AA'^2)^2.$$

The fourth example concerns the elimination of x, y, z between the three equations

$$\left.\begin{aligned} Ax^2 + By^2 + Cz^2 + 2A'yz + 2B'zx + 2C'xy &= 0 \\ Lx^2 + My^2 + Nz^2 + 2L'yz + 2M'zx + 2N'xy &= 0 \\ Px^2 + Qy^2 + Rz^2 + 2P'yz + 2Q'zx + 2R'xy &= 0 \end{aligned}\right\} .$$

Using each of the three multipliers x, y, z with each of the three equations, we obtain nine equations linear in the ten quantities,

$$x^3,\ y^3,\ z^3,\ x^2y,\ x^2z,\ y^2x,\ y^2z,\ z^2x,\ z^2y,\ xyz .$$

Another such equation is thus necessary for success. Sylvester obtains it very ingeniously by writing the given equations in the form

$$\left.\begin{aligned} (Ax + B'z + C'y)x + (By + C'x + A'z)y + (Cz + A'y + B'x)z &= 0 \\ (Lx + M'z + N'y)x + (My + N'x + L'z)y + (Nz + L'y + M'x)z &= 0 \\ (Px + Q'z + R'y)x + (Qy + R'x + P'z)y + (Rz + P'y + Q'x)z &= 0 \end{aligned}\right\} ,$$

and then eliminating x, y, z. The work is not continued further.

We may ourselves note, in conclusion, that the fourth example includes in a sense the three others, but that it does not follow therefrom that by giving the requisite special values to the coefficients in the result of the general example, we should obtain the results for the particular examples in the forms already reached. Indeed, it is on account of this apparent non-agreement that the dialytic method is valuable to the theory of determinants, some very remarkable identities being arrived at by its aid. An explanation is also thus afforded of the trouble we have taken to elucidate its history.

CRAUFURD, A. Q. G.* (February 1841).

[On a method of algebraic elimination. *Cambridge Math. Journal*, ii. pp. 276–278.]

In Craufurd we have an independent discoverer of the dialytic method. A full account of his paper is quite unnecessary: the few

* Only the initials A. Q. G. C. are appended to the article. There can be little doubt, however, that they belong to Craufurd, whose name in full appears elsewhere in the *Journal*.

lines dealing with his introductory example will suffice to establish the fact. He says :—

"Let it be required to eliminate x from the equations

$$x^2+px+q=0,$$
$$x^2+p'x+q'=0.$$

Multiply each of the proposed equations by x, and you obtain

$$x^3+px^2+qx=0,$$
$$x^3+p'x^2+q'x=0.$$

These two combined with the two given equations make a system of four equations containing three quantities to be eliminated, viz., x, x^2, x^3; and they are of the first degree with respect to each of these quantities. We may, therefore, eliminate x, x^2, x^3 by the rules for equations of the first degree. The result is . . . . "

He enunciates a general rule, and then takes up the analogous subject in Differential Equations, where successive differentiation takes the place of successive multiplication by x. In a postscript he acknowledges Sylvester's priority which the editor had pointed out to him. He knew nothing of determinants.

CAUCHY (March 8, 1841).

[Note sur la formation des fonctions alternées qui servent à résoudre le problème de l'élimination. *Comptes Rendus . . . . Paris*, xii. pp. 414–426; or *Œuvres Complètes d'Augustin Cauchy*, 1re Sér., vi. pp. 87–99.]

Recalling the fact that the final equation, resulting from the elimination of several unknowns from a set of linear equations, has for its first member "une fonction alternée," and pointing out the further fact that the same holds good in regard to the elimination of one unknown from two equations of any degree, "puisque les methodes de Bezout et d'Euler reduisent ce denier problème au premier," Cauchy affirms the importance of being able easily to write out the full expansion of such functions. There can be little

doubt, however, that it was the second fact alone,—in other words, the discoveries of Jacobi, Sylvester, and Richelot,—which influenced the veteran Cauchy to return to a subject practically untouched by him for thirty years.

The opening part of the paper is, of course, necessarily old matter. One thing to be noted is that Cauchy tacitly discards the term *determinant*, which he was the means of introducing, using uniformly the more general expression *fonction alternée* instead. Another is that he adopts the rules of signs which makes use of the number of *interchanges*. From this his own peculiar rule of signs is deduced, and made the starting point for the fresh investigation which forms the main portion of the paper. The exposition of his rule, which differs from that of 1812, is worthy of a little attention, both on its own account and because otherwise the matter following would be scarcely intelligible. In the case of any term ("terme" or "produit") of the determinant

$$\Sigma \pm a_{0,0}a_{1,1}a_{2,2}a_{3,3}a_{4,4}a_{5,5}a_{6,6},$$

say the term

$$a_{0,1}a_{1,0}a_{2,5}a_{3,3}a_{4,6}a_{5,4}a_{6,2},$$

there is an underlying separation of the indices 0, 1, . . , 6 into groups ("groupes"), by reason of the system of pairing; that is to say, since an index is found paired along with one index and not with another, there arises the possibility of looking upon those which happen to be paired with one another as belonging to the same family group. Thus, attending to the first a of the term, we see that 1 and 0 belong to the same group, and as on scanning the rest of the term, we find neither of them associated with any other index, we conclude that the group is *binary* ("un groupe *binaire*"). Again, we see that 2 is paired with 5, 5 with 4, 4 with 6, and 6 with 2; this gives us the quaternary group (2, 5, 4, 6). Lastly, 3 is seen to be paired with 3, and thus forms a group by itself. Now, if we wish to find how many interchanges of the second indices are necessary in order to obtain the given term

$$a_{0,1}a_{1,0}a_{2,5}a_{3,3}a_{4,6}a_{5,4}a_{6,2}$$

from the typical term

$$a_{0,0}a_{1,1}a_{2,2}a_{3,3}a_{4,4}a_{5,5}a_{6,6},$$

we may do the counting piecemeal, attending at one time to only

that part of the term which corresponds to one of the groups of indices. In the case of the group (3), the number of interchanges s 0; in the case of the binary group (0, 1) it is 1; and in the case of the quaternary group it is 3—the number of interchanges being "évidemment" one less than the number of indices in the group. If, therefore, for a given term there be in all m groups, viz. f groups of one index each, g groups of two indices each, h of three, k of four, &c., the number of necessary interchanges will be

$$0.f + 1.g + 2.h + 3.k + \ldots,$$

which

$$= f + 2.g + 3.h + 4.k + \ldots,$$
$$-(f + g + h + k + \ldots),$$
$$= n - m;$$

and consequently the sign of the term will be + or − 1 according as $n - m$ is even or odd. (III. 28)

The first step of the new investigation is to define "termes semblables ou de même espèce." *Two terms are said to be alike or of the same species when the one may be obtained from the other by subjecting* both *sets of indices in the latter to one and the same substitution or permutation.* Thus recurring to the term above used,

$$a_{0,1}a_{1,0}a_{2,5}a_{3,3}a_{4,6}a_{5,4}a_{6,2},$$

and substituting in both of its sets of indices 6, 0, 1, 4, 3, 2, 5, instead of 0, 1, 2, 3, 4, 5, 6 respectively,—in other words, and with the notation of the memoir of 1812, performing the substitution

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 0 & 1 & 4 & 3 & 2 & 5 \end{pmatrix},$$

we obtain the like term

$$a_{6,0}a_{0,6}a_{1,2}a_{4,4}a_{3,5}a_{2,3}a_{5,1}. \quad \text{(LV.)}$$

The groups in two like terms are evidently similar, the values of $f, g, h, \ldots$ for the one being the same as those for the other. Indeed, since it is in this matter of groups or cycles that the terms have any likeness at all, the expression "*cyclically alike*" would have been a better term for Cauchy to use.

From the definition there arises the self-evident proposition—*Terms which are cyclically alike have the same sign.* (III. 29)

Also, *the full expansion of a determinant may be represented by writing a term of each cyclical species, and prefixing to each such typical term the symbol* Σ *with its proper sign,* + *or* − . (LV. 2)

To obtain a term of any given cyclical species, that is to say, corresponding to given values of $f, g, h, \ldots$, all the preparation that is necessary is to write the indices

$$0,\ 1,\ 2,\ 3,\ \ldots,\ (n-1),$$

enclose each of the first f of them in brackets, enclose in brackets each of the next g pairs, then each of the next h triads, and so on. This gives the groups of the term, and the term itself readily follows. For example, if we desire in the case of the determinant $\Sigma \pm a_{00}a_{11}a_{22}a_{33}a_{44}a_{55}a_{66}$ a term corresponding to $f=2$, $g=1$, $h=1$* we take the indices

$$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6\,;$$

bracket them thus

$$(0),\ (1),\ (2,\ 3),\ (4,\ 5,\ 6)\,;$$

and with the help of this, write finally

$$a_{0,0}\,a_{1,1}\,a_{2,3}\,a_{3,2}\,a_{4,5}\,a_{5,6}\,a_{6,4}\,. \qquad \text{(II. 7)}$$

The number of different cyclical species of terms in a determinant of the nth order is evidently equal to the number of positive integral solutions of the equation

$$f+2g+3h+\ \ldots\ +nl=n\,. \qquad \text{(LV. 3)}$$

Cauchy's illustration of this is clearness itself. He says (p. 419):—

"Si, pour fixer les idées, on suppose $n=5$, alors, la valeur de n pouvant être présentée sous l'une quelconque des formes,

$$1+1+1+1+1,$$
$$1+1+1+2,$$
$$1+2+2,$$
$$1+1+3,$$
$$2+3,$$
$$1+4,$$
$$5,$$

les systèmes de valeurs de

$$f,\ g,\ h,\ k,\ l,$$

se réduiront à l'un des sept systèmes

* It would be convenient to say, *a term of the cyclical species* $2(1)+1(2)+1(3)$.

$$\begin{array}{lllll} f=5, & g=0, & h=0, & k=0, & l=0, \\ f=3, & g=1, & h=0, & k=0, & l=0, \\ f=1, & g=2, & h=0, & k=0, & l=0, \\ f=2, & g=0, & h=1, & k=0, & l=0, \\ f=0, & g=1, & h=1, & k=0, & l=0, \\ f=1, & g=0, & h=0, & k=1, & l=0, \\ f=0, & g=0, & h=0, & k=0, & l=1\,; \end{array}$$

et par suite, une fonction alternée du cinquième ordre renfermera sept espèces de termes."

The next question considered is as to the number of terms of a given cyclical species which exist in any determinant of the nth order. The species being characterised by f groups of one index each, g groups of two indices each, h groups of three indices each, &c., the required number of terms is denoted by

$$\mathrm{N}_{f,\,g,\,h,\,\ldots,\,l}\ .$$

Now all the terms of the species will certainly be got if we write in succession the various permutations of the n indices 0, 1, 2, 3, ... , $n-1$, and then in the usual way mark off each permutation into the specified groups, viz., first f groups of one index each, then g groups of two indices each, and so on. As a rule, however, each term of the species will, in this way, be obtained more than once. For, if we examine in its grouped form the particular permutation, which was the first to give rise to a certain term, we shall find that changes are possible upon it without entailing any change in the term. For example, the set of groups

$$(0),\ (1),\ (2,\,3),\ (4,\,5,\,6),$$

instanced above as corresponding to the term

$$a_{0,0}a_{1,1}a_{2,3}a_{3,2}a_{4,5}a_{5,6}a_{6,4}\ ,$$

might be changed into

$$(1),\ (0),\ (2,\,3),\ (4,\,5,\,6)$$

or $$(1),\ (0),\ (3,\,2),\ (6,\,4,\,5)$$

or . . . . . .

which, while still corresponding to the term

$$a_{0.0}\,a_{1.1}\,a_{2.3}\,a_{3.2}\,a_{4.5}\,a_{5.6}\,a_{6.4}$$

are derivable from different permutations of the seven indices 0, 1, 1, 3, 4, 5, 6. In fact, the f groups of one index each may be permuted among themselves in every possible way, so may the g binary groups, the h ternary groups, &c. Further, with like immunity to the term, each separate group may be written in as many ways as there are indices in it,—the group (4, 5, 6), for example, being safely changeable into (5, 6, 4) or (6, 4, 5). The number, therefore, of different permutations of 0, 1, 2, 3, 4, 5, 6, which will give rise to any particular term, is

$$(1.2.3\ldots f \times 1.2.3\ldots g \times 1.2.3\ldots h \times \ldots \times 1.2.3\ldots l) \times (1^f 2^g 3^h \ldots n^l),$$

or say,

$$(f!\,g!\,h!\ldots l!)(1^f 2^g 3^h \ldots n^l).$$

There thus results the equation

$$(f!\,g!\,h!\ldots l!)(1^f 2^g 3^h \ldots n^l)\,\mathrm{N}_{f,g,h,\ldots,l} = n!,$$

whence

$$\mathrm{N}_{f,g,h,\ldots,l} = \frac{n!}{(f!\,g!\,h!\ldots l!)(1^f 2^g 3^h \ldots n^l)}. \qquad \text{(LV. 4)}$$

Following this interesting result a few deductions and verifications are given. First of all it is pointed out that since the total number of terms of all species is $n!$ we must conclude that

$$n! = \Sigma \frac{n!}{(f!\,g!\,h!\ldots l!)(1^f 2^g 3^h \ldots n^l)},$$

where

$$f + 2g + 3h + \ldots + nl = n.$$

Cauchy says (p. 423):—

> "Cette dernière formule paraît digne d'être remarquée. Si, pour fixer les idées, on prend $n = 5$ l'équation donnera
>
> $$1.2.3.4.5 = \mathrm{N}_{5,0,0,0,0} + \mathrm{N}_{3,1,0,0,0} + \mathrm{N}_{1,2,0,0,0} + \mathrm{N}_{2,0,1,0,0} + \mathrm{N}_{0,1,1,0,0} + \mathrm{N}_{1,0,0,1,0} + \mathrm{N}_{0,0,0,0,1},$$
>
> et par suite
>
> $$1.2.3.4.5 = 1 + 10 + 15 + 20 + 20 + 30 + 24 = 120,$$
>
> ce qui est exact."

Again, since the number of positive terms in a determinant is equal to the number of negative terms, and since the terms, whose number $\mathrm{N}_{f,g,h,\ldots,l}$, has just been found, have all the sign-factor

$$(-1)^{n-(f+g+h+\ldots+l)},$$

we have on leaving out the common factor $(-1)^n$ the identity

$$0=\Sigma(-1)^{f+g+h+\ldots+l}\frac{n\,!}{(f\,!\,g\,!\,h\,!\ldots l\,!)(1^f2^g3^h\ldots n^l)},$$

which like its companion may be illustrated by the case of $n=5$, viz.,

$$0=1-10+15+20-20-30+24.*$$

Lastly, attention is directed to the fact that when n is a prime, and therefore not exactly divisible by any integer less than itself, the number

$$\frac{n\,!}{(f\,!\,g\,!\,h\,!\ldots l\,!)(1^f2^g3^h\ldots n^l)}$$

must be exactly divisible by n, except in the case

$$f=n,\ g=0,\ h=0,\ \ldots\ l=0,$$

when it has the value 1, and in the case

$$f=0,\ g=0,\ h=0,\ \ldots\ l=1,$$

when it has the value $(n-1)\,!$ It, therefore, follows from either of the two preceding identities, that the sum of these two values must be divisible by n,—which is Wilson's theorem.

The remaining two pages are occupied with the expansion of a determinant of special form, viz., that afterwards known by the name *axisymmetric.*

JACOBI (1841).

[De formatione et proprietatibus Determinantium. *Crelle's Journal*, xxii. pp. 285–318.]

The value which Jacobi attached to determinants as an instrument of research has already become well known to us: we have

* In connection with this and in illustration of a previous remark regarding a mode of expressing the full expansion of a determinant, we have

$$\begin{aligned}\Sigma\pm a_{00}a_{11}a_{22}a_{33}a_{44} = {} & a_{00}a_{11}a_{22}a_{33}a_{44} - \Sigma a_{00}a_{11}a_{22}a_{34}a_{43}\\ & + \Sigma a_{00}a_{12}a_{21}a_{34}a_{43} + \Sigma a_{00}a_{11}a_{23}a_{34}a_{42}\\ & - \Sigma a_{01}a_{10}a_{23}a_{34}a_{42} - \Sigma a_{00}a_{12}a_{23}a_{34}a_{41}\\ & + \Sigma a_{01}a_{12}a_{23}a_{34}a_{40}.\end{aligned}$$

found him, indeed, in almost constant employment of the functions. In the memoir now reached, however, we have still stronger evidence of his interest in the subject, and of his opinion as to its importance. Knowing of no succinct and logically arranged exposition of their properties readily accessible to mathematicians, he deliberately set himself the task of preparing a memoir to supply the want. In his few words of preface he says :—

> "Sunt quidem notissimi Algorithmi, qui aequationum linearium litteralium resolutioni inserviunt. Neque tamen video eorum proprietates praecipuas, ita breviter enarratas atque in conspectum positas esse, quantum optare debemus propter earum in gravissimis quaestionibus Analyticis usum. Scilicet illae proprietates quamvis elementares non omnes ita tritae sunt, ut quas indemonstratas relinquere deceat, et valde molestum est earum demonstrationibus altiorum ratiociniorum decursum interrumpere. Cui defectui hic supplere volo quo commodius in aliis commentationibus ad hanc recurrere possim ; neutiquam vero mihi propono totam illam materiam absolvere."

While Jacobi was aware, as we have already partly seen, of the labours of Cramer, Bezout, Vandermonde, Laplace, Gauss, and Binet, his main source of inspiration is Cauchy. Of all the writers since Cauchy's time, indeed, he is the first who gives evidence of having read and mastered the famous memoir of 1812. It scarcely needs be said, however, that his own individuality and powerful grasp are manifest throughout the whole exposition.

At the outset there is a reversal of former orders of things ; Cramer's rule of signs for a permutation and Cauchy's rule being led up to by a series of propositions instead of one of them being made an initial convention or definition. This implies, of course, that a new definition of a signed permutation is adopted, and that conversely this definition must have appeared as a deduced theorem in any exposition having either of these rules as its starting point.

The new definition has its source in Cauchy, and rests on the well-known agreement as to a definite mode of forming the product P of the differences of an ordered series of quantities. This being settled to be

$$(a_1 - a_0)(a_2 - a_0)(a_3 - a_0) \ldots . (a_n - a_0)$$
$$(a_2 - a_1)(a_3 - a_1) \ldots . (a_n - a_1)$$
$$(a_3 - a_2) \ldots . (a_n - a_2)$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$(a_n - a_{n-1})$$

for the quantities $a_0, a_1, a_2, \ldots . a_n$, while in the order here written, the definition stands as follows (pp. 285–286):—

> "Vocemus eas indicum 0, 1, . . . , n permutationes, pro quibus P valorem eundem servat, *positivas;* eas pro quibus P valorem oppositum induit, *negativas;* sive priores dicamus pertinere ad *classem positivam permutationum*, posteriores ad *classem negativam.*"

This implies of course that the original permutation 0, 1, 2, ... , n is to be considered positive; and, such being the case, there seems to be a certain appropriateness in applying the term *negative* to a permutation whose corresponding difference-product is of the opposite sign from the difference-product corresponding to 0, 1, 2, ... , n.

The propositions which lead from the definition to Cramer's rule may be enunciated as follows :—

(*a*) One permutation performed upon another gives rise to a third, and the combined effect produced by performing the second and first in succession is the same as the effect of performing the third.

(*b*) Two given permutations belong to the same class or to opposite classes according as the permutation by means of which the one is obtained from the other belongs to the positive or negative class.

(*c*) If the same permutation be performed on a number of permutations which all belong to one class, the resulting permutations will still all belong to one class, viz., the same or the opposite according as the operating permutation is positive or negative.

(*d*) The order of compounding a set of permutations is, as a rule, not immaterial.

(*e*) The permutations which arise by compounding a set of permutations in every possible order belong all to the same class. (III. 31)

(*f*) The interchange of two indices is equivalent to the performance of a negative permutation.

(*g*) The interchange of two indices causes all the positive permutations to become negative, and all the negative to become positive.

Definition.—Two permutations may be called reciprocal which being performed in succession do not alter the order existing before the operations. (XXIV. 2)

(*h*) Reciprocal permutations belong to the same class.

In the original, it must be borne in mind, these are not separated and numbered, but appear merely as consecutive sentences in a paragraph. The words "classem negativam" of the definition above given are followed in the same line by

> "Binis propositis permutationibus quibuscunque, certa exstabit permutatio, qua post alteram adhibita altera prodit. Pertinebunt duæ permutationes propositæ ad classem eundem aut ad classes oppositas, prout permutatio, qua altera ex altera obtinetur, ad classem positivam aut negativam pertinet," &c.

—that is to say, by the propositions which have been paraphrased into (*a*), (*b*), &c.

The most essential point to be considered in connection with them is the probable meaning of the expression "permutationem adhibere," or the free English translation of it, "to perform a permutation." An example will make it clear. To perform the permutation 35412 would seem to be the operation of removing the 3rd member of a series of five things to the first place, the 5th member to the second place, the 4th member to the third place, and so on. With this explanation the proposition (*a*) is self-evident, an example of it being (if we may improvise a symbolism)

$$(35412)(41352) = (32541),$$

where 35412 is the operating permutation. Cauchy's usage, it may

be remembered, was to speak of "applying a substitution to a permutation."*

Of the proposition (*b*) a proof is given, which may be paraphrased as follows:—Let the three permutations referred to change P, the original product of differences, into e_1P, e_2P, e_3P, respectively, the *e*'s of course being either +1 or −1. Then as the performance of the first two permutations in succession will result in the change of P into $e_1.e_2$P, we must have

$$e_1 \cdot e_2 = e_3 ,$$

so that e_1 and e_3 have the same or opposite signs according as e_2 is +1 or −1; and this is virtually the proposition to be proved. (III. 30).

A demonstration of (*d*) is also given. The two permutations being A and B, l the first index of A, and m the first index of B, the performance of A on B implies that the lth index in B is to take the first place, and the performance of B on A that the mth index of A is to take the first place. The resulting permutations will consequently not agree in the first index, unless the lth index of B is the same as the mth index of A, which manifestly need not be the case.†

To prove (*f*) is of course the same as to prove that the interchange of two indices r and s, r being the greater, alters the sign of the product of differences; and this is done by separating the product into three portions, viz., (1) the portion which contains neither a_r nor a_s; (2) the single factor which contains both, $a_r - a_s$; and (3) the product of all the factors having either one or the other for a term. It is then asserted that the interchange of r and s cannot alter the last of these, because it is symmetrical with respect to a_r and a_s; also, that no alteration is possible in the first, and consequently that the change in the second accounts for the validity of the proposition. (III. 32)

* He says, for example (*Jour. de l'Éc. Polyt.*, x. p. 10), "Si en appliquant successivement à la permutation A, les deux substitutions $\begin{pmatrix} A_2 \\ A_3 \end{pmatrix}$ et $\begin{pmatrix} A_4 \\ A_5 \end{pmatrix}$, on obtient pour résultat la permutation A_6; la substitution $\begin{pmatrix} A_1 \\ A_6 \end{pmatrix}$ sera équivalente au produit des deux autres et j' indiquerai cette équivalence comme il suit

$$\begin{pmatrix} A_1 \\ A_6 \end{pmatrix} = \begin{pmatrix} A_2 \\ A_3 \end{pmatrix} \begin{pmatrix} A_4 \\ A_5 \end{pmatrix} ."$$

† This also is a paraphrase of Jacobi's proof.

As for the permutations which are called reciprocal they are, exactly those whose existence we have seen noted by Rothe, and called by him "verwandte Permutationen." Jacobi's definition, however, presents them in a slightly different light, the property involved in it being readily deducible from Rothe's. The latter's illustrative example was, as may be seen on looking back,

$$\left.\begin{matrix} 3, & 8, & 5, & 10, & 9, & 4, & 6, & 1, & 7, & 2 \\ 8, & 10, & 1, & 6, & 3, & 7, & 9, & 2, & 5, & 4 \end{matrix}\right\} \begin{matrix} A \\ B \end{matrix}.$$

Now the performance of either A on B or B on A* gives rise to

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10,$$

the original arrangement: consequently A and B satisfy Jacobi's definition. The proposition (*h*) is also Rothe's.

After these propositions, as already intimated, the subject of other rules of signs is taken up, the first rule considered being Cramer's. Since in the product of differences corresponding to any permutation every factor in which an index is preceded by a smaller index would require the sign-factor -1 to be annexed to it in order that the said product might be transformed into the original product of differences, it is clear that the determination of the class to which the permutation belongs is reduced to counting the number of such inversions. But the pairs of indices in the product of differences corresponding to the given permutation are exactly the pairs of indices to be examined in applying Cramer's rule. The identity of the two rules is thus apparent. (III. 33)

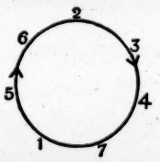
To the demonstration Jacobi adds "quam regulam olim cel. Cramer dedit ill. Laplace demonstravit." The last assertion is notable for two reasons: first, because the rule like Jacobi's own is incapable of proof being a definition, postulate, or convention according to the mode in which it is expressed: secondly, because an examination of Laplace's memoir shows that there is no ground for the statement. The fitness of the rule for the determination of the signs of the numerators and denominators of the unknowns in a set of simultaneous linear equations may of course be demonstrated, and perhaps this was in Jacobi's mind, but prior to the statement the abstract subject of permutations had alone been discussed.

* In the compounding of reciprocal permutations the order is immaterial. This is the exception hinted at in (*d*).

The other rule of signs dealt with is Cauchy's, in which permutation-cycles are counted instead of inversions. The existence of such cycles is the first point to be established, that is to say, it has to be shown that *any permutation* of 1 2 3 . . . *n may be obtained from any other by the performance of one or more cyclical permutations.* Let 3271654 be the permutation sought,* and 2647513 the permutation from which it is to be derived. Placing the former under the latter, thus

2 6 4 7 5 1 3
3 2 7 1 6 5 4,

we see that 2 has to be changed into 3, then seeking 3 in the upper line we see that it has to be changed into 4, similarly that 4 has to be changed into 7, 7 into 1, 1 into 5, 5 into 6, and 6 into 2, the element with which we started. Now the proof turns upon the simple fact that the elements in the two lines being exactly the same, by following a string of changes like this we are bound sooner or later to reach in the second line the element we started within the first. It may be that as here one cycle



suffices for the second transformation; but if not, as in the case of the two permutations

2 6 4 7 5 1 3
4 1 5 7 2 3 6,

where the short cycle 245 is obtained, we turn to the remaining elements, and knowing that those in the first line are of necessity the same as those in the second, we see that the application of the same process to them must, for the same reason as before, lead to a cycle. The possibility of arriving at any permutation by means of cyclical permutations alone is thus made manifest. The next point to be established is that *a cyclical permutation of* r *elements can be accomplished by* r − 1 *interchanges of pairs of elements.* Little more than the statement of this is necessary. For if the elements of the

* This is a paraphrase of Jacobi's demonstration, which is not so simple as it might have been. The notation of substitutions, which Jacobi did not follow Cauchy in using, is here a great help toward clearness.

cycle be $a_1, a_2, a_3, \ldots, a_r$, it is clear that to change a_1 into a_2, a_2 into a_3, &c., has the same effect as to interchange a_1 and a_2, then a_1 and a_3, then a_1 and a_4 and so on, the final interchange being that of a_1 and a_r; and there are in all $r-1$ interchanges. This being proved, the final step is taken as in Cauchy's Note of 8th March. (III. 34)

This rule of Cauchy's Jacobi deservedly characterises as beautiful. It is important, however, to take note that it possesses the other quality of usefulness in as marked a degree; and such being the case one is surprised to find that it has not received the attention which was its due. Any reader who will make a comparison of it and Cramer's by actual application of them to a number of examples will soon find that Cramer's is more lengthy and requires more care to be given to it to avoid errors.*

The preliminary subject of permutations having been thus dealt with, determinants are taken up. In the first section regarding them there is little noteworthy. Cauchy's word "terme" is supplanted by the fitter word *element*, and *term* ("terminus") is put to a more appropriate use; that is to say, $a_k^{(s)}$ is called an element of the determinant $\Sigma \pm a a'_1 a''_2 \ldots a_n^{(n)}$ and $a_k a'_{k'} a''_{k''} \ldots a_{k^{(n)}}^{(n)}$ a term. Further, the word *degree* is employed in place of Cauchy's more suitable word *order*, "ipsum R dicam determinans $n+1^{\text{ti}}$ *gradus*."

A section of two pages is given to considering the effect produced upon the aggregate of terms by the vanishing of certain of the elements. The propositions enunciated, with the exception of one made use of at an earlier date by Scherk, are as follows (pp. 291, 292):—

"I. Quoties pro indicis k valoribus 0, 1, 2, . . . , $m-1$ evanescant elementa $a_k^{(m)}, a_k^{(m+1)}, \ldots . a_k^{(n)}$, determinans

$$\Sigma \pm a a_1' a_2'' \ldots . a_n^{(n)}$$

abire in productum a duobus determinantibus

$$\Sigma \pm a a_1' \ldots . a_{m-1}^{(m-1)} \cdot \Sigma \pm a_m^{(m)} a_{m+1}^{(m+1)} \ldots . a_n^{(n)}. \quad \text{(XIV. 6)}$$

* The best way perhaps of applying Cauchy's rule is to write the primitive permutation, 123456789 say, above the given permutation, 683192457 say, draw the pen through 1 and the figure below it, seek 6 in the upper line and draw the pen through it and the figure below it, and so on, marking down 1 on the completion of every cycle.

"II. Evanescentibus elementis omnibus,

$$a_k^m,\ a_k^{(m+1)},\ \ldots,\ a_k^{(n)}$$

in quibus respective index inferior k indicibus superioribus $m, m+1, \ldots, n$ minor est, fieri (VI. 7)

$$\Sigma \pm a a_1' a_2'' \ldots a_n^{(n)} = a_m^{(m)} a_{m+1}^{(m+1)} \ldots a_n^{(n)} \cdot \Sigma \pm a a_1' \ldots a_{m-1}^{(m-1)}.$$

"IV. Evanescentibus elementis omnibus,

$$a_k^{(m)},\ a_k^{(m+1)} \ldots a_k^{(n)},$$

in quibus indices inferiores superioribus minores sunt, si insuper habetur,

$$a_m^{(m)} = a_{m+1}^{(m+1)} \ldots = a_n^{(n)} = 1,$$

fit $$\Sigma \pm a a_1' a_2'' \ldots a_n^{(n)} = \Sigma \pm a a_1' \ldots a_{m-1}^{(m-1)}.$$" (VI. 7)

As immediate deductions from the definition these are somewhat out of place, the trouble of demonstrating the first of them being virtually thrown away. The trouble taken by Jacobi, too, was less than required, the question of sign, for example, being inadequately discussed.

In the course of the next section which deals with what we have called the recurrent law of formation, and with the vanishing aggregate connected with this law, Jacobi gives an expression for the complete differential of a determinant, the elements being viewed as independent variables. The passage is (p. 293) :—

"Determinans R est singularum quantitatum $a_k^{(i)}$ respectu expressio linearis, atque ipsius $a_k^{(i)}$ coefficientem, qua in determinante R afficitur, vocavimus $A_k^{(i)}$; unde adhibita differentialium notatione ipsum $A_k^{(i)}$ exhibere licet per formulam,

$$3.\quad A_k^{(i)} = \frac{\partial R}{\partial a_k^{(i)}}.$$

Hinc si quantitatibus $a_k^{(i)}$ incrementa infinite parva tribuimus,

$$da_k^{(i)},$$

simulque R incrementum dR capit, fit

$$4.\quad dR = \Sigma A_k^{(i)} da_k^{(i)},$$ (LVI.)

siquidem sub signo summatorio utrique indici i et k valores 0, 1, 2, . . . , n conferuntur."

The recurrent law of formation and its dependent neighbour formula he is enabled, by means of (3), to view as the partial differential equations which the determinant must satisfy. His words are (p. 295) :—

"Substituendo formulas (3), inventas formulas sic quoque exhibere licet :

$$9.\quad \mathrm{R} = a^{(i)}\frac{\partial \mathrm{R}}{\partial a^{(i)}} + a_1^{(i)}\frac{\partial \mathrm{R}}{\partial a_1^{(i)}} + \ldots + a_n^{(i)}\frac{\partial \mathrm{R}}{\partial a_n^{(i)}},$$

$$= a_k\frac{\partial \mathrm{R}}{\partial a_k} + a'_k\frac{\partial \mathrm{R}}{\partial a'_k} + \ldots + a_k^{(n)}\frac{\partial \mathrm{R}}{\partial a_k^{(n)}},$$

$$10.\quad 0 = a^{(i')}\frac{\partial \mathrm{R}}{\partial a^{(i)}} + a_1^{(i')}\frac{\partial \mathrm{R}}{\partial a_1^{(i)}} + \ldots + a_n^{(i')}\frac{\partial \mathrm{R}}{\partial a_n^{(i)}},$$

$$0 = a_{k'}\frac{\partial \mathrm{R}}{\partial a_k} + a'_{k'}\frac{\partial \mathrm{R}}{\partial a'_k} + \ldots + a_{k'}^{(n)}\frac{\partial \mathrm{R}}{\partial a_k^{(n)}}.$$

Quae sunt aequationes differentiales partiales quibus determinans R satisfacit."

Passing over a section (7) on simultaneous linear equations, and a short section (8) in which Laplace's expansion-theorem is enunciated, we come to two sections dealing with what at a later time would have been called the secondary minors. No name is given to them by Jacobi ; they only appear as co-factors of the product of a pair of elements, the aggregate of the terms containing $a_g^{(f)}\, a_{g'}^{(f')}$ as a factor being denoted by

$$a_g^{(f)}\, a_{g'}^{(f')}.\ \mathrm{A}_{g,g'}^{f,f'}. \qquad \text{(XLI. 8)}$$

From observing that the interchange of f and f' or of g and g' alters R into $-\mathrm{R}$ and cannot alter $\mathrm{A}_{g,g'}^{f,f'}$, it is concluded that

$$\mathrm{A}_{g',g}^{f,f'} = \mathrm{A}_{g,g'}^{f',f} = -\mathrm{A}_{g,g'}^{f,f'},$$

and that the full co-factor of $\mathrm{A}_{g,g'}^{f,f'}$ is $a_g^{(f)}\, a_{g'}^{(f')} - a_{g'}^{(f)}\, a_g^{(f')}$ in accordance with the expansion-theorem of the previous section. The

remark that $A_g^{(f)}$ can be expressed in terms of n of the quantities $A_{g,g'}^{f,f'}$ leads up to a curious set of equations the determinant of which belongs to the special class of determinants known afterwards as zero-axial skew determinants. The passage is (pp. 300, 301):—

"Designemus br. causa per (k, k') expressionem

$$10. \qquad A_{k,k'}^{f,f'} = (k, k'),$$

ita ut sit $$(k, k') = -(k', k).$$

Fit e (8) ipsi g substituendo numeros 0, 1, 2, . . , n:

$$11. \begin{cases} A^{(f)} = \quad * \quad + a_1^{(f')}(0,1) + a_2^{(f')}(0,2) \; . \; . \; + a_n^{(f')}(0,n) \\ A_1^{(f)} = a^{(f')}(1,0) + \quad * \quad + a_2^{(f')}(1,2) \; . \; . \; + a_n^{(f')}(1,n) \\ A_2^{(f)} = a^{(f')}(2,0) + a_1^{(f')}(2,1) + \quad * \quad . \; . \; + a_n^{(f')}(2,n) \\ \quad . \; . \\ A_n^{(f)} = a^{(f')}(n,0) + a_1^{(f')}(n,1) + a_2^{(f')}(n,2) \; . \; . \; + \quad * \end{cases}$$

Similes formulae e (9) derivari possunt. In aequationibus (11) ipsorum $a^{(f')}$, $a^{(f')}$, etc. coëfficientes in diagonali positi evanescunt, bini quilibet coëfficientes diagonalis respectu symmetrice positi valoribus oppositis gaudent. Quae est species aequationum linearium memorabilis in variis quaestionibus analyticis obveniens." (LVII.)

The simple step from the expression of A_g^f as a differential coefficient to the similar expression for $A_{g,g'}^{f,f'}$ is next made (p. 301):—

"Ex ipsa enim aggregati $A_{g,g'}^{f,f'}$ definitione eruimus formulas

$$1. \quad A_{g,g'}^{f,f'} = \frac{\partial^2 R}{\partial a_g^{(f)} \partial a_{g'}^{(f')}} = -\frac{\partial^2 R}{\partial a_g^{(f')} \partial a_{g'}^{(f)}},$$

$$= \frac{\partial . A_g^{(f)}}{\partial a_{g'}^{(f')}} = \frac{\partial . A_{g'}^{(f')}}{\partial a_g^{(f)}} = -\frac{\partial . A_g^{(f')}}{\partial a_{g'}^{(f)}} = -\frac{\partial . A_{g'}^{(f)}}{\partial a_g^{(f')}}."$$

By taking the identities

$$0 = aA_k + a'A'_k + \ldots + a^{(n)}A^{(n)}_k,$$

$$0 = a_1A_k + a_1A'_k + \ldots + a^n_1A^{(n)}_k,$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$R = a_kA_k + a'_kA'_k \ldots + a^{(n)}_kA^{(n)}_k,$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$0 = a_nA_k + a_nA_k + \ldots + a^{(n)}_nA^{(n)}_k;$$

using the multipliers

$$A^{i,i'}_{0,k'},\ A^{i,i'}_{1,k'}, \ldots\ldots, A^{i,i'}_{k,k'}, \ldots\ldots, A^{i,i'}_{n,k'},$$

and adding, there is obtained

$$4. \qquad R \,.\, A^{i,i'}_{k,k'} = A^{(i)}_kA^{(i')}_k - A^{(i)}_{k'}A^{(i')}_k,$$

—a result at once recognisable as a case of the theorem regarding a minor of the adjugate. Next by starting with Bezout's identity connecting any eight quantities, the particular eight taken being

$$A^{(i)}_k,\ A^{(i)}_{k'},\ A^{(i)}_{k''},\ A^{(i)}_{k'''},$$
$$A^{(i')}_k,\ A^{(i')}_{k'},\ A^{(i')}_{k''},\ A^{(i')}_{k'''},$$

and making six substitutions of the kind

$$A^{(i)}_kA^{(i')}_{k'} - A^{(i)}_{k'}A^{(i')}_k = R \,.\, A^{i,i'}_{k,k'},$$

just seen to be valid, there arises the identity

$$A^{i,i'}_{k,k'}A^{i,i'}_{k''k'''} + A^{i,i'}_{k,k''}A^{i,i'}_{k''',k'} + A^{i,i'}_{k,k'''}A^{i',i'}_{k',k''} = 0. \quad \text{(XXIII. 11)}$$

This clearly belongs to the class of vanishing aggregates of products of pairs of determinants; but in order that its true character may be seen, and comparison made possible between it and others of the same class already obtained, a more lengthy notation is necessary. Taking for shortness the case where the primitive determinant is of the 8th order, but writing it in the form

$$|a_1b_2c_3d_4e_5f_6g_7h_8|$$

and making

$$i, i' = 3, 6 \quad \text{and} \quad k, k', k'', k''' = 5, 6, 7, 8,$$

we find the identity to be

$$|a_1b_2d_3e_4g_7h_8|.|a_1b_2d_3e_4g_5h_6| - |a_1b_2d_3e_4g_6h_8|.|a_1b_2d_3e_4g_5h_7| + |a_1b_2d_3e_4g_6h_7|.|a_1b_2d_3e_4g_5h_8| = 0,$$

a glance at which suffices to show that it is nothing more than the extensional of

$$|g_7h_8|.|g_5h_6| - |g_6h_8|.|g_5h_7| + |g_6h_7|.|g_5h_8| = 0,$$

the very identity of Bezout which was taken as a basis for it. As the same extensional has already been found among those of Desnanot, any new interest in it is due to the peculiar way in which Jacobi obtained it. By the same method, viz., by substituting for secondary minors an expression (4) involving primary minors and the primitive determinant, he shows that

$$A_k^{(i)}A_{k',k''}^{i,i'} + A_{k'}^{(i)}A_{k''k,}^{i,i'} + A_{k''}^{(i)}A_{k,k'}^{i,i'} = 0. \quad \text{(XXIII. 12)}$$

This being translated in the same manner as the preceding, becomes

$$|a_1b_2d_3e_4f_6g_7h_8|.|a_1b_2d_3e_4g_5h_8| - |a_1b_2d_3e_4f_5g_7h_8|.|a_1b_2d_3e_4g_6h_8| + |a_1b_2d_3e_4f_5g_6h_8|.|a_1b_2d_3e_4g_7h_8| = 0,$$

and is thus seen to be another of Desnanot's results, viz., the extensional of

$$|f_6g_7|.g_5 - |f_5g_7|.g_6 + |f_5g_6|.g_7 = 0. \quad \text{(XXIII. 12)}$$

The deduction

$$\partial . \frac{A_{k'}^{(i)}}{A_k^{(i)}} \Big/ \partial\, a_{k''}^{(i')} = -\frac{A_{k''}^{(i)}A_{k,k'}^{i,i'}}{A_k^{(i)}A_k^{(i)}}, \qquad \partial . \frac{A_k^{(i)}}{A_k^{(i')}} \Big/ \partial\, a_{k'}^{(i'')} = -\frac{A_k^{(i'')}A_{k,k'}^{i,i'}}{A_k^{(i)}A_k^{(i)}},$$

is made from it by substituting appropriate differential coefficients for the primary and secondary minors involved in it. (LVIII.)

The eleventh section is devoted to the establishment of the general theorem which includes the theorem

$$R.\, A_{k,k'}^{i,i'} = A_k^{(i)}A_{k'}^{(i')} - A_{k'}^{(i)}A_k^{(i')}$$

of the preceding section, and which, as we have seen, Jacobi had first enunciated in 1833. To start with it is repeated that the system of equations

$$\left.\begin{aligned}
&a\ t + a_1\ t_1 + \ldots + a_k\ t_k + a_{k+1} t_{k+1} + \ldots + a_n\ t_n = u,\\
&a'\ t + a_1'\ t_1 + \ldots + a_k'\ t_k + a_{k'+1} t_{k+1} + \ldots + a_n'\ t_n = u_1,\\
&\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&a^{(k)} t + a_1^{(k)}\ t_1 + \ldots + a_k^{(k)} t_k + a_{k+1}^{(k)} t_{k+1} + \ldots + a_n^{(k)} t_n = u_k,\\
&\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&a^{(n)} t + a_1^{(n)} t_1 + \ldots + a_k^{(n)} t_k + a_{k+1}^{(n)} t_{k+1} + \ldots + a_n^{(n)} t_n = u_n,
\end{aligned}\right\}$$

gives rise to the system

$$\left.\begin{aligned}
&\mathrm{A}\ u + \mathrm{A}'\ u_1 + \ldots + \mathrm{A}_k\ u_k + \mathrm{A}_{k+1} u_{k+1} + \ldots + \mathrm{A}_n u_n = \mathrm{R}\,.\,t\\
&\mathrm{A}'\ u + \mathrm{A}_1'\ u_1 + \ldots + \mathrm{A}_k'\ u_k + \mathrm{A}'_{k+1} u_{k+1} + \ldots + \mathrm{A}'_n u_n = \mathrm{R}\,.\,t_1\\
&\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\mathrm{A}^{(k)} u + \mathrm{A}_1^{(k)} t_1 + \ldots + \mathrm{A}_k^{(k)} u_k + \mathrm{A}_{k+1}^{(k)} u_{k+1} + \ldots + \mathrm{A}_n^{(k)} u_n = \mathrm{R}\,.\,t_k\\
&\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&\mathrm{A}^n\ u + \mathrm{A}_1^{(n)} u_1 + \ldots + \mathrm{A}_k^{(n)} u_k + \mathrm{A}_{k+1}^{(n)} u_{k+1} + \ldots + \mathrm{A}_n^{(n)} u_n = \mathrm{R}\,.\,t_n
\end{aligned}\right\}$$

in which

$$\mathrm{R} = \Sigma \pm a a'_1 \ldots a_n^{(n)}, \qquad \mathrm{A}_n^{(n)} = \Sigma \pm a a'_1 \ldots a_{(n-1)}^{(n-1)}.$$

Then taking only the first $k+1$ equations of the first system and eliminating $t, t_1, \ldots, t_{k-1}$, there is obtained

$$\mathrm{C}_k t_k + \mathrm{C}_{k+1} t_{k+1} + \ldots + \mathrm{C}_n t_n = \mathrm{D} u + \mathrm{D}_1 u_1 + \ldots + \mathrm{D}_k u_k, \quad \text{(X)}$$

where the multipliers $\mathrm{D}, \mathrm{D}_1, \ldots, \mathrm{D}_k$, by which the elimination is effected, are

$$\begin{aligned}
&(-1)^k \Sigma \pm a' a''_1 \ldots a_{k-1}^k,\\
&(-1)^{k+1} \Sigma \pm a a''_1 \ldots a_{k-1}^k,\\
&\quad\ldots\ldots\ldots\ldots\\
&\qquad + \Sigma \pm a a'_1 a''_2 \ldots a_{k-1}^{(k-1)},
\end{aligned}$$

and consequently by $\mathrm{C}_k, \mathrm{C}_{k+1}, \ldots, \mathrm{C}_n$ are denoted

$$\begin{aligned}
&\Sigma \pm a a'_1 a''_2 \ldots a_{k-1}^{(k-1)} a_k^{(k)},\\
&\Sigma \pm a a'_1 a''_2 \ldots a_{k-1}^{(k-1)} a_{k+1}^{(k)},\\
&\quad\ldots\ldots\ldots\ldots\\
&\Sigma \pm a a'_1 a''_2 \ldots a_{k-1}^{(k-1)} a_n^{(k)}.
\end{aligned}$$

Similarly, taking only the last $n-k+1$ equations of the second system and eliminating $u_{k+1}, u_{k+2} \ldots, u_n$ there is obtained

$$Eu + E_1u_1 + \ldots + E_ku_k = RF_kt_k + RF_{k+1}t_{k+1} + \ldots + RF_nt_n, \quad (Y)$$

where the multipliers $F_k, F_{k+1}, \ldots, F_n$ by which the elimination is effected are

$$\Sigma \pm A_{k+1}^{(k+1)} A_{k+2}^{(k+2)} \ldots A_n^{(n)},$$

$$-\Sigma \pm A_k^{(k+1)} A_{(k+2)}^{(k+2)} \ldots A_n^{(n)},$$

$$\ldots\ldots\ldots\ldots$$

$$(-1)^{n-k} \Sigma \pm A_k^{(k+1)} A_{k+1}^{(k+2)} \ldots A_{n-1}^{(n)},$$

and consequently by $E, E_1, \ldots, E_k$ are denoted

$$\Sigma \pm A_k A_{k+1}^{(k+1)} \ldots A_n^{(n)},$$

$$\Sigma \pm A_k' A_{k+1}^{(k+1)} \ldots A_n^{(n)},$$

$$\ldots\ldots\ldots\ldots$$

$$\Sigma \pm A_k^{(k)} A_{k+1}^{(k+1)} \ldots A_n^{(n)}.$$

These two derived equations (X), (Y), however, must be identical, because they may be both viewed as giving t_k in terms of $t_{k+1}, t_{k+2} \ldots, t_n, u, u_1, \ldots, u_k$, and, as the first system of equations shows, this can only be done in one way. We thus have the deduction

$$\frac{D_k}{C_k} = \frac{E_k}{R.F_k},$$

$$i.e. \quad \frac{\Sigma \pm a a_1' a_2'' \ldots a_{k-1}^{(k-1)}}{\Sigma \pm a a_1' a_2'' \ldots a_k^{(k)}} = \frac{\Sigma \pm A_k^{(k)} A_{k+1}^{(k+1)} \ldots A_n^{(n)}}{R\Sigma \pm A_{k+1}^{(k+1)} A_{k+2}^{(k+2)} \ldots A_n^{(n)}}.$$

This is the keystone of the demonstration. The simple continuation of it may for sake of historical colour be given in Jacobi's own words (p. 304): *—

"In hac formula generali ipsi k tribuendo valores

$$n-1, n-2, n-3, \ldots, 1,$$

prodit:

* The demonstration in the original is considerably disfigured by misprints.

$$\begin{cases} \dfrac{\Sigma \pm a a_1' \quad \ldots \quad a_{n-2}^{(n-2)}}{\Sigma \pm a a_1' \ldots . \; a_{n-1}^{(n-1)}} = \dfrac{\Sigma \pm A_{n-1}^{(n-1)} A_n^{(n)}}{R A_n^{(n)}}, \\ \dfrac{\Sigma \pm a a_1' \ldots . \; a_{n-3}^{(n-3)}}{\Sigma \pm a a_1' \ldots . \; a_{n-2}^{(n-2)}} = \dfrac{\Sigma \pm A_{n-2}^{(n-2)} A_{n-1}^{(n-1)} A_n^{(n)}}{R \Sigma \pm A_{n-1}^{(n-1)} A_n^{(n)}}, \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ \dfrac{a}{\Sigma \pm a a_1'} = \dfrac{\Sigma \pm A_1' A_2'' \ldots . \; A_n^{(n)}}{R \Sigma \pm A_2'' A_3''' \ldots . \; A_n^{(n)}}. \end{cases}$$

"Harum aequationum prima suppeditat,

$$\Sigma \pm A_{n-1}^{(n-1)} A_n^{(n)} = R \Sigma \pm a a_1' \ldots . \; a_{n-2}^{(n-2)} = R A_{n-1, n}^{n-1, n},$$

quae cum formula (4) § pr. convenit. Deinde aequationum (10) duas, tres, quatuor etc. primas inter se multiplicando, prodit formularum systema hoc:

$$\begin{cases} \Sigma \pm A_{n-1}^{(n-1)} A_n^{(n)} = R \, \Sigma \pm a a_1' \ldots . \; a_{n-2}^{(n-2)}, \\ \Sigma \pm A_{n-2}^{(n-2)} A_{n-1}^{(n-1)} A_n^{(n)} = R^2 \Sigma \pm a a_1' \ldots . \; a_{n-3}^{(n-3)}, \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ \Sigma \pm A_1' A_2'' \ldots . \; A_n^{(n)} = a R^{n-1}. \end{cases}$$

"Quas formulas amplectitur formula generalis,

$$\Sigma \pm A_{k+1}^{(k+1)} A_{k+2}^{(k+2)} \ldots . \; A_n^{(n)} = R^{n-k-1} \Sigma \pm a a_1' \ldots . \; a_k^{(k)}." \qquad \text{(xx. 5)}$$

Cauchy's theorem

$$\Sigma \pm A A_1' \ldots . \; A_n^{(n)} = R^n,$$

which may be viewed as the ultimate case of this, Jacobi arrives at by expressing $\Sigma \pm A A_1' \ldots A_n^{(n)}$ in terms of $A, A_1, \ldots, A_n$ and their cofactors, substituting for the said cofactors their equivalents as just obtained, viz.

$$a R^{n-1}, \quad a_1 R^{n-1}, \quad a_2 R^{n-1}, \quad \ldots . \, , \quad a_n R^{n-1},$$

and then using the identity

$$A a + A_1 a_1 + \ldots + A_n a_n = R.$$

Passing over the twelfth section, which relates to certain special

systems of equations, we come to two sections devoted to the multiplication-theorem. Of the five formally enunciated propositions which they contain, two, the second and fourth, need not be more than referred to, as their substance comes from Binet and Cauchy, and as the mode in which they are established will be sufficiently understood from the treatment of one of the others. The general problem of the two sections is the investigation of the determinant

$$\Sigma \pm cc_1 c_2'' \ldots . c_n^{(n)},$$

where

$$c_k^{(i)} = a^{(i)}a^{(k)} + a_1^{(i)}a_1^{(k)} + \ldots . + a_p^{(i)}a_p^{(k)}.$$

Taking a single term of the determinant, we have of course

$$\begin{aligned} cc_1'c_2'' \ldots c_n^{(n)} = {} & (aa + a_1a_1 + \ldots . + a_p a_p) \\ & \times (a'a' + a_1'a_1' + \ldots . + a_p'a_p') \\ & \ldots \ldots \ldots \ldots \\ & \times (a^{(n)}a^{(n)} + a_1^{(n)}a_1^{(n)} + \ldots . + a_p^{(n)}a_p^{(n)}), \end{aligned}$$

and we see that if the multiplications indicated on the right be performed there must arise a series of $(p+1)^{n+1}$ terms of the type

$$a_r a_r . a_s'a_s' . a_t''a_t'' \ldots \ldots a_w^{(n)}a_w^{(n)},$$

or by alteration of the order of the factors

$$a_r a_s'a_t'' \ldots a_w^{(n)} . a_r a_s'a_t'' \ldots a_w^{(n)},$$

where each of the inferior indices $r,s,t, \ldots, w$ may be any member of the series $0,1,2, \ldots, p$. If we bear in mind the meaning which we thereby assign to the summatory symbol S we may write this in the form

$$cc_1'c_2'' \ldots . c_n^{(n)} = \mathrm{S}(a_r a_s'a_t'' \ldots a_w^{(n)} \cdot a_r a_s'a_t'' \ldots a_w^{(n)}).$$

The next point to consider is the transition from the single term $cc_1'c_2'' \ldots c_n^{(n)}$ to the full aggregate $\Sigma \pm cc_1'c_2'' \ldots c_n^{(n)}$. A glance at the sum of terms denoted by $c_k^{(i)}$ shows that by permuting the superior indices of $cc_1'c_2'' \ldots c_n^{(n)}$, the superior indices of the a's are subjected to the same permutation, and that, on the other hand, when we permute the inferior indices of $cc_1'c_2'' \ldots c_n^{(n)}$ it is the a's

that are affected, the like permutation being given to the superior indices. Making the choice of the *superior* indices of the c's, let us permute them in every possible way, and to each term thus derived from $cc_1'c_2'' \ldots c_n^{(n)}$ prefix the sign + or − according as its superior indices constitute a positive or negative permutation. By so doing the left-hand side of our identity becomes $\Sigma \pm cc_1'c_2'' \ldots c_n^{(n)}$; and, owing to the consequent permutation of the superior indices of the a's, each term on the right-hand side gives rise to $1.2.3 \ldots (n+1)$ terms whose signs are the same as the signs of the terms corresponding to them on the left hand side;—in other words, each term $a_r a_s' a_t'' \ldots a_w^{(n)} \cdot a_r a_s' a_t'' \ldots a_w^{(n)}$ gives rise to the compound term

$$a_r a_s' a_t'' \ldots a_w^{(n)} \cdot \Sigma \pm a_r a_s' a_t'' \ldots a_w^{(n)}.$$

We thus reach the result

$$\Sigma \pm cc_1'c_2'' \ldots c_n^{(n)} = \mathrm{S}\left(a_r a_s' a_t'' \ldots a_n^{(n)} \cdot \Sigma \pm a_r a_s' a_t'' \ldots a_w^{(n)}\right).$$

Although the number of terms on the right is the same as before, viz. $(p+1)^{n+1}$, arising from giving to each of the $n+1$ indices $r, s, t, \ldots, w$ any one of the $p+1$ values $0, 1, 2, \ldots, p$, it has now to be noticed that a goodly proportion of them must vanish because of the fact that $\Sigma \pm a_r a_s' a_t'' \ldots a_n^{(n)} = 0$ when any two of its inferior indices are alike. The right-hand side will thus not be altered in substance if the summatory symbol be now taken to mean that $r, s, t, \ldots, w$ are to be any $n+1$ of the $p+1$ indices $0, 1, 2, \ldots, p$. If p be less than n it will be impossible to have $r, s, t, \ldots, w$ all different, so that in that case the right-hand side must be 0. This is Jacobi's first proposition, and it constitutes his addition to the multiplication-theorem. His formal enunciation of it is (p. 309):—

"Sit

$$c_k^{(i)} = a^{(i)}a^{(k)} + a_1^{(i)}a_1^{(k)} + \ldots + a_p^{(i)}a_p^{(k)},$$

quoties $p < n$ evanescit determinans

$$\Sigma \pm cc_1'c_2'' \ldots c_n^{(n)}."$$ (XVIII. 6)

The consideration of the case when $p=n$ leads to his second proposition. The natural addendum is then made regarding the multiplication of more than *two* determinants of the same degree (p. 310):—

> "Datis quotcunque eiusdem gradus determinantibus, eorum productum ut eiusdem gradus exhiberi posse determinans, cuius elementa expressiones sint rationales integrae elementorum determinantium propositorum." (XVIII. 7)

The equally natural transition to the subject of the multiplication of two determinants of different degrees results in the proposition (p. 311):—

> "Sit pro indicis i valoribus 0, 1, 2, ... , m,
>
> $$c_k^{(i)} = a^{(i)}a^{(k)} + a_1^{(i)}a_1^{(k)} + \ldots + a_n^{(i)}a_n^{(k)},$$
>
> pro indicis i valoribus maioribus quam m,
>
> $$c_k^{(i)} = a_i^{(k)} + a_{i+1}^{(i)}\,a_{i+1}^{(k)} + a_{i+2}^{(i)}a_{i+2}^{(k)} + \ldots + a_n^{(i)}a_n^{(k)}:$$
>
> erit
>
> $$\Sigma \pm a a_1' \ldots a_m^{(m)} . \Sigma \pm a a_1' \ldots a_n^{(n)} = \Sigma \pm c c_1' c_2'' \ldots c_n^{(n)}.$$" (XVII. 8)

Proposition IV. concerns the case where $p>n$. Proposition V. is but a corollary to the combined propositions I., II., IV., its subject being the effect of the specialisation

$$a_k^{(i)} = a_k^{(i)}.$$

The enunciation is as follows (p. 312):—

> "Posito
>
> $$c_k^{(i)} = c_i^{(k)} = a^{(i)}a^{(k)} + a_1^{(i)}a_1^{(k)} + \ldots + a_p^{(i)}a_p^{(k)},$$
>
> sit determinans
>
> $$\Sigma \pm c c_1' \ldots c_n^{(n)} = \mathrm{P};$$
>
> ubi $p<n$ fit
>
> $$\mathrm{P} = 0;$$
>
> ubi $p=n$ fit
>
> $$\mathrm{P} = \left\{\Sigma \pm a a_1' \ldots a_n^{(n)}\right\}^2;$$
>
> ubi $p>n$ fit
>
> $$\mathrm{P} = \mathrm{S}\left\{\Sigma \pm a_m a'_{m'} \ldots a_{m^{(n)}}^{(n)}\right\},$$

siquidem pro indicibus inferioribus m, m' &c. sumuntur quilibet $n+1$ diversi e numeris 0, 1, 2 . . . . p." (XVIII. 7)

The two remaining sections (15 and 16) deal with a special system of simultaneous linear equations, interesting application being made to the theory of the Method of Least Squares.

It is important to note, in conclusion, that from one point of view Jacobi's memoir was but the introduction to two others of really greater importance, both treating of a special class of determinants. The first concerns determinants of the kind afterwards deservedly associated with his name, and bears the title "*De determinantibus functionalibus.*" It occupies the forty-one pages (pp. 319–359) immediately following the general memoir. The other, with the title "*De functionibus alternantibus earumque divisione per productum e differentiis elementorum conflatum,*" treats of those determinants, first considered by Cauchy, in which the members of one set of indices represent powers, and to which the name *alternants* afterwards came to be assigned. It extends to twelve pages (pp. 360–371). The three memoirs together constitute an excellent treatise on the subject, and are known to have been markedly influential in spreading a knowledge of it among mathematicians.

CAUCHY (1841).

[Note sur les diverses suites que l'on peut former avec des termes donnés. *Exercices d'analyse et de phys. math.*, ii. pp. 145–150.]

[Mémoire sur les fonctions alternées et sur les sommes alternées. *Exercices d'analyse et de phys. math.*, ii. pp. 151–159.]

[Mémoire sur les sommes alternées, connues sous le nom de résultantes. *Exercices d'analyse et de phys. math.*, ii. pp. 160–176.]

[Mémoire sur les fonctions différentielles alternées. *Exercices d'analyse et de phys. math.*, ii. pp. 176–187.]

From internal evidence there can be little doubt that this series of papers, containing the fundamental conceptions and salient pro-

positions of the theory of determinants, was prompted by the appearance of Jacobi's memoirs, and by the consequent conviction that the work of 1812 had begun to bear fruit. The first paper, called a "note," is introductory, on the subject of signed permutations; the three others, called "memoirs," correspond to Jacobi's,—the first of them to Jacobi's third, the second to Jacobi's first, and the third to Jacobi's second.

The note, although on so trite a subject as the division of permutations into positive and negative, is most interesting. Cauchy's original stand-point with regard to the subject is so far unaltered that the rule of signs specially known by his name is made fundamental, and all others deduced from it. The explanations preparatory for the rule are, however, on the lines of his paper of 1840, that is to say, it is *groups* and not *circular substitutions* that are spoken of. The preference is a little difficult to justify; for notwithstanding Cauchy's assertion that groups come naturally into evidence, the idea is far-fetched as compared with that of circular substitutions. He says (p. 145):—

> "Si l'on compare une quelconque des nouvelles suites* à la première, on se trouvera naturellement conduit par cette comparaison à distribuer les divers termes
>
> $$a,\ b,\ c,\ d\ \ldots\ .$$
>
> en plusieurs groupes, en faisant entrer deux termes dans un même groupe, toutes les fois qu'ils occuperont le même rang dans la première suite et dans la nouvelle, et en formant un groupe isolé de chaque terme qui n'aura pas changé de rang dans le passage d'une suite à l'autre."

The question of the natural order of ideas and the best mode of presentment is really, however, of small importance, for in application a *group* and a *circular substitution* are essentially the same. The difference is entirely one of stand-point, nomenclature, and notation. The permutation

$$e,\ a,\ b,\ d,\ c,\ g,\ f,$$

* *I.e.*, permutations of $a,b,c,d,\ \ldots$

being in question, and comparison between it and the primitive permutation,

$$a,\ b,\ c,\ d,\ e,\ f,\ g,$$

having been instituted, we are directed to form the members ("termes") of the permutation into groups, commencing to form a group with e and a, because they occupy like positions in the two permutations, putting b in the same group because it occupies the same position in the second permutation as one already in the group occupies in the first permutation, putting c in for the same reason, making d constitute a group by itself, and finally putting f and g together to form a third group. We are directed further, to write the members of each group in such an order that any member and the one following it may be found to occupy like positions in the primitive and derived permutations respectively. The result thus is

$$\begin{aligned} &(a,\ e,\ c,\ b),\quad (d),\quad (f,\ g),\\ \text{or}\quad &(e,\ c,\ b,\ a),\quad (d),\quad (g,\ f),\\ \text{or}\quad &\ldots\ldots\ldots, \end{aligned}$$

it being possible to write the first group in four ways, and the last in two. Now all this is nothing more than an unreasoning way of arriving at the circular substitutions which are necessary for the derivation of the given permutation from the primitive one. Cauchy himself, indeed, in pointing out that there would only be one way of writing a group if the members were disposed in a circumference instead of in a straight line, says:—"C'est par ce motif que dans le tome x du *Journal de l'École Polytechnique* j'ai désigné sous le nom de *substitution circulaire* l'opération qui embrasse le système entier des remplacements indiqués par un même groupe." It must be borne in mind, however, that not only the operation, but the symbol of the operation, was so denoted, and such being the case, we may then very pertinently ask, What is a group in Cauchy's usage but the symbol of a circular substitution?

The peculiarity of using the number of groups to separate the various permutations of $a, b, c, d, \ldots$ into two classes makes its appearance in the following sentence (p. 147):—

"De plus, ces mêmes suites ou arrangements se partageront en deux classes bien distinctes, la comparaison de chaque nouvel arrangement au premier

$$a,\ b,\ c,\ d,\ \ldots\ .$$

pouvant donner naissance à un nombre pair ou à un nombre impair de groupes."

Of course, the primitive permutation is looked upon as having its groups also, viz., one for every letter in the permutation.

Then comes the important proposition—*The interchange of two letters increases or diminishes the number of groups (substitution-cycles) by unity.* In proving it the two letters are first taken in different groups,

$$(a,b,c,\ \ldots\ ,h,k),\quad (l,m,n,\ \ldots\ ,r,s);$$

and since any member of a group may occupy the first place, the letters a and l are fixed upon. Now what the groups imply is that the letters

$$a,\ b,\ c,\ \ldots\ .\ h,\ k,\ l,\ m,\ n,\ \ldots\ .\ r,\ s$$

in the primitive permutation are changed into

$$b,\ c,\quad \ldots\ .\ k,\ a,\ m,\ n,\quad \ldots\ .\ s,\ l$$

respectively to form the given permutation. If therefore in the given permutation the letters a and l be interchanged, the new permutation so obtained will be got from the primitive by changing

$$a,\ b,\ c,\ \ldots\ ,\ h,\ k,\ l,\ m,\ n,\ \ldots\ ,\ r,\ s$$

into

$$b,\ c,\ \ldots\ .\ k,\ l,\ m,\ n,\ \ldots\ .\ ,\ s,\ a;$$

that is to say, by the changes indicated by the single group

$$(a,b,c,\ \ldots\ ,\ h,k,l,m,n,\ \ldots\ ,\ r,s).$$

The interchange of two letters belonging to different groups is thus seen to reduce the number of groups by one. On the other hand, it is clear that had this single group belonged to the given permutation, the interchange of two letters, a and l say, would have the effect of breaking up the group into two,

$$(a,b,c,\ \ldots\ ,h,k)\ \text{and}\ (l,m,n,\ \ldots\ ,r,s).$$

The theorem is thus established. (III. 35)

A Revision of the Genus Coscinodiscus and some Allied Genera.

By John Rattray, M.A., B.Sc., F.R.S.E. (With Three Plates.)

(Read June 17, 1889.)

The present paper is a continuation of my monographs already published by the Royal Microscopical Society, London, in their Journals for June and December 1888. It has been carried out under similar conditions in the Botanical Department of the British Museum (Natural History), South Kensington, but I am especially indebted to Edmund Grove, Esq., F.R.M.S., for much valuable co-operation, and to Julien Deby, Esq., for the readiness with which he has placed at my command the resources alike of his library and cabinet. Herr E. Weissflog and Dr James Rae, R.N., have also furnished me with many excellent preparations.

COSCINODISCUS, Ehrb. emend., *Ehrb. Abh. Ber. Ak.*, 1838, p. 128.

Valve circular, rarely regularly or bluntly and irregularly angular. Surface flat, often somewhat depressed at centre, and convex towards border, or with alternate subcentral elevations and depressions, more rarely with a sharply-defined elevated zone, or with one or more concentric low undulations. Colour transparent or smoky grey, sometimes with concentric zones of different brilliant hues. Central space angular or round, hyaline, or with a few isolated granules; rarely apiculate, sometimes indistinct, or replaced by a conspicuous or less evident rosette. Markings round and granular or angular, sometimes punctiform; the central papillæ prominent, obscure, or absent; rows radial or subradial, often in straight or curved fasciculi, those in each fasciculus parallel to that at its centre or side; secondary rows often oblique, curved or straight and decussating, more rarely regularly or irregularly concentric; the opposite valves of a frustule sometimes dissimilar; interspaces of varying size, usually largest towards centre or opposite the shorter rows, often absent. Apiculi few or many, scattered at irregular intervals over the surface, or forming one or more circlets near the border; some-

times only one present, robust and spine-like, with blunt extremities inserted at or near inner edge of border, more rarely at some distance from it; a small hyaline space sometimes present around the base of each. Border narrow or broad, hyaline or striated, the striæ sometimes faint.—*Symbolophora*, Ehrb., *pro parte*, *Mon. Ber. Ak.*, 1844, p. 74; *Endictya*, Ehrb., *ibid.*, 1845, p. 71; *Odontodiscus*, Ehrb., *pro parte*, *ibid.*, 1845, p. 72; *Heterostephania*, Ehrb., *Microg.* (plates), p. 15; *Cestodiscus*, Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 48; *Cosmiodiscus*, Grev., *ibid.*, 1866, p. 79; *Stoschia*, Janisch., van Heurck, *Syn. Diat. Belg.*, Explan. pl. cxxviii. fig. 6; *Janischia?* Grun., Van Heurck, *ibid.*, Explan. pl. xcv. *bis* figs. 10, 11; *Micropodiscus*, Grun., *Denk. Wien. Ak.*, 1884, p. 79; *Willemoesia*, Cstr., *Diat. Chall. Exped.*, 1886, p. 165; *Ethmodiscus*, Cstr., *ibid.*, p. 166.

§ I. Inordinati.

Round or elongately elliptical. No rosette; a central space rarely present, sometimes excentric. Markings punctiform, granular or areolate, without order.

C. exasperans, sp. n. Sch., *Atl.*, pl. lviii. fig. 9 (no name).—Diam. about ·018 mm. Central space and rosette absent. Markings small, round, granular, slightly smaller near the border; interspaces hyaline, largest towards the centre; non-apiculate. Border sharply defined, hyaline.

Habitat.—Monterey (Schmidt).

C. dimorphus, Cstr., *Diat. Chall. Exped.*, p. 157, pl. xvii. fig. 6.—Diam. ·0225 mm. Central space absent. Markings punctiform, irregular, sometimes most crowded towards the centre, and few on a narrow band around the border or conversely, those of the two valves of a frustule dissimilarly arranged. Border sometimes distinct.—Sch., *Atl.*, pl. lvii. fig. 1 (no name).

Distinguished from *C. marginulatus* by the smaller number of markings and the absence of apiculi. Some valves approach the minute *Cyclotella pumila*, Cleve (Van Heurck, *Syn. Diat. Belg.*, pl. xciv. fig. 16).

Habitat.—South Atlantic, H.M.S. Challenger (Castracane); Sandwich Islands (Schmidt).

C. subnitidus, sp. n. Sch., *Atl.*, pl. lviii. fig. 16.—Diam. about ·11 mm. Surface convex. Central space absent. Markings large, rounded, granular, with subequal hyaline interspaces. Border broad, not sharply defined on inner edge; striæ coarse, 4 to 5 in ·01 mm.

Distinguished from *C. nitidus* by the more convex surface and the long striæ upon the border.

Habitat.—Springfield deposit, Barbados (Schmidt).

C. confusus, sp. n. Sch., *Atl.*, pl. lxiv. fig. 15 (no name).—Diam. ·053 mm. Central space small, indefinite, rounded, rosette absent. Markings small, rounded, granular, most crowded towards the border, with interspaces irregular, largest towards the centre; disposed without order, but short irregularly oblique, straight, or slightly curved rows manifest. Border sharply defined, about $\frac{1}{12}$ of radius broad, hyaline.

Habitat.—Campeachy Bay (Schmidt).

C. sphæroidalis, sp. n.—Diam. ·045 mm. Central space absent; a subcircular space, having a few isolated round granules and about ·0075 mm. broad, placed close to the border at one side of the valve. Markings round, granular, 5 in ·01 mm., in rows radiating and diverging around the excentric space, separated by hyaline lines and not traceable at a distance greater than the radius from the edge of this space, on the lunate area at the opposite side of the valve smaller, and in curved oblique rows, towards the the border punctiform in evident regular oblique, decussating rows. Border narrow, striæ 8 to 10 in. ·01 mm.—(Pl. I. fig. 15.)

Habitat.—Monterey (Weissflog !).

Var. *cincta*. Sch., *Atl.*, pl. lviii. fig. 6.—Diam. about ·07 mm. No distinct excentric hyaline space. Markings on one half of valve less sharply separated from those on remaining half, the rows of larger granules more straight. Border broad, sharply defined; oblique decussating rows evident.

Schmidt has regarded this as a probable abnormality, but it is clearly to be associated under one species with the similar valves from Monterey.

Habitat.—Springfield deposit, Barbados (Weissflog).

C. inexpectatus, sp. n. *Coscinodiscus* (?) sp. (?), Cstr., *Diat. Chall. Exped.*, p. 163, pl. x. fig. 10.—Elliptical; major axis ·143 mm. about $1\frac{1}{2}$ times minor. Central space absent. Markings rounded, granular, 4 to 5 in ·01 mm. A distinct hyaline band adjacent to the border, about $\frac{1}{25}$ of major axis broad, its inner edge somewhat irregular. Border narrow, hyaline.

This species forms the transition to the genus *Willemœsia*, Cstr. (*ibid.*, p. 165, pl. viii. fig. 8), which is not sufficiently distinct from *Coscinodiscus* to be separated from it.

Habitat.—Zebu, Philippine Islands (Rae).

C. tenuisculptus, sp. n. *Stoschia? punctata*, Grove and Sturt, *Journ. Quek. Micr. Cl.*, 1887, p. 145, pl. xiv. fig. 52.—Elongately elliptical, the sides subuniformly curved, or with evident local constrictions; length, ·1375 mm., about 7 times the greatest breadth; surface slightly convex. Central space absent. Markings small, free round brilliant granules, with wide hyaline unequal interspaces disposed subuniformly over the general surface. Border narrow, but sharply defined.

Habitat.—Oamaru (Grove !).

C. humilis, sp. n. *Willemœsia* sp., Cstr., *Diat. Chall. Exped.*, 1886, p. 165, pl. viii. figs. 8, 8*a*, 8*b*.—Elongately elliptical; the sides slightly concave to convex; extremities unequally obtuse; length ·0625 to ·125 mm., from 7 to 11 times greatest breadth. Surface slightly convex. Central space absent. Markings small, round, granular, most crowded towards the border, sometimes leaving an irregular elongate hyaline area at the middle, or punctiform and closely arranged. Border narrow, hyaline.

Distinguished from *C. tenuisculptus* by the general appearance and arrangement of the markings.

Habitat.—(?) (Castracane).

C. cinctus, Kütz. *Bacil.*, p. 131, pl. i. fig. 17.—Diam. ·0835 mm. Surface flat towards the centre, convex near the border. Central space absent. Markings round, granular, crowded towards the centre; elsewhere remote, scattered. Border distinctly defined, striæ interrupted at the middle.—Ralfs, in *Pritch. Inf.*, p. 831;

C. patina,* Bail., *Amer. Journ. Sci.*, 1843, vol. xlii. p. 96, pl. ii. figs. 13 *a*, *b*.

Seychelle specimens, ·048 mm. diam., have 10 striæ in ·01 mm. on the border.

Habitat.—Richmond deposit, Va. (Bailey); mud from Elbe, at Cuxhaven (Kützing); Seychelles, Mauritius, and Tonga Islands (Cleve and Grunow).

C. impolitus, sp. n. *C. antarcticus*, Cstr. (non Grunow), *Diat. Chall. Exped.*, p. 157, pl. xii. fig. 10.—Diam. ·055 mm. Central space absent. Markings punctiform, closely and uniformly disposed; apiculi minute but evident, scattered over the surface at wide subregular intervals. Border narrow, hyaline.

This species approaches very close to *Melosira Borreri*, Grev. (= *M. moniliformis* and *M. lineata*, Ag.) (Van Heurck, *Syn. Diat. Belg.*, pl. lxxxv. fig. 7).

Habitat.—Heard Island, H.M.S. Challenger (Castracane).

C. insutus, sp. n. Sch., *Atl.*, pl. lvii. fig. 2 (no name).—Diam. ·024 mm. Central space absent. Markings punctiform, irregular, not crowded; interspaces wide, hyaline; a circlet of long narrow curved apiculi inserted a considerable distance within border. Border broad, sharply defined; striæ robust, 4 in ·01 mm.

Habitat.—Sansego (Schmidt).

C. granulosus, Grun. *Kongl. Sv. Vet.-Ak. Handl.*, Stockholm, 1880, No. 2, p. 113, pl. vii. fig. 130.—Diam. ·016 to ·033 mm.; Surface not depressed at centre. Central space absent. Markings punctiform; most crowded, largest, and most evident towards the centre; apiculi at the border minute, 3 in ·01 mm. Border relatively broad—about ·0025 mm., sharply defined; striæ 17 to 18 in ·01 mm.—*Odontodiscus granulosus*, Grun., *ibid.*, p. 113. (Pl. I. fig. 23.)

This species approaches *C. cinctus*, Kütz., but in the latter the central puncta are coarser, the apiculi are absent, and the border striæ stronger. Finmark specimens have been observed ornamented towards the centre with radiating curved faint lines instead of isolated granules, thus forming a transition to *C. marginulatus*.

* Name preoccupied by Ehrenberg in 1838.

There is systematic convenience in retaining *C. granulosus*, *C. marginulatus*, and *C. cinctus* as distinct species, though transitions occur.

Habitat.—Adria, Seychelles, Finmark (Cleve and Grunow); Quarnero (Van Heurck); Maasö, Finmark (Cleve and Möller !); Greenland (Cleve !); Kara (Cleve).

Var. *conspicua*, nov. Sch., *Atl.* pl. lvii. fig. 3 (without name).—Diam. ·049 mm. Markings irregular, granular, smaller, and somewhat more crowded towards the border, interspaces wider; apiculi at border conspicuous, large. Border striæ 12 to 13 in ·01 mm.—Cleve and Grunow, *ibid.*, p. 113.

Habitat.—Campeachy Bay (?) (Schmidt).

Var. *distincta*, nov.—Diam. ·05 mm. Apiculi absent. Border much more evident; striæ coarse $3\frac{1}{2}$ to 4 in ·01 mm.

Habitat.—Crescent City, Cal. (Weissflog !).

C. hirtulus, sp. n. *Cestodiscus* (*pulchellus*, var.?) *hirtulus* Grun., Van Heurck, *Syn. Diat. Belg.*, pl. cxxvi. fig. 3.—Diam. ·03 mm. Surface with the central portion extending to about $\frac{2}{3}$ of radius, its outer edge irregularly rounded, sharply defined. Central space and rosette absent. Markings on the central portion round, granular, irregular; a few larger triangular dark specks towards the centre, the outer portion with evident subregular striæ, 9 in ·01 mm.; apiculi 8, at subequal intervals, inserted at the outer edge of the central portion. Border sharply defined, about $\frac{1}{5}$ of radius broad.—*Cestodiscus pulchellus*, Habirsh., *Cat. Diat.*, ed. 2, 1885, § *Cestodiscus*.

Habitat.—Naparima deposit, Trinidad (Van Heurck).

C. subareolatus, sp. n.—Diam. about ·125 mm. Surface flat. Central space absent. Markings faint, areolate from the centre to about the semiradius, beyond this only indistinctly visible at intervals; on the outer $\frac{1}{5}$ of the radius closely disposed, distinct, radial slightly flexuous, sometimes irregular, subpruinose lines with clear linear interspaces evident. Border ?—(Pl. I. fig. 10.)

Fragmentary, the outer portion of the valve being removed close to the border.

Habitat.—Gazelle Expedition (Weissflog !).

C. turgidus, sp. n. *C. velatus*, Sch., *Atl.*, pl. lxii. fig. 10.—Diam. ·0375 to ·0625 mm. Surface flat at the centre, convex towards the border. Central space absent. Markings polygonal, largest at the centre, decreasing rapidly towards the border; at the centre 3, at the border 6 in ·01 mm. Border with inner edge indistinct; striæ coarse, irregular.

Habitat.— Springfield deposit, Barbados (Schmidt, Firth!)* Richmond, Va. (Schmidt); Cambridge deposit, Barbados (Kinker, O'Meara!) "Barbadoes" (Johnson!† O'Meara); Bridgewater, Barbados (Johnson!).†

C. anastomosans, Grun., *Denk. Wien. Ak.*, 1884, p. 75.—Diam. ·18 mm. Central space hyaline, irregular. Markings unequal, varying from ·002 to ·02 mm. in length, and forming an irregular network of anastomosing costæ.

Grunow justly regards this as probably abnormal. He is inclined to associate it with *C. oculus-iridis*, or some allied species.

Habitat.—Santa Monica deposit (Grunow).

C. irregularis, sp. n. *C. ? pacificus*, ‡ Cstr., *Diat. Chall. Exped.*, p. 158, pl. viii. fig. 5; pl. xxii. fig. 1.—Diam. ·087 mm. Central space absent. Markings polygonal, unequal, from 2½ to 3 in ·01 mm.; smaller towards the border. Border distinct, about ¼ of radius broad; the striæ distant, evident, 4 to 5 in ·01 mm.

Distinguished from *C. marginatus* by the less robust irregular markings. Similar irregularities are found in *C. antarcticus*, from Kerguelen. The affinity to *Endictya*, founded on the character of the border, as pointed out by Castracane, is remote.

Habitat.—Pacific Ocean, H.M.S. Challenger (Castracane).

C. luxuriosus, sp. n.—Diam. ·205 mm. Surface slightly convex from the centre for about ¼ of the radius, thence flat to the border. Central space absent. Markings 4–6-, mostly 6-, angled, 2½ to 3 in ·01 mm.; central papillæ evident; a few large irregular areolæ with a distinct median line scattered irregularly among the others, and forming an indistinctly defined band adjacent to the border, and

* In the collection of Dr F. W. Griffin.

† In the collection of Dr Greville.

‡ Preoccupied by Grunow in 1884, for what he regarded a doubtful var. of *C. oculus-iridis*.

about $\frac{1}{5}$ of the radius broad; rows irregular. Border sharply defined, with widely placed subcostate striæ hardly extending to the circumference.—(Pl. I. fig. 18.)

Habitat.—Manilla (Firth !).

C. megacoccus, Cstr. *Diat. Chall. Exped.*, p. 162, pl. xvii. fig. 2. —Diam. ·041 mm. Central space absent. Markings polygonal, $1\frac{1}{4}$ to $1\frac{3}{4}$ in ·01 mm., unequal. Border formed by a single band of 4-angled areolæ, with inner edges somewhat convex inwards, 3 in ·01 mm.

In the large size of the markings this species approaches *Stephanopyxis*. From *C. marginatus* it is distinguished by its more irregular marking and the marginal band of areolæ.

Habitat.—Pacific Ocean (Castracane).

C. nottinghamensis, Grun. Van Heurck., *Syn. Diat. Belg.*, pl. cxxix. fig. 2.—Diam. ·0165 mm. Central space absent. Markings polygonal, about $3\frac{1}{2}$ in ·01 mm., decreasing slightly outwards. Border about $\frac{1}{5}$ of radius broad, with closely placed radial striæ; apiculi long, delicate, numerous, inserted at inner edge of border, but extending beyond the circumference.

Habitat.—Nottingham deposit (Van Heurck).

C. antediluvianus, sp. n.—Diam. ·1125 mm. Surface almost flat, slightly convex towards the border. Central space absent. Markings irregularly areolate, 4 to 5 in ·01 mm., on the outer third of radius decreasing uniformly but somewhat rapidly to 8 in ·01 mm. at the border; apiculi prominent close to the border, inserted at unequal intervals, varying from ·005 to ·015 mm. Border indistinct, striæ 6 to 8 in ·01 mm.—(Pl. I. fig. 12.)

Habitat.—Santa Monica deposit (Grove !).

C. spinulosus, Ehrb. *Mon. Ber. Ak.*, 1845, p. 154.—Elliptical. Diam. ·047 mm. Surface flat, slightly convex. Central space absent. Markings angular, their edges slightly spinulose—about 6 in ·01 mm.; around the border a fringe of prominent closely disposed apiculi.—Ehrb., *Mikrog.*, pl. xxxviii. B. xxii. fig. 9.

This is probably a *Stephanopyxis*.

Habitat.—Fossil at Port Desire, Patagonia (Ehrenberg).

C. (?) *venulosus*, Cstr. *Diat. Chall. Exped.*, p. 162, pl. xvii. fig. 1.—Diam. ·034 mm. Central space rounded, about $\frac{1}{8}$ of diam. broad. Markings consisting of a few wavy lines diverging outwards from the central space, and confined to the central half of the valve.

Habitat.—South of Kerguelen Island (Castracane).

§ II. Cestodiscoidales.

Cestodisci, Pant., *Fossil. Bacil. Ung.*, p. 73.—Circular, rarely elliptical. Central space small or absent; no rosette. Markings round or angular, cestodiscoid; rows fasciculate or radial, a distinct band adjacent to the border frequent; valves sometimes dissimilar; apiculi few or more numerous. Border striæ usually distinct.

C. proteus, Rattray. *Cestodiscus proteus*, Hardman; Van Heurck, *Syn. Diat. Belg.*, pl. cxxvi. fig. 8.—Diam. ·035 to ·09 mm. Surface slightly convex. Central space minute. Markings rounded, granular, decreasing slightly from the centre outwards, towards the centre 6 in ·01 mm., on a marginal zone about $\frac{1}{5}$ to $\frac{1}{6}$ of radius broad, sub-punctiform; rows fasciculate, those composing each fasciculus parallel to the radial row at its middle, the interfasciculate radial rows most prominent between the centre and semi-radius, secondary oblique decussating rows on the marginal zone only and conspicuous; interspaces hyaline, largest opposite the ends of the shorter rows; apiculi 6 to 12 interfasciculate, large, inserted a short distance from the border. Border indistinctly defined, striæ evident uniform, 8 in ·01 mm.

Habitat.—Naparima deposit, Trinadad (Van Heurck, Grove !).

C. stokesianus, Grun. Pant., *Fossil. Bacil. Ung.*, p. 73.—Diam. ·075 mm. Central space absent. Markings rounded, granular; about 4 in ·01 mm., subequal; interspaces narrow, most evident at origin of shorter rows; rows radial, straight; apiculi 6, symmetrical, free ends blunt, inserted near inner edge of border. Border sharply defined, about $\frac{1}{10}$ of radius broad, striæ evident, 6 in ·01 mm.—*Cestodiscus stokesianus*, Grev., *Trans. Mic. Soc. Lond.*, 1866, p. 123, pl. xi. fig. 4; *Coscinodiscus stokesianus forma minor*, Grun.,

ibid., p. 73, pl. xxvii. fig. 257; *Coscinodiscus stokesianus forma baldjikiana*, Grun., *ibid.*, p. 73.

Distinguished from *C. superbus* (= *Cestodiscus pulchellus*, Grev.) by its smaller and more crowded markings, and more distant, less numerous apiculi.

Habitat.—Moron deposit (T. G. Stokes, Greville !).

C. moronensis, Rattray. *Cestodiscus moronensis*, Grev. MS.—Subcircular. Diam. ·075 mm. Central space absent. Markings angular towards the centre 8, gradually increasing outwards to 6, at $\frac{3}{4}$ of radius on a distinct band adjacent to border punctiform, 10 to 12 in ·01 mm.; central papillæ small, but prominent; subulate spaces faint, most evident near the marginal band; rows radial, straight; secondary subconcentric bands well marked within the marginal band. Apiculi prominent at intervals of about ·01 mm. Border narrow.—(Pl. II. fig. 16.)

Habitat.—Moron deposit (Johnson !).*

C. johnsonianus, Rattray. *Cestodiscus johnsonianus*, Grev., *Trans. Mic. Soc. Lond.*, 1865, p. 48, pl. v. fig. 8.—Diam. ·08 mm. Central space small, indistinct. Markings rounded, granular, 6 in ·01 mm., smaller and subpunctiform towards the border; interspaces evident, largest opposite origin of shorter rows; rows radial, straight, most crowded towards the border; apiculi many, stout, spine-like, with blunt extremities inserted near border at intervals of about ·01 to ·015 mm. at the middle of small, rounded, hyaline spaces. Border distinctly defined, striæ 6 to 8 in ·01 mm. evident.

This species forms a transition to *Aulacodiscus*, from which it is distinguished by the absence of primary rays, and the more minute and simple character of the processes.

Habitat.—Moron deposit (Johnson ! Greville).

C. superbus, Hardman MS. *Cestodiscus pulchellus* (?), Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 123, pl. xi. fig. 5.—Diam. ·0875 to ·1 mm. Surface convex towards centre, a distinctly defined band adjacent to border. Central space small, indefinite. Markings dissimilar on opposite valves of same frustule: on the one rounded, pearly, within submarginal band, with distinct hyaline interspaces;

* In the collection of Dr Greville.

largest towards centre, 3 in ·01 mm. on the submarginal band punctiform, 8 in ·01 mm.; rows straight, radial, secondary oblique, decussating rows evident on the submarginal band: on the other subquadrate or pentagonal, ·6 in ·01 mm.; crowded interspaces absent, secondary rows concentric, evident. Apiculi on both valves prominent, inserted at inner edge of marginal band at intervals of ·0075 to ·01 mm. Border sharply defined, striæ evident, 8 in ·01 mm.—*Cestodiscus pulchellus*, Cleve and Möll., *Diat.*, No. 162.

The name *pulchellus* cannot be adopted, there being already a *Coscinodiscus pulchellus*, Grev. (see p. 469). I have therefore used the name of Hardman's MS. species *superbus*, which seems to answer to Greville's description of the type form of *Cestodiscus pulchellus*.

Habitat.—Newcastle Estate, Barbados, abundant (Grove! Rae! Hardman!); Nancoori (Cleve and Möller! Cleve!).

Var. *nova-zealandica*, Grove MS.—Diam. ·0675 mm. Surface without a defined band adjacent to the border. Markings rounded, granular, 3 in ·01 mm., somewhat smaller towards border; rows radial, obscure, irregular concentric rows manifest towards border; interspaces hyaline, largest towards centre; apiculi evident at intervals of ·0075 to ·0175 mm. Border evident; striæ distinct, 8 in ·01 mm.—(Pl. II. fig. 15.)

Habitat.—Oamaru deposit (Grove!).

Var. *moravica*, nov. *C. pulchellus*, var. *moravica*, Grun., Pant. *Fossil. Bacil. Ung.*, p. 73, pl. xxvii. fig. 260.—Diam. ·07 mm. Central space absent. Markings rounded, granular, decreasing rapidly from the centre outwards, towards the border punctiform, most crowded; rows radial, alternately longer and shorter; apiculi large, numerous at regular intervals, inserted a short distance from the border.

Habitat.—Alsó-, Felsö-Esztergaly, Kékkö, Szakal, Szent Peter deposits; Brünn Tegel; Newcastle deposits, Barbados (Pantocsek!).

C. pusillus, Grove MS.—Diam. ·08 mm. Surface with a distinctly defined band adjacent to border. Central space absent. Markings angular, subequal, 8 in ·01 mm., on the band adjacent to the border punctiform, in radial striæ 10 in ·01 mm.; minute subulate clear spaces opposite origin of shorter rows, evident within semiradius;

rows straight, fasciculate, those in each fasciculus parallel to the central row; apiculi evident, inserted at inner edge of submarginal band at intervals of ·01 to ·0125 mm. Border sharply defined, narrow, striæ obscure, 12 to 14 in ·01 mm. (Pl. II. fig. 10.)

Habitat.—Straits of Macassar, recent (Grove !).

C. ovalis, Rattray. *Cestodiscus ovalis*, Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 49, pl. v. fig. 9.—Elliptical or diamond-shaped, with angles obtuse. Major axis ·0625 to ·1 mm., $1\frac{1}{4}$ to $1\frac{1}{2}$ times minor. Central space absent. Markings towards the centre rounded 6, at about $\frac{3}{5}$ radius angular 8, on a sharply defined zone adjacent to the border punctiform, 12 in ·01 mm.; rows radial, straight, the secondary oblique, decussating rows most evident on the median portion, interspaces most evident towards the centre; apiculi prominent at intervals of about ·0125 mm., inserted at inner edge of marginal band.

In the Nancoori deposit, from the Nicobar Islands, as well as in that of Richmond, Va., are very similar specimens, but possessing a distinct pseudonodule, and to these Grunow has given the name *Actinocyclus ellipticus* (Van Heurck, *Syn. Diat. Belg.*, pl. cxxiv. fig. 10). The specimen figured by Witt from the Archangelsk Polirschiefer (*Ü. d. Polirschief. von Archangelsk-Kurojedowo im Gouv. Simbirsk*, 1885, p. 23, pl. viii. fig. 11), as *Cestodiscus ovalis*, var.? an *Actinocyclus?* is to be associated with this species of Grunow, the pseudonodule being quite distinct.

Habitat.—Moron deposit (T. G. Stokes ! Hardman !).

§ III. EXCENTRICI, Pant., *Fossil. Bacil. Ung.*, p. 72.

Central space and rosette absent. Markings angular, decreasing gradually or more rapidly from the centre outwards; radial rows obscure, oblique decussating rows evident, frequently somewhat curved towards the border; apiculi only near border, sometimes absent.

C. minuens, Rattray. *C. decrescens*,* Cstr., *Diat. Chall. Exped.*, p. 159, pl. xii. fig. 14.—Diam. ·115 mm. Markings polygonal, decreasing from the centre to the border, towards the centre 2, towards

* Name preoccupied by Grunow.

the border 4 to 6 in ·01 mm.; radial rows 6, inconspicuous, the oblique decussating rows straight or slightly curved; apiculi numerous, prominent, forming a circlet close to the border at short intervals.

The general arrangement of the markings resembles somewhat that of *C. excentricus*, but is less regular, the areolæ too are larger, and decrease more rapidly to the border.

Habitat.—Philippine Islands, H.M.S. Challenger (Castracane).

C. antimimos,* sp. n.—Diam. ·08 mm. Markings round or bluntly angular, subpearly, towards the centre $2\frac{1}{2}$, decreasing gradually outwards to 3 in ·01 mm.; central papillæ obscure; radial rows few, inconspicuous; secondary oblique, decussating rows distinctly concave towards the border; non-spiculate. Border striæ evident, 6 in ·01 mm., short.—Cleve and Möll., *Diat.*, No. 257.—(Pl. II. fig. 11.)

Distinguished from *C. excentricus* by its much larger subpearly markings.

Habitat.—North Carolina (Cleve and Moller !).

C. antiquus. C. (*excentricus*, var.?) *antiquus*, Grun., *Denk. Wien. Ak.*, 1884, p. 84., pl. iv. (D), fig. 24.—Diam. ·0565 mm. Markings hexagonal, conspicuous, decreasing from the centre outwards; towards the centre 3, towards the border 4 in ·01 mm.; oblique, decussating rows substraight or slightly concave outwards; non-apiculate. Border sharply defined; striæ evident, distant, 3 to 4 in ·01 mm.—*C. concavus*, Ehrb., *Mikrog.*, pl. xxi. fig. 4 (not pl. xviii. fig. 38); *C. concavus*, Greg., var., Sch., *Atl.*, pl. lix. fig. 16.

Differs from *C. excentricus* in its larger markings and more distinct border.

Habitat.—Mors deposit, Monterey tripoli (Grunow).

C. excentricus. Ehrb., *Abh. Ber. Ak.*, 1839, p.146.—Diam. ·0525 to ·0825 mm. Surface flat. Markings polygonal, towards the centre 4, gradually decreasing towards the border to 8 in ·01 mm., at the centre a single areola surrounded by 5 to 8 similar ones, whence inconspicuous radial rows proceed to the border, the oblique decus-

* ἀντίμιμος, imitating.

sating rows distinctly concave outwards; apiculi distinct, numerous at subequal distances apart, sometimes absent.—Ehrb., *ibid.*, 1841, p. 323, 371, pl. iii. 7, fig. 5; *Mikrog.*, pl. xviii. fig. 32, pl. xxi. fig. 6; W. Sm., *Syn. Brit. Diat.*, i. p. 23, pl. iii. fig. 38; Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer*, Kiel, 1874., ii. p. 94, pl. iii. figs. 36–38; *Atl.*, pl. lviii. figs. 46 to 49; Van Heurck, *Syn. Diat. Belg.*, p. 217, pl. cxxx. figs. 4, 7, 8; Janisch,* *Gazelle Exped.*, taf. ii. fig. 3; vi. figs. 3, 7–11; H. L. Sm., *Diat. Sp. Typ.*, No. 93; *Coll. Kütz. Diat.*, No. 285; Van Heurck, *Typ. Syn. Diat. Belg.*, Nos. 529, 530; Cleve and Möller, *Diat.*, Nos. 148, 150, 183, 207, 210, 211, 215, 228, 257, 258, 276. *Odontodiscus excentricus.* Ehrb., *Mon. Ber. Ak.*, 1845, p. 79; Sch., *ibid.*, 1874, ii. p. 94; Raben., *Algen. Europas*, No. 2149, 2263, 2437, 2483, 2484, 2485, 2486, 2487, 2558; *C. lineatus*, var., Habir., *Cat. Diat.*; § *Coscinodiscus, C. minor*, Sch., *Atl.*, pl. cxiii. fig, 9.

In Ehrenberg's early figures apiculi are not indicated; these are first noted by W. Smith. Janisch (*Abh. Sch. Ges. väter. Cult.*, 1862, p. 4), erroneously states that the markings are round. Distinguished from *C. sol*, Wallich, by the presence on the latter of a wide slightly siliceous broad external zone.

Habitat.—Chalk marl, Oran, Richmond and Petersburg, Va.; lat. 71° 19′ N., long. 11° 28′ W., 1319 fms., in fine yellowish-grey mud; lat. 63° 40′ N., long. 5° 28′ E., 569 fms., in fine light mud; Cuxhaven, North Sea; Tjörn, Cattegat, Vera Cruz (Ehernberg); Barbados (Cleve and Möller !); Bohuslan, Hvidingsoe (Schmidt); Poole Bay, near Lewes, and stomach of scallop (W. Smith); Blankenberge, Antwerp (Van Heurck); San Benito deposit, California (Grove !); Firth of Forth (Rattray !); Porto Seguro (Hardman !);† San Francisco

* To the specimens figured in Janisch's *Gazelle Expedition* plates, which, though still unpublished, have had considerable private circulation, the following names have, according to Herr E. Weissflog, been given by Janisch, namely:—Pl. I. figs. 1–5, *Stoschia admirabilis*, C. Jan., gen. et sp. n. Pl. II. fig. 1, *Coscinodiscus gigas*? var.; figs. 4, 5, *C. nodulifer*, A.S. sp. n. Pl. III. fig. 2, *C. africanus*, C. Jan., sp. n.; fig. 4, *C. praetextus*, C. Jan., sp. n.; fig. 6, *C. tumidus*, C. Jan., sp. n. Pl. IV. figs. 1, 2, *C. lentiginosus*, C. Jan., sp. n.; figs. 3, 5, *C. arafuraensis.* Pl. V. figs. 2, 3, *C. gyratus*, C. Jan., sp. n.; fig. 6, *C. atlanticus*, Grun.; fig. 7, *C. lentiginosus*, C. Jan., sp. n. Pl. VI. fig. 12, *C. bullatus*, C. Jan., sp. n.; fig. 13, *C. nobilis*, Grun.

† In the collection of Julien Deby.

(Firth !) Peruvian guano (Kinker ! Rae !); Campeachy Bay (Hardman ! Cleve and Moller ! Grove !); Singapore (Hardman !);* Virginia (Hardman !); Sand Heads, Bengal (Greville !); Cresswell (Donkin !);† anchor ground, Reikjavik, Iceland; mud from Glückstadt; Vera Cruz; anchor ground, Laguna Harbour, 20 miles N. of Laguna, in the sea; Elbe, above Cuxhaven (Rabenhorst and Schwartz !); Carral, near Valdiva (Rabenhorst and Gerstenberger !); German Ocean (Kützing!); coast of St Paul Island, South Sea (v. Frauenfeld !);‡ stomach of scallop, locality? (Greville ! W. Smith !) Baltic Sea (Van Heurck !); Teignmouth (Arnott !);† soundings off Kurile Islands, 1329 fms. (H. L. Smith !); Cumbrae (Arnott !);† Santa Monica deposit (Hardman !);* Sawa Nada, Japan (Hardman !);* Lamlash Bay (Dickie !); Corrighills (Dickie !); Humber (O'Meara !); Ascidia, Hull (Van Heurck ! Greville !); marine deposit, Fiji Islands (Grove !); Mejillones deposit (Cleve ! O'Meara !); Los Angelos deposit (O'Meara !); Knight Errant Expedition (Grove !); Macassar Straits (Grove !); Marstrand (Kinker !); Aegina (Schmidt); Kamtschatka Sea, 1700 fms. (Bailey !); Rovigno, Balearic Islands; Hampshire; Patagonia; Antarctic Ocean; Delaware; Nottingham deposit, North Carolina; Pensacola, California (Cleve and Möller !); west coast Florida, U.S. Survey (Febiger !); Bohuslan; Elbing; West Prussia; Cape Wankarema; Saldanha Bay guano, Patagonian guano, Schleswig-Holstein; mud from Savannah rice fields; Mors deposit; Nankoori deposit; between Aden and Bab-el-mandeb (Cleve !); off Ascension Island, in Globigerina ooze (Grove !).

Var. *micropora*, Grun. Cleve and Möll., *Diat.*, No. 114.—Diam. ·0175 to ·03 mm. Markings 12 in ·01 mm.; apiculi absent.

Habitat.—Mascara (Cleve and Möller !).

Var. *perpusilla*, Grun. *Denk. Wien. Ak.*, 1884, p. 84, pl. iv. (D), fig. 7.—Diam. ·0075 mm. Markings minute, 24 in ·01 mm.; apiculi hardly conspicuous.

O'Meara has confounded this var. with the very distinct *C. apiculatus.*

* In the collection of Julien Deby.
† In the collection of Dr Greville.
‡ Collected during the Expedition of the "Novara."

Habitat.— Franz Josef's Land (Grunow); Ascidia, Kinsale (O'Meara !).

Var. *punctifera*, Grun. *ibid.*, p. 84.—Diam. ·1 mm. Markings 5 to 9 in ·01 mm.; apiculi absent or minute, a distinct point—the rudiment of a bristle—between the central and one of the surrounding areolæ.

Habitat.—Southern Ocean (Grunow).

Var. *hyalina*, nov. *Eupodiscus excentricus*, O'Meara, *Quart. Jour. Micr. Sci.*, 1867, p. 245, pl. vii. fig. 2.—Diam. about ·035 mm; a broad hyaline band adjacent to border; apiculi many, inserted at middle of this hyaline band at subequal intervals.

Habitat.—Dredged off Arran Islands, co. Galway (O'Meara).

Var. *zebuensis*, nov. *C. zebuensis*, Grun. MS.—Diam. ·0275 to ·07 mm. Markings towards the centre 4 or 5, decreasing outwards to 8 in ·01 mm., a large prominent central nodule; apiculi distinct, numerous, close to border at intervals of about ·0075 mm. Border sharply defined.

Habitat.—Campeachy Bay (Grove !), Zebu, Philippine Islands (Weissflog !).

C. decipiens, Grun. *Denk. Wien. Ak.*, 1884, p. 85.—Diam. ·024 to ·03 mm. Markings polygonal, decreasing uniformly but somewhat rapidly from the centre outwards; at the centre 5 to 6, towards the border 10, in ·01 mm.; secondary oblique decussating rows concave outwards, subarcuate; apiculi many, minute but distinct, at short unequal distances.—Van Heurck, *Syn. Diat. Belg.*, pl. xci. fig. 10; Van Heurck, *Typ. Syn. Diat. Belg.*, No. 471; *C. excentricus*, var.? *decipiens*, Grun., *Sitzungsb. naturw. Ges. Isis.*, Dresden, 1878, p. 28; pl. iv. fig. 18; *C. minor*, Anglor (non Ehrb.); *Orthosira angulata*, Greg,* *Trans. Roy. Soc. Edin.*, 1857, p. 498, pl. x. figs. 43, 43*b*; *C. minor*, W. Sm., *Syn. Brit. Diat.*, i. p. 23, pl. iii. fig. 36 (excl. *C. decipiens*, Grun.; Sch. *Atl.*, pl. lix. figs. 18, 19.)

Distinguished from *C. excentricus* by the more irregular markings, which decrease more towards the border, and by the more prominent apiculi.

* "*Melosira angulata*, Greg," *fide* Grun., *ibid.*, 1884, p. 85.

Habitat.—Baku Harbur, Caspian Sea (Grunow); Lamlash (Gregory); Woolwich (Walker Arnott, Van Heurck !).

C. minor. Ehrb., *Abh. Ber. Ak.*, 1838, p. 129, pl. iv. fig. 12*e*. —Diam. ·0225 mm. Markings polygonal, minute, decreasing gradually from the centre outwards; at the centre 6, at the border 9 to 10 in ·01 mm; rows less regular, the oblique decussating rows curved but slightly towards the border. Apiculi minute, close to the border, in the intervals delicate striæ.—Ehrb., *ibid.*, 1839, p. 147, pl. iii. fig. 2; *ibid.*, 1841, p. 371, pl. ii. 4. fig. 8; pl. ii. 6. fig. 17, pl. iii. fig. 3; *Mikrog.*, pl. xviii. fig. 31; pl. xix. fig. 3; pl. xx. 1. fig. 28; pl. xxi. fig. 5; pl. xxii. fig. 7; pl. xxxiii. 14. fig. 4 (?); pl. xxxix. 2. fig. 22 (?); Ehrb., *Die II^te Deutsch. Nordpolarf.*, 1869–70, Leipzig, 1874, p. 455, pl. ii. figs. 20, 23; Ralfs in *Pritch Inf.*, p. 831; Janisch, *Abh. Sch. Ges. väter., Cult.*, 1862, p. 4, pl. ii.A, fig. 6; Grun., *Sitzungsb. naturw. Ges. Isis*, Dresden, 1878, p. 28; Sch., *Atl.*, pl. lviii. figs. 39, 40; pl. lix. fig. 8, 9; pl. cxiii. fig. 10; Raben., *Alg. Europ.*, Nos. 2261, 2481, 2487 (excl. *C. minor*, W. Sm., *Syn. Brit. Diat.*, i. p. 23, pl. iii. fig. 36 = *Melosira nivalis* and *C. minor*, Anglor, non Ehrb. = *C. decipiens*, Grun.)

The early figures of Ehrenberg are unsatisfactory, and may indicate more than one species. The rows of smaller markings are more irregular than in *C. excentricus* and *C. sol.* Distinguished from *C. decipiens* by the less rapid diminution of the markings outwards and the less evident apiculi. The Aegina valve figured by Schmidt (*Atl.*, pl. cxiii. fig. 10) shows a transition to *C. tumidus* in the arrangement of the markings.

Habitat.—Caltanisetta (Ehrenberg, Grunow, Schmidt); Oran, Zante; Aegina, Richmond; Norwich, Con. (Ehrenberg); Peruvian and Ichaboe guanos (Janisch); Cuxhaven, S. Domingo, Haiti, Cuba, Vera Cruz (Ehrenberg); Girgenti (Grunow); Table Bay (Schmidt), lat. 71° 19′ N., long. 11° 28′ W., 1319 fms., in yellowish-grey mud; lat. 73° 16′ N., long. 15° 48′ W., 1300 fms., in fine dark brown mud; lat. 63° 40′ N., long. 5° 28′ E., 569 fms., in fine light grey mud; lat. 74° 18′ N., long. 19° 24′ W., 13 fms. (Ehrenberg); Davis Straits, deep water (O'Meara !); Nancoori (Hardman !); Cumbrae and Lamlash (Dickie !); Humber (O'Meara !); marine deposit, Fiji Islands (Grove !); Aegina (Schmidt); Algeria (Arnott !); Springfield deposit, Barbados

(Schmidt); Vera Cruz, among *Sertularia;* Elbe, above Cuxhaven (Rabenhorst and Schwarz !); Carral, near Valdiva (Rabenhorst and Gerstenberger !).

C. circumdatus. Sch., *Atl.*, pl. lix. fig. 3.—Diam. ·07 mm. Markings polygonal, decreasing slightly from the centre towards the border; towards the centre 4, towards the border 6 in ·01 mm.; radial rows 6 to 8, inconspicuous; the oblique decussating rows almost straight. Border about $\frac{1}{11}$ of radius broad, inner $\frac{2}{3}$ hyaline between distant subregular narrow radial lines, outer $\frac{1}{3}$ with evident striæ, 8 to 10 in ·01 mm.—Van Heurck, *Syn. Diat. Belg.*, pl. cxxxi. fig. 4.

Distinguished by the border.

Habitat.—Yokohama (Gründler).

C. sol. Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 38, pl. ii. figs. 1, 2.—Diam. ·0625 to ·15 mm. Markings on the central highly silicified portion distinct, decreasing somewhat rapidly from the centre outwards; towards the centre $4\frac{1}{2}$ to 5, towards the border 6 to 7 in ·01 mm., the central areola surrounded by 6 or 7 similar areolæ, whence a corresponding number of inconspicuous radial rows pass to the border; the oblique decussating rows distinctly concave outwards, non-apiculate, the outer portion from $\frac{1}{3}$ to $\frac{1}{2}$ of radius broad, scarcely siliceous, with numerous but distinct subuniform costæ.—Ralfs, in *Pritch. Inf.*, p. 830; Sch., *Atl.*, pl. lviii. figs. 41, 42, 45; Grun., *Denk. Wien. Ak.*, 1884, p. 84; Cleve and Möll., *Diat.*, Nos. 145, 146. *Cestodiscus sol*, Wallich, in Van Heurck, *Syn. Diat. Belg.*, pl. cxxix. fig. 6.

Habitat.—From *Salpa spinosa*, South Sea (Weissflog !); surface, Gulf of Guinea (Rattray !); Cambridge deposit, Barbados (Greville !); Java Sea (Kitton, Cleve and Möller !).

§ IV. LINEATI, Pant., *Fossil. Bacil. Ung.*, p. 72.

Central space absent, rarely small; a rosette rare. Markings angular in contact; radial rows obscure; oblique, decussating rows straight. Apiculi sometimes present, a larger apiculus inserted at a greater distance from border somewhat rare.

C. subconcavus, Grun. Sch., *Atl.*, p. lix. figs. 12. 13.—Diam

·0275 mm. Surface slightly convex. Central space and rosette absent. Markings hexagonal, central dots sometimes evident, decreasing but slightly from the centre outwards, about $2\frac{1}{2}$ in ·01 mm. Border narrow, showing evident short radial lines.

Habitat.—Simbirsk (Schmidt).

Var. *tenuior*, nov. *C. subconcavus*, Grun., var.? Sch., *Atl.*, pl. lix. fig. 15.—Diam. ·038 mm. Markings smaller, decreasing more distinctly outwards, towards the centre 3, towards the border 4 in ·01 mm.; a single band of smaller areolæ adjacent to the border. Border smooth, narrow.

Habitat.—Simbirsk (Schmidt).

C. vigilans. Sch., *Atl.*, pl. cxiv. figs. 11, 12.—Diam. ·0455 to ·068 mm. Surface slightly convex. Central space and rosette absent. Markings round, robust, pearly, smooth, largest at the centre, decreasing rapidly near the border; at the centre 2, at the border 4 in ·01 mm.; central papillæ faint, interspaces hyaline. Border sharply defined, formed by one or two bands of rounded granules in contact laterally, and 6 or 7 in ·01 mm.

Distinguished from *C. subvelatus* by the larger size and by the form of the markings and the distinct character of the border.

Habitat.—Archanjelsk-Kurojedowo (Schmidt).

C. Mölleri. Sch., *Atl.*, pl. lix. fig. 17.—Diam. ·0625 to ·08 mm. Surface convex, and more opaque at the centre. Central space and rosette absent. Markings hexagonal, decreasing regularly but somewhat rapidly from the centre outwards, at the centre most distinct 2 to $2\frac{1}{4}$, towards the border more faint, 3 in ·01 mm.; at the border a single band of quadrate equal areolæ with longer axis at right angles to the radius, 3 in ·01 mm.; oblique decussating rows obvious, straight or slightly concave towards the border. Border narrow, hyaline.

Habitat.—Mors deposit (Schmidt, Cleve! Grunow, Möller!).*

Var. *macroporus.* Grun., *Denk. Wien. Ak.*, 1884, p. 84, pl. iv. (D), fig. 25.—Diam. ·063 to ·125 mm. Markings at the centre 2, at the border 4 in ·01 mm., their surface obsoletely

* In the collections of Julien Deby and of Prof. Cleve.

and irregularly punctate, the central areola without a prominent papilla.

Habitat.—Franz Josef's Land (Grunow); Simbirsk deposit, Kitton !).*

C. heteromorphus, sp. n. Sch., *Atl.*, pl. lxv. fig. 17 (no name). —Diam. ·053 mm. Central space and rosette absent. Markings angular, about 3 in ·01 mm.; central papillæ absent; adjacent to the border a broad zone, consisting of a single row of large cylindrical areolæ, with rounded outer and inner extremities, their sides straight or concavo-convex of somewhat unequal length. Border narrow.

Similar forms, but with more robust markings, occur in the Oamaru deposit, and have been regarded by Mr E. Grove either as abnormal forms of *Stephanophyxis*, or as probably belonging to *Liradiscus*. The former opinion, adopted by Schmidt (*Atl.*, pl. cxxx. fig. 5) seems to me to be the more tenable.

Habitat.—Piscataway (Weissflog).

C. splendidus. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 44, pl. v. fig. 3.—Diam. ·065 to ·125 mm. Surface convex. Central space absent. Markings polygonal, central papillæ towards the centre $1\frac{1}{2}$, decreasing outwards to 2 or $2\frac{1}{4}$ in ·01 mm., those forming the outermost band larger. Border hyaline.—Sch., *Atl.*, pl. lxv. fig. 11.

The markings and border are similar to those of *Stephanophyxis superba*, Grun. (*Denk. Wien. Ak.*, 1884, p. 91), but the latter is provided with spines.

Habitat.—Cambridge deposit, Barbados (Johnson !);† "Barbados" (Greville !); St Vincent, Austral. (Weissflog !).

C. macraeanus. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 46, pl. v. fig. 4.—Diam. ·0525 to ·13 mm. Surface convex towards the centre. Central space absent. Markings polygonal, $2\frac{1}{2}$ to 3 in ·01 mm., subequal or somewhat smaller near the border; apiculi numerous, large, clavate, attached irregularly at the inner edge of the border. Border hyaline, sharply defined, $\frac{1}{13}$ of radius broad.

Habitat.—"Guano" (Macrae !);† Indian Ocean (Macrae, *fide* Greville); Bahia (Kitton !).

* In the collection of Julien Deby.
† In the collection of Dr Greville.

C. pulchellus. Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 3, pl. i. fig. 7.—Diam. from ·06 to ·09 mm. Surface convex. Central space absent. Markings mostly hexagonal, $2\frac{1}{2}$ in ·01 mm., decreasing gradually to the border, at wide irregular intervals narrower and more distinct areolæ are scattered irregularly on the middle $\frac{1}{3}$, those forming the outermost band longer but not wider, their long axis radial. Border sharply defined; striæ distinct, remote, 4 in ·01 mm.

This species approaches *C. splendidus*, Grev., and *C. macraeanus*, Grev., but is distinguished from the former by the striated border and from the latter by the marginal apiculi. Grunow, who has justly reduced the genus *Cestodiscus* to a section of *Coscinodiscus* (Pant., *Fossil. Bacil. Ung.*, p. 73), names *Cestodiscus pulchellus*, Grev. (*Trans. Micr. Soc. Lond.*, 1866, p. 123, pl. ii. fig. 5), *Coscinodiscus pulchellus*, though distinct from Greville's *Coscinodiscus pulchellus*, which was published somewhat earlier. *Coscinodiscus* (*pulchellus*, var.?) *hirtulus*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. cxxvi. fig. 3), and *Coscinodiscus pulchellus*, var. *Trinitatis*, Grun. (Van Heurck, *ibid.*, pl. cxxvi. fig. 4), are related to Greville's *Cestodiscus pulchellus.*

Habitat.—Cambridge deposit, Barbados (Johnson!).*

C. zonulatus, sp. n.? Sch., *Atl.*, pl. lix. fig. 6.—Diam. ·024 mm. Central space and rosette absent. Markings regularly angular, subequal, 10 in ·01 mm.; adjacent to the border a distinct band of large imperfect areolæ, with inner edge absent, and outer slightly convex outwards, about 5 in ·01 mm. Border sharply defined about $\frac{1}{7}$ of radius broad, hyaline.

Habitat.—Cape of Good Hope (Schmidt).

C. aphrastos,† sp. n. Sch., *Atl.*, pl. lxv. fig. 18.—Diam. ·0755 mm. Central space and rosette absent. Markings pentagonal or hexagonal, about $1\frac{1}{2}$ in ·01 mm.; at wide intervals, a few minute areolæ interspersed. Central papillæ absent. Border prominent, sharply defined, hoop-like; striæ robust, about $1\frac{1}{2}$ in ·01 mm.

Habitat.—Campeachy Bay (Gründler).

C. concavus. Greg., *Trans. Roy. Soc. Edin.*, 1857, pl. x. fig. 47. —Diam. ·07 to ·2375 mm. Surface subplain, with a prominent

* In the collection of Dr Greville. † ἄφραστος, unexpected.

border. Central space and rosette absent. Markings subpearly, robust, pentagonal or hexagonal, each with a faint central dot, 2½ to 3½ in ·01 mm., decreasing but little towards the border. Border sharply defined, subopaque, broad; striæ evident, 3½ to 4 in ·01 mm.; a series of closely placed concentric lines sometimes visible, the outer edge rugose. Girdle aspect,* with smaller areolæ, 4 to 5 in ·01 mm., forming straight or slightly flexuous rows parallel to the edge of the girdle; girdle narrow.—Cleve, *Bih. k. Sv. Vet.-Ak. Handl. Stockh.*, 1873, No. 11, p. 4; Sch., *Atl.*, pl. lxii. fig. 8; *C. concavus*, Ehrb., *pro parte*, *Abh. Ber. Ak.*, 1841, p. 412; *Mikrog.*, pl. xxi. fig. 4 † (excl. pl. xviii. fig. 38); *Endictya oceanica*, Ehrb., *Mon. Ber. Ak.*, 1845, p. 76; *Mikrog.*, pl. xxxv. A, 18, figs. 6, 7; Ralfs in *Pritch. Inf.*, p. 831, pl. v. fig. 70; Möll., *Typ. Pl.* 4, 4, 8, Cleve and Möll., *Diat.*, No. 110, 259; Raben., *Alg. Europ.*, No. 2556; H. L. Sm., *Diat. Sp. Typ.*, No. 148; Sch., *Atl.*, pl. lxv. figs. 10, 12, 13, 15. *Coscinodiscus concavus*, var. *africanus*. Kütz, *Sp. Alg.*, p. 125; *C. oceanicus*, Kütz., *ibid.*, p. 126; *Melosira cribrosa*, de Bréb., W. Sm. in *Ann. and Mag. Nat. Hist.*, 1857, p. 11, pl. ii. fig. xv.; *Orthosira oceanica*, Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 96; *Endictya minor*, Sch., *Atl.*, pl. lxv. figs. 14, 16; *Melosira oceanica*, Habirsh, *Cat. Diat.*, ed. 2, § *Endictya*.

The specimen named *C. concavus* by Ehrenberg (*Mikrog.*, pl. xviii. fig. 38), from Richmond, approaches the unnamed organism figured by Gregory from the Glenshira Sand (*Trans. Micr. Soc. Lond.*, 1857, p. 85, pl. i. fig. 52), but is distinct from the present species. *C. concavus*, var., Sch. (*Atl.*, pl. lix. fig. 16) is *C. antiquus*, Grun. W. Smith followed de Brébisson's determination of *Melosira cribrosa* provisionally, separating it from *Coscinodiscus* only because he believed that the frustules might occur concatenated. *C. concavus*, var. *africanus*, from Oran, was first differentiated by Ehrenberg as *C. concavus africæ* (*Mon. Ber. Ak.*, 1844, p. 79), with 3½ to 4 markings in ·01 mm., but this is inadequate on which to establish a variety. Cork specimens, authenticated by Grunow as *Endictya minor* (Sch., *Atl.*, *supra*), agree with Gregory's *C. concavus*. Specimens sometimes named *Endictya oceanica* differ from *C. concavus* only in showing the markings somewhat more irregular.

* This applies to specimens hitherto named *Endictya*.
† Specimen not typical.

Habitat.—Oran deposit (Ehrenberg); Oamaru deposit (Grove and Sturt); Mejillones guano (Deby! Hardman! Firth!); Peruvian guano (Cleve! Johnson!); Chincha Island guano (Arnott!); Arica and Saldanha Bay guanos (Ehrenberg); off Bermuda, 1075 fathoms (Rae!): Loch Fyne (Greville!); Lamlash (Gregory!) Biarritz, Bay of Biscay (Brébisson!); Black Sea (W. Smith); Java (Cleve); Amboina shell scrapings (Kinker!); Ascidia, Roundstone Bay, county Galway (O'Meara!); S. America (Möller!);* Lough Hym, county Cork (Grove!); Kirkwall, Orkney (Grove!); Edible seaweeds, India (Macrae!); Locality? (Barnett?† Weissflog!); Valparaiso (Schmidt); Villefranche, Trinidad (Cleve and Möller!); Campeachy Bay (Grove!); Monterey Stone (Cleve!); Balearic Islands, Pabillan di Pico guano, Bolivian guano (Cleve!); Port William, Falkland Islands (Rabenhorst and Schwarz!).

C. bisculptus, sp. n. *C. labyrinthus*, Roper, var.? Sch., *Atl.*, pl. lix. fig. 14.—Diam. ·035 mm. Surface somewhat convex. Central space and rosette absent. Markings large hexagonal or pentagonal, unequal areolæ, 2 to $2\frac{1}{2}$ in ·01 mm., somewhat smallest towards the centre; within these more minute, faint, angular areolæ—the larger in obscure radial, but more evident oblique substraight decussating lines, the smaller without order. Border sharply defined, about $\frac{1}{7}$ of radius broad; striæ coarse, 3 in ·01 mm.

Habitat.—Peruvian guano (Schmidt).

C. labyrinthus. Roper, *Quart. Jour. Micr. Soc.*, 1858, p. 21, pl. iii. figs. 2*a*, 2*b*.—Diam. ·0625 to ·0875 mm. Central space absent. Markings hexagonal towards the centre 4, decreasing gradually to the border to 6 in ·01 mm., punctate, forming straight or slightly curved oblique decussating rows and distinct secondary subequal hexagonal, areolæ from ·0025 to ·0085 mm. broad; minute apiculi sometimes present at the border. Border indistinct; striæ 7 to 8 in ·01 mm.—Ralfs in *Pritch. Inf.*, p. 831; Cleve and Möll., *Diat.*, No. 276 (excl. *C. labyrinthus*, Roper, var.? Sch., *Atl.*, pl. lix. fig. 4).

The smaller hexagonal markings recall those of *C. excentricus*, Ehrb., and of *C. sol.*, Wallich. At the centre a faint stellette is

* In the collection of Julien Deby.
† In the collection of Dr Griffin.

sometimes found. Not a *Pyxidicula*, as suggested by Grunow (*Denk. Wien. Ak.*, 1884, p. 73).

Habitat.—Californian guano (Norman !) *; stomach of Ascidia, Hull (Greville !); Lamlash (Greville !); Caldy, Pembrokeshire (Roper !); Hull (Firth !) †; Humber (Dickie !); California (Cleve and Möller !).

C. bipartitus, sp. n. Sch., *Atl.*, pl. lix. fig. 35.—Diam. about ·0875 mm. Central space absent, rosette large, surrounding a single small circular areola. Markings hexagonal, $2\frac{1}{2}$ subequal, for about $\frac{3}{4}$ of radius, on outer $\frac{1}{4}$ 6 in ·01 mm., forming a distinct band. Central papillæ absent, radial rows on inner $\frac{3}{4}$ of radius obscure, the oblique decussating straight rows manifest, on outer $\frac{1}{4}$ the radial rows evident, the secondary oblique decussating rows uniformly curved. Border narrow, hyaline; beyond the border 4 large unsilicified subrugose blunt protuberances.

Habitat.—Java (Gründler).

C. blandus. Sch., *Atl.*, pl. lix. figs. 36, 37.—Diam. about ·07 mm. Central space small, rosette large, at the inner angles of its component areolæ distinct minute round granules. Markings hexagonal, 3 in ·01 mm., somewhat smaller towards the periphery; the central papillæ faint, a distinct band sometimes present adjacent to the border, upon this the markings rounded granular and irregular apiculi numerous, inserted at inner edge of border. Border narrow; striæ evident, 6 in ·01 mm.

Habitat.—Gulf of Mexico (Schmidt).

C. lineatus. Ehrb., *Abh. Ber. Ak.*, 1838, p. 129.—Rarely angular. Diam. ·05 to ·15 mm. Surface towards the centre flat, slightly convex near the border. Central space and rosette absent. Markings hexagonal, $2\frac{1}{2}$ to 4 in ·01 mm., subequal, or sometimes at border 6 in ·01 mm., their central dots distinct; apiculi small, sometimes absent. Border distinct, consisting of a few concentric rows of contiguous granules, 8 or 9 in ·01 mm.—Ehrb., *ibid.*, 1841, p. 371, pl. i. 3. fig. 20; pl. iii. 7. figs. 7, 8; *Mikrog.*, pl. xviii. fig. 33; pl. xxii. figs. 6*a*, *b*; pl. xxxv.A, 16. fig. 3; pl. xxxv.A, 17. fig. 7;

* In the collection of Dr Greville. † In a slide prepared by Mr Norman.

Raben., *Alg. Europ.*, Nos. 2481, 2482, 2483, 2484, 2485, 2486; Van Heurck, *Syn. Diat. Belg.*, p. 217, pl. cxxxi. fig. 3; Janisch, *Gazelle Exped.*, taf. iv. fig. 8; taf. xx. fig. 14; Sch., *Atl.*, pl. lix. figs. 27–30; H. L. Sm., *Diat. Sp. Typ.*, No. 98; Cleve and Möll., *Diat.*, Nos. 57, 114, 148, 150, 162, 207, 276. *C. lineatus*, var.? Sch., *Atl.*, pl. lix. figs. 31, 32. *C. Ehrenbergii*, O'Me., *Proc. Roy. Irish Ac.*, 1875, p. 264, pl. xxvi. fig. 24. Sp. n. Sch.. *Atl.*, pl. cxiv. fig. 13.

Coscinodiscus lineatus (Weisse, *Bull. Ac. Imp. Sci. St Petersb.* 1855, p. 276, pl. i. figs. 2*a*, *b*) is perhaps *Dictyopyxis subtilis*, Ehrb., according to Grunow (*Denk. Wien. Ak.*, 1884, p. 92). Schmidt separates the specimen figured in his *Atl.*, pl. cxiv. fig. 13, because of its convexity.

Habitat.—Richmond, Va. (Ehrenberg, Bailey, Hardman! Cleve and Moller!); Caltanisetta, Peruvian and African guanos (Ehrenberg); Patos guano (Kinker!); Moron (Schmidt); marine deposit, Fiji Islands (Grove!); Sta Monica deposit, Sta Maria deposit (Grove!); Cambridge deposit, Barbados (Hardman!); Barbados (Cleve and Möller!); Californian guano (Norman!);* Rappahannock (Bailey!);* Nancoori (Cleve and Möller! Cleve! Hardman!);* Ningpo (Kinker!); Mascara (Cleve and Moller!); Kamtschatka Sea, 1700 fathoms (Bailey!); Indian Ocean, sounding by Capt. Pullen, 2200 fathoms (Greville!); Japan (H. L. Smith!); Singapore (Hardman!); Yokohama and Brazil (Schmidt); Mejillones (Cleve! O'Meara); Cambodia (Hardman!); † Monte Gubbio (Grove!); edible sea-weeds, Indian Ocean (Macrae!); Campeachy Bay (Cleve and Möller! Grove!); Cannibal Islands (Greville!); Andaman Islands (Macrae!); Cuxhaven (Bailey); Malahide and Dollymount, county Dublin; Ascidia, Roundstone Bay, county Galway (O'Meara); Patagonia, California (Cleve and Möller!); west coast, Florida, U.S. Survey (Febiger!); Yeddo; Patagonian guano; near Elbing, West Prussia; Mors deposit; Labuan; Kusu; between Aden and Bab-el-Mandeb (Cleve!); Archangelsk (Cleve); Vera Cruz, among *Sertularia;* Laguna, Mexico, on stones among Algæ; anchor ground, Laguna Harbour, 20 miles N. of Laguna in the sea (Rabenhorst and Schwarz!); Simbirsk Polirschiefer (Hardman!);† Faeroe Channel (Grove!).

* In the collection of Dr Greville.
† In the collection of Julien Deby.

C. marginato-lineatus. Sch., *Atl.*, pl. lix. fig. 33.—Diam. ·0335 mm. Central space absent. Markings hexagonal, equal $3\frac{1}{2}$ to 4 in ·01 mm. Border about $\frac{1}{7}$ of radius broad; striæ distinct, 6 to 8 in ·01 mm., the inner half separated from the outer by a distinct narrow line.

Distinguished by the regularity of the markings and border.

Habitat.—Campeachy Bank (Schmidt).

C. peruanus, Grun. Sch., *Atl.*, pl. lviii. fig. 43.—Diam. ·0425 mm. Central space absent. Markings polygonal, decreasing slightly from the centre outwards; towards the centre 5, towards the border 6 in ·01 mm.; the oblique decussating rows straight or slightly curved outwards, well-marked; apiculi numerous, distinct, close to the border. Border hyaline.—Grun., *Denk. Wien. Ak.*, 1884, p. 85.

Distinguished from *C. excentricus* by the size of the markings, the apiculi, and border.

Habitat.—Peru guano (Schmidt).

C. sublineatus. C. (*excentricus*, var.?) *sublineatus*, Grun., *Denk. Wien. Ak.*, 1884, p. 85, pl. iv. (D), figs. 21, 22.—Diam. ·032 to ·053 mm. Central space and rosette absent. Markings hexagonal, gradually decreasing towards the border; at the centre 5, at the border 9 in ·01 mm.; the oblique slightly bent, decussating rows distinct, non-apiculate. Border narrow, hyaline.

Distinguished from *C. excentricus* by having the markings at the border smaller in proportion to the others, and from *C. lineatus* by the less uniform markings.

Habitat.—Franz Josef's Land, White Sea (Grunow); Simbirsk Polirschiefer (Grunow).

C. anguste-lineatus. Sch., *Atl.*, pl. lix. fig. 34.—Diam. ·0275 to ·0455 mm. Central space and rosette absent. Markings polygonal, subequal, 6 in ·01 mm. Apiculi minute, sometimes indistinct, at the border. Border narrow, hyaline.—Janisch, *Gazelle Exped.*, taf. iii. fig. 6. *C. lineatus*, var. *tenera*, Tru. & Witt., *Jeremie Diat.*, p. 14, pl. ii. fig. 2; Cleve and Möll., *Diat.*, No. 154.

Truan and Witt's *C. lineatus*, var. *tenera*, differs chiefly, according to the figure in the somewhat more distinct appearance of fasciculi

between the radial rows. It is not strictly defined by the authors.

Habitat.—Yokohama (Schmidt), Cambodia (Hardman !) * Mejillones (Hardman);* Los Angelos (O'Meara !); marine deposit, Fiji Islands (Grove!); rice fields, Georgia (Greville!); Zebu, Philippine Islands (Weissflog !); Jeremie deposit, Hayti (Truan and Witt); Indian Ocean, sounding by Capt. Pullen, 2200 fathoms (Greville !); Balearic Islands (Cleve and Möller !).

C. pseudo-lineatus. Pant., *Fossil. Bacil. Ung.*, p. 73, pl. ix. fig. 77. —Diam. ·08 to ·125 mm. Centre occupied by a single circular areola, ·0025 mm. broad. Markings hexagonal, 8 to 9 in ·01 mm., decreasing somewhat outwards; within the border a narrow punctate band, somewhat irregular on its inner side. Apiculi minute, numerous, forming a circlet at the border, one larger inserted somewhat further inwards. Border striæ, delicate, 12 in ·01 mm., merging into the adjacent band on the inner side.

Habitat.—Dolje deposit (Pantocsek !).

C. cristatus, sp. n.—sp. n.? Sch., *Atl.*, pl. lix. fig. 4.—Diam. ·0305 mm. Central space and rosette absent. Markings regularly angular, 10 in ·01 mm., a narrow hyaline band adjacant to the border; apiculi numerous, their outer ends obtuse, placed at the inner edge of the hyaline band. Border distinct, hyaline.

Habitat.—Peruvian guano (Schmidt).

Var. *distans.* (?) Sch., *Atl.*, pl. lix. fig. 5.—Diam. ·02 mm. Markings 10 to 12 in ·01 mm., secondary oblique rows distinctly curved outwards; no hyaline band adjacent to border; apiculi similar, but more distant.

Habitat.—Kings Mill (Schmidt).

C. tumidus, Janisch. Sch., *Atl.*, pl. lix. figs. 38, 39.—Diam. ·164 to ·2 mm. Surface convex towards the centre. Central space absent. Markings hexagonal, towards the centre 4 to 4½ in ·01 mm., increasing outwards to the border to 3 in ·01 mm.; oblique decussating rows, straight, or with slight bendings. Border striæ, 4 in ·01 mm.—Cleve and Möll., *Diat.*, Nos. 125, 207.

Habitat.—Table Bay (Schmidt, Weissflog !); surface, Antarctic

* In the collection of Julien Deby.

Ocean, H.M.S. Challenger (Rae! Cleve and Möller!); Patagonia (Cleve and Möller!); Patagonia, 1375 fathoms, H.M.S. Challenger (Cleve!).

Var. *fasciculata*, nov.—Diam. ·2 to ·28 mm. Similar to the type, but markings in fasciculi, the rows in each parallel to that at its centre, the fasciculi few and wide, or numerous and narrow.

The fasciculi when wide resemble those of *C. polyradiatus*, Cstr., but the border is quite unlike that of the latter.

Habitat—Surface, Antarctic Ocean, H.M.S. Challenger (Rae!); Gazelle Expedition (Weissflog!).

C. leptopus, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxxi. figs. 5 and 6.—Diam. ·1 mm. Central space absent. Markings hexagonal, decreasing but slightly at the border, and showing numerous round granules; towards the centre 4, towards the border 5 in ·01 mm.; on a narrow irregular zone adjacent to the border minute round, granular; the oblique decussating rows sub-straight, evident; apiculi well defined, forming a circle close to the border at subregular intervals of about ·005 mm., one larger inserted somewhat farther inwards. Border sharply defined; striæ delicate 8 to 10 in ·01 mm.—Cleve and Möll., *Diat.*, No. 114; Janisch, *Gazelle Exped.*, taf. v. fig. 4; *C. lineatus*, Sch., *Atl.*, pl. lix. fig. 26; *C. macraeanus*, Grun. (non Grev.), *fide* Sch., *Atl.*, *ibid.*

The large apiculus is not unlike that of *Podosira oliverana*, Grun. (Van Heurck, *ibid.*, pl. cxviii. fig. 5), found abundantly at Kerguelen by H.M.S. Challenger.

Habitat.—Off Ascension Island, S.S. Buccaneer (Grove!); Mascara (Cleve and Möller!); Californian guano (Greville!); Patos Island guano (Greville!); Mejillones guano, Balearic Islands (Van Heurck); Santa Marta deposit (Doeg!); Los Angelos (Cambridge!); Macassar Straits (Grove!). Trawled by H.M.S. Challenger, lat. 34° 37′ N., long. 140° 32′ E. (Rae!); Indian Ocean soundings, Capt. Pullen, 2200 fathoms (Greville!).

Var. *discrepans*, nov.—Diam. ·175 mm. Markings hexagonal, at the centre of the valve an inequilateral 4-sided area, bounded by a narrow irregular band of dissimilar angular areolæ, which are continued outwards unequally from the angles towards the border;

the rows in the intervals slightly bent, oblique, and decussating; the band adjacent to the border sub-regular, with punctiform striæ 8 to 10 in ·01 mm. Apiculi on border at intervals of ·005 to ·0075 mm.; the single large apiculus with a knob-like free end, inserted at inner edge of band adjacent to the border, about ·0055 mm. long.—(Pl. II. fig. 3.)

Habitat.—Gazelle Exped. (Weissflog !).

§ V. Fasciculati, Grun., *Denk. Wien. Ak.*, 1884, p. 80; Pant., *Fossil. Bacil. Ung.*, p. 71.

Markings fasciculate, the fasciculi sometimes indistinct, or recognisable only on outer portion of valve, the rows composing each parallel to that at its centre or side; apiculi frequent.

C. vetustissimus.. Pant., *Fossil. Bacil. Ung.*, p. 71, pl. xx. fig. 186. —Diam. ·075 to ·1 mm. Central space and rosette absent. A small slightly excentric nodule, distinct. Markings hexagonal, increasing slightly to about the semiradius, again decreasing somewhat to the border, towards the centre and border 5, at the semiradius $4\frac{1}{2}$, in ·01 mm., central papillæ distinct; irregular, on a small subcircular somewhat excentric area, about ·006 mm. broad, and surrounded by an indistinct narrow, irregular, hyaline band; elsewhere in obscurely fasciculate substraight or in subradial rows, those in each fasciculus parallel to that at one of its edges, and most obvious when the papillæ are in pairs; non-apiculate. Border narrow, indistinct; striæ delicate, 8 to 10 in ·01 mm.—Cleve and Möll., *Diat.*, Nos. 57, 155, 162, 164; Grun., *Bot. Centralbl.*, Bd. xxxiv. Nos. 2, 3, p. 35; *C. inæqualis*, Grove and Sturt., *Jour. Quek. Mic. Cl.*, 1887, p. 68. —(Pl. II. fig. 17.)

In the fasciculation this species recalls *C. curvatulus*, but is distinguished by the excentric arrangement of its markings. It approaches *C. africanus*, var. *wallichiana*, Grun., in the latter respect, but Grunow's var. is non-fasciculate. In the excentric arrangement of the much smaller markings, as well as in their subfasciculate disposition, this species may be easily distinguished from *C. nodulifer*, Janisch.

Habitat.—Oamaru deposit (Grove ! Cleve!); Yokohama(Schmidt); Cambridge deposit, Barbados (Kinker ! Johnson!); Richmond, Va.

(Cleve and Möller!); Balearic Islands; Nancoori; Sta Monica deposit (Cleve and Möller!); between Aden and Bab-el-Mandeb; Mejillones guano (Cleve!); Alsó-, Felsó-, Esztergaly, Kékkö and Szakal deposits (Pantocsek!).

Var. *curvatuloides*, Grove MS.—Diam. ·1 mm. Markings irregular, and smallest on a small round excentric area, elsewhere subequal, 4½ in ·01 mm., and in evident fasciculate rows, those in each fasciculus parallel to that at its edge; apiculi minute, interfasciculate.—Cleve and Möll., *Diat.*, Nos. 57, 164.

Through this var. *C. vetustissimus* is allied to *C. curvatulus*, var. *genuina*.

Habitat.—Jackson's Paddock, Oamaru deposit (Grove!); Richmond, Va.; Sta Monica deposit (Cleve and Möller!).

C. atlanticus. Cstr., *Diat. Chall. Exped.*, p. 158, pl. v. fig. 8.—Diam. ·046 mm. Central space and rosette absent. Markings round, granular, without order, and with hyaline interspaces from the centre to a little beyond the semiradius, thence polygonal, subequal, 10 in ·01 mm., and in radial subfasciculate rows to the border. Border distinct, hyaline.

Habitat.—South Atlantic, H.M.S. Challenger (Castracane).

Var. *striatula*, nov. *C. atlanticus*, var., Cstr., *ibid.*, p. 158, pl. iii. fig. 7.—Diam. ·0715 mm. Markings round, granular, and irregular from the centre to about ¼ of radius beyond this polygonal in evident fasciculi; those composing each fasciculus parallel to that at its centre, subequal, 6 in ·01 mm. Border about ⅓ of radius broad, striæ 8 to 10 in ·01 mm.

Habitat.—(?) (Castracane).

C. nitidus. Greg., *Trans. Roy. Soc. Edin.*, 1857, p. 499, pl. x. fig. 45.—Diam. ·03 to ·075 mm. Surface almost flat. Central space absent. Markings small, rounded, subpearly, with hyaline interspaces, largest towards the centre, decreasing slightly to the border, irregular, sometimes in inconspicuous radial subfasciculate rows around the border. Border striæ, 6 in ·01 mm., distinct.—Ralfs in *Pritch. Inf.*, p. 831, pl. viii. fig. 18; Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 94, pl. iii. fig. 32;

Sch., *Atl.*, pl. lviii. fig. 18 (excl. figs. 16, 17); Pant., *Fossil Bacil. Ung.*, p. 73, pl. xviii. fig. 166; Cleve and Möller's *Diat.*, No. 210; Janisch, *Gazelle Exped.*, taf. v. figs. 12, 14–16; *C. nitidus*, Greg., var., Cleve. and Möll., *Diat.*, Nos. 150, 154, 155, 208, 257, 311; *C. foraminosus*, Grev. MS. in Coll. Brit. Mus.

Habitat.—Kékkö, Mogyorod, Szakal and Szent Peter deposits (Pantocsek); sand washings, Cumbrae (Arnott!);* Lamlash (Greville! Gregory!); Ascidia, Roundstone Bay, county Galway; Malahide, county Dublin; Restrevor, county Down; Kilkee, county Clare (O'Meara); Hvidingsoe (Schmidt); Manilla shells (Greville!); Tahiti (Kinker!); Numea Algæ (Kinker!); Manilla (Firth!);† Tamatave (Hardman!);‡ Rio Janeiro (Hardman!); Oamaru deposit (Grove!); Campeachy Bay (Weissflog! Cleve and Möller!); Monterey (Weissflog!);§ Gazelle Expedition (Weissflog!); Andaman Islands (Macrae!);‖ coral washings, locality? (Doeg!); shell cleanings, locality? (Doeg!); Rovigno, North Carolina, Balearic Islands, Gripp (Cleve and Möller!); Galapagos Islands, Labuan, Virgin Islands (Cleve!).

Var. *minor*. Cleve and Möll., *Diat.*, No. 154.—Diam. ·025 to ·0325 mm. Markings angular, 4 in ·01 mm. Central papillæ prominent, rows evident near border; interspaces subobsolete.

Habitat.—Balearic Islands (Cleve and Möller!).

Var. *sparsa*. *C. nitidus*, Sch., *Atl.*, pl. lviii. fig. 17.—Diam. ·035 mm. Markings round, isolated granules, with wider interspaces, smaller near the border. Border striæ more evident and longer.

Habitat.—Campeachy Bank (Schmidt).

Var. *tenuis*, nov. *C. nitidus*, Greg., var. Sch., *Atl.*, pl. lviii. fig. 19. —Diam. about ·04 mm. Markings minute, with smaller hyaline interspaces, on a narrow band adjacent to the border punctiform, and forming regular radial striæ.—Sch., *Atl.*, pl. lvii. fig. 45 (?).

Habitat.—Campeachy Bank (Schmidt).

* In the collection of Dr Greville.

† In the collection of Dr F. W. Griffin.

‡ In the collection of Julien Deby.

§ This specimen has, on what seems to me insufficient grounds, been named on Weissflog's slide *C. nitidus*, var. by Grunow.

‖ In the collection of Dr Greville.

Var. *moronensis.* Grun. MS.—Diam. ·0875 mm. Markings with central papillæ more prominent, scabrous; secondary, irregularly, curved rows, subconcentric within the semiradius; around the border the subradial rows short, inconspicuous. Border narrow; striæ 6 or 7 in ·01 mm.—(Pl. I. fig. 21.)

Habitat.—Moron deposit (Weissflog !).

C. nitidulus, Grun. Sch., *Atl.*, pl. lviii. figs. 20, 21.—Sometimes trilobate. Diam. ·04 to ·1175 mm. Central space absent. Markings small, round, granular, decreasing slightly towards the border; about 4 in ·01 mm.; rows radial, beyond the semiradius subfasciculate; interspaces hyaline. Border distinct, narrow; striæ 6 to 8 in ·01 mm.—Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 2; Pant., *Fossil. Bacil. Ung.*, p. 73, pl. xxiv. fig. 214; Janisch, *Gazelle Exped.*, taf. v. fig. 13.

Distinguished from *C. nitidus* by the smaller size of the markings, which decrease less towards the border.

Habitat.—Campeachy Bay (Van Heurck, Schmidt.); Szakal, Szent Peter and Dolje deposits (Pantocsek); Hong Kong (Hardman !);* Cambodia (Firth !); Springfield deposits, Barbados (Firth !); Sta Maria deposit (Grove !); Oamaru deposit (Firth !); Rio Janeiro (Hardman !);* Macassar Straits (Grove !); between Aden and Bab-el-Mandeb (Cleve !).

Var. *subradians*, nov.—Diam. ·05 mm. Markings round, granular, largest at the centre, decreasing gradually outwards, punctiform at the border; interspaces wide, smallest towards the border; rows subradial, non-fasciculate, crowded on a distinct zone at the border about $\frac{1}{8}$ of the radius broad.

Habitat.—Aegina (Schmidt).

C. suspectus, Janisch. Sch., *Atl.*, pl. lix. fig. 2.—Diam. ·106 mm. Central space and rosette absent. Markings polygonal, about 7 in ·01 mm., decreasing slightly towards the border; rows radial or oblique and decussating, the former forming inconspicuous narrow fasciculi most evident beyond the semiradius, the latter straight or slightly curved outwards. Border narrow, hyaline.—Grun., *Denk. Wien. Ak.*, 1884, p. 85.

* In the collection of Julien Deby.

Distinguished from *C. circumdatus* by the greater irregularity of the rows about the centre and the simple border.

Habitat.—San Francisco, Cal. (Schmidt).

C. Kützingii. Sch., *Atl.*, pl. lvii. figs. 17, 18.—Diam. ·0635 mm. Central space absent. Markings polygonal, about 6 in ·01 mm., subequal or decreasing slightly towards the border; rows distinctly fasciculate beyond the semiradius; those in each fasciculus parallel to one another, secondary oblique decussating rows evident, and curved towards the border. Border sharply defined, bearing crowded oblique decussating rows of areolæ.—Grun. *Denk. Wien. Ak.*, 1884, p. 84; *C. marginatus*, Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 94, pl. iii. fig. 35.

Distinguished from *C. suspectus* by the more evident fasciculi and border, and from *C. subtilis* in the absence of apiculi. The relationship to *C. excentricus* referred to by Grunow is more remote.

Habitat. — Cuxhaven, Firth of Tay (Schmidt); Arctic and Antarctic (Grunow).

Var. *glacialis.* Grun., *Denk. Wien. Ak.*, 1884, p. 84, pl. iv. (D), fig. 18.—Diam. ·045 mm. Markings 10 in ·01 mm., decreasing slightly towards the border; rows less distinctly fasciculate, a circlet of minute apiculi inserted at the border. Border striæ radial, distinct, 8 to 10 in ·01 mm.

Habitat.—Franz Josef's Land; Cape Wankarema, North Siberia; Kerguelen (Grunow).

C. subglobosus. Cleve and Grun., *Denk. Wien. Ak.*, 1884, p. 84, pl. iv. (D), figs. 19, 20.—Diam. ·025 to ·04 mm. Surface somewhat convex. Central space absent. Markings polygonal, decreasing slightly towards the border, 8 in ·01 mm.; rows radial, on outermost portion of valve parallel and subfasciculate; secondary oblique rows curved outwards, most evident towards the border.—Sch., *Atl.*, pl. lviii. fig. 44 (no name); Cleve and Möller, *Diat.*, Nos. 114, 172, 302, 319.

Distinguished from *C. Kützingii* by the more irregular markings on the central portion.

Habitat.—Arctic, Davis Straits; Franz Josef's Land, N. Siberia; Antarctic (Grunow); Davis Straits (Cleve and Moller!); Mascara, Cape Wankarema (Cleve! Cleve and Möller!); Greenland (Cleve!).

C. inclusus, sp. n. Sch., *Atl.*, pl. lvii, fig. 47 (no name).—Diam. about ·07 mm. Central space distinct, rounded, slightly excentric. Markings rounded, granular, about 5 in ·01 mm., smaller towards the border; rows fasciculate, those in each fasciculus parallel to the central radial row, non-apiculate. Border sharply defined, striæ 6 to 8 in ·01 mm.

Habitat.—Richmond deposit, Virginia (Schmidt).

C. tuberculatus. Grev., *Trans. Micr. Soc. Lond.* 1861, p. 42, pl. iv. fig. 6.—Diam. ·0375 to ·0975 mm. Surface almost flat. Central space irregular, small. Markings around the central space minute, rounded, granular; beyond this polygonal 4½ in ·01 mm., subequal to the zone of the apiculi; at the border 6 in ·01 mm.; rows radial or obscurely fasciculate, straight; apiculi distinct placed between the fasciculi. Border striæ delicate, 8 to 10 in ·01 mm.—Sch., *Atl.*, pl. lvii. fig. 42; Grun., *Denk. Wien. Ak.*, 1884, p. 82.

This species cannot be united with *Aulacodiscus*, as suggested, with some doubt, in the second edition of Habirshaw's Catalogue, § *Coscinodiscus.* Small specimens in Weissflog's collection have been distinguished as *forma minor*, but these are quite normal. Distinguished from *C. subtilis* by its larger markings and more robust apiculi.

Habitat. — Barbados deposit (Greville! Hardman! Firth! Cleve!); Chalky mound, Barbados (Weissflog! Firth!); Cambridge deposit, Barbados (Firth! Johnson! Hardman!).

Var. *Monicæ.* Grun. *ibid.*, p. 82, pl. iii. (C), fig. 29.—Diam. ·0625 mm. Markings 3 to 5 in ·01 mm., smaller close to the border; a few rows in each inconspicuous fasciculus; the apiculi interfasciculate, more evident.—*C. tuberculatus*, Grev. var.? Sch., *Atl.*, pl. lvii. figs. 40, 41.

The markings in the Barbados valves are larger than those in Sta Monica specimens. According to Schmidt, there are no pro-

cesses: "die dunkeln Flecke am Rande entstehen dadurch, dass sich je 2 bis 3 Zellchen mit Schalensubstanz füllen."

Habitat.—Sta Monica deposit (Grunow); Cambridge deposit Barbados (Johnson!).

C. isoporus. Ehrb., *Mikrog.*, pl. xxxiii. 17. fig. 3.—Diam. about ·055 mm. Central space and rosette absent. Markings large, 3½ to 4 in ·01 mm., distinct, subequal to the circumference; rows radial, subfasciculate; secondary concentric rows evident.—Ralfs in *Pritch. Inf.*, p. 830.

Differs from *C. concavus* in the absence of a distinct border, and the concentric arrangement of the markings, and from *C. patina*, Ehrb., by the reduction in size, in the latter, of the markings around the evident clear border and their less conspicuous concentric arrangement.

Habitat.—Rappahannock Cliff, Virginia (Ehrenberg).

C. Payeri. Grun., *Denk. Wien. Ak.*, 1884, p. 80, pl. iii. (C), figs. 12, 13.—Diam. ·024 to ·03 mm. Central space small, about ⅓ of diam. broad, irregular. Markings around the central space rounded, elsewhere angular, often quadrilateral; central papillæ distinct, towards the central space 5 or 6, at the border 9 in ·01 mm.; rows radial, in inconspicuous fasciculi.

Habitat.—Franz Josef's Land (Grunow).

Var. *subrepleta.* Grun., *ibid.*, 1884, p. 80, pl. iii. (C), figs. 14, 15. —Diam. ·034 mm. Central space subobsolete, or with small round isolated granules. Markings smaller, 4½ to 5 in ·01 mm., a band of small granules adjacent to the border.

Habitat.—Franz Josef's Land (Grunow).

C. hyalinus. Grun., *Denk. Wien. Ak.*, 1884, p. 108, pl. iii. (C), fig. 28.—Diam. ·025 mm. Central space minute, inconspicuous, bearing isolated puncta. Markings punctiform, subequal, 24 in ·01 mm.; rows radial to subparallel in inconspicuous fasciculi; apiculi numerous, distinct, in a single circlet. Border broad, hyaline.—Cleve and Möll., *Diat.*, No. 315; *Odontodiscus hyalinus*, Grun., *Kon. Sv. Vet.-Ak. Handl. Stockh.*, 1879, p. 113.

Distinguished from *C. bioculatus* by the absence of the two conspicuous central granules and the more numerous apiculi.

Habitat.—From iceberg, lat. 74° 48′ 4″ N., long. 54° 52′ 8″ E., August 1872 (Grunow); Cape Wankarema (Cleve and Möller! Cleve !); Tindingen, Franz Josef's Land; Kara (Cleve).

C. capensis. Grun., *Denk. Wien. Ak.*, 1884, p. 86, pl. iv. (D), fig. 29.—Diam. ·032 mm. Surface with slight circular undulation about the semiradius. Central space circular, about $\frac{1}{13}$ of diam. broad, distinct, with a few isolated granules at its centre. Markings punctiform, smallest towards the border; rows straight or with slight bendings, inconspicuously fasciculate; apiculi numerous, distinct, frequently arranged in a double row. Border sharply defined, hyaline.—Cleve and Möller, *Diat.*, No. 197.

Distinguished from *C. biplicatus*, *C. pellucidus*, and *C. bengalensis* by its central space, subfasciculate rows, and apiculi.

Habitat.—Brackish water, Baaken River, S. Africa (Grunow).

C. bioculatus. Grun., *Denk. Wien. Ak.*, 1884, p. 107, pl. iii. (C), fig. 30; pl. iv. (D), fig. 1.—Diam. ·02 to ·03 mm. Surface convex, with faint concentric undulations. Central space subcircular, bearing two large conspicuous round granules, with evident central dot. Markings rounded, punctiform, least crowded towards the centre; rows subparallel in inconspicuous fasciculi, 18 to 22 in ·01 mm.; apiculi small, but evident, close to the border, numerous, subregular.

In specimens from the Kara Sea and Cape Wankarema, N. Siberia, the markings and apiculi are more distant.

Habitat.—Franz Josef's Land (Grunow).

Var. *exigua.* Grun., *ibid.*, p. 108, pl. iv. (D), fig. 2.—Diam. ·012 to ·015 mm. Central space minute, the large round granule single, or sometimes two of unequal sizes, the one indistinct. Markings less evident; rows 24 to 26 in ·01 mm.; apiculi minute, 4 in ·01 mm.

Habitat.—Franz Josef's Land (Grunow).

C. semipennatus. Grun., *Denk. Wien. Ak.*, 1884, p. 83.—Diam. about ·0305 mm. Central space absent or subobsolete. Markings rounded, or obtusely angular, granular; towards the centre 4, decreasing uniformly but considerably towards the border; rows fasciculate,

slightly curved towards the same direction; those in each fasciculus parallel to the corresponding and more conspicuous side rows; interspaces hyaline.—Sch., *Atl.*, pl. lvii. fig. 32 (no name).

Habitat.—Springfield deposit, Barbados (Schmidt); Cambridge deposit, Barbadoes (Kinker !).

C. Grunowii. Pant., *Fossil. Bacil. Ung.*, p. 74, pl. ix. fig. 74.—Diam. ·062 to 0·75 mm. Surface flat, slightly convex near the border. Central space small, indistinct, almost filled in by isolated round granules. Markings obtusely angular, subequal, 7 to 8, towards the border more crowded, 9 to 10 in ·01 mm.; slightly smaller at the centre than on the adjacent area, the central papillæ prominent; within the border from 10 to 23 small clear rounded subregular spaces; rows fasciculate, straight, and radial between the centre and the clear spaces near the border, the others parallel to these in the corresponding fasciculus. Border striæ delicate, 16 in ·01 mm.

Habitat.—Alsö-, and Felsö-, Esztergály deposits (Pantocsek !).

Var. *minor.* *C. Grunowii forma minor*, Pant., *ibid.*, p. 74, pl. xiv. fig. 25.—Diam. ·024 to ·036 mm. Markings punctiform, 10 to 15 in ·01 mm.; the fasciculi more distinct, the clear spaces within the border more irregular, sometimes larger.

Habitat.—Felsö-Esztergály deposit (Pantocsek).

C. odontodiscus. Grun., *Denk. Wien. Ak.*, 1884, p. 81, pl.iii. (C), fig. 23.—Diam. ·0455 to ·1125 mm. Central space absent, but a narrow hyaline ring close to the centre. Markings hexagonal, 6 to 7 in ·01 mm., decreasing slightly outwards, punctiform in a narrow zone adjacent to the border; irregular at the centre within the clear band, elsewhere in subparallel rows; secondary oblique decussating rows evident; fasciculi evident, the rows composing each parallel to that at one of its edges; apiculi minute, interfasiculate.—Cleve and Möll., *Diat.*, No. 57, 162; *C. subtilis*, var., Sch., *Atl.*, pl. lvii. figs. 15, 16; *Odontodiscus spica*, Ehrb., *Mon. Ber. Ak.*, 1845, p. 79; *O. uranus*, Ehrb., *ibid.*, 1845, p. 79; *C. odontodiscus*, var. *parva-tenuistriata*, Cleve and Möll., *Diat.*, No. 276.

Habitat.—Richmond, Va. (Schmidt, Grove! Cleve and Möller !);

Patos Island guano (Norman !);* Nancoori; California; Sta Maria deposits (Grove !); Balearic Islands; Pabillan di Pico guano; Successful Bay, Kerguelen (Cleve !).

Var. *subsubtilis*, nov. *C. subtilis*, Sch., *Atl.*, pl. lvii. fig. 14.—Diam. ·125 mm. Markings sometimes increasing slightly from the centre outwards or subequal; at the centre 10, at the border 8 in ·01 mm.; rows straight, radial, fasciculate or subfasciculate towards the border. A narrow hyaline band adjacent to the border; apiculi minute, irregular, but interfasciculate, sometimes absent. Border striæ distinct, 6 in ·01 mm.

Habitat.—Peruvian guano (Schmidt); Monterey (Kinker !); Nancoori (Hardman !); † Lobos di Afuera guano (Grove !); marine deposit, Fiji Islands (Grove !); Inland Sea, Japan, H.M.S. Challenger (Grove !); Springfield deposit, Barbados (Doeg !).

C. curvatulus, Grun. Sch., *Atl.*, pl. lvii. fig. 33.—Diam. ·045 to ·07 mm. Central space absent or indistinct, with numerous rounded granules. Markings polygonal, subequal, 6 in ·01 mm.; rows in gentle curves, fasciculate; those composing each fasciculus parallel to that at its convex edge; the curves on the two valves of a frustule in opposite directions, secondary oblique decussating rows obvious; apiculi absent.—Janisch, *Gazelle Exped.*, taf. ii. fig. 7; taf. v. figs. 2, 3, 8; taf. vi. fig. 2; taf. xx. fig. 17; *C. curvatulus*, var. *inermis*, Grun., *Denk. Wien. Ak.*, 1884, p. 83, pl. iv. (D), figs. 11, 12; Sch., *Atl.*, pl. cxiii. fig. 6; Cleve and Möll., *Diat.*, Nos. 57, 154, 162, 276, 319; *C. curvatulus*, var. *densius-striata* (?), Sch., *Atl.*, pl. lvii. fig. 35; *Odontodiscus curvatulus*, Cleve, *Vega Exped. Vetensk. Jakttag. Stockh.*, Bd. iii. 1885, p. 488. In H. L. Sm., *Diat. Spec. Typ.*, No. 99.

This species is sometimes confounded with the very distinct *C. symmetricus*, Grev.

Habitat.—Los Angelos, Cal. (Cambridge !); Richmond deposit (Rae ! Cleve and Möller !); Caltanisetta deposit; Barbados deposit (Johnson !); Peruvian guano, Franz Josef's Land (Grunow); Bolivian guano (Cleve !); Infusorial deposit, "Algeria" (Greville !);

* In the collection of Dr Greville.
† In the collection of Julien Deby.

Yszee (Kinker !); Table Bay (Weissflog !);* Virginia (Hardman !); Nancoori, Balearic Islands (Cleve and Möller !); Melville Bay, lat. 75° 27′ N., long. 64° 34′ W. (O'Meara !);* Faeroe Isles (Grove !); marine deposit, Fiji Islands (Grove !); Japan (H. L. Smith !); Monterey (Cleve, Weissflog !); Ægina (Schmidt); Cape Wankarema, California (Cleve and Möller !); Patagonian guano (Cleve !); Sta Monica deposit (Grove !).

Var. *latius-striata.* Sch., *Atl.*, pl. lvii. figs. 30, 34.—Diam. ·1 mm. Central space and rosette absent. Markings 3½ to 4 in ·01 mm., increasing gradually outwards to about semiradius, again decreasing similarly to the border; fasciculi distinct, rows sometimes slightly curved; apiculi absent. Border striæ, 6 in ·01 mm.

Distinguished by the large size of the markings. Sometimes associated with *C. subtilis*, to which it bears no affinity.

Habitat.—Cambridge deposit, Barbados (Hardman !);† Barbados (Cleve !).

Var. *minor.* Grun., *ibid.*, 1884, p. 83, pl. iv. (D), figs. 8, 10.—Diam. ·03 to ·04 mm. Central space small and granular, or absent. Markings in more straight, less distinctly fasciculate rows; secondary oblique rows indistinct, non-apiculate (excl. *C. minor*, Ehrb.).

The union of *C. minor*, as figured in Schmidt's *Atlas*, pl. lviii. fig. 40, to the present var., as proposed by Grunow, is erroneous.

Habitat.—Girgenti and Caltanisetta deposits (Grunow); Nancoori deposit (Hardman !).‡

Var. *genuina.* Grun., *ibid.*, 1884, p. 83, pl. iv. (D), fig. 13.—Diam. ·0325 to ·125 mm. Central area small, circular, with but few granules. Markings 8 to 10 in ·01 mm.; apiculi minute, interfasciculate. Border sharply defined; striæ delicate, 14 to 16 in ·01 mm.—Van Heurck, *Typ. Syn. Diat. Belg.*, No. 534; Sch., *Atl.*, pl. lvii. fig. 36 (no name) and 37. *C. Szontaghii*, Pant., *Fossil. Bacil. Ung.*, p. 72, pl. xv. fig. 133.

Specimens from Bolivia pass into the var. *subocellata*, Grun.

* Procured by M'Clintock.
† In the collection of Julien Deby.
‡ *Ibid.*

Pantocsek's figure does not show the fasciculation, though this is distinct in his specimens.

Habitat.—From ice in lat. 74° 48′ 4″ N., long. 54° 52′ 8″ E., Aug. 2, 1872 (Grunow); San Francisco, Cal. (Firth !); Los Angelos, Cal. (Griffin !) ;* Oran, Algeria (Van Heurck !); soundings off Kurile Islands, 1329 fathoms (H. L. Smith !) ;† Barbados (Johnson !) ;‡ Richmond deposit, Va. (Rae !); Kekkö deposit (Grove !); Jack's Ranch, Cal. (Macrae !); Szakal and Szent Peter deposits (Pantocsek !).

Var. *kariana.* Cleve and Grun., *Kong. Sv. Vet.-Ak. Handl Stockh.*, 1880, p. 113, pl. vii. fig. 129.—Diam. ·023 to ·024 mm. Central space absent. Markings distinct, 11 to 12 rows in each fasciculus and 13½ to 14 in ·01 mm.; apiculi interfasciculate, distinct.—*Odontodiscus curvatulus*, var. *kariana*, Cleve and Grun., *ibid.*, 1880, p. 113.

Habitat.—Kara Sea (Cleve and Grunow); Finmark (Cleve).

Var. *subocellata.* Grun., *ibid.*, 1884, p. 83, pl. iv. (D), fig. 15, from Bolivian guano and Cape of Good Hope (Grunow); Kerguelen Island and Challenger dredgings off Vancouver Island (Grove !), belongs to *Actinocyclus.*

Var. *recta*, nov. *C. curvatulus*, var., Cstr., *Diat. Chall. Exped.*, p. 160, pl. iii. fig. 10.—Diam. ·03 to ·0875 mm. Markings 4 to 5 in ·01 mm., arranged in straight fasciculi; apiculi distinct, interfasciculate, sometimes absent. Border with slight indentations opposite the apiculi.—Cleve and Möll., *Diat.*, Nos. 57, 164, 276; Janisch, *Gazelle Exped.*, taf. i. fig. 6; taf. iv. fig. 4.

This var. approaches var. *minor.* Specimens have sometimes been confounded with *C. barbadensis*, Grev., or regarded as fasciculate forms of *C. oculus-iridis.*

Habitat.—H.M.S. Challenger (Castracane); Richmond; Sta Monica deposit, California (Cleve and Möller !); Barbados deposit (Cleve !); Yokohama (Cleve !); Monterey stone (Cleve !); San Benito deposit, California (Grove !); Marstrand (Kinker !).

* In the collection of Dr F. W. Griffin.
† In H. L. Sm. *Diat. Spec. Typ.*, No. 93.
‡ In the collection of Dr Greville.

I have not seen *C. curvatulus*, var. *barbadensis*, Cleve MS., from Barbados, nor *C. curvatulus*, var. Cleve MS., from Yokohama. *C. curvatulus*, var. *frigida*, Grun. (*Vega Exped., Vetensk. Jakttag. Stockh.*, Bd. iii. 1883, p. 488), remains undefined. Specimens so named were procured in Franz Josef's Land.

C. crenulatus, Grun. Sch., *Atl.*, pl. lvii. fig. 38.—Diam. ·0605 mm. Central space absent. Markings polygonal, subequal, about 5 in ·01 mm.; irregular on a small area at the centre, elsewhere in fasciculate rows; those composing the fasciculi parallel to that at their corresponding sides; apiculi distinct, inserted at inner edge of border and interfasciculate. Border sharply defined, its outer edge crenate, the indentations corresponding in position to the spines, its inner half with distinct striæ 6 in ·01 mm., its outer half smooth. —Grun., *Denk. Wien. Ak.*, 1884, p. 83, pl. iv. (D), fig. 17.

Habitat.—Seychelles, Bolivian guano, ex *Salpa spinosa*, Southern Ocean, Balearic Islands (Grunow); Kings-Mill Islands (Schmidt).

C. æginensis. Sch., *Atl.*, pl. cxiii. figs. 13, 14.—Diam. ·061 to ·085 mm. Surface flat. Central space subcircular, $\frac{1}{10}$ to $\frac{1}{14}$ of diam. broad, bearing a few isolated round granules, one much larger than the others. Markings around the central space round, subequal, elsewhere polygonal, increasing outwards to about the semiradius, thence decreasing gradually to the border; towards the centre $4\frac{1}{2}$, at the semiradius $3\frac{1}{2}$ in ·01 mm.; rows radial, subfasciculate beyond the semiradius, secondary oblique rows evident. Border narrow, striæ 6 in ·01 mm.

Habitat.—Aegina (Schmidt).

C. simbirskianus. Grun., *Denk. Wien. Ak.*, 1884, p. 81.—Diam. ·1 to ·225 mm. Central space absent, sometimes minute; rosette inconspicuous or subobsolete, sometimes distinct. Markings hexagonal, increasing slightly outwards to about the semiradius, thence decreasing gradually to the border; towards the centre 4, at the semiradius 3 to $3\frac{1}{2}$, at the border 6 in ·01 mm.; central papillæ evident; rows fasciculate, those in each fasciculus subparallel to that at its centre; oblique decussating rows evident. Border narrow; striæ 4 to 5 in ·01 mm.—Sch., *Atl.*, pl. cxiii. figs. 11, 12.

Distinguished from *C. radiatus* by the fasciculate arrangement of the markings.

Habitat.—Simbirsk (Grunow, Grove !); Ananino deposit (Grove ! Kinker ! Rae ! Deby !); Archangelsk-Kurojedowo (Schmidt, Cleve !).

C. symmetricus. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 68, pl. viii. fig. 2.—Diam. ·075 to ·175 mm. Central space small, rounded, $\frac{1}{2\frac{1}{3}}$ of diam. broad, sometimes absent. Markings subpearly; round or subangular, granular, 4 in ·01 mm., subequal or slightly smaller at the border; interspaces hyaline; rows straight, fasciculate, those composing a fasciculus parallel to that at its centre; around the border a narrow hyaline clear space, irregular on its inner side, sometimes bearing a few isolated granules. Border striæ evident, 6 in ·01 mm.—Excl. *C. symmetricus*, Sch., *Atl.*, pl. lvii. figs. 25–27; Grun., *Denk. Wien. Ak.*, 1884, p. 81.

Narrow hyaline radial spaces sometimes, in larger valves, run outwards for some distance from the central space. Quite distinct from *C. subtilis* in the size and arrangement of the markings. *C. ? clypeus*, Ehrb. (*Mikrog.*, pl. xi. fig. 6), from Bilin deposit, Bohemia, is too minute for identification. The fragments show round granular fasciculate markings with hyaline interspaces. Ehrenberg did not define the species, and regarded it as probably a fragment of *Campylodiscus clypeus*. The figure may represent a small piece of *Coscinodiscus symmetricus*, Grev.

Habitat.—Cambridge deposit, Barbados (Johnson ! Greville ! Firth !); Manilla (Firth !); Newcastle deposit, Barbados (Firth !); "Barbados" (Kinker !).

C. planiusculus, sp. n.—Diam. ·0825 mm. Surface flat. Central space rounded, indistinct, about $\frac{1}{17}$ of diam. broad. Markings rounded or oval, with long axis radial; outlines faint; central papillæ evident; about $3\frac{1}{2}$ in ·01 mm., subequal, on a distinct band around the border, about $\frac{1}{8}$ to $\frac{1}{9}$ of radius broad, moniliform; rows fasciculate, those in each fasciculus parallel to that at its middle. Border crenate, a minute dark speck at each indentation at intervals of about ·005 mm.—Janisch, *Gazelle Exped.*, taf. vi. fig. 12. (Pl. I. fig. 22.)

Differs from *C. subtilis* in the shape of the markings, the monili-

form band adjacent to the border and the crenate appearance of the latter.

Habitat.—Gazelle Expedition (Weissflog !).

C. fasciculatus. O'Me., *Quart. Jour. Micr. Sci.*, 1867, p. 245, pl. vii. fig. 1.—Diam. ·0825 mm. Central space circular, distinctly defined. Markings areolate, subequal to the border; rows fasciculate, 9 composing each fasciculus and parallel to that at its centre, the central radial row only extending to the central space, each adjacent pair successively terminating farther and farther from it; interspaces at origin of shorter rows hyaline; apiculi absent.

The markings are intermediate in size between those of *C. symmetricus* and of *C. Normani* or *C. subtilis.*

Habitat.—Dredged off Arran Islands, co. Galway (O'Meara).

C. echinatus, sp. n. Sch., *Atl.*, pl. lviii. figs. 35, 36 (no names).—Diam. about ·03 mm. Central space minute, rosette absent. Markings angular, subequal or decreasing gradually from the centre outwards, towards the centre 4½ to 5, towards the border sometimes 6 in ·01 mm.; rows fasciculate, those in each fasciculus parallel to the central row; fasciculi few, 3 to 5. Apiculi large, spine-like, interfasciculate, inserted some distance from border. Border sharply defined, striæ 6 in ·01 mm., sometimes obscure.

Habitat.—Moron deposit (Schmidt).

C. lentiginosus, Janisch. Sch., *Atl.*, pl. lviii. fig. 11.—Diam. ·075 mm. Central space absent. Markings round, granular, least crowded and with narrow hyaline interspaces towards the centre, angular and more crowded around the border; towards the centre 6, at the border 8 in ·01 mm.; irregular on a small indistinct central area, elsewhere in fascisculate rows, those in each fasciculus parallel to that at its middle; a minute apiculus close to the border, readily overlooked. Border striæ delicate, 16 in ·01 mm.—Cleve and Möll., *Diat.*, No. 207; *C. lentiginosus*, var. *maculatus*, Grun., Cleve and Möll., *Diat.*, No. 183.

A similar spine occurs in *C. leptopus* and *C. kryophilus.* Its character is distinct from those of *Eupodiscus*, to which on its account it has sometimes been proposed to unite Janisch's species.

Habitat.—Patagonia, Antarctic Ocean (Cleve and Möller !);

Kerguelen (Grove!); Antarctic ooze 1950 fms., H.M.S. Challenger (Rae! Kinker! Hardman! Grunow); lat. 46° 46′ N., long. 45° 31′ E., 1375 fms. H.M.S. Challenger (Castracane!); Oamaru deposit (Doeg! Marshall!);* Table Bay (Schmidt); Challenger dredgings, off Vancouver's Island, and Globigerina ooze, off Ascension, S.S. Buccaneer (Grove).

C. kryophilus. Grun., *Denk. Wien. Ak.*, 1884, p. 81, pl. iii. (C), fig. 21.—Diam. ·044 mm. Central space rounded, indistinct, about $\frac{1}{15}$ of diam. broad, bearing isolated rounded granules. Markings polygonal, minute; rows fasciculate, those composing each fasciculus parallel to the radial row at its middle; a single prominent spine inserted near the border, outside of this a circlet of numerous, closely placed minute apiculi.—Janisch, *Gazelle Exped.*, taf. iii. fig. 3.

Distinguished from *C. lentiginosus*, Janisch, by the larger spine and by the apiculi, and from *C. polyacanthus* by the smaller but more numerous apiculi.

Habitat.—Cape Wankarema, N. Siberia (Grunow).

C. symbolophorus. Grun., *Denk. Wien. Ak.*, 1884, p. 82, pl. iv. (D), figs. 3–6.—Diam. ·085 to ·175 mm. Surface convex. Colour at centre steel grey, an opaque ring towards the semiradius, whence dark radial bands proceed outwards. Central space small, and usually branching into a distinct star with 3 to 5 rays, sometimes absent, but around the central area a similar star with 3 to 5 lanceolate rays. Markings polygonal, decreasing regularly outwards; towards the centre 6, towards the border 8 to 9 in ·01 mm.; fasciculate, those composing each fasciculus parallel to the radial row at its middle; secondary oblique decussating rows faint, often somewhat curved outwards. Border narrow, hyaline.—*Symbolophora*, sp. Ehrb., Grun., *ibid.*, 1884, p. 82; *S. microtrias*, Ehrb., *Mon. Ber. Ak.*, 1884, p. 205; *Mikrog.*, pl. xxxv. A. 21. fig. 16; *S. tetras*, Ehrb., *ibid.*, 1844; p. 205; *Abh. Ber. Ak.*, 1872, pl. xii. 2. fig. 1; *S. pentas*, Ehrb., *ibid.*, 1844, p. 205; *Mikrog.*, pl. xxxv. A. 22. fig. 19; *S. hexas*, Ehrb.,

* In the collection of F. W. Griffin. Mr Grove remarks that he has never seen this species in the Oamaru deposit, and greatly doubts its presence there, it being a form which may easily adhere to tubes and beakers.

1844, p. 205; *S. microtetras*,* Ehrb., *Mon. Ber. Ak.*, 1855, p. 302; *S. micropentas*,* Ehrb., *ibid*, 1855, p. 302; *S. microhexas*,* Ehrb., *ibid.*, 1855, p. 302; not *S. trinitatis*, Ehrb., *ibid.*, 1844, p. 88; Ralfs in *Pritch. Inf.*, p. 833, pl. xi. fig. 36, as indicated in the second edition of Habirshaw's Catalogue, § *Symbolophora.*

By Grunow this species is brought near *C. subtilis*, but it is readily distinguished from the latter by the central star and the absence of apiculi. *Symbolophora acuta*, Ehrb., from Hollis Cliff, Virginia, of which but a fragment is figured (*Mikrog.*, pl. xxxiii. 15. fig. 21), probably belongs to the present species. *S. euprepia*, Ehrb., from Licuare River, coast land Mozambique, remains a *nomen nudum*, but may come here (see Ehrb., *Mikrog.*, p. 228). Small specimens of *C. symbolophorus* approach *C. excentricus.*

Habitat.—Nottingham deposit, Mors deposit (Cleve! Grunow); Simbirsk Polirschiefer (Ehrenberg, Grunow); Kékkö, Szakal and Szent Peter deposits (Pantocsek); Franz Josef's Land (Grunow); pancake ice, Antarctic Ice Barrier, and sounding of 190 fms., lat. 78° 10′ S., long. 162° W.; sounding 270 fms., lat. 63° 40′ S., long. 55° W. (Hooker); Ananino deposit (Grove! Kinker! Hardman!);† Lumfiord, Jutland (Hardman!); Bermuda tripoli (Greville!); Oamaru deposit (Grove! Marshall!); Ananino deposit (Rae! Deby!).

C. stellaris. Roper, *Quart. Jour. Micr. Sci.*, 1858, p. 21, pl. iii. fig. 3.—Diam. ·075 to ·1 mm. Surface slightly convex. Markings, around the centre 5 or 6 large areolæ, at subequal distances apart, and tapering towards both ends, elsewhere scarcely visible, 16 to 20 in ·01 mm., angular, most evident towards the centre, the rows forming fasciculi, those in each fasciculus parallel to the radial row at its middle.—*C. stellaris*, var. Cstr., *Diat., Chall. Exped.*, p. 155, pl. iii. fig. 2; *C. stellaris*, var. *fasciculata*, Cstr., *Diat., Chall. Exped.*, 1886, p. 158, pl. v. fig. 9.

The colour when dry is brownish. Castracane's Antarctic frustule seems to differ only in having four markings at the centre. Transitional forms to *C. symbolophorus* occur.

Habitat.—Caldy, Pembrokeshire (Roper! Rev. J. Guillemard);

* *Nomina nuda*, probably identical with *S. tetras, S. pentas, S. hexas.*

† In the collection of Julien Deby.

Tenby Bay (Roper!);* oyster shells, Dublin Bay (O'Meara); Mediterranean (Grunow); Balearic Islands (Weissflog!); Antarctic Ice Barrier, H.M.S. Challenger (Castracane).

Var. *Mejillonis.* Grun., *Denk. Wien. Ak.*, 1884, p. 82.—Diam. ·23 mm. Markings 18 in ·01 mm., central rosette with 8 large areolæ.

Habitat.—Mejillones guano (Grunow, Grove!).

C. minutellus, sp. n.—Diam. ·0225 mm. Surface flat. Central space and rosette absent; a minute, somewhat excentric triangular clear area (·0025 mm. broad) with a single central dot evident. Markings polygonal, 10 in ·01 mm., still smaller towards the border; rows faintly fasciculate, subradial, the secondary oblique outwardly concave, decussating rows more distinct; apiculi at the border prominent, at subequal intervals of ·0076 mm., or somewhat less.—Border narrow, hyaline.—(Pl. II. fig. 5.)

Habitat.—From *Salpa spinosa*, locality? (Weissflog!).

C. subtilis. Ehrb., *Abh. Ber. Ak.*, 1841, p. 412, pl. i. 3. fig. 18; pl. iii. 7. fig. 4.—Diam. ·0425 to ·1125 mm. Central space and rosette absent. Markings polygonal, 6 to 10 in ·01 mm., decreasing somewhat towards the border, without order on a small central area, elsewhere in fasciculate rows, about 12 forming each fasciculus at its outer extremity, and arranged parallel to the central radial row; secondary oblique decussating rows evident; apiculi sometimes present at the border, interfasciculate. Border striæ delicate, faint, 12 to 14 in. ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 35, *a.b.*; pl. xxxiii. 13. fig. 4; pl. xxxiii. 16. fig. 7; pl. xxxiv. 7. fig. 6; pl. xxxv. 22. fig. 5; pl. xxxv. 23. fig. 5. Grev., *Quart. Jour. Micr. Sci.*, 1859, p. 81; Ralfs in *Pritch. Inf.*, p. 830; Jan., *Abh. Sch. Ges. väter. Cult.*, 1862, Heft ii. p. 4, pl. i. A, fig. 2; Janisch, *Gazelle Exped.*, taf. ii. fig. 8; iv. figs. 1, 2; v. figs. 5, 7; vi. figs. 1, 5; xx. fig. 5; Grun., *Sitzungsb. naturw. Ges. Isis., Dresden*, 1878, p. 124; Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, ii. p. 94; Sch., *Atl.*, pl. lvii. figs. 11, 13, 28, 29 (no name); pl. lviii. fig. 37 (no name); Grun., *Denk. Wien. Ak.*, 1884, p. 81,

* In the collections of Dr Greville and E. Grove.

pl. ii. (C), fig. 26; Raben., *Alg. Europ.*, Nos. 2142, 2487, 2558; Van Heurck, *Syn. Diat. Belg.*, p. 218, pl. cxxxi. fig. 1; *Typ. Syn. Diat. Belg.*, Nos. 519, 520, 532, 533; Cleve and Möll., *Diat.*, Nos. 57, 125, 162, 164, 207, 211, 257, 258, 319; H. L. Smith, *Diat. Spec. Typ.*, No. 100; *C. subtilis?*, Sch., *Atl.*, pl. lvii. fig. 12 (excl. *C. radiolatus?* = *C. subtilis*, Ehrb., *Mikrog*, pl. xxii. fig. 4).

According to Ehrenberg's original definition, there should be 12 markings in ·01 mm. Fasciculi were hardly indicated in his figures, several of which approach his *C. intermedius*. There is no close affinity to *C. punctatus*, Ehrb., as he at one time believed (*Mon. Ber. Ak.*, 1844, pp. 186, 188–191). The species approaches *C. Normani*, Greg., but in the latter the markings are less regular, the fasciculi at the border consist of about 6 rows instead of 12, and the lines between the fasciculi are less distinct. Janisch erroneously describes the markings as round. Schmidt, in 1874, erroneously refers to the fasciculi as branched towards the same side by a bent ray. Specimens are sometimes confounded with *C. odontophorus*, Grun., and *C. Rothii*, Grun., and are distinguished from *C. symmetricus*, Grev., and *C. denarius*, Sch., only by their smaller markings, to which transitional forms occur, Grunow having observed forms from Monterey and Australia with 5½ to 7 markings in ·01 mm.

Habitat.—Stratford (Ehrenberg); Richmond, Va. (Erhenberg, Rae! Cleve and Möller!); Caltanisetta (Ehrenberg); Moron deposit (Schmidt); Peruvian guano (Janisch, Schmidt); Angamos guano (Janisch); Bolivian guano (Grunow); Patos guano (Greville!); Assistance Bay, San Francisco (Ehrenberg); lat. 71° 19′ N., long. 11° 28′ W., 129 fms., in yellowish-grey mud; lat. 73° 16′ N., long. 15° 48′ W., 1300 fms., in dark greyish-brown mud; lat. 63° 40′ N., long. 5° 28′ E., 569 fms., in light grey fine mud; lat. 74° 11′ N., long. 15° 19′ W., 224 fms., in fine greyish-brown mud; lat. 74° 33′ N., long. 18° 39′ W., 90 fms., in coarse sandy mud (Ehrenberg);* pancake ice, Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W.; melted ice, lat. 75° S., long. 170° W.; snow and ice, near Vancouver Island, lat. 70° S., long. 165° W.; floating ice, lat. 64° S., long. 160° W.; Gulf of Erebus and Terror, lat. 63° 40′ S., long. 55° W., 207 fms.; ex *Salpâ*, lat. 66° S., long. 157° W.

* Specimens procured by the second German North Polar Expedition.

(Hooker); Canton (Ehrenberg); Yokohama and Arica (Schmidt); *Ascidia*, Hull (Greville !), rice fields, Georgia (Grove ! Bailey,* Greville !) Yszee (Kinker !); dredged in 28 fms., Royal Sound, Kerguelen, by H.M.S. Challenger (Rae !);† "Antarctic Ocean" (Cleve and Möller !); rice field mud, Savannah (Cleve ! H. L. Smith !) Cambodia (Hardman !‡ Firth !); Port Elizabeth (Hardman !); Humber (O'Meara !); Richmond Tunnel (O'Meara !); "India" (Macrae !); § Holstein (Van Heurck !); Maryland (O'Meara !); Japan (H. L. Smith !); Los Angelos deposit (O'Meara !); Cannibal Islands (Greville !); Gazelle Expedition (Weissflog !); Macassar Straits (Grove !); mud from Glückstadt; Elbe, above Cuxhaven (Rabenhorst and Schwarz !); Archangelsk (Cleve); Campeachy Bank, Gulf of Mexico (Rabenhorst and Gerstenberger !); Rappahannock River, Va. (Rogers ! Greville !); on Dutch rushes, Hull (Norman !); Virginia (Greville !); Indian Ocean soundings, Capt. Pullen, 2200 fms. (Greville !); Kannahack, Cannibal Islands (Greville !); Woodlark Island (Roberts !); Bass Straits (Greville !); Kamtschatka, 1700 fms. (Greville !); Jersey (Wallich !); Patos Island guano (Greville !); shell cleanings from Singapore (Hardman !); Nicobar shell cleanings (Doeg !); Santa Marta deposit (Doeg !); coral washings, Mauritius (Doeg !); Newcastle deposit, Barbados (Doeg !); N. Atlantic, lat. 51° 20′ N., long. 52° 25′ W., 232 fms. (O'Meara !); Wexford (O'Meara !); Faeroe Isles (Grove !); Monkstown (O'Meara !); Jack's Ranch California (Macrae !); Monterey (Schmidt); Upolu (Weissflog); Nancoori; Sta Monica deposit; Patagonia; Delaware; North Carolina; Pensacola; Cape Wankarema (Cleve ! Cleve and Möller !); Balearic Islands (Cleve !); Greenland, Yeddo, Franz Josef's Land, Peruvian guano, Pabillan di Pico guano, Patagonian guano, Schleswig Holstein; Elbing, West Prussia; Labuan, Virgin Islands (Cleve !).

Var. *siberica*. Grun., *Kongl. Sv. Vet.-Ak. Handl. Stockh.*,

* Found in various localities in United States in 1850. Its presence, with other brackish or salt water species in the rice fields, has been held to ndicate the presence of salt water much further up the river formerly than at present.

† In the collection of Dr F. W. Griffin.

‡ In the collection of Julien Deby.

§ In the collection of Dr Greville.

1880, No. 2, p. 115.—Diam. ·044 mm. Markings more delicate, 15 to 16 rows in ·01 mm., irregular on a small central area; fasciculi numerous, each consisting of about 12 rows; apiculi absent.

This forms transition to *C. glacialis.*

Habitat.—Jenissey (Cleve and Grunow).

Var. *lineolata,* nov.—Diam. ·0575 mm. Central space subcircular, from this a single narrow straight line passing to the border and at intervals of about $\frac{1}{3}$ of the surface, two other pairs of straight rows in contact with one another. Markings in irregular inconspicuous fasciculi; apiculi at the border, inter-fasciculate; indistinct.—(Pl. I. fig. 16.)

Grunow has regarded this var. as an abnormal form of *C. subtilis.*

Habitat.—Mors Island (Weissflog!).

Var. *scabra,* nov.—Diam. ·0375 to ·055 mm. Central space absent. Markings 12 to 14 in ·01 mm., somewhat smaller towards the border; fasciculi inconspicuous; apiculi few, indistinct at wide intervals, not between each pair of fasciculi; minute scattered hyaline specks (apiculi?) most numerous towards the centre.—(Pl. III. fig. 6.)

Habitat.—Nancoori (Weissflog!).

C. whampoensis. Grove M.S.—Diam. ·075 mm. Surface with a distinct narrow elevated ring about $\frac{2}{3}$ of the radius from the centre. Central space and rosette absent. Markings hexagonal, decreasing slightly towards the border; towards the centre 8 to 10, towards the border 12 in ·01 mm.; irregular on a minute round central area, elsewhere in substraight fasciculate rows, deflected slightly near the border, those in each fasciculus parallel to that at or near its middle; apiculi minute, inserted at the border at wide intervals of ·0175 mm. or more, between or upon the fasciculi. Border narrow, hyaline.—(Pl. I. fig. 24.)

The elevated zone, the less regular appearance of the fasciculi at their outer ends, and the distant apiculi, distinguish this species from *C. subtilis* and *C. Rothii.*

Habitat.—Canton River, Whampoa (Grove!).

C. odontophorus. *C.* (*subtilis* var.?) *odontophorus*, Grun., *Denk. Wien. Ak.*, 1884, p. 82, pl. iii. (C), fig. 24.—Diam. ·05 to ·175 mm. Surface slightly concave at the centre, and convex towards the border. Central space and rosette absent. Markings polygonal, 6 in ·01 mm.; punctiform on a distinct narrow zone extending outwards from the apiculi, thence gradually becoming more delicate and indistinct to the border; rows irregularly fasciculate, those in each fasciculus subparallel to that at its centre, or to a row near one of its edges; secondary oblique decussating rows distinct, most evident on the narrow zone at the apiculi; apiculi prominent, inserted at unequal intervals upon or between the outer ends of the fasciculi, at a considerable distance from the border. Border distinct, narrow; striæ delicate, 8 in ·01 mm.

Distinguished from *C. subtilis* by the markings and apiculi.

Habitat.—California deposits (Grunow); "chalk beds between White Plains and Hot Spring Station," California (Grove! Kinker!).

C. glacialis. *C.* (*subtilis*, var.?) *glacialis*, Grun., *Denk. Wien. Ak.*, 1884, p. 82, pl. iii. (C), fig. 27.—Diam. ·0215 mm. Surface slightly convex. Central space and rosette absent. Markings polygonal, 15 to 16 in ·01 mm., on a small rounded central area, irregular, elsewhere forming 8 broad radiating fasciculi; apiculi at border minute, interfasciculate.

Habitat.—Under side of iceberg, lat. 74° 48′ 4″ N., long. 54° 52′ 8″ E., Aug. 1872 (Grunow).

C. polyacanthus. Grun., *Kong. Sv. Vet.-Ak. Handl. Stockh.*, 1880, No. 2, p. 112, pl. vii. fig. 127.—Diam. ·026 mm. Central space absent. Markings polygonal, minute, irregular on a small indistinct central area, elsewhere in fasciculate rows, the rows composing each fasciculus 12, parallel to that at the middle, 15 to 16 in ·01 mm.; secondary oblique decussating rows faint, apiculi distinct, numerous, about 5 in ·01 mm., placed upon and between the outer ends of the fasciculi. Border narrow, hyaline.—*Odontodiscus polyacanthus*, Grun., *ibid.*, 1880, p. 112.

Distinguished from *C. Rothii* and *C. odontodiscus* by the more numerous and stronger apiculi.

Habitat.—Jamal (Cleve and Grunow); Franz Josef's Land

(Grunow); Baltic Sea (Grunow); Tindingen (Cleve!); San Benito deposit, California? (Grunow!?).

Var. *davisiana.* Grun., *Denk. Wien. Ak.*, 1884, p. 81, pl. iii. (C), fig. 19.—Diam. ·035 mm. Markings larger, hexagonal, 10 in ·01 mm.; fasciculi less evident, secondary oblique rows more indistinct; apiculi closer to the border, arranged in two concentric rows.

Habitat.—Davis Straits, Franz Josef's Land (Grunow).

Var. *intermedia.* Grun., *ibid.*, 1884, p. 81, pl. iii. (C), fig 25.—Diam. ·06 mm. Markings in evident fasciculi, the secondary oblique rows distinct; apiculi distinct, less numerous, at some distance from the border, forming a single row, and inserted at the middle of and between the fasciculi.

Habitat.—Cape Wankarema, N. Siberia (Grunow).

Var. *baltica.* Grun., *ibid.*, 1880, No. 2, p. 112.—*C. polyacanthus*, Grun., *Sitzungsb naturw. Ges. Isis, Dresden*, 1878, p. 125.—Diam. ·03 to ·1 mm. Central space minute and irregular, sometimes distinct, about $\frac{1}{21}$ of diam. broad, having minute isolated granules. Markings somewhat larger, rows fasciculate, 12 to 14 in ·01 mm., straight and regular; apiculi some distance from the border, of variable size, especially among larger specimens, arranged usually in two subconcentric indistinct rows.—Cleve and Möller, *Diat.*, No. 237; *C. polyacanthus*, var.? *baltica*, Grun., *ibid.*, 1884, p. 81, pl. iii. (C), figs. 17*a*, *b*; *C. balticus*, Grun., in Cleve's Coll.

Though this var. was originally named *C. polyacanthus* in 1878, it remained imperfectly defined, *C. polyacanthus* being subsequently diagnosed from Jamal valves; the original *C. polyacanthus* being at the same time (1880) reduced to var. *baltica.*

Habitat.—Waxholm, Hernösand; Rathan, Baltic Sea (Cleve! Grunow); Baltic bottom clay, black clay from Roslagen, silt from Ronneby, Dalarö, Furusund, Norrteljie (Juhlin-Dannfelt).

C. divisus. Grun., *Sitzungsb. naturw. Ges. Isis, Dresden*, 1878, p. 125.—Diam. ·08 mm. Central space indistinct, rounded, with numerous isolated granules, about $\frac{1}{10}$ of diam. broad. Markings round, granular about the central space, elsewhere polygonal; about 10 in ·01 mm., decreasing slightly outwards, on a distinct band

around the border, minute, 15 to 16 in ·01 mm.; rows fasciculate, the fasciculi consisting of 8 to 10 rows, their sides almost straight or more curved; apiculi interfasciculate, minute, inserted at inner edge of marginal band.—*C.* (*curvatulus*, var.?) *divisus*, Grun., *Denk. Wien. Ak.*, 1884, p. 83, pl. iv. (D), fig. 16).

In 1878 Grunow distinguished as var. *arcuata* specimens with the edges of the fasciculi somewhat bent, those of the type being straight.

Habitat.—Peruvian guano, on *Macrocystis* and *Lessonia* from the coast of Peru (Grunow).

C. Normani, Greg., in Grev., *Quart. Jour. Micr. Sci.*, 1859, p. 80, pl. vi. fig. 3.—Diam. ·0625 to ·1125 mm. Surface slightly convex. Central space and rosette absent. Markings polygonal, about 8 in ·01 mm., decreasing somewhat towards the border; rows radial, fasciculate, converging slightly towards the periphery, towards the border 6 rows composing each fasciculus; apiculi absent or minute. Border with delicate striæ of closely placed punctiform markings.—Janisch, *Gazelle Exped.*, taf. v. fig. 6; *C. fasciculatus*; Sch.,* *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer., Kiel*, 1874, ii. p. 94; Sch., *Atl.*, pl. lvii. figs. 9, 10; *Odontodiscus subtilis*, Grun., in Sch., *ibid.*, 1874, p. 95; *C. subtilis*, var. *Normanii*, Van Heurck, *Syn. Diat. Belg.*, p. 218; *C. normanicus*, Van Heurck, *ibid.*, Explan. pl. cxxxi. fig. 1; in Van Heurck, *Typ. Syn. Diat. Belg.*, No. 532; *C. subtilis*, Ehrb., *Eul. Diat. Sp. Typ.*, No. 115 (*fide* Van Heurck); *C. curvatulus*, var. *Normanii*, Cleve, *Vega Exped. Vetensk. Jakttag. Stockh.*, 1883, Bd. iii. p. 488.

An undulation of the surface, resulting from greater prominence of the lines between the fasciculi, as referred to by Greville, is not constant. Well-preserved specimens of *C. fasciculatus*, Sch., show indications of minute processes.

Habitat.—Ex *Ascidiis*, Hull (Norman); Roundstone Bay, co. Galway (O'Meara); Dutch rushes from Holland (?); Arran Islands, co. Galway (O'Meara); Cuxhaven (Schmidt); marshy ground, Wedel (Grunow); locality ? (Kinker !); Richmond tunnel (O'Meara !); Holstein (Van Heurck !); Richmond, Va. (Rae !); Californian guano (Norman !);† Arafura Sea, H.M.S. Challenger (Doeg !).

* Not *C. fasciculatus*, O'Me., *Quart. Jour. Micr. Sci.*, 1867, p. 245, pl. vii. fig. 1.

† In the collection of Dr Greville.

C. marginulatus, Rattray. *C. marginulatus*, var. *gallopagensis*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. xciv. fig. 30.—Diam. ·0315 mm. Central space and rosette absent. Markings punctiform; rows straight, inconspicuous, fasciculate; those in each fasciculus parallel to the central row; at intervals evident, radial striæ not quite reaching the centre; apiculi at the border minute, at somewhat regular intervals. Border broad, striæ evident, 8 in ·01 mm.

Habitat.—Galapagos Islands (Van Heurck).

Var. *curvato-striata*, Grun. Sch., *Atl.*, pl. lvii. fig. 5.—Diam. ·045 to ·05 mm. Central space $\frac{1}{18}$ to $\frac{1}{20}$ of diam. broad, hyaline. Fasciculi numerous, almost straight or curved; apiculi minute, sometimes hardly visible. Border striæ 8 to 10 in ·01 mm.—Van Heurck, *Syn. Diat. Belg.*, pl. xciv. fig. 32.

In Moron (?) valves, in Weissflog's collection, the striæ on the border are still more delicate, 12 to 14 in ·01 mm.

Habitat.—Campeachy Bay (Weissflog! Van Heurck); Moron (?) deposit (Weissflog?).

Var. *stellulifera*, Grun. Van Heurck, *ibid.*, pl. xciv. fig. 34.—Diam. ·02 mm. Central space as in var. *curvato-striata*. Fasciculi separated around the central space by short straight lines, indistinct on the outer half; apiculi less evident. Border striæ more delicate.

Habitat.—Campeachy Bay (Von Heurck).

Var. *sparsa*, Grun. Van Heurck, *ibid.*, pl. xciv. fig. 31.—Diam. ·035 mm. Central space absent. Markings irregular; apiculi absent. Border striæ distinct.

The var. *campechiana*, Grun. (Van Heurck, *ibid.*, pl. xciv. fig. 33), from Campeachy Bay, differs only in having evident apiculi at the border.

Habitat.—Campeachy Bay (Van Heurck).

C. angulatus, Grev., *Trans. Micr. Soc. Lond.*, 1864, p. 9, pl. ii. fig. 11.—Diam. ·075 mm. Surface flat, showing an octagonal figure at inner edge of border. Central space and rosette absent. Markings polygonal, $3\frac{1}{2}$ to 4 in ·01 mm., slightly smaller at the border; the rows straight parallel to those passing from the centre to the apiculi, secondary oblique rows less evident. Apiculi

obvious, placed at the angles of the octagon. Border with its inner edge distinct, about $\frac{1}{15}$ of radius broad, striæ evident, 4 to 5 in ·01 mm.

Habitat.—Cambridge deposit, Barbados (Cleve, Greville!); Oamaru deposit (Grove!).

C. Rothii. Grun., *Denk. Wien. Ak.*, 1884, p. 29, pl. iii. (C), figs. 20*a*, 20*b*, 22.—Diam. ·07 to ·175 mm. Surface with faint undulations. Central space absent. Markings polygonal, 6 to 8 in ·01 mm., decreasing slightly towards the border, irregular on a small central area, elsewhere in straight fasciculate rows, subparallel to that at centre of each fasciculus or subradial; apiculi small, placed at the middle of the outer ends of each fasciculus. Border distinct, with uniform striæ, 14 in ·01 mm.—Cleve and Möll., *Diat.*, No. 57; *C. Rothii forma minor*, Grun.; Van Heurck, *Typ. Syn. Diat. Belg.*, No. 533; *C. symmetricus*, Kitton and Weissflog (not Grev.), Sch., *Atl.*, pl. lvii. figs. 25, 26, 27; *Heterostephania Rothii* (α) *octonaria*, Ehrb., *Mikrog.*, pl. XXXV. A, 13B. fig. 4; *H. Rothii* (β) *denaria*, Ehrb., *ibid.*, pl. XXV. A, 13B. fig. 5; *H. Rothii*, Ralfs in *Pritch. Inf.*, p. 833., pl. v. fig. 33.

Ehrenberg established but did not define *Heterostephania*, of which the only known species was *H. Rothii;* his forms *octonaria* and *denaria* founded only on the number of fasciculi may be abandoned. *C. Rothii* is sometimes distinguished from *C. subtilis* by the smaller number of rows in each fasciculus, and especially by the position of the apiculi. *C. Rothii forma minor*, Grun., differs only in its small size, ·025 to ·0375 mm. in diam.

Habitat.—Elbe Tertiary mud (Ehrenberg).* Ceylon (Macrae!);† Caspian Sea (Grunow); locality? (Rae!). Porto Seguro (Hardman!); Manilla (Firth!); Amboina shell scrapings (Kinker!); surface, Arafura Sea, H.M.S. Challenger (Rae!); Oamaru deposit (Marshall!);‡ Chalky Mt., Barbados (Firth!);‡ Para River, S. America (Hardman!);§ India (Macrae!);‖ Antwerp (Van Heurck!); Oamaru deposit (Grove!); Whampoa (Grove!); rice fields, Georgia (Greville!);¶ Cambridge deposit, Barbadoes (Johnson!);‖ Rio Janeiro

* This diluvial formation was discovered by Roth.
† In the collection of Dr Greville. ‡ In the collection of Dr F. W. Griffin.
§ In the collection of Julien Deby. ‖ In the collection of Dr Greville.
¶ Rare, amongst abundance of *C. subtilis.*

(Weissflog !); Curtis Straits (Roberts !); Richmond (Cleve and Möller !); Successful Bay, Kerguelen (Cleve !); Virgin Islands (Cleve !).

Var. *singaporensis*, nov.—Diam. ·085 mm. Central rosette evident, large. Markings 4 in ·01 mm. Adjacent to the border a sharply defined broad band with smaller markings, 6 in ·01 mm. Apiculi large, with a median constriction and rounded extremities, inserted at inner edge of the marginal band. Border narrow, striæ 6 in ·01 mm.

Habitat.—Singapore (Schmidt).

Var. *actinocycloides*. *C. actinocycloides*, Pant., *Fossil. Bacil. Ung.*, p. 71, pl. ix. fig, 72.—Diam. ·075 to ·1125 mm. Surface flat towards centre, slightly convex towards the border. Central space small, subcircular, punctate. Markings 6 towards the border, gradually diminishing to 8 in ·01 mm.; rows parallel to that at centre of each fasciculus, secondary oblique decussating rows straight, the fasciculi separated by evident rows of small subulate hyaline interspaces; apiculi distinct.

Habitat—Kékkö, Szakal, Felsö-Esztergály deposits (Pantocsek !); Kékkö deposit (Grove !); Szent Peter deposit (Grove !).

Var. *grandiuscula*, nov. Sp. n.? Sch., *Atl.*, pl. lvii. fig. 23.—Diam. ·04 mm. Markings 6 in ·01 mm. Apiculi prominent, placed at a considerable distance from the border.

Habitat.—Rio de Janeiro (Schmidt).

C. doljensis. Pant., *Fossil. Bacil. Ung.*, p.72, pl. xii. p. 10 5. —Diam. ·036 to ·1 mm. Surface slightly convex towards the border. Central space minute, indefinite, with minute isolated or subangular granules. Markings delicate, 12 to 15 in ·01 mm., somewhat less crowded towards the centre, towards the border punctiform; rows radial and subparallel, obscurely fasciculate; minute subulate hyaline spaces opposite origin of the shorter rows, on a distinct band adjacent to the border, the oblique decussating rows more manifest. Apiculi prominent at intervals of ·006 to ·01 mm. Border narrow, hyaline.

Habitat.—Dolje Klebschiefer (Pantocsek !).

C. barbadensis. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 43, pl. iv. fig. 9.—Diam. ·035 mm. Surface flat. Central space absent. Markings polygonal, 8 in ·01 mm., 9 subsymmetrical prominent rows proceeding from the centre to the border, the intervening rows subradial.

This species, like *C. senarius*, Sch. (*Atl.*, pl. lvii. fig. 24), forms the transition to *Aulacodiscus*. In the second edition of Habirshaw's Catalogue, the unnamed Springfield valves figured by Schmidt (*Atl.*, pl. lvii. fig. 32), subsequently justly separated by Grunow as *C. semipennatus* (*Denk. Wien. Ak.*, 1884, p. 83), are erroneously associated with this species.

Habitat.—Barbados deposit (Greville !).

C. Gregorii. O'Me., *Proc. Roy. Ir. Acad.*, 1875, p. 263, pl. xxvi. fig. 23.—Central space small, angular. Markings subquadrangular, smaller and more equal than in *C. nitidus*, Greg.; rows radial from the angles of the central space, fasciculate; those in each fasciculus parallel to that at its centre or subradial.

This is not *Campylodiscus?* an *Coscinodiscus?* Greg., from Glenshira Sand (*Trans. Micr. Soc. Lond.*, 1857, p. 84, pl. i. fig. 50), as stated by O'Meara, Gregory's specimens being devoid of a central space, and having large rounded sparsely placed markings in rows partly parallel and partly radial, among the rows a broad rectangular cross being faintly visible.

The *C. semipennatus*, Grun. (Sch., *Atl.*, pl. lvii. figs. 32, 32*), from Barbados, is not so close to *C. Gregorii*, O'Me., as it is to Gregory's valves. *C. Gregorii* differs from *C. senarius*, Sch., in the presence of a central space, and in having less evident rays between the fasciculi.

Habitat.—Arran Island; *Ascidia*, Roundstone Bay, co. Galway; *Ascidia*, co. Clare (O'Meara).

C. denarius. Sch., *Atl.*, pl. lvii. figs. 19, 20, 21.—Diam. ·053 to ·0755 mm. Central space absent. Markings polygonal, equal, $3\frac{1}{2}$ to 4 in ·01 mm., rows fasciculate, those of each fasciculus parallel to the radial row at its centre, secondary straight oblique decussating rows obvious. Border striæ sometimes distinct, 8 in ·01 mm.—*C.*

* In the collection of Dr Griffin, and procured in the original sample sent to Firth by Kitton.

denarius, var., Sch., *Atl.*, pl. lvii. fig. 22; *C.* (*symmetricus*, var.) *denarius*, Sch., Cleve and Grun. in Moll., *Diat.*, No. 183.

The fasciculi sometimes do not reach the centre, because of the extension inwards of the rows belonging to the adjacent fasciculi. Distinguished from *C. subtilis* by the larger more uniform markings.

Habitat.—Springfield deposit, Barbados (Doeg! Schmidt!); Campeachy Bank, Sansego (Schmidt); Cambridge deposit, Barbados (Greville!); Chalky Mount, Barbados (Firth!);* Antarctic Ocean (Cleve and Möller!).

Var. *variolata.* *C. variolatus*, Cstr., *Diat. Chall. Exped.*, p. 155, pl. ii. fig. 5.—Diam. ·068 mm. Surface spotted at wide irregular intervals with small groups of more prominent granules. Border narrow, hyaline.

Habitat.—Phillipine Islands, H.M.S. Challenger (Castracane).

Coscinodiscus? Ehrb., *Abh. Ber. Ak.*, 1871, p. 140, pl. i. (B), fig. 20.—A minute fragment showing closely placed angular markings in straight fasciculate rows, which are parallel to that at the centre of the fasciculus. Border hyaline, distinct.

In the second edition of Habirshaw's Catalogue, this is associated with a valve indicated as *Coscinodiscus*, sp.? Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 95, pl. iii. fig. 42), but it is more nearly allied to *C. denarius*, var., Sch. (*Atl.*, pl. lvii. fig. 19), from Sansego.

C. senarius. Sch., *Atl.*, pl. lvii. fig. 24.—Diam. ·04 mm. Central space absent. Markings polygonal, equal, 4 in ·01 mm.; rows fasciculate, those composing each fasciculus parallel to that at its middle, the interfasciculate rows most prominent; secondary oblique decussating rows straight, non-apiculate. Border narrow, striæ delicate, 12 to 14 in ·01 mm.—Janisch, *Gazelle Exped.*, taf. vi. fig. 5.

Habitat.—Springfield deposit, Barbados (Schmidt).

C. partitus. Grove and Sturt MS.—Diam. ·05 mm. Central space minute, rosette absent. Markings subactinocycloid, rounded, granular, towards centre 6, towards border more crowded, subpunctiform, 8 in ·01 mm.; rows straight, fasciculate, inconspicuous,

those in each fasciculus parallel to the central row; interfasciculate radial rows most evident, secondary oblique decussating rows most obvious towards border; interspaces minute, most evident towards centre; apiculi distinct, inserted a short distance from border. interfasciculate. Border narrow, striæ delicate, 10 in ·01 mm.—Cleve and Möll, *Diat.*, Nos. 114, 162.—(Pl. III. fig. 5.)

Habitat.—Totara, Oamaru (Grove !); Mascara, Nancoori (Cleve and Möller !).

C. extravagans. Sch., *Atl.*, pl. lviii. fig. 33.—Diam. ·053 mm. Central space distinct, circular, about $\frac{1}{16}$ of diam. broad. Markings small, granular, about 6 in ·01 mm., more crowded, somewhat smaller on a distinct marginal zone about $\frac{1}{8}$ to $\frac{1}{9}$ of radius broad; rows radial, fasciculate, those between the fasciculi most prominent; those composing each fasciculus parallel to the radial row at its middle, secondary oblique rows evident on the marginal zone; interspaces hyaline at the inner ends of the shorter rows; apiculi large, conical, interfasciculate, inserted at inner edge of marginal zone. Border hyaline.

Habitat.—Yokohama (Schmidt).

C. interlineatus, sp. n.—Diam. ·06 mm. Surface flat. Central space and rosette absent. Markings hexagonal, 8 to 10 in ·01 mm., somewhat smaller towards the border, rows fasciculate, those in each fasciculus parallel to that at its middle; secondary oblique decussating rows straight, slightly flexuous or concave outwards, obvious; between each fasciculus a distinct radial row, the fasciculi 7, unequal; apiculi evident, interfasciculate, inserted at the border. Border indistinct.—(Pl. I. fig. 6.)

Differs from *C. senarius*, Sch., by the more delicate markings and apiculi.

Habitat.—Newcastle deposit, Barbados (Weissflog !); Nancoori deposit (Cleve !).

C. actinosus, Grove MS.—Diam. ·06 mm. Surface slightly convex near the border. Central space inconspicuous, round, with rounded granules at its middle. Markings actinocycloid, round, granular, faint; towards the border angular and in contact, towards the centre 8, towards the border 10 in ·01 mm.; interspaces small and

hyaline towards the centre, most evident opposite origin of shorter rows; rows fasciculate, straight, those in each fasciculus parallel to that at its centre, interfasciculate radial rows evident; secondary oblique decussating rows most evident towards the border. Border distinct, striæ delicate, 14 to 16 in ·01 mm.—(Pl. II. fig. 7.)

Habitat.—Manilla Algæ (Grove !).

C. obnubilus, Rattray. *C. umbonatus*,* Cstr., *Diat., Chall. Exped.*, p. 156, pl. ii. fig. 8.—Diam. ·077 mm. Surface rising steeply from the centre for about $\frac{1}{3}$ of radius, thence descending rapidly outwards, becoming slightly concave, near the border flat. Central space subcircular, about $\frac{1}{12}$ of diameter broad, with several isolated granules at its middle. Markings punctiform, subequal, about 8 in ·01 mm., most crowded towards the border; rows fasciculate, those composing each fasciculus parallel to the radial rows at one of its sides or subradial; interspaces small, hyaline, opposite the origin of the shorter rows; apiculi distinct, inserted at the border, interfasciculate. Border distinct, narrow.

Habitat.—Pacific Ocean, 2900 fms., H.M.S. Challenger (Castracane).

§ VI. Radiati, Grun., *Denk. Wien. Ak.*, 1884, p. 70; Pant., *Fossil. Bacil. Ung.*, p. 69. *Pseudostephanodiscus*, Grun., *ibid.*, p. 85. *Clivosi*, Pant., *ibid.*, p. 72. *Eleganti*, Pant., *ibid.*, p. 73.

Surface flat, rarely undulate, centre sometimes depressed. Markings rounded or areolate; rows radial, sometimes faintly fasciculate towards the border; apiculi sometimes present.

C. diversus. Grun., *Denk. Wien. Ak.*, 1884, p. 72.—Diam. ·07 to ·135 mm. Central space absent. Markings rounded, pearly, with hyaline interspaces from the centre for $\frac{1}{2}$ to $\frac{3}{5}$ of radius, increasing slightly outwards; on the outer portion polygonal, $2\frac{1}{2}$ to 3, at the border 6 in ·01 mm.; central papillæ distinct; secondary oblique rows indistinct. Border with inner edge indistinct; striæ obvious, radial or subradial, about 6 in ·01 mm.—Sch., *Atl.*, pl. lxii. figs. 13–15 (without name); *C. caraibicus*, Tru. and Witt., *Jeremie Diat.*, p. 13, pl. ii. fig. 3.

* Name preoccupied by Gregory for a different species.

Grunow considers that this may be an abnormal form of *C. radiatus*. To me it seems to be more allied to *C. marginatus*.

Habitat.—Springfield deposit, Barbados, and Cambridge deposit, Barbados (Schmidt, Hardman !);* "Barbados deposit" (Rae ! Greville !).

Var. *completa.*—Diam. ·1125 to ·14 mm. Central space small, angular, ·0035 to ·005 mm. broad; the bounding areolæ inconspicuous. Markings polygonal and in contact to the central space, towards the centre $3\frac{1}{2}$, increasing outwards to $2\frac{1}{2}$ in ·01 mm. at about $\frac{3}{4}$ of the radius, again decreasing to the border, punctate; secondary oblique rows obvious.

Habitat.—Barbados deposit (Rae !).

C. profundus. Ehrb., *Mon. Ber. Ak.*, 1854, p. 238.—Diam. ?. Central space and rosette absent. Markings somewhat larger at centre, about semiradius $6\frac{1}{2}$ to 7 in ·01 mm., near border smaller and more crowded; interspaces distinct, most evident opposite the shorter rows, at about $\frac{2}{3}$ of radius from centre.—Ehrb., *Mikrog.*, pl. xxxv. B.B. fig. 8; Ralfs in *Pritch. Inf.*, p. 830.

Ehrenberg's figure shows the markings as subcircular and as decreasing gradually from the centre outwards, but more rapidly near the border.

Habitat.—Atlantic Ocean, 2000 fms.; lat. 62° 6′ N., long. 32° 21′ W., 1540 fms.; lat. 59° 12′ N., long. 50° 38′ W.; lat. 58° 3′ N., long. 51° 50′ W.;† northern and equatorial zone, 16 to over 2000 fms. (Ehrenberg).‡

C. antarcticus. *C.* (*subglobosus*, var.?) *antarcticus*, Grun., *Denk. Wien. Ak.*, 1884, p. 84, pl. iv. (D), fig. 23.—Diam. ·03 mm. Central space and rosette absent. Markings irregular, polygonal, increasing from the centre to about the semiradius, again diminishing towards the border; at the centre 8, at the semiradius 4, towards the border 8 in ·01 mm.; rows indistinct; on outer half of valve inconspicuous, irregular, concentric bands; apiculi numerous, inserted close to the border, Border with inner edge indistinct; striæ irregular, 10 to 12 in ·01 mm.

* In the collection of Dr Greville.
† *Mon. Ber. Ak.*, 1861, p. 280.
‡ *Abh. Ber Ak.*, 1872, p. 263.

The valve named *C. decipiens*, Grun. (Sch., *Atl.*, pl. lix. fig. 18), from Table Bay, is distinct, though Grunow has proposed to unite them; in the latter the markings are largest at the centre, and the apiculi are more prominent.

Habitat.—Antarctic, Kerguelen (Grunow).

C. lanceolatus. Cstr., *Diat. Chall. Exped.*, p. 164, pl. xvii. fig. 19.—Elliptical to subdiamond-shaped. Major axis ·0775 mm., about $2\frac{1}{8}$ times minor. Central space and rosette absent. Markings polygonal, largest and subequal on a small indefinite central area, thence decreasing to the border; at the centre 3, at the border 6 in ·01 mm.; irregular or in faint radial rows.

This species forms a transition to the untenable genus *Stoschia*.

Habitat.—South of Australia, H.M.S. Challenger (Castracane).

C. velatus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 78.—Diam. about ·055 mm. Central space and rosette absent. Markings angular, robust, pearly, about $2\frac{1}{2}$ in ·01 mm., punctate; rows obscurely radial, subregularly concentric.—Ehrb., *Mikrog.*, pl. xviii. fig. 37; Ralfs in *Pritch. Inf.*, p. 830.

Ehrenberg regarded this species as probably belonging to *Eupodiscus*. With this, owing to the absence of processes, it cannot be united. It is closely allied to *Stephanopyxis*. It also approaches *C. marginatus*.

Habitat.—Richmond deposit, Va. (Ehrenberg).

C. marginatus. Ehrb., *Abh. Ber. Ak.*, 1841, p. 142.—Diam. from ·0375 to ·15 mm. Central space absent. Markings polygonal, somewhat pearly, with large, round central papillæ; towards the centre 2 to $2\frac{1}{2}$ in ·01 mm., decreasing gradually towards the border. Border distinct, ·0025 to ·0075 mm. broad, with coarse striæ, 4 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 44; pl. xxxiii. 12. fig. 13; pl. xxxviii. B. 22. fig. 8; Cleve and Möll., *Diat.*, No. 114, 164, 215; H. L. Sm., *Diat. Sp. Typ.*, Nos. 94, 95; Sch., *Atl.*, pl. lxii. figs. 1, 2, 3, 4, 5, 9, 11, 12; pl. lix. fig. 11 (no name); *C. fimbriatus limbatus*,* Ehrb., *Mikrog.*, pl. xix. fig. 4; Sch., *Atl.*, pl. lxv. figs. 3–6; pl. cxiii. fig. 2; *C. limbatus*, Ehrb., *Mon. Ber. Ak.*, 1840, p. 206;

* Quoted "*C. fimbriato-limbatus*," by Grunow, *Denk. Wien. Ak.*, 1884, p. 72.

Mikrog., pl. xx. figs. 29*a*, *b*; Sch., *Atl.*, pl. lxv. fig. 7;* Raben., *Alg. Europ.*, Nos. 2484, 2485; *C. radiatus f. heterosticta*, Grun. in Pant., *Fossil. Bacil. Ung.*, p. 70, pl. xx. fig. 184; *C. oculus-iridis*, Sch. in *Atl.*, pl. cxiii. fig. 2; *C. subconcavus forma major*, Sch., *Atl.*, pl. lxii. fig. 7;† (excl. *C. limbatus*, Jan. et Raben.—Raben., *Beitr. Kennt. u. Verbreit. Alg.*, *Leipz.*, 1863, p. 7, pl. iv. fig. 1; and *C. marginatus*, Kütz., *Bacil.*, p. 131, pl. i. fig. 7).

Hardman's original Monterey specimen, on which *C. robustus*, Grev., was founded, is not in the Grevillean Collection of the British Museum, but two specimens on a slide of Bermuda tripoli, labelled by Greville "*C. robustus*," and now in this collection, belong to *C. marginatus*. The *C. limbatus*, Jan. et Raben., has a central space, markings increasing to about the semiradius, and again decreasing to the border. Forms occur in Cambridge deposit, Barbados, similar to that figured by Schmidt (*Atl.*, pl. lxv. fig. 7). Stokes has labelled specimens of this species *C. ambiguus*.

Schmidt is only prohibited from uniting the specimens figured on pl. lxii. figs. 11, 12, with *C. velatus* by the small size of the markings and the absence of the fine puncta which, according to Grunow, cover the surface of that species.

Habitat.—Richmond, Va.; tripoli from Columbia River Oregon, Patagonian tufa; plastic clay, Aegina (Ehrenberg, Schmidt); Nottingham, U.S. (Hardman!);‡ San Diego, San Pedro (H. L. Smith!); Bajtha, Elesd Alsö-, Felsö-Esztergally, Kékkö, Mogyorod, Szakal, and Dolje deposits (Pantocseck!); sounding from 2950 fms., H.M.S. Challenger (Rae!); "California" (Deby! Cleve!); Cambridge deposit, Barbados (Hardman!);† Monterey (Stokes!§ Firth! Greville! Cleve!); Sta Barbara deposit (Kinker!); Moron deposit (Kinker!); Faeroe Channel (Grove!); Nagy-Kurtos deposit, Hungary (Rae! Deby!); Nancoori (Hardman!); soundings off Kurile Islands, 1329 fms. (H. L. Smith!); Behring Sea, 1681 fms. (H. L. Smith!); Los Angelos (Hardman! Cambridge!);‖ Oamaru deposit (Grove!);

* Quoted "*C. fimbriatus*, Sch. (*nec.* Ehrb.)" by Grunow, *ibid.*, p. 72.

† Grunow (*Denk. Wien. Ak.*, 1864, p. 72) refers to this simply as *C. subconcavus*, Grun., and proposes to name it *C. marginatus*, var. *subconcava*, or better, to unite it to *C. robustus*, Grev., the markings towards the centre being 2, towards the border 3 to 3½ in ·01 mm. The central papillæ are prominent, as in *C. robustus*.

‡ In the collection of Julien Deby. § In the collection of E. O'Meara.

‖ In the collection of Dr Griffin.

"Barbados" (Johnson! Cleve!); Rappahannock River, Va. (Rogers!);* Kékkö and Sta Maria deposits (Grove!); Szent Peter deposit (Pantocsek! Grove!); San Benito deposit, California (Grove!); Kamtschatka Sea, 1700 fms. (Greville! Bailey!); Atlantic Telegraph soundings (Roper!);* Piscataway deposit (Greville!); Santa Monica deposits (Cleve and Möller! Firth!);* King's Mill (Schmidt); Nicobar Islands (Schmidt); Mascara (Cleve and Möller!); Nottingham deposit, Md. (Cleve and Möller!); Brünn Tegel (Cleve!); Holland's Cliff (Cleve!); anchor ground, Laguna Harbour, 20 miles N. of Laguna in the sea (Rabenhorst and Schwarz!); Marstrand (Kinker!).

Var. *decussata*, nov.—Diam. ·115 mm. Markings 3 in ·01 mm., subequal to the zone at the border, the secondary oblique decussating rows obvious; radial rows not differentiated. Border more sharply defined.

Habitat.—Roundstone Bay, Ireland (O'Meara!).

Var. *latemarginata.* Pant., *Fossil. Bacil. Ung.*, p. 70, pl. xxii. fig. 201.—Diam. ·057 mm. Markings subequal, 3 in ·01 mm. Border sharply defined. Striæ more distant.

Habitat.—Elesd deposit, Hungary (Pantocsek!).

Var. *intermedia*, Rattray. *C. robustus*, var. *intermedia*, Grun., *Denk. Wien. Ak.*, 1884, p. 72.—Diam. ·165 mm. Markings increasing slightly outwards, those at the centre somewhat larger than those on the adjoining area; near the centre 2, towards the border $1\frac{1}{3}$, at the border 3 to $3\frac{1}{2}$ in ·01 mm.—*C. robustus*, Sch. (not Grev.), *Atl.*, pl. lxii. fig. 6.

C. robustus. Grev., *Trans. Micr. Soc.*, Lond., 1866, p. 3, pl. i. fig. 8.—Diam. from ·0825 to ·325 mm. Surface slightly convex towards centre. Central space and rosette absent. Markings pearly, $1\frac{1}{2}$ to 2 in ·01 mm., subequal for about $\frac{2}{3}$ of radius, smaller towards the border, at intervals smaller areolæ sometimes distinct among the larger, the central papillæ prominent; radial rows inconspicuous, sometimes secondary, subconcentric or short oblique decussating rows visible. Border prominent, sharply defined, from $\frac{1}{11}$ to $\frac{1}{20}$ of radius broad; striæ evident, 4 to 6 in ·01 mm.—Sch.,

* In the collection of Dr Greville.

Atl., pl. lxii. figs. 16, 17 ; Grun. *Denk. Wien. Ak.*, 1884, p. 72; H. L. Sm., *Diat. Spec. Typ.*, No. 99 ; Janisch, *Gazelle Exped.*, taf. iv. figs. 10, 11 ; *C. marginatus*, var. *submarginata*, Grun., *ibid.*, p. 72 ; *C. subvelatus*, Grun.—Sch., *Atl.*, pl. lxv. fig. 9 ; *C. kinkerianus*, Tru. and Witt, *Jeremie Diat.*, p. 13, pl. iii. fig. 1.

In a Santa Monica form ·13 mm. in diam., discovered by Dr Rae, the usual striated border was surrounded by a second more sharply defined but narrower band, with a slightly convex surface, and bearing delicate striæ, 8 to 10 in ·01 mm.; at one place this band is interrupted and somewhat more convex on the two sides of the break. This gives it the appearance of an elastic spring enveloping the valve.

Habitat.—Santa Monica deposit (Kinker ! Hardman ! Weissflog ! Rae ! Firth !) ; Santa Maria deposit (Rae ! Grove !) ; Santa Marta deposit (Doeg !) ; Nagy-Kurtos deposit, Hungary (Rae ! Deby !) ; Monterey (Weissflog ! Hardman !) ; Mejillones (O'Meara ! Hardman !) ; Jeremie deposit (Truan and Witt); Los Angelos (O'Meara !); Japan (H. L. Smith !) ; Sea of Kamtschatka, 1700 fms. (Bailey !) ; Brünn Tegel (Cleve) ; San Pedro (Grove !).

Var. *kittoniana*, nov.—Diam. ·1125 to ·225 mm. Markings $1\frac{1}{2}$ to $1\frac{3}{4}$ in ·01 mm., central papillæ prominent, forming transversely truncated cones, with finely but evidently and closely furrowed sides.

Habitat.—Holothurians, China (Macrae !).

Var. *fragilis*, nov.—Diam. ·1875 mm. Markings more minute ; at the centre $2\frac{1}{2}$, at the border 3, in ·01 mm.; adjacent to border a single band of markings elongated radially ; central papillæ more faint, puncta distinct ; oblique decussating rows more evident.

Habitat.—Santa Maria deposit (Rae !).

C. implicatus, sp. n.—Diam. ·15 to ·25 mm. Surface somewhat convex towards the centre. Central space and rosette absent. Markings hexagonal, decreasing but very slightly outwards ; at the centre $3\frac{1}{3}$, towards the border $4\frac{1}{2}$ in ·01 mm.; rows irregular, oblique, straight or curved, forming short, inconspicuous broad fasciculi, that are interrupted by those meeting them at variable angles.

Border sharply defined, $\frac{1}{14}$ to $\frac{1}{16}$ of radius broad; striæ distinct, 4 in ·01 mm.—(Pl. III. fig. 1.)

This species does not strictly belong either to the *Radiati* or to the *Fasciculati*. It is placed here for convenience, since it approaches *C. robustus* in general appearance.

Habitat.—Sta Maria deposit (Rae !); Sta Marta deposit (Doeg !).

Var. *picturata*, nov.—Diam. ·3 mm. Markings angular or subrotund; surface mottled with dark, mostly quadrangular spots, which are somewhat more crowded on the flattened central portion than towards the border. Border striæ 6 in ·01 mm.—(Pl. III. fig. 11.)

Habitat.—Santa Monica deposit (Thum !).*

C. glaberrimus, sp. n.—Diam. ·1 to ·1075 mm. Surface flat from the centre to about semiradius, thence sloping somewhat steeply to the border. Central space and rosette absent. Markings polygonal, pearly, subequal to semiradius, largest about $\frac{2}{3}$ of radius from the centre, thence decreasing to the border, towards the centre $3\frac{1}{2}$, at $\frac{2}{3}$ of radius 3, in ·01 mm.; secondary oblique decussating rows faint. Border broad, with inner edge indistinct; striæ 4 to 5 in ·01 mm. —(Pl. I. fig. 19.)

Distinguished from *C. diversus* by the presence of a central space and the polygonal outline of the markings on the central half of the valve.

Habitat.—Cambridge deposit, Barbados (Rae !).

C. obscurus. Sch., *Atl.*, pl. lxi. fig. 16.—Diam. ·09 to ·165 mm. Central space minute, sometimes absent. Markings subpearly, with central dots evident, increasing but slightly outwards; towards the centre $2\frac{1}{2}$, at about $\frac{4}{5}$ of radius from centre 2, at the border 4 to 5 in ·01 mm.; secondary oblique rows inconspicuous, at the origin of the shorter rows are small clear spaces, readily overlooked. Border striæ coarse, 4 to 6 in ·01 mm.—Grun., *Denk. Wien. Ak.*, 1884, p. 74; *Cestodiscus obscurus*, Van Heurck, *Syn. Diat. Belg.*, pl. cxxix. fig. 4.

This species is intermediate in the appearance of its markings between *C. crassus*, Bail., and *C. radiatus*, Ehrb., on the one hand,

* In the collection of Julien Deby.

and *C. marginatus*, Ehrb., on the other. Van Heurck's figure is from a photograph by Dr Woodward.

Habitat.—Moron deposit (Greville! Grunow); Sta Monica deposit (Rae!); Szent Peter and Dolje deposits (Pantocsek); sounding, lat. 3° 1′ S., long. 33° 50′ W., H.M.S. Challenger (Rae!); Virginia (Greville!); Sta Maria deposit (Grove!).

Var. *minor*, nov. *C. obscurus*, var.? Sch., *Atl.*, pl. lxi. figs. 17, 18.—Diam. ·05 to ·06 mm. Markings smaller, 3 in ·01 mm. Border striæ longer.—Grun., *ibid.*, 1884, p. 74.

Habitat.—Moron deposit (Greville!).

C. radiatus. Ehrb., *Abh. Ber. Ak.*, 1839, p. 148, pl. iii. figs. 1 *a–c* (excl. *d*).—Diam. ·0675 to ·18 mm. Central space absent. Markings polygonal, 2 to 2½ in ·01 mm., subequal from the centre for about $\frac{6}{7}$ of radius, thence decreasing sometimes to 6 in ·01 mm., in inconspicuous radial sometimes subfasciculate rows, central dots faint.—Ehrb., *Mikrog.*, pl. xix. fig. 1; pl. xxii. fig. 3; pl. xxxiii. 13. figs. 2, 2*; pl. xxxiii. 16. fig. 6; pl. xxxv. A. 17. fig. 6 (excl. pl. xx. 1. fig. 27; pl. xxi. fig. 1); Ralfs in *Pritch. Inf.*, p. 831, pl. xi. figs. 39, 40; Sch., *Atl.*, pl. lx. figs. 5, 6, 9; pl. lxii. fig. 18; pl. lxv. fig. 8; Grun., *Denk. Wien. Ak.*, 1884, p. 71; pl. iii. (c), figs. 4, 7; in H. L. Smith, *Diat. Spec. Typ.*, No. 99; Cleve and Möll., *Diat.*, Nos. 57, 114, 155, 164, 207, 211, 215, 257; Raben., *Alg. Europ.*, Nos. 2263, 2437, 2487; Coll., *Kütz. Diat.*, No. 902; *C. caspius*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 170, pl. xii. fig. 14; *C. argus*, Sch. (non Ehrb.), *Atl.*, pl. lxi. fig. 13 (excl. *C. radiatus*, Weisse, *Bull. Acad. Imp. St Petersb.*, 1868, p. 122, pl. i. fig. 25; and *C. radiatus*, Bail, *Amer. Jour. Sci.*, xlii. 1842, p. 95, pl. ii. fig. 14).

Schumann (*Schrift. Phys. Oek. Ges. Königsberg*, 1867, p. 61) proposed to break up this species restricting *C. radiatus* for those forms in which the markings are angular, and decrease from the centre outwards from about 6 to 7½ in ·01 mm. Other forms from the Baltic, with round markings and fine short furrows within the border, 16½ in ·01 mm., he named *C. vicinus;* but the definition of this last species, which is not accompanied by a figure, is inadequate, and the name may be abandoned.

His *C. fallax* (*ibid.*, p. 62, pl. iii. fig. 76) with, in the dry state, round markings, between which smaller faint granules, each resolvable under high powers into two, occur, is also *C. radiatus*, Ehrb. In balsam *C. fallax*, like *C. radiatus*, showed hexagonal markings, and by good illumination smaller granules at their angles. *C. caspius*, Ehrb., was only distinguished by having the rows inconspicuously radial and the secondary oblique curved rows evident. Oamaru specimens show transitions to *C. argus*, and have the rows subfasciculate, with sometimes a distinct central rosette. Schmidt misinterprets Ehrenberg's *C. argus*, in overlooking the increase of the markings outwards (*Atl.*, pl. lxi. fig. 13).

Habitat.—White chalk marl, Caltanisetta; Polirschiefer, Stratford Cliff, Va.; Zante, Plattenmergel; plastic clay, Aegina; tripoli, San Francisco, Cal. (Ehrenberg); Ichaboe guano (O'Meara!); dredged in 1319 fathoms, lat. 71° 19′ N., long. 11° 28′ W., in yellowish-grey mud; in 1300 fathoms, lat. 73° 16′ N., long. 15° 48′ W., in dark greyish-brown mud; and in 569 fathoms, lat. 63° 40′ N., long. 5° 28′ W., in clear grey fine mud (Ehrenberg); Caspian Sea, 14 to 422 fathoms; North Sea, at Cuxhaven; Baltic, at Wismar (Ehrenberg); Mors deposit (Schmidt and Cleve!); Vera Cruz, Mexico (Ehrenberg); off Ascension Island, 1845 fathoms, S.S. Buccaneer (Grove!); Cambridge deposit, Barbados (Hardman!); sounding, lat. 3° 1′ S., long. 33° 50′ W., H.M.S. Challenger (Rae!); "Atlantic Ocean" (Schmidt); Mascara (Cleve and Möller!); "Virginia" (Hardman!); Kékkö deposit (Grove!); Oamaru deposit (Grove! Doeg!); Hong Kong and Monte Gubbio (Grove!); marine deposit, Fiji Islands (Grove!); Japan (H. L. Smith!); "Barbados" (Johnson!); Newcastle deposit, Barbados (Grove!); marshy ground from Wedel (Schmidt); Aegina (Schmidt); San Benito deposit, California (Grove!); Balearic Islands, Sta Monica deposit, Patagonia, Delaware, North Carolina (Cleve and Möller!); Yedo, Mejillones guano, Bohuslan, mud from Elbing, West Prussia, Saldanha Bay guano, Patagonian guano, Schleswig-Holstein, Labuan, Nancoori deposit, Cape Wankarema, between Aden and Bab-el-Mandeb (Cleve!); Grip, 70 fathoms; Kiel; Brünn Tegel (Cleve); anchor ground, Reikjavik, Iceland; mud from Glückstadt; Elbe, above Cuxhaven (Rabenhorst and Schwarz!); coast of St Paul Island, South Sea (von Frauenfeld!); Oran marl

(Ehrenberg, Kützing!); coasts of Britain (Grove! Rattray!); Kirkwall and Faeroe Islands (Grove!); Marstrand (Kinker!).

Var. *subæqualis.* Grun., *ibid.*, p. 72, pl. iii. (C), fig. 3.—Diam. ·12 mm. Markings as in the type, but subequal almost to the border, around which on a narrower zone they are larger than in the type.—*C. radiatus*, var. *abyssalis*, Cstr., *Diat. Chall. Exped.*, p. 165, pl. xxix. figs. 2, 11, 15; Sch., *Atl.*, pl. cxiii. fig. 15 (no name).

Castracane's var. *abyssalis*, which is not sufficiently characterised, is provisionally placed here from his note that the markings gradually diminish in size to the border.

Habitat.—Oran, Monterey and Nancoori Island deposits (Grunow); Atlantic Ocean, H.M.S Challenger (Castracane); Monkstown, in tide pool (O'Meara!); Cambridge deposit, Barbados (Greville!); San Diego (Gründler).

Var. *glacialis.* Grun., *ibid.*, Expl. pl. iii (C), fig. 1; *C. borealis*, Ehrb., *Mon. Ber. Ak.*, 1861, p. 294 (not *C. borealis*, Bail., *Amer. Jour. Sci.*, 1856, vol. xxii. p. 3).—Diam. ·1 to ·15 mm. Surface flat; central rosette absent. Markings subequal, 3 to 3½ decreasing to 4 in ·01 mm. at the border; central papillæ delicate.—*C. radiatus*, var. *borealis*, Grun., *ibid.*, p. 72; Sch., *Atl.*, pl. cxiii. fig. 8; *C. radiatus*, Sch., *Atl.*, pl. cxiii. fig. 8.

The varietal name *glacialis* given, by Grunow in the explanation of his plate, is better than *borealis*, as it avoids confusion with Bailey's species.

Habitat.—Lat. 62° 40′ N., long. 29° W., 1000 fathoms; lat. 62° 6′ N., long. 32° 21′ W., 1540 fathoms; lat. 59° 12′ N., long. 50° 38′ W., 1833 fathoms; lat. 58° 3′ N., long. 51° 50′ W., 1840 fathoms; lat. 60° 5′ N., long. 50° 27′ W., 2090 fathoms (Ehrenberg); Franz Josef's Land (Grunow! Cleve!); Aegina (Schmidt).

Var. *media.* Grun., *ibid.*, p. 72, pl. iii. (C), fig. 2.—Diam. ·075 to ·14 mm. Markings 3½ to 4 in ·01 mm., gradually decreasing towards the border, where they are 6 to 6½ in ·01 mm.—Sch., *Atl.*, pl. cxiii. fig. 21; *C. radiatus*, Sch., *Atl.*, pl. lx. fig. 10; *C. radio-*

latus, Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 94.

Schmidt, in 1878, stated that this form is the traditional type of *C. radiatus*, Ehrb., but in Ehrenberg's original definition the markings are given as about 2 in ·01 mm. Los Angelos specimens have been observed by Grunow occasionally to have on their valves groups of larger markings, forming 4- to 6-angled rosettes.

Habitat.—Atlantic sounding for telegraph cable (Greville!); Oran, Nancoori and Los Angelos deposits (Grunow); King's Bay, Spitzbergen, 160 fathoms (Cleve); Davis Straits (Cleve); Franz Josef's Land (Grunow); Baltic (Schumann); Gulf of California (H. L. Smith!); Sussex (Dickie!); Cambridge deposit, Barbados (Hardman!);* Peruvian guano (Hardman!);* Lumfiord, Jutland (Hardman!);* Nottingham, U.S. (Hardman!);* Compeachy Bay (Hardman!); Rio Janeiro (Hardman!); Californian guano (Norman!);† Teignmouth (Arnott!);† Melville Bay, lat. 75° 27′ N., long. 64° 34′ W. (O'Meara!); Nottingham deposit (Hardman!);* Lamlash Bay (Dickie! Gregory!);† Ascidia, Belfast (O'Meara!); Maryland (O'Meara!); Mejillones deposit (O'Meara!); rice fields, Georgia (Greville!); Gulf of Mexico (Schmidt); Algeria (Arnott!); Indian Ocean soundings, Capt. Pullen, 2200 fathoms (Greville! Roper!);† shell cleanings from Singapore (Hardman!);† Bay of Bengal (Macrae!)

Var. *minor*. *C. radiatus f. minor*, Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 94, pl. iii. fig. 34.—Diam. ·03 to ·0525 mm. Markings 4 in ·01 mm. at the centre, decreasing to 8 to 9 in ·01 mm. at the border, the radiating rows less obvious. Border striæ, 6 to 8 in ·01 mm.—*C. radiatus*, var. *parva*, Grun., *Sitzungsb. Naturw. Ges. Isis, Dresden*, 1878, p. 124, pl. iv. fig. 16; *C. devius*, Sch., *Atl.*, pl. lx. figs. 1–4; Van Heurck, *Syn. Diat. Belg.*, pl. cxxx. fig. 3; Cleve and Möller, *Diat.*, No. 150.

Habitat.—Peruvian guano, Rio de Janeiro, Santos, Campeachy Bay, Japan, and Baku Harbour, Caspian Sea (Grunow); Hvidingsoe (Schmidt); Nancoori (Hardman!);* Manilla (Hardman!).*

* In the collection of Julien Deby.
† In the collection of Dr Greville.

Var. *irregularis*, Grun. Van Heurck, *Syn. Diat. Bely.*, pl. cxxix. fig. 1.—Obtusely triangular, sometimes elliptical. Diam. about ·105 mm. Markings 5½ to 6 in ·01 mm., subequal almost to the border; radial rows straight or curved, evident.

This var. is distinguished from var. *glacialis* by the size of the markings and the arrangement of the rows. Transitional to the untenable genus *Stoschia*.

Habitat.—Naparima deposit (Van Heurck).

Var. *crenulata*, Rattray. *C. radiatus*, var., Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 48, pl. ii. fig. 22.—Diam. about ·025 mm. Markings sub-equal. Border crenate.

Habitat.—From *Salpæ*, Indian Ocean (Wallich).

C. luctuosus, Grove MS.—Diam. ·0875 to ·125 mm. Surface rising gradually from centre to about semiradius, thence descending with a similar slope to border, convex. Central space and rosette absent. Markings at centre obtusely angular, soon becoming acutely angular and areolate, subequal, 3½ in ·01 mm.; rows straight. Border sharply defined, $\frac{1}{15}$ to $\frac{1}{25}$ of radius broad; striæ obvious, 5 to 6 in ·01 mm.—(Plate III. figs. 8, 9.)

Habitat.—Troublesome Gully, Oamaru (Grove !).

C. compositus, Rattray. Sp. n.? Sch., *Atl.*, pl. lix. fig. 10.—Diam. ·023 mm., central space and rosette absent. Markings angular, towards the centre about 6 in ·01 mm., decreasing slightly towards the border, central dots absent; rows inconspicuous, secondary oblique rows obscure. Border sharply defined, about $\frac{1}{7}$ of radius broad; striæ evident, about 4 in ·01 mm.

Habitat.—Nottingham (Schmidt).

C. egregius, Rattray. Sp. n? Sch., *Atl.*, pl. lvii. fig. 39.—Diam. ·03 mm. Central space and rosette absent. Markings angular, increasing to about semiradius, thence decreasing gradually to the border; towards the centre 4, at semiradius 3, towards border 3½ in ·01 mm; central dots evident, radial rows inconspicuous, secondary curved rows evident; a distinct sharply defined band with faint striæ 6 in ·01 mm. adjacent to border, prominent truncate, but small markings

(processes?) at intervals of about ·01 mm. inserted at inner edge of marginal band. Border narrow, hyaline.

Habitat.—Table Bay (Schmidt).

C. pectinatus, Rattray. *C. decipiens*, Grun.; Sch., *Atl.*, pl. lix. figs. 18, 19.—Diam. ·024 to ·0515 mm. Central space and rosette absent. Markings angular, subequal, or increasing somewhat to about semiradius, again decreasing to border; towards centre 4½ to 5, about semiradius 3½, near border 4, in ·01 mm.; secondary oblique decussating rows manifest; apiculi prominent, long acicular, inserted at inner edge of border, and reaching outwards to its outer edge. Border distinct; striæ faint, 6 in ·01 mm.

The name *decipiens* cannot be adopted here, having been already applied to a distinct form.

Habitat.—Table Bay (Schmidt).

C. bulliens. Sch., *Atl.*, pl. lxi. figs. 11, 12.—Diam. ·05 to ·1075 mm. Central space absent. Markings polygonal; at the centre 2½ to 3, increasing to the semiradius to 1½ or 2, again gradually decreasing to the border to 6, in ·01 mm.; the largest areolæ forming a conspicuous zone. Border indistinctly defined; striæ short, inconspicuous, 6 in ·01 mm.—Cleve and Möll., *Diat.*, No 215. *C. ebulliens*, var. Cstr., *Diat. Chall. Exped.*, p. 159, pl. v. fig. 1.

Some small specimens from Cambridge deposit show only a single band of large markings. This species has sometimes been confounded with *C. heteroporus.*

Habitat.—Springfield deposit, Barbados (Schmidt, Grunow); Maryland (Kinker!); Szent Peter deposit (Pantocsek); Cambridge deposit, Barbados (Greville! Johnson! Hardman!);* Oamaru deposit (Grove!); Barbados (Johnson!);† Nottingham deposit (Cleve and Möller!); Maryland (Cleve!).

C. asperulus. Grun., *Denk. Wien. Ak.*, 1884, p. 73.—Diam. ·088 to ·093 mm. Central space absent. Surface somewhat curvex, but with slight slope to the border. Markings polygonal; towards the centre 3 to 3½, at the border 4, in ·01 mm.; distinctly punctate.

* In the collection of Julien Deby.
† In the collection of Dr Greville.

Distinguished from *C. radiatus* by the more convex surface and the evident punctation of the markings.

Habitat.—Church Hill, Richmond (Grunow); Dolje deposit (Pantocsek !).

C. subangulatus. Grun., *Denk. Wien. Ak.*, 1884, p. 73.—Outline irregular, obtusely angular. Diam. ·06 mm. Surface very convex at the border. Markings polygonal; towards the centre 3, at the border 4 to 4½ in ·01 mm., distinctly punctate. Border striæ evident, radial or oblique, 4 to 5 in ·01 mm., its inner edge indistinct.

Habitat.—Moron deposit (Grunow, Greville !).

C. nodulifer, Janisch. Sch., *Atl.*, pl. lix. figs. 21–23.—Diam. ·065 to ·1 mm. Central space and rosette absent, but one (rarely a few) evident nodules present. Markings angular, 3½ to 4 in ·01 mm., decreasing slightly around the border; radial rows inconspicuous, the oblique decussating rows more distinct. Border sharply defined; striæ evident 4 to 6 in ·01 mm.—Cleve and Möll., *Diat.*, No. 57, 155; Janisch, *Gazelle Exped.*, taf. ii. figs. 4–5.

Habitat.—Richmond, Va. Balearic Islands (Cleve and Möller !) Sta Monica deposit (Grove !); California, Gazelle Exped. (Schmidt); Islay, Peru (Kitton !); Isle of Muntok, near Sumatra (Grove !); Macassar Straits (Grove !); Indian Ocean sounding, Capt. Pullen, 2200 fathoms (Greville !); coral washings, locality ? (Doeg !); Atlantic Ocean, lat. 3° 3′ N., long. 15° W. (O'Meara !); off Ascension Island, S.S. Buccaneer (Rattray ! Grove !); Galapagos Island (Cleve !) Patagonian guano; between Aden and Bab-el-Mandeb (Cleve !).

Var. *apiculata*, nov. *C. nodulifer* (Janisch), Sch., *Atl.*, pl. lix. fig. 20.—Diam. ·0685 to ·15 mm. Central nodule single, larger. Markings decreasing more around the border; apiculi numerous, subregular, placed at inner side of border.

Habitat.—Campeachy Bay (Schmidt); trawled at lat. 12° 42′ N., long. 152° 1′ W., in 2900 fathoms, by H.M.S. Challenger (Rae !).

C. radiosus, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 7—Diam ·09 to ·11 mm. Surface almost flat, or somewhat convex towards the centre. Central space absent. Markings poly-

gonal; towards the centre 6 to 9, gradually decreasing towards the border to 9 or 10 in ·01 mm.; secondary oblique rows evident; minute, subulate spaces at origin of the shorter rows.—Grun., *Denk. Wien. Ak.*, 1884, p. 72; Janisch, *Gazelle Exped.*, taf. v. fig. 9; taf. vi. fig. 4.

This species was formerly associated by Grunow with *C. radiolatus*, Ehrb., a species that cannot be determined with certainty. The specimens referred to from Macassar Straits are more convex towards the centre, and have been authenticated by Grunow.

Habitat.—Monterey and Barbados deposits (Grunow); South Sea (Grunow); Los Angelos (O'Meara!); Macassar Straits (Grove!).

Var. *kerguelensis.* Grun., *Denk. Wien. Ak.*, 1884, p. 73. Diam. ·047 mm. Markings towards the centre 6, at the border 9 in ·01 mm.; close to the border a circlet of minute apiculi.

Habitat.—Kerguelen (Grunow).

C. subaulacodiscoidalis, sp. n. Sch., *Atl.*, pl. lvii. fig. 8.—Diam. ·0425 mm. Surface convex. Central space absent, rosette minute. Markings angular, 6 to 8 in ·01 mm., decreasing gradually towards the border; secondary oblique decussating rows evident; apiculi, six large, with a slight median constriction inserted some distance from the border at subuniform intervals. Border narrow, hyaline, sharply defined.

This species in its general characters approaches *Aulacodiscus concinnus*, Kitton, but there are no primary rays, and the processes are reduced to stout apiculi.

Habitat.—Baldjik (Schmidt).

C. Baileyi, Rattray. *Cestodiscus Baileyi*, H. L. Sm., *Amer. Quart. Jour. Microscopy*, 1878, p. 16, pl. iii. fig. 8.—Diam. ·04 to ·0925 mm. Central space small, rounded, indistinct, bearing isolated granules. Markings 12 in ·01 mm.; rows radial, straight; distinct hyaline subulate spaces opposite origin of shorter rows, secondary oblique rows evident; apiculi distinct, at wide intervals inserted a short distance within border; inner layer of valve with a clear central space surrounded by a zone of closely disposed costæ 6 in ·01 mm., outside the latter a second broad hyaline zone

adjacent to border.—*Cestodiscus Baileyi*, H. L. Sm., *Diat. Spec. Typ.*, No. 67.

This species was first collected by Lieut. Williamson (*Explor. and Surveys for Railroad Route from Mississippi River to Pacific Ocean*, vol. vi. pt. 2, "Geology," chap. iv.). Prof. H. L. Smith, in his remarks on the species, first throws doubt on the validity of the genus *Cestodiscus*.

Habitat.—Lost River, lower Klamath Lake, Oregon fossil (H. L. Smith !).

C. fragilissimus, Grun., in Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 4.—Diam. ·3165 mm. Central space minute, indistinct, rounded. Markings minute, 12 in ·01 mm.; secondary oblique rows manifest; apiculi distinct, scattered at wide unequal intervals, most crowded towards the border. Border narrow, hyaline.—*Ethmodiscus convexus*, Cstr., *Diat. Chall. Exped.*, 1886, p. 167, pl. iii. fig. 9.

Habitat.—Arafura Sea (Van Heurck, Castracane !).

C. asteroides. Tru. and Witt, *Jeremie Diat.*, p. 13, pl. iii. fig. 2. —Diam. ·15 to ·2 mm. Surface usually showing a circlet of six to twelve small shallow depressions at a distance of $\frac{1}{3}$ to $\frac{1}{4}$ of the radius from the centre. Central space absent. Markings hexagonal, on a small central area 2 to $2\frac{1}{2}$ in ·01 mm., decreasing somewhat suddenly at about $\frac{1}{5}$ of the radius to 3 or $3\frac{1}{2}$, thence increasing gradually outwards to $1\frac{1}{2}$ in ·01 mm., again becoming somewhat smaller at the border. Central papillæ distinct; secondary oblique, curved, decussating rows manifest. Border narrow.

In Maryland specimens the shallow depressions are not found.

Habitat.—Monte Gubbio (Grove !); Jeremie deposit (Truan and Witt !); Cove, Calvert County, Maryland (Greville !); South Naparima, Trinidad (Greville !); Nottingham deposit, Maryland (Johnson !); * Rappahannock, Va (Greville !).

C. lunatus. Grove MS.—Diam. ·09 to ·15 mm. Central space minute; a rosette frequently distinct, occasionally subobsolete. Surface with an evident lunate unilateral depression, its long axis at right angles and subequal to or somewhat longer than the radius,

* In the collection of Dr Greville.

about twice its greatest breadth, its outer edge more distinct, convex towards the border, its inner less curved, the extremities obtuse; the slope to the border gentle. Markings areolate, subequal, 3 to 3½ in ·01 mm. on the depression round, granular; with hyaline interspaces, the central papillæ prominent; rows radial, straight; secondary oblique decussating rows uniformly curved, manifest. Border relatively narrow, with coarse evident, subradial striæ about 6 in ·01 mm.

Habitat.—Santa Barbara County, California (Grove !).

C. excavatus. Grev., Ralfs in *Pritch. Inf.*, p. 829, pl. viii. fig. 26. —Diam. from ·1 to ·255 mm. Surface with 1 to 3 rounded or subcuneate elevations, and alternate depressions around the centre, elsewhere subplain. Central rosette sometimes distinct. Markings hexagonal, increasing regularly outwards, but becoming somewhat smaller around the border; near the centre 4, towards the border 1½, at the border 2, in ·01 mm., the central dots faint; secondary oblique decussating rows evident. Border striæ 4 in ·01 mm.—Grun., *Denk. Wien. Ak.*, 1884, p. 73.

Habitat.—Piscataway deposit (Dallas !* Rae !); Newcastle deposit, Barbados (Rae !); Holland's Cliff (Cleve); "Artesian Well" (Febiger).

Var. *genuina.* Grun., *ibid.*, p. 73.—Rarely elliptical. Diam. greater than that of the other vars., from ·25 to ·3 mm. Surface elevations and depressions 3.—*C. excavatus*, Grev., Sch., *Atl.*, pl. lxv. fig. 1.

Habitat.—Piscataway deposit (Dallas !* Rae ! Grunow, Deby !); Naparima (Grunow); Newcastle deposit, Barbados (Grunow, Rae !); Naparima, Trinidad (Firth ! Kinker !); Richmond, Va. (Kitton); Rappahannock (Rogers !).*

Var. *quadriocellata.* Grun, *ibid.*, p. 73.—Circular or roundly elliptical. Diam. ·15 to ·19 mm. Surface elevations and depressions 2. Central rosette inconspicuous. Markings more uniform; towards the centre 4, towards the border 2, in ·01 mm. Border striæ, 6 in ·01 mm.—*C. diophthalmus*, Cstr., *Diat. Chall. Exped.*, p. 163, pl. xvi. fig. 4.

* In the collection of Dr Greville.

Habitat.—Newcastle deposit, Barbados (Rae! Weissflog! Febiger!* Grunow); trawled by H.M.S. Challenger, in 2900 fathoms, lat. 12° 42′ N., long. 152° 1′ W.; "Barbados" (Firth! Febiger!).

Var. *biocellata.* Grun, *ibid.*, 1884, p. 73.—Diam. ·0875 to ·15 mm. Surface elevation and depression opposite, roundly elliptical, edges abrupt. Markings on the elevation 6 towards its central edge, 3 towards the peripheral, in ·01 mm., in radial, diverging rows; on the depression more equal and larger, $2\frac{1}{2}$ in ·01 mm., elsewhere as in var. *quadriocellata.*—*C. diophthalmus*, var. *monophthalma*, Cstr., *Diat. Chall. Exped.*, p. 163, pl. xvi. fig. 7.

Habitat.—Newcastle deposit, Barbados (Rae! Kitton! Weissflog! Firth! Febiger!);* Cambridge deposit, Barbados (Rae!); Hardman!) † "Barbados" (Febiger!); trawled by H.M.S. Challenger, in 2900 fathoms, lat. 12° 42′ N., long. 152° 1′ W. (Rae!).

Var. *semilunaris.* Grun., *ibid.*, 1884, p. 73.—Diam. ·1 to ·1175 mm. Surface elevation semilunate, with rounded ends, sometimes short and broad, uniformly disposed with respect to the centre round which it curves, the depression slight, half inclosed by the elevation. Central rosette inconspicuous. Markings towards the centre 4, towards the border 3 in ·01 mm.—*C. semilunaris*, Grun., *ibid*, 1884, p. 71.

The vars. *quadriocellata*, *biocellata*, and *semilunaris* approach *C. crassus*, Bail., in the character of the markings; those of var. *genuina* recall *C. gigas*, Ehrb.

Habitat.—Newcastle deposit, Barbados, (Weissflog! Grove!).

Var. *deliquescens*, nov.—Diam. ·0475 mm. Surface elevation and depression opposite, the former indistinct, the latter evident, but elliptical. Markings on the depression round free granules, with hyaline interspaces, elsewhere angular, $4\frac{1}{2}$ in ·01 mm.; towards the border smaller, rounded; between the outer ends of the radial rows

* In the collection of Herr E. Weissflog.
† In the collection of Julien Deby.

narrow hyaline areas bearing a few minute markings, attenuating inwards; secondary oblique rows obvious.—In H. L. Sm., *Diat. Spec. Typ.*, No. 99 (no name).—(Pl. III. fig. 7.)

Habitat.—Japan (H. L. Smith !).

C. decrescens, Grun. Sch., *Atl.*, pl. lxi. figs. 7 to 9, 10 (?).—Diam. ·038 to ·05 mm. Central space and rosette absent. Markings polygonal, with evident central papillæ; at the centre 3 in ·01 mm., decreasing rapidly on outer $\frac{1}{3}$ of radius to the border; rows on the outer portion sometimes subfasciculate, secondary oblique sometimes outwardly curved decussating rows distinct towards the border.—Grun., *Denk. Wien. Ak.*, 1884, p. 80.

Distinguished from *C. marginatus* by the rapid decrease in size of the markings on the outer third of the valve. Sometimes obtusely triangular specimens occur.

Habitat.—Springfield deposit, Barbados (Grunow); Dolje (Pantocsek); Barbados (Cleve !); west coast "Florida" U.S. Survey (Febiger !); Faeroe Channel (Grove !).

Var. *irregularis.* Grun., *ibid.*, 1884, p. 80.—Obtusely triangular, two of the angles more evident than the third. Diam. ·068 mm. Markings increasing slightly from the centre to about the semiradius.

Habitat.—Springfield deposit, Barbados (Grunow).

Var. *venusta.* Grun., *ibid.*, 1884, p. 80; *C. heteroporus*, Ehrb., *forma major*, Grun., Sch., *Atl.*, pl. lxi. fig. 6.—Diam. ·1135 mm. Central space minute. Markings increasing distinctly from central space for about $\frac{1}{2}$ to $\frac{5}{7}$ of radius, thence decreasing rapidly to the border; towards the centre 4, increasing to $2\frac{1}{2}$ in ·01 mm.—*C. argus*, Grun. (non Ehrb.) in Sch., *Atl.*, pl. cxiii. fig. 7.

The central space and increase of the markings outwards bring this var. near to *C. heteroporus*, but the appearance of the markings and the arrangement around the border bring it nearer to *C. decrescens.* The transition from specimens like that shown in Schmidt's *Atlas*, pl. cxiii. fig. 7, to *C. bulliens*, A. S., is easy.

Habitat.—Springfield, Barbados (Grunow); Ægina (Schmidt); "Barbados earth" (Greville !).

Var. *valida.* Grun., *ibid.*, 1884, p. 80; *C. decrescens*? * Sch., *Atl.*, pl. lxi. fig. 15.—Diam. ·1 mm. Central space small, about $\frac{1}{3\,3}$ of diam. broad, angular. Markings increasing but little from centre for $\frac{1}{3}$ of radius, from the semiradius decreasing rapidly to the border; at the central space $2\frac{1}{2}$, from $\frac{1}{3}$ to $\frac{1}{2}$ of radius $1\frac{1}{2}$ to 2 in ·01 mm.; rows radial, irregularly concentric bands indistinct.

Habitat.—Springfield deposit, Barbados (Schmidt, Grunow).

Var. *polaris.* Grun., *ibid.*, 1884, p. 80, pl. iii. (C), fig. 11.—Diam. ·047 to ·055 mm. Central space small, about $\frac{1}{19}$ of diam. road. Markings increasing slightly outwards to the semiradius to about 3 in ·01 mm.; rows subfasciculate, and secondary subconcentric rows evident.

Habitat.—Franz Josef's Land (Grunow); Monterey deposits (Hardman !)† Barbados deposits (Greville !).

Var. *repleta.* Grun., *ibid.*, 1884, p. 80, pl. iii. (C), fig. 18.—Diam. ·0325 to ·0825 mm. Surface convex towards the centre. Central space absent. Markings 3 to $3\frac{1}{2}$, sometimes 4, in ·01 mm.; secondary concentric rows faint, oblique decussating rows undifferentiated near the border.—In H. L. Sm., *Diat. Spec. Typ.*, No. 99.

Habitat.—Franz Josef's Land (Grunow); Oamaru deposit (Grove !); Japan (H. L. Smith !); Macassar Straits (Grove !).

C. epiphanes, sp. n.—Diam. ·165 to ·21 mm. Surface rising slightly from the centre for about $\frac{3}{4}$ of the radius, here descending abruptly, and continuing thence on one plain to the border. Central space absent, rosette distinct. Markings hexagonal, increasing slightly from the centre to the highest zone, here decreasing suddenly, and from this subequal to the border; towards the centre 3, at the highest zone $2\frac{1}{2}$, towards the border 3 in ·01 mm.; central papillæ faint, secondary oblique decussating rows evident. Border narrow; striæ faint, 8 to 10 in ·01 mm.—(Pl. II. fig. 14.)

Habitat.—Richmond deposit (Deby !). ‡

* Quoted "*C. decrescens*, var.?" by Grunow (*Denk. Wien. Ak.*, 1884, p. 80).

† In the collection of Julien Deby.

‡ In a Coscinodiscus type-plate by Thum, in the collection of Julien Deby.

C. patina. Ehrb., *Abh. Ber. Ak.*, 1839, p. 147, pl. iii. figs. 3 *a–e.* —Diam. ·035 to ·1125 mm. Surface flat. Central space and rosette absent. Markings angular, subequal, somewhat smaller near the border; rows concentric, obvious, radial rows undifferentiated. Border narrow, hyaline.—Ehrb., *Mikrog.*, pl. xx. 1. fig. 31; Ralfs in *Pritch.* Inf., p. 830; Janisch, *Gazelle Exped.*, taf. v. fig. 1; *C. patina*, Ehrb., *pro parte*, *Abh. Ber. Ak.*, 1838, p. 129, pl. iv. figs. 10–12 *d.* (Excl. *C. patina*, Bail., *Amer. Journ. Sci.*, 1842, vol. xlii. p. 96, pl. ii. fig. 13.)

The Simbrisk valve, figured by Weisse (*Bull. Acad. Imp. St Petersb.*, 1855, p. 276, pl. i. fig. 6) has round free markings, with a more prominent circlet at the border, and is probably distinct. Ehrenberg at first embraced in *C. patina* his *C. radiatus* (*conf. Abh. Ber. Ak.*, 1839, p. 148). The specimen figured by Janisch (*Gazelle Exped.*) shows the concentric arrangement of the markings most clearly towards the border; the areolæ are at intervals irregular.

Habitat.—Zante, Caltanisetta, Oran and Grecian deposits, Cuxhaven (Ehrenberg).

C. argus. Ehrb., *Abh. Ber. Ak.*, 1838, p. 129.—Diam. from ·0675 to ·175 mm. Central space absent, a rosette sometimes present. Markings polygonal, increasing gradually outwards; at the centre 4, near the border 2 to 3, on a narrow zone adjacent to the border 4 or 5, in ·01 mm.; secondary oblique rows indistinct or undifferentiated.—Ehrb. *ibid.*, 1839, p. 145, *Mon. Ber. Ak.*, 1844, p. 79; *Mikrog.*, pl. xxi. fig. 2 (excl. pl. xxii. fig. 5, 8); Grun., *Denk. Wien. Ak.*, 1884, p. 72; *C. irradiatus*, Harting, *Verh. Kon. Ak. Wetensch, Amsterdam*, 1864, No. ii. p. 8, pl. i. fig. 1); *C. radiatus*, Ehrb., *Mikrog.*, pl. xxi. fig. 1; *C. Woodwardii*, Sch., (not Eul.) *Atl.*, pl. lxi. fig. 2; *C. heteroporus*, Grun., in Sch., *Atl.*, pl. lxi. fig. 2.

Ehrenberg regarded this as probably a var. of *C. radiatus;* Brightwell and Grunow more correctly accept it as a distinct species. Harting points out the affinity of his *C. irradiatus* to *C. radiatus*, Ehrb., but its markings increase towards the border as in *C. argus*, to which it seems rather to belong. Some specimens belonging to this species were labelled by O'Meara *C. sinensis.*

Cleve has named *C. argus*, var. *subimpressa*, some Oamaru specimens that differ from more typical valves only in showing a subfasciculate arrangement of the markings, chiefly visible when the papillæ are in focus.

Habitat.—Oran deposit (Ehrenberg, Grunow, Greville!); Caltanisetta deposit (Ehrenberg); Richmond, Va. (Kützing); Szent Peter deposit (Pantocsek); Cuxhaven (Ehrenberg); Carpentaria Bay (Schmidt); N. America (Grunow); Banda Sea, 1200 fms. (Harting); locality? (Deby!); Japan (Kinker!); Cambodia (Hardman!); Mejillones (O'Meara!); stomach of oysters at Howth (O'Meara!); Los Angelos deposit, Cal. (O'Meara!); Cambridge deposit "Barbados" (Johnson!* Weissflog! Hardman!); Canton River, Whampoa (Grove!); Indian Ocean sounding, by Captain Pullen, 2200 fathoms (Greville!); Maryland (O'Meara!); Oamaru deposit (Cleve! Grove!).

Var. *subtraducens*, nov.—Diam. ·15 to ·225 mm. Central space absent or minute, rosette absent or obscure. Markings hexagonal, increasing from the centre almost to the border; towards the centre 4, near the border 3 in ·01 mm.; central papillæ evident; secondary oblique curved decussating rows distinct. Border narrow; striæ, 5 to 6 in ·01 mm.—(Pl. I. fig. 20.)

Transitional between *C. argus* and *C. traducens*. Specimens have sometimes been erroneously associated with *C. fimbriatus*. *C. intermedius*, Ehrb. (*Mikrog.*, pl. xxxiii. 13. fig. 3), may perhaps belong to this variety, its markings being figured as more delicate than those of *C. argus* (*Mikrog.*, pl. xxi. fig. 2). Here may also come the valve figured by Ehrenberg as *C. radiolatus?* (*Mikrog.*, pl. xxxix. 2. fig. 18), but *C. radiolatus*, Ehrb. (*Mikrog.*, pl. xxii. fig. 4), is distinct (see *infra*).

Habitat.—Jackson's Paddock, Oamaru deposit (Grove!).

C. traducens, sp. n.—Diam. ·1 mm. Surface flat. Central space and rosette absent; a small central area surrounded by a subcircular hyaline line evident. Markings hexagonal, gradually increasing in size from the central area outwards; towards the centre 8, at the border 6 in ·01 mm.; irregular on the central area,

* In the collection of Dr Greville.

secondary oblique curved decussating rows distinct; a narrow hyaline band adjacent to the border. Border narrow, sharply defined, with small evident granules 6 in ·01 mm.—*C. nebula*, Ehrb.? *Abh. Ber. Ak.*, 1872, p. 167, pl. xii. fig. 15. Sp. n.? Sch., *Atl.*, pl. lviii. fig. 12.

C. nebula, Ehrb., is an imperfectly defined species, approaching *C. radiolatus*, Ehrb., and *C. intermedius*, Ehrb.; its insertion here is provisional.

Habitat.—Railway cutting, Oamaru (Grove!) S.E. of Birjutskaja Kossa, Sea of Azof (Ehrenberg).

Var. *hispida*, nov. Sch., *Atl.*, pl. lviii. fig. 38 (no name).—Diam. about ·035 mm. Markings 6 to 7 in ·01 mm; apiculi prominent, numerous, at intervals of about ·0075 mm., inserted some distance within border.

Habitat.—Yokohama (Gründler).

C. exutus, sp. n.—Diam. ·0775 mm. Central space and rosette absent. Markings polygonal, increasing slightly outwards to the marginal band; at the centre 6 to 7, about the semiradius 5 to $5\frac{1}{2}$, on the sharply defined marginal band, 10 in ·01 mm., this band about $\frac{1}{8}$ of radius broad; rows radial from the centre to the marginal band, upon the latter the oblique decussating rows more evident. Border narrow, distinct.

Habitat.—Los Angelos (Hardman!).*

C. debilis, sp. n.—Diam. ·3 mm. Surface almost flat, a gentle slope near the border. Central space and rosette absent. Markings hexagonal, 5 to 7 in ·01 mm., slightly smaller at the centre, and towards the border submoniliform; central papillæ distinct; secondary curved oblique decussating rows evident; minute subulate areas at the origin of the shorter rows. Border sharply defined, usually opaque, about $\frac{1}{30}$ of radius broad; its broad or inner portion closely and irregularly punctate; the outer with evident striæ, 6 or 7 in ·01 mm.—(Pl. I. fig. 4.)

Habitat.—Jackson's Paddock, Oamaru deposit (Grove!).

C. dubiosus, Grun. MS.—Diam. ·0925 to ·15 mm. Central

* In the collection of Julien Deby.

space and rosette absent. Markings hexagonal, minute, subpunctiform, smallest and most crowded towards the border; towards the centre 8 to 10, towards the border 14 to 16 in ·01 mm.; secondary rows slightly oblique or irregularly subconcentric, the latter more evident near the centre, narrow radial subulate clear lines opposite the origin of the shorter rows; clear irregularly disposed puncta sometimes evident. Border narrow; striæ, 10 to 12 in ·01 mm. —Cleve and Möll., *Diat.*, No. 164; Janisch, *Gazelle Exped.*, taf. v. figs. 10, 11.

This species sometimes approaches *Podosira hormoides*, Mont. (Van Heurck, *Syn. Diat. Belg.*, pl. lxxxiv. figs. 3–6), but the markings are not in fasciculate rows. In *Podosira hormoides* Grunow has noted that the markings are grouped in almost radial lines (*Sitzungsb. naturw. Ges. Isis, Dresden*, 1878, p. 33).

Habitat.—Oamaru deposit (Grove! Doeg!); California (Arnott!);* Monterey deposit (Hardman!* Cleve!); Sta Monica deposit (Cleve and Möller!); Java (Cleve!); Successful Bay, Kerguelen (Cleve!).

Var. *curvans*, nov.—Diam. ·11 mm. Markings similar, but numerous hyaline subulate spaces towards the centre; the rows uniformly curved between centre and border, radial; the secondary rows but slightly oblique or almost parallel to the border.

Habitat.—Troublesome Gully, Oamaru (Grove!).

C. plicatus, Grun. Sch., *Atl.*, pl. lix. fig. 1.—Diam. ·0425 to ·05 mm. Surface with a short transverse central plication. Central space absent. Markings polygonal, increasing from the centre outwards, again decreasing slightly at the border; towards the centre 7, towards the border 6 in ·01 mm.; a circlet of evident, but small, apiculi at the border.—Grun., *Denk. Wien. Ak.*, 1884, p. 73, pl. iii. (C), fig. 10.; Cleve and Möll., *Diat.*, Nos. 114, 276.

Habitat.—Polycistinous rock, Nancoori (Cleve! Grunow, Schmidt, Hardman!);† Mascara and California (Cleve and Möller!).

C. corolla. Sch., *Atl.*, pl. lviii. fig. 32.—Diam. ·047 mm. Central space and rosette absent. Markings polygonal, 8 to 10 in ·01

* In the collection of Dr Greville.
† In the collection of Julien Deby.

mm., decreasing slightly to the border; irregular on a small central area, elsewhere the rows radial, sometimes subfasciculate; secondary oblique decussating rows obvious; apiculi numerous, forming a circlet placed some distance from the border. Border distinctly defined, about $\frac{1}{12}$ of radius broad; striæ distinct, 5 or 6 in ·01 mm.

Habitat.—Yokohama (Schmidt).

C. denticulatus. Cstr., *Diat. Chall. Exped.*, p. 155, pl. iii. fig. 8.—Diam. ·13 mm. Surface slightly convex, on outer $\frac{1}{6}$ of radius. Central space and rosette absent. Markings polygonal, subequal, 8 in ·01 mm.; central dots distinct; apiculi scattered over the surface at irregular wide intervals, distinct. Border sharply defined; striæ 8 to 10 in ·01 mm.

This species is nearly allied to *C. radiosus*, but differs in the more uniform markings and in the apiculi. Compare also *Podosira hormoides*, Mont. (= *Melosira nummuloides*, Ehrb.) from Lima (Van Heurck, *Syn. Diat. Belg.*, pl. lxxxiv. fig. 3).

Habitat.—Pacific Ocean (Castracane).

C. impressus, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 5.—Diam. ·08 mm. Surface with a long depression near the centre. Central space minute, irregular, about $\frac{1}{21}$ of diameter broad. Markings increasing gradually from the margin of the central space to the border; towards the centre 8, toward the border 7, in ·01 mm.; secondary oblique rows indistinct. Border striæ, subregular, distinct, 8 to 10 in ·01 mm.—Grun., *Denk. Wien. Ak.*, 1884, p. 73.

Habitat.—Sta Monica deposit (Grunow).

C. concinnus. W. Sm. *Syn.*, *Brit. Diat.*, ii. p. 85.—Diam ·062 to ·35 mm. Surface somewhat convex. Central rosette of large but delicate areolæ, sometimes inconspicuous, ·0075 to ·01 mm. broad. Markings polygonal, delicate, most evident towards the centre, where there are 7 or 8 in ·01 mm., decreasing outwards to 12 in ·01 mm.; rows obscurely fasciculate; near the border at subequal intervals short narrow radial clear areas, whence faint subhyaline lines proceed inwards; a circlet of apiculi at the border, minute; 2 larger apiculi unsymmetrical close to the border, a few long acicular apiculi also sometimes present on a zone within the processes.—Roper, *Quart.*

Jour. Micr. Sci., 1858, p. 20, pl. iii. figs. 12, 12*a*; Ralfs in *Pritch. Inf.*, 828, pl. v. fig. 89; Janisch, *Gazelle Exped.*, taf. ii. fig. 6; H. L. Sm., *Amer. Jour. Micr.*, 1877, No. 8, p. 102; Sch., *Atl.*, pl. cxiii. figs. 8, 9; Cleve and Möll., *Diat.*, No. 215, 319; *C.? tenuis*, Bail., *Boston Jour. Nat. Hist.*, 1862, p. 333, pl. vii. fig. 9; *C. centralis*, Schulze, *fide* Grunow, *Jour. Roy. Micr. Soc.*, 1879, p. 688 (excl. *Eupodiscus gregorianus*, de Bréb., *Jour. Quek. Micr. Cl.*, 1870. p. 41).

Bailey has seen specimens from Para River with 3 processes, but this species cannot be united to *Eupodiscus*, as suggested by Bailey, since the character of the processes of the latter is distinct.

C. concinnus, var. *kerguelensis*, Grun., differs in the markings, decreasing outwards from 5 to 7 in ·01 mm., and *C. concinnus*, var. *arafurensis*, Grun., in having a small clear circular central space and the markings 9 to 12 in ·01 mm., the clear radii being very long. *Eupodiscus gregorianus*, de Bréb., is *Eup. subtilis*, Greg. (Rattray, *Jour. Roy. Mic. Soc. Lond.*, 1888, p. 915). *C. concinnus* agrees with *C. centralis*, Ehrb., in its two large unsymmetrical apiculi, but differs in the degree of fineness and arrangement of the markings.

Habitat.—Peruvian guano (Schmidt); Para River (Bailey); Kerguelen, 25 fathoms (O'Meara, Grunow, Rae!); Baltic (Flögel); stomach of Pecten, Sussex coast and Kinsale Bay (W. Smith!); Firth of Clyde (Hennedy); Cumbrae (Arnott!); Hull (Ralfs); Loch Fyne and Inveraray (Gregory!); *Ascidia*, Hull (Greville! Gregory!); Gorleston (Roper!); Caldy, Pembrokeshire (Rev. J. Guillemard); Humber dredgings (Norman); seaweeds, Ballybrack, and oyster shells, Dublin Bay (O'Meara); San Francisco, Cal. (Firth! Schmidt); Helder Algæ (Kinker!); Mejillones deposit (O'Meara!); "Barbados" (Johnson!);* stomach of Pecten, Penzance (Montgomery!);* Nottingham deposit, Cape Wankarema (Cleve and Möller! Grunow!); Schleswig-Holstein (Cleve!); Heligoland (Schulze!* Weissflog!); Firth of Forth (Grove! Rattray!); Sheerness (Grove!); Marstrand (Kinker!).

Var. *jonesiana*, Rattray. *Eupodiscus jonesianus*, Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 22, pl. ii. fig. 3.—Rarely triangular. Diam. ·21 to ·45 mm. Markings coarser, and more sharply defined; towards

* In the collection of Dr Greville.

the centre 6, towards the border 8, in ·01 mm.; the unsymmetrical processes 2, larger, obtusely conical, and placed towards the same side of the valve; the clear radial lines less distinct; long apiculi sometimes present within the processes, as in the type.—*C. concinnus*, H. L. Sm., *Diat. Spec. Typ.*, No. 92; *Eup. jonesianus*, Grev.; Cleve, *Bih. Sv. Vet.-Ak. Handl.*, 1873, No. 11, p. 5, H. L. Smith, *Diat. Spec. Typ.*, No. 163; *Eup.? commutatus*, Grun., and *Coscinodiscus commutatus*, Grun., *Denk. Wien. Ak.*, 1884, p. 79; Van Heurck, *Typ. Syn. Diat. Belg.*, No. 490; *Eup. concinnus*, var. *triangularis*, *ibid.*

Greville erroneously states that the processes are 3. In triangular specimens they occur at the middle of two of the sides.

Habitat.—Peruvian guano (Grunow); Hong Kong (H. L. Smith! Hardman! Grove! Palmer!* Greville!); Yokohama (Hardman!); Canton River, Whampoa (Grove!); Schleswig (Van Heurck!); shell cleanings, Singapore (Hardman!);* Sand Heads, Bay of Bengal, Ceylon, and edible seaweeds, India (Macrae!);* Port Elizabeth (Hardman!); Java Sea (Cleve, Grunow); surface, Arafura Sea, H.M.S. Challenger (Rae!); Cherbourg (H. L. Smith!); Cuxhaven, Brazil, and China (Grunow); North Sea (Griffin!); trawled at lat. 34° 36′ N., long. 140° 22′ E., by H.M.S. Challenger (Rae!);† Kusu (Cleve! O'Meara!); from Pecten, Penzance (Greville!); Bay of Bengal (Macrae!); Tindingen, Greenland (Cleve!); Java (O'Meara! Cleve!); Cape Wankarema (Cleve!); lat. 4° 20′ S., long. 105° 22′ E. (Cleve!).

Var. *Moseleyi*, Rattray. *C. Moseleyi*, O'Me., *Quart. Jour. Micr. Sci.*, 1875, p. 330.—Diam. ·28 to ·55 mm. Colour iridescent when dry. Central rosette distinct, of large unequal areolæ. Markings towards the centre 5 to 6, at the border 8, in ·01 mm.; rows obscurely fasciculate, unsymmetrical; processes 2, minute; apiculi obscure. Border narrow.—O'Meara, *Jour. Lin. Soc.* (Bot.), 1877, p. 57, pl. i. fig. 6; Cstr., *Diat. Chall. Exped.*, p. 153; *C. concinnus*, var. *kerguelensis*, Grun., *Denk. Wien. Ak.*, 1884, p. 79.

Habitat.—Kerguelen, at 25 fathoms (Rae! Hardman!‡ Peal!‡ O'Meara, Grunow); Royal Sound, Kerguelen (Rae!).

* In the collection of Dr Greville.
† In the collection of Dr F. W. Griffin.
‡ In the collection of Julien Deby.

Var. *arafurensis.* Grun., *Denk. Wien. Ak.*, 1884, p. 79. *C. papuanus*, Cstr., *Diat. Chall. Exped.*, p. 154, pl. iii. fig. 3.—Diam. ·152 to ·475 mm. Central space minute, surrounded by a minute and inconspicuous rosette, sometimes hardly differentiated. Markings 9 to 12 in ·01 mm.

This form is frequent in the Arafura Sea. The central space is smaller than figured by Castracane in all the specimens I have observed, and is not of specific value, as he maintains. He has overlooked the distinct marginal processes characteristic of the species.

Habitat.—Arafura Sea, H.M.S. Challenger (Rae! Castracane); between Kerguelen and Heard Island, H.M.S. Challenger (Rae!).

C. africanus, Janisch. Sch., *Atl.*, pl. lix. figs. 24, 25.—Subcircular or roundly elliptical. Diam. ·035 to ·088 mm. Central space and rosette absent. Markings polygonal, increasing gradually outwards, and again smaller at the border; towards the centre 6, towards the border 4 in ·01 mm.; irregular on an indistinctly defined somewhat excentric area, elsewhere the rows radial, straight, or slightly bent, sometimes indistinctly subfasciculate towards the border. Border regularly striated, sometimes double; the inner portion about $\frac{1}{3}$ of the breadth of the outer. Sometimes also with more evident striæ; oblique curved, more distant and more distinct lines.—Janisch, *Gazelle Exped.*, taf. iii. fig. 2.

This species is readily distinguished by the character of its border. From *C. vetustissimus*, Pant., it differs by the absence of a nodule from the excentric area.

Habitat.—Gazelle Expedition (Janisch); Newcastle, Barbados (Firth); off Ascension Island, S.S. Buccaneer (Grove!).

Var. *wallichiana*, Grun. Cleve and Möll., *Diat.*, No. 183, 207.—Diam. ·05 to ·0575 mm. Central space irregular, excentric, with a few isolated rounded granules. Markings rounded, granular; towards the excentric area 8, about the semiradius 5 to $5\frac{1}{2}$, at the border 6 in ·01 mm.; rows radial, straight; at irregular intervals hyaline, narrow radial spaces passing inwards for a short distance from border. Border distinct; striæ delicate, 10 to 12 in ·01 mm.—*C. africanus*, var. *rotunda*, Cstr., *Diat. Chall. Exped.*, 1886, p. 159, pl. xxiv. fig. 3.—(Pl. II. fig. 4.)

Habitat.—Antarctic Ocean, Patagonia (Cleve and Möller !); Table Bay (Cleve !).

C. mirificus. Cstr., *Diat. Chall. Exped.*, p. 154, pl. iii. figs. 6, 6*a*.—Diam. ·326 mm. Central space irregularly subcircular, about $\frac{1}{11}$ of diam. broad. Markings hexagonal, their sides composed of closely placed round granules, the central dots distinct, decreasing but slightly to the border; towards the centre 6, at the border 8, in ·01 mm. Border formed by a simple line.

In its large central space, this species approaches *C. asteromphalus* var. *pabellanica*, Grun., but is distinguished by the absence of a distinct band around the central space, and by the greater uniformity of the markings.

Habitat.—Hong Kong, in the sea, H.M.S. Challenger (Castracane).

C. Hauckii, Grun. Van Heurck., *Syn. Diat. Belg.*, pl. xciv. fig. 29.—Diam. ·0365 mm. Central space absent. Markings obscure, punctiform, irregular, and with hyaline interspaces on the central portion, which extends outwards for $\frac{2}{3}$ of radius, on outer $\frac{1}{3}$ more minute, closely disposed in crowded radial lines, 16 in ·01 mm.; apiculi sometimes evident, inserted near border about 2 in ·01 mm.—Cleve and Möll., *Diat.*, No. 210.

Habitat.—Rovigno (Van Heurck, Cleve and Möller !); Green land (Cleve !); Labuan ? (Cleve !).

C. liocentrum. Ehrb., *Abh. Ber. Ak.*, 1870, p. 53, pl. ii. 2. fig. 9. —Diam. ·075 mm. Central space extending almost to semiradius, smooth. Markings polygonal, delicate, increasing slightly outwards; rows distinct.

This species is at once characterised by the large size of the central space, in which it approaches *C. mesoleius*, Cleve.

Habitat.—Humboldt Valley deposit, Cal. (Ehrenberg).

C. vacuus, sp. n. *Melosira ?* Sch., *Atl.*, pl. lviii. fig. 29.—Diam. about ·03 mm. Central space large, extending outwards for about $\frac{2}{3}$ of radius. Markings minute, punctiform, forming closely disposed radial rows, on outer $\frac{1}{3}$ of radius. Border narrow, indistinct.

This is distinguished from *C. Hauckii*, Grun. (Van Heurck, *Syn.*

Diat. Belg., pl. xciv. fig. 24), by the absence of punctiform granules on the central space. In Schmidt's figure of *C. vacuus*, a minute round central areola (nodule?) is faintly indicated.

Habitat.—Cape of Good Hope (Schmidt).

C. mesoleius. Cleve, *Vega Exped. Jakttag. Stockh.*, Bd. iii., 1883, p. 503, pl. xxxviii. fig. 2.—Diam. ·03 mm. Central portion hyaline, extending outwards to about $\frac{2}{3}$ of the radius, sharply defined. Markings on outer portion punctiform, 28 in ·01 mm., forming radial striæ.

The valve is very thin and transparant. It approaches *C. Hauckii*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xciv. fig. 29); but in the latter the central portion is covered with irregular scattered puncta. Both approach the genus *Cyclotella.*

Habitat.—Labuan, near Borneo, among Algæ * (Cleve!).

C. lutescens, sp. n.—Diam. ·125 mm. Surface flat. Central space circular, minute, surrounded by a subobsolete band of large cuneate areolæ. Markings hexagonal, distinct, increasing outwards for about $\frac{6}{7}$ of the radius, thence decreasing to the border; towards the centre 5, at $\frac{6}{7}$ of radius $3\frac{1}{2}$, in ·01 mm.; central papillæ faint; rows straight, at intervals with the markings oblique; apiculi minute, inserted at the border at intervals of about ·005 mm. Border narrow; striæ faint, 8 to 10 in ·01 mm.—(Pl. II. fig. 2.)

Distinguished from *C. cribrosus*, Tru. and Witt, by the gradual increase of the markings outwards.

Habitat.—Jeremie deposit, Hayti (Rae!).

C. modestus, sp. n.—Diam. ·325 mm. Surface flat, becoming convex towards the border. Central space round, indistinct, about $\frac{1}{4\,3}$ of diam. broad. Markings delicate and faint for inner $\frac{2}{3}$ of radius, more evident on the outer $\frac{1}{3}$, increasing from the central space outwards, towards the centre punctiform 8, beyond the semi-radius distinctly hexagonal, with evident central papillæ, 4 in ·01 mm.; rows around the central space of unequal lengths, and separated by hyaline interspaces, towards the border subfasciculate, the fasciculi separated by fossil subhyaline radial lines; apiculi 2, unsymmetrical, separated by an interval equal to about $\frac{1}{3}$ of the

* Collected by Dr Kjellman.

circumference, small but distinct; a circlet of more minute apiculi at the border absent. Border narrow.—(Pl. I. fig. 3.)

Habitat.—Rio Janeiro (Kitton !); * Peruvian guano (Macrae !).

C. patellæformis. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 80, pl. x. fig. 4.—Diam. ·07 to ·1 mm. Surface slightly convex. Central space and rosette absent. Markings polygonal, in contact and without order for about $\frac{1}{3}$ of radius, thence obtusely angular, pearly, 4 in ·01 mm., increasing slightly outwards and forming straight rows; secondary subconcentric rows also evident; at the origin of the shorter rows are sharply defined, irregular, elongate hyaline spaces; a narrow hyaline band within the border. Border with a single row of granules, 4 in ·01 mm.—*C. detritus*, Sch., *Atl.*, pl. lviii. fig. 15.

In Greville's specimen, the undulating concentric lines crossed by stronger radial costæ—as shown in his figure—do not occur.

Habitat.—Springfield deposit, Barbados (Hardman !).†

C. oblongus. Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 4, pl. i. figs. 9, 10.—Elliptical or oval, major axis ·07 to ·145 mm., from $2\frac{1}{3}$ to 3 times minor. Surface depressed at the centre. Central area roundly or elongately elliptical, with rounded granules irregular, or in rows parallel to the major axis. Markings round, granular, increasing slightly from the edge of the central area outwards, again decreasing towards the border; at the semiradius 4, near the border 6, in ·01 mm.; rows straight only along the major and minor axes, between these slightly curved towards the major axis.—Sch., *Atl.*, pl. lxvi. figs. 10, 11; *C. oblongus forma typica*, Truan and Witt, *Jeremie Diat.*, p. 14, pl. ii. fig. 16.

Distinguished from *C. punctatus* by the presence of a central depression, the absence of a central space, and of oblique decussating rows near the border.

Habitat.—Chalky Mount., Barbados (Firth !); Barbados (Weissflog! Deby! Hardman! Kinker! Cleve! Greville!); Springfield deposits, Barbados (Johnson! Firth! ‡ Hardman); § Pacific Ocean,

* In the collection of Herr E. Weissflog.
† In the collection of Dr Greville.
‡ In the collection of Dr F. W. Griffin.
§ In the collection of Julien Deby.

H.M.S. Challenger (Castracane); Jeremie deposit, Hayti (Truan and Witt).

C. ellipticus. Grun., *Reise. d. Novara Wien.*, 1870 (Bot. Th.), p. 104, pl. i. A. fig. 18 *a*, *b*.—Elliptical, major axis ·04 to ·075 mm., minor ·02 to ·035 mm. Central space absent. Markings rounded, granular; on the central portion large, subradial or irregular, decreasing slightly outwards, on a well-defined band at the border minute, in radiating delicate crowded striæ.

Distinguished from *Cestodiscus ovalis*, Grev., by the absence of apiculi, and from *C. oblongus*, Grev. (*Trans. Micr. Soc. Lond.*, 1866, p. 4, pl. i. figs. 9, 10), in the relatively larger size of the central area, and the greater diminution of the markings on the marginal band. In the explanation of Cleve and Möller's *Diat.*, No. 57, *C. ellipticus* is given as equivalent to *C. lewisianus*, var.; with this opinion I am unable to agree.

Habitat.—Polycistinous Rock, Nancoori (Grunow, Cleve!).

C. obovatus. Cstr., *Diat. Chall. Exped.*, p. 160, pl. viii. fig. 4; pl. xviii. fig. 7; pl. xxii. fig. 9.—Roundly elliptical or oval, major axis ·099 mm., about $1\frac{1}{4}$ times minor. Surface almost flat. Central space and rosette absent. Markings polygonal; towards the centre 4, near the border decreasing to 6 or 8 in ·01 mm.; rows straight and parallel to the major axis on the central portion, which is distinctly defined, and extends to about $\frac{2}{5}$ of the radius from the centre, beyond this in radial rows.

Habitat.—Pacific Ocean (Castracane!).

Var. *circularis*, nov.—Circular. Diam. ·0625 mm. Markings similar, but the central area less distinctly defined, the rows radiating from its outer edge to the border sometimes slightly curved.

Specimens of this var. have been labelled *C. subtilis* by O'Meara.

Habitat.—Humber (O'Meara!); stomach of oysters, at Howth (O'Meara!); locality? (O'Meara!); Atlantic Ocean, lat. 3° S., long. 15° W. (O'Meara!); off Ascension Island, lat. 0° 1·6′ S., long. 15° 56·5′ W., 1845 fathoms, S.S. Buccaneer (Grove! Rattray!).

C. dubius, sp. n. Sch., *Atl.*, pl. lxi. fig. 14.—Diam. ·094 mm. Surface flat from centre for about $\frac{2}{3}$ of radius, thence convex to the

border. Central space angular, about $\frac{1}{10}$ of diam. broad. Markings rounded, adjacent to the central space, elsewhere polygonal, $2\frac{1}{2}$ to 3 in ·01 mm., subequal for about $\frac{2}{3}$ of radius, thence decreasing slightly to the border, smooth; secondary oblique rows inconspicuous. Border indistinctly defined; striæ somewhat irregular, 4 to 6 in ·01 mm.—*C. crassus*, Bail. var.? Grun., *Denk. Wien. Ak.*, 1884, p. 74.

Habitat.—Springfield deposit, Barbados (Schmidt).

C. cingulatus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 200.—Diam. ·049 mm. Central space small, hyaline, indistinct. Markings minute, about 13 in ·01 mm. Border smooth, distinct.—Ehrb. *Mikrog.*, pl. xxxv. A. 21. fig. 6; Ralfs in *Pritch. Inf.*, p. 829.

In Antarctic diatomaceous gatherings I have not observed specimens similar to this. Is it possible that Ehrenberg was dealing with a valve of *C. subtilis*?

Habitat.—Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W. (Hooker!).*

C. crassus. Bail., *Amer. Jour. Sci.*, 1856, pl. xxii. p. 4.—Diam. ·12 to ·14 mm. Central space small, angular to elliptical. Markings subpearly; towards the centre 3, towards the border 2 to $2\frac{1}{4}$ in ·01 mm., at the border suddenly smaller, secondary oblique rows inconspicuous. Border striæ 4 in ·01 mm.—Ralfs in *Pritch Inf.*, p. 830; Grun., *Denk. Wien. Ak.*, 1884, p. 74. *C. crassus*, var., Sch., *Atl.*, pl. lxi. fig. 19.

Grunow, in his synopsis of this genus (*Denk. Wien. Ak.*, 1884, p. 71), states correctly in his section 15 that the markings become smaller both towards the centre and towards the border, but erroneously states in the following section (16) that only the outermost markings are smaller, and by this means he differentiates this species.

Habitat.—Soundings, Sea of Kamtschatka (Bailey); Monterey (Bailey); Kékkö, Szakal and Dolje deposits (Pentocsek!); Barbados (Cleve!).

Var. *morsiana.* Grun., *ibid.*, 1884, p. 74.—Diam. ·174. Mark-

* *Fide* Ehrenberg.

ings towards the centre 3, increasing outwards to 2, at the border 8 in ·01 mm., forming 3 concentric zones.

Habitat.—Mors deposit (Grunow).

Var. *gelida.* Grun., *ibid.*, 1884, p. 74, pl. iii. (C), fig. 6.—Diam. ·114 mm. Markings increasing, but little from the centre outwards; towards the centre 3, towards the border $2\frac{3}{4}$, at the border 7, in ·01 mm.

Distinguished from var. *morsiana* by the markings towards the border, and from *C. apiculatus*, var. *Woodwardii*, by the increase of the markings outwards.

Habitat.—Franz Josef's Land (Grunow).

Var. *algida.* Grun., *ibid.*, 1884, p. 74, pl. iii. (C), fig. 5.—Diam. ·094 mm. Central smooth area small. Markings subequal, 2 to $2\frac{1}{2}$ in ·01 mm., at the border $3\frac{1}{2}$ in ·01 mm.

This var. approaches *C. marginatus* in the character of the markings.

Habitat.—Franz Josef's Land (Grunow).

C. heteroporus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 265.—Diam. ·072 to ·1125 mm. Central space small, or replaced by a small rosette. Markings at the centre $3\frac{1}{2}$ to 4, increasing outwards to an annular, somewhat elevated area about the semiradius to 2 or $2\frac{1}{2}$, again decreasing to the border to 6 or 7 in ·01 mm.; rows sometimes indistinctly radial; secondary oblique rows irregular, obscure. Border striæ evident, 6 in ·01 mm.—Ralfs in *Pritch. Inf.*, p. 831; Grun., *Denk. Wien. Ak.*, 1884, p. 74. *C. heteroporus*, var. Grun., in Sch., *Atl.*, pl. lxi. fig. 4.

Recent Manilla valves show a transition to *C. apiculatus*, var. *Woodwardii.* It differs from *C. crassus* in the less robust appearance of the markings and the distinctly striated border.

Habitat.—Nottingham and Monterey deposits (Grunow); Elephant Point, Bengal (Grunow); Piscataway deposit (Griffin!); Santa Monica deposit (Rae!); Delaware, Maryland (O'Meara!); Manilla (Grove!); Labuan (Cleve!); Maryland (Cleve).

Var. *moronensis.* Grun., *Denk. Wien. Ak.*, 1884, p. 75.—Diam. ·1 mm. Central rosette distinct. Markings on the elevated ring

more prominent; rows distinctly radial, secondary rows more obvious. Border more sharply defined.—*C. heteroporus*, var. Grun., Sch. in *Atl.*, pl. lxi. fig. 1.

Habitat.—Moron deposit (Grunow).

C. boliviensis. Grun., *Denk. Wien. Ak.*, 1884, p. 76.—Diam. ·15 to ·22 mm. Central space small, irregular. Markings hexagonal, non-punctate, increasing gradually outwards for about $\frac{2}{3}$ of the radius; towards the centre 4, at about $\frac{2}{3}$ of radius 3 in ·01 mm., thence decreasing rapidly to the border to 6 in ·01 mm.; secondary oblique rows inconspicuous.—*C. Woodwardii.* var.? Sch., *Atl.*, pl. lx. fig. 8.

Distinguished from *C. apiculatus*, var. *ambigua*, by the increase of the markings outwards, and from *C. gigas* by their arrangement in contact around the centre. The increase in the markings outwards is greater in some specimens than in others.

Habitat.—Bolivian guano, Sta Monica deposit (Grunow); Darien (Grunow, Schmidt); Cambridge deposit, Barbados (Hardman!* Kinker!); Lobos de Afuera guano (Grove!).

Var. *spinulosa.* Grun., *ibid.*, 1884, p. 76; *C. boliviensis*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 4.—Diam. ·16 to ·17 mm. Central space large, circular, about $\frac{1}{10}$ of diam. broad. Markings towards the centre 5, towards the border $3\frac{1}{2}$ in ·01 mm.; a circlet of numerous minute apiculi close to the border.

Habitat.—Bolivian guano (Grunow).

C. gigas. Ehrb., 1841, *Abh. Ber. Ak.*, 1841, p. 412.—Diam. ·159 to ·31 mm., easily seen by the naked eye. Central space subcircular, about $\frac{1}{20}$ to $\frac{1}{24}$ of diam. broad. Markings obtusely angular, and least crowded towards the centre, with central dots faint, soon becoming hexagonal, and increasing gradually outwards; towards the centre 4, towards the border $1\frac{3}{4}$ to 2 in ·01 mm., at the border again small; secondary oblique decussating rows distinct. Border narrow; striæ radial, about 4 in ·01 mm., subregular.—Ehrb., *Mikrog.*, pl. xviii. fig. 34; Ralfs in *Pritch. Inf.*, p. 829; Jan., *Sch. Ges. väter. Cult.*, 1862, Heft ii. p. 3, pl. 1A. fig. 12; *Gazelle*

* In the collection of Julien Deby.

Exped., taf. iii. fig. 4; vi. fig. 13; Sch., *Atl.*, pl. lxiv. fig. 1; Grun., *Denk. Wien. Ak.*, 1884, p. 76; Cleve and Möll., *Diat.*, Nos. 57, 162, 164; *C. radiatus*, Bail., *Amer. Jour. Sci.*, 1842, vol. xlii. p. 95, pl. ii. fig. 14.

In all the specimens there is a central space, not indicated in the earlier figures. Ralfs first noted the striated border, Janisch the more robust character of the markings towards the border, and Grunow the delicate puncta on the markings and their small size at the border.

Habitat.—Richmond, Va. (Ehrenberg, Kinker! Cleve and Möller!); Nancoori (Schmidt, Cleve!); Sta Monica deposit (Weissflog! Grove!); locality? (Deby!); Hong Kong (Hardman!); Peruvian guano (Cleve! Hardman!); Macabees guano (Firth!); Crescent City (Hardman!); Sea of Java (O'Meara!); Maryland (O'Meara!); Los Angelos deposit (O'Meara!); Cove Calvert county, Maryland (Greville!); Bay of Bengal (Macrae!); Sta Monica deposit, Patagonian guano (Cleve!); Marstrand (Kinker!).

Var. *punctiformis*, nov. *C. gigas*, var., Grun., *ibid.*, 1884, p. 76.—Diam. ·151 mm. Markings free towards the centre, from the semiradius to the border polygonal, punctiform, almost invisible markings at origin of shorter radii.—*C. Woodwardii*, Eul., var., Grun. in Sch., *Atl.*, pl. lxv. fig. 2.

According to Schmidt, this is intermediate between *C. gigas* and *C. diorama*, Sch.

Habitat.—Aegina (Schmidt).

Var. *diorama*. Grun., *ibid.*, 1884, p. 76. *C. diorama*, Sch., *Atl.*, pl. lxiv. fig. 2.—Diam. ·15 to ·25 mm. Surface slightly convex. Markings towards the central space rounded 4 in ·01 mm., increasing gradually but to a less degree outwards, subequal—12½ in ·01 mm.—on outer half of valve, again decreasing gradually near the border.—*C. gigas*, var. *Montereyi*, Grun., *ibid.*, 1884, p. 76.

Grunow's var. *Montereyi* differs only in having somewhat smaller markings, 3½ in ·01 mm.

Habitat.—Santa Monica deposit (Schmidt, Cleve!); Monterey deposit (Grunow, Cleve!); Isle of Muntok, Indian Archipelago (Kitton!); Pabillan de Pico guano (Cleve!); Patagonian guano (Cleve!).

Var. *duplicata.* Grun., *ibid.*, 1884, p. 76.—Diam. ·0215 mm. Markings hexagonal, placed obliquely so that their lower and upper ends are united by oblique walls.—Cleve and Möll., *Diat.*, No. 57.

Habitat.—Shokoe Hill deposit, Richmond, U.S. (Grunow); Richmond Va. (Greville ! Cleve and Möller !).

Var. *californica*, nov. *C. californicus*, O'Me. MS.—Diam. ·22 mm. Markings towards the central space 6 to 8, increasing outwards to 4, again decreasing to 8, in ·01 mm. at the border.

Habitat.—" Guano " (O'Meara !); Medway (Dallas !).*

Var. *guineensis*, Rattray. *C. guineensis*, Grun., *Denk. Wien. Ak.*, 1884, p. 76.—Diam. ·05 to ·1 mm. Central space distinct. Markings hexagonal; towards the centre rounded, granular, $5\frac{1}{2}$ in ·01 mm., not distinctly punctate; the interspaces provided with small hexagonally arranged puncta.

Habitat.—Brackish water, Lagos (Grunow).

Var. *laxa*, nov. *C. guineensis*, Grun., *ibid.*, p. 76.—Diam. ·13 mm. Central space round with irregularly placed puncta. Markings free.

Habitat.—Monterey deposit (Grunow).

C. Janischii. Sch., *Atl.*, pl. lxiv. figs. 3, 4.—Diam. ·16 to ·245 mm. Central space subcircular, hyaline, $\frac{1}{22}$ to $\frac{1}{24}$ of diam. broad. Markings faint, but sharply defined, on a narrow band at the border, obtusely angular, increasing but slightly outwards, $3\frac{1}{2}$ to 4 in ·01 mm., non-punctate, the central dots indistinct; rows radial, secondary rows obscure.—Grun., *Denk. Wien. Ak.*, 1884, p. 76; Janisch., *Gazelle Exped.*, taf. iv. figs. 3, 5; *C. marginatus*, Janisch. (*non* Ehrb.), *Abh. Sch. Ges. väter. Cult.*, 1882, p. 3, pl. i. A. fig. 20.

This species has sometimes been confounded with *C. gigas.*

Habitat.—" Guanos " (Grunow ! Macrae ! Firth !); Ichaboe guano (Janisch, Joshua ! Greville !); Felsö-Esztergály, Kékkö, Szent Peter deposits (Pantocsek !); Peruvian guano (Firth !);† soundings, Gulf of California (H. L. Smith !);‡ Richmond (O'Meara !); Chincha guano (Grove !); Saldanha Bay guano (Cleve, Greville !); Cambridge deposit (?) (O'Meara !); Java, (Cleve !).

* In the collection of Dr Greville.
† In the collection of Dr F. W. Griffin.
‡ H. L. Smith, *Diat. Spec. Typ.*, No. 91.

Var. *arafurensis.* Grun., *ibid.*, 1884, p. 76.—Diam. from ·29 to ·425 mm. Markings increasing more from the central space outwards; towards the centre $3\frac{1}{2}$, towards the border $2\frac{1}{2}$ in ·01 mm. —*C. arafurensis* var., Cstr., *Diat. Chall. Exped.*, p. 153, pl. ii. fig. 4; *C. craspedodiscus*, O'Me., *Quart. Jour. Micr. Sci.*, 1877, p. 463.

The presence of the large central space referred to by O'Meara in his *Coscinodiscus craspedodiscus* excludes it from *Coscinodiscus craspedodiscus*, Kütz., where there is a central rosette, and identifies it with the present var. The hoop-like appearance of the valve under a low power referred to by Castracane is not shown in his figure (*Diat. Chall. Exped.*, p. 152–154, pl. iii. fig. 5).

Habitat.—Arafura Sea, H.M.S. Challenger (Grunow, Weissflog! Rae!); Gazelle Expedition (Weissflog!); Bay of Bengal (Macrae!); "Atlantic Ocean" (Cleve).

C. entoleion. Grun. Sch., *Atl.*, pl. cxiv. fig. 3.—Diam. ·25 to ·3 mm. Surface slightly convex near the border. Central space circular, $\frac{1}{20}$ to $\frac{1}{24}$ of diam. broad. Markings hexagonal, increasing from the central space outwards, again decreasing near the border; towards the centre most delicate, 3 to $2\frac{1}{2}$, near the border 2 to $2\frac{1}{2}$ in ·01 mm.; oblique decussating rows evident. Border sharply defined, narrow; striæ 4 to 5 in ·01 mm.

Distinguished from *C. gigas* by the more delicate and smaller markings and the absence of narrow hyaline interspaces radiating outwards from the central space. Specimens have sometimes been confounded with *C. perforatus*, var. *cellulosa*, from which they differ by the marked increase of the areolæ outwards.

Habitat.—Thames mud, at Southend (Dickie!); Hungarian marl (Thum).*

C. flexilis, sp. n. Sch., *Atl.*, pl. cxiv. fig. 6.—Diam. ·15 to ·12 mm. Surface almost flat. Central space distinct, $\frac{1}{16}$ to $\frac{1}{12}$ of diam. broad, surrounded by an inconspicuous band of larger areolæ. Markings polygonal, mostly hexagonal, increasing for a short distance outwards from the band surrounding the central space, again gradually decreasing towards the border; towards the centre $4\frac{1}{2}$ to 5, about the semiradius 4, at the border 6, in ·01 mm.; rows

* *Fide* A. Schmidt.

straight, secondary oblique decussating rows evident. Border narrow, sharply defined; striæ faint, 6 in ·01 mm.

Distinguished from *C. apiculatus* by the more delicate markings that are devoid of prominent central papillæ, and by the distinct central space.

Habitat.— ? (Griffin !); Chincha guano (Schmidt).

C. conformis, sp. n. Sch., *Atl.*, pl. cxiv. fig. 4.—Diam. ·2 mm. Surface somewhat depressed at the centre. Central space circular, about $\frac{1}{26}$ of diam. broad, surrounded by an inconspicuous band of areolæ subobsolete on their central side. Markings 4- to 6-angled, without puncta at the angles, increasing gradually to the semiradius, again decreasing to the border; towards the central space $4\frac{1}{2}$, at the semiradius $3\frac{1}{2}$, at the border 5 or 6 in ·01 mm.; central papillæ indistinct; delicate puncta at the origin of the shorter rows; secondary oblique rows short, substraight, evident. Border narrow; striæ 8 in ·01 mm.

Habitat.—Arica (Schmidt).

C. josefinus. Grun., *Denk. Wien. Ak.*, 1884, p. 75, pl. iii. (C), fig. 16.—Diam. ·08 mm. Surface convex. Central space small, subcircular. Markings smooth, decreasing but slightly near the border, 7 to 8 in ·01 mm. Border striæ delicate, 14 in ·01 mm., about $\frac{1}{16}$ of radius broad, at its middle a sharp line concentric with the outer edge.

Distinguished from *C. radiosus*, Grun., and *C. fimbriatus*, Ehrb., by the character of the border.

Habitat.—Franz Josef's Land (Grunow).

C. nobilis. Grun., *Jour. Roy. Micr. Soc. Lond.*, 1879, p. 687, pl. i. fig. 1.—Diam. ·375 to ·54 mm. Central space distinct, hyaline. Markings minute, about 7 in ·01 mm., hexagonal towards the border, separated into obscure fasciculi by inconspicuous radial lines.—Cleve and Möll., *Diat.*, Nos. 145, 146, 162; *C. regius*, Grun., *Sitzungsb. naturw. Ges. Isis., Dresden*, 1878, p. 124.

Sometimes mistaken for *C. concinnus*, but distinguished by its large central area and more distinct radial rows of markings.

Habitat.—In *Noctiluca*, at Gorleston Pier, Suffolk, Harwich (Grunow !); *Ascidia*, Hull (Greville !); Hong Kong and Arafura Sea

(Grunow ! Rattray !) ; Java Sea (Grunow ! Cleve and Möller !) ; lat. 4° 12′ 7″ N., long. 3° 57′ 5″ E, 1460 fms. (Rae !) ; Isle of Muntok, Indian Archipelago (Kitton !) ;* Nancoori (Cleve and Möller !) ; Java (Cleve and Möller ! Cleve !); lat. 4° 20′ S., long. 105° 22′ E. (Cleve !); surface, Gulf of Guinea, S.S. Buccaneer Exped. (Grove !).

C. Gazellæ. Janisch., *Jour. Roy. Micr. Soc., Lond.*, 1879, p. 688.—Diam. 1·8 to 1·9 mm. Central space circular, about ·0375 mm. broad, hyaline, bearing at its centre a group of irregular evident apiculi, and having at its boundary a circlet of similar apiculi at wide unequal intervals. Markings delicate, punctiform, 6 to 7 in ·01 mm.; rows straight, short secondary transverse or oblique rows obvious ; adjacent to the border a distinct narrow (about ·003 mm. broad) hyaline zone.† Border sharply defined, about ·005 mm. broad, hyaline.—*Ethmodiscus tympanum*, Cstr., *Diat. Chall. Exped.*, 1886, p. 170, pl. xiv. fig. 3; *E.*, sp. (*fragmenta*) Cstr., *ibid.*, p. 170, pl. xiv. figs. 4*a–c*; *E. gigas*, Cstr., *ibid.*, p. 169, pl xiv. fig. 5.

E. wyvilleanus, Cstr. (*ibid.*, p. 170, pl. xiv. fig. 6), differs only in having the angles of the valves rounded, and may be a var. of the present species. To the same var. belongs *E. sphæroidalis*, Cstr., *ibid.*, p. 170, pl. xxii. fig. 10 ; in the specimen figured division has recently been completed.

Habitat.—Gazelle sounding No. 125, lat. 30° 53′ S., long. 177° 6′ E., depth 4151 metres (Janisch !); and sounding No. 96, lat. 9° 57′ S., long. 121° 52′ E. (Weissflog !) ; H.M.S. Challenger, station 265, depth 2900 fms. (Grunow !) ; Nottingham deposit in fragment (Grunow).

C. imperator, Janisch MS.—Diam. ? Central space and rosette ? Markings minute, delicate, angular, 8, towards the border recognised with greater difficulty, 10 to 12 in ·01 mm.; rows straight ; the oblique decussating rows faint ; hyaline band adjacent to the border absent. Border narrow, hyaline.—(Pl. I. fig. 5.)

Habitat.—Gazelle Expedition, sounding No. 96, lat. 9° 57′ S., long. 121° 52′ E. (Weissflog !).

C. praetor, Grove MS.—Diam. ? Surface flat. Central space

* In the collection of E. Grove.

† This also occurs in *C. rex.*

absent, a distinct central area ·04 mm. broad, bounded by a narrow ring of closely disposed minute apiculi. Markings angular recognised with difficulty; within the central area 6, at the borde 8 to 9 in ·01 mm.; secondary oblique decussating rows short, obvious; no hyaline band at the border. The connecting zone bears straight parallel rows of short striæ, 10 in ·01 mm., the rows separated by narrow hyaline lines.—(Pl. III. figs. 2 and 3.)

Habitat.—"Barbados" (Grove!).

C. punctatus. Ehrb., *Mon. Ber. Ak.*, 1884, p. 78.—Elliptical or subdiamond-shaped, major axis ·0625 to ·12 mm., $1\frac{1}{3}$ to $1\frac{2}{3}$ times minor. Central space small, circular sometimes bearing a few isolated round granules. Markings rounded, granular, 6 to 8 in ·01 mm.; interspaces hyaline, largest towards the centre, towards the border more crowded; rows radial, straight; oblique decussating rows evident near the border. Border indistinct on inner side; striæ delicate, 8 to 10 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. figs. 40, 41; Ralfs in *Pritch. Inf.*, p. 830; H. L. Sm., *Diat. Spec. Typ.*, No. 97; Cleve and Möll., *Diat.*, No. 57.

This species is sometimes mistaken for *C. lewisianus*, but it differs in the arrangement of the markings. It differs from *Cestodiscus ovalis*, Grev., by the absence of obtuse submarginal processes. In the original definition Ehrenberg gives the markings as 12 to 13 in ·01 mm.

Kitton has found frustules with very dissimilar valves in the Richmond deposit, Virginia. In one case one valve was normal, whilst the second showed a large rounded central area, with round isolated granules disposed without order, the zone adjacent to the border only appearing normal.

Grove inclines to associate this form with *Actinocyclus*, as his forms in H. L. Smith's series show pseudonodules.

Habitat.—Richmond, Va. (Greville! Bailey!* Grove! Kitton! Cleve and Möller!); Nancoori (Hardman!); Crescent City, Cal. (Weissflog!); Paita deposit, Peru (H. L. Smith!).

Var. *rhombica*, nov. *C. rhombicus*, Cstr., *Diat. Chall. Exped.*, p. 164, pl. xxii. fig. 11.—Major axis ·119 mm., about $2\frac{1}{2}$ times

* In the collection of Dr Greville.

minor. Central space absent. Markings without order, but around the border smaller, and forming short radial lines.

Habitat.—Sea of Japan, H.M.S. Challenger (Castracane).

C. reniformis. Cstr., *Diat. Chall. Exped.*, p. 160, pl. xii. fig. 12.—Reniform, somewhat broader at one end than at the other. Major axis ·1725 mm., about $2\frac{1}{2}$ times the greatest breadth. Central space and rosette absent. Markings polygonal, gradually increasing from the centre outwards; towards the centre 8, at the border 6, in ·01 mm.; rows straight.

Grunow correctly notes (*Bot. Centralb.*, Bd. xxxiv. p. 40) that Janisch had named this form *Stoschia admirabilis* in his still unpublished manuscript of his report on the Diatoms of the Gazelle Expedition.

Habitat.— ? (Castracane).

C. sarmaticus. Pant., *Fossil. Bacil. Ung.*, p. 74, pl. viii. fig. 62. —Elliptical, major axis ·016 to ·025 mm., about $1\frac{1}{4}$ times minor. Central space and rosette absent. Markings delicate, punctiform, least crowded and most evident towards the centre; radial rows obscure; apiculi 2, minute, inserted at the extremities of the minor axis and close to the border, sometimes absent. Border narrow but distinct, smooth.

Habitat.—Dolje deposit (Pantocsek !).

C. biangulatus. Sch., *Atl.*, pl. lxiii. fig. 13.—Diam. ·125 to ·175 mm. Surface slightly convex. Central space and rosette absent. Markings decreasing gradually from the border outwards; towards the centre $2\frac{1}{2}$ to 3, towards the border 4, in ·01 mm; central papillæ distinct; rows straight, secondary oblique curved decussating rows evident. Border sharply defined, from $\frac{1}{13}$ to $\frac{1}{18}$ of radius broad, its inner edge with 2 unsymmetrical deep constrictions at an interval from each other about equal to the radius; striæ coarse, moniliform, 4 or 5 in ·01 mm.—*C. asteromphalus*, var. *biangulata*, Cleve and Möll., *Diat.*, No. 215.

Habitat.—Nottingham deposit, Md. (Schmidt, Cleve and Möller !); Bermuda tripoli (Greville !); Calvert, county Md. (Cleve); Nagy-Kurtós deposit, Hungary (Rae ! Deby !); Moron deposit (Grove).

C. asteromphalus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 77.—Diam. ·085 to ·3 mm. Surface slightly depressed at the centre, and convex towards the border. Central space small, obtusely angular, surrounded by a distinct rosette. Markings polygonal, robust, punctate, $3\frac{1}{2}$ to 4 in ·01 mm., increasing slightly towards the border to $2\frac{1}{2}$ or 3 in ·01 mm.; central dots distinct. Border distinct; striæ obvious, coarse, about 4 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 45; pl. xxxiii. 15. fig. 7; Ralfs in *Pritch. Inf.*, p. 828; Grun., *Denk. Wien. Ak.*, 1884, p. 78; Pant., *Fossil. Bacil. Ung.*, p. 71; Sch., *Atl.*, pl. cxiii. fig. 23; Van Heurck, *Typ. Syn. Diat. Belg.*, No. 508; Cleve and Möll., *Diat.*, Nos., 57, 164; Janisch, *Gazelle Exped.*, taf. iv. fig. 9; *C. asteromphalus*, var. *genuina*, Grun., *Denk. Wien. Ak.*, 1884, p. 78; *C. asteromphalus*, var. *conspicua*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxx. figs. 1, 2, 5, 6; Grun., *Denk. Wien. Ak.*, 1884, p. 78; sp. n.? Sch., *Atl.*, pl. lxiii. fig. 5.

This species is distinguished by the evident puncta on the markings. Grunow has observed in Richmond specimens a finely punctate layer detached. The places corresponding to the angles of the markings have coarse, mostly triangular dots, the round central dots are only found in perfect valves.

Habitat.—Richmond (Ehrenberg, Cleve and Möller !); and Holles Cliff deposit, Va. (Ehrenberg); guano (Grunow); Sta Monica deposit (Weissflog! Rae !); Fernando Noronha guano (Rattray !); New York, in the sea (Grunow); Arica (Schmidt).

Var. *eximia*, Grun., *ibid.*, 1884, p. 78.—Diam. ·475 mm. Central space irregular, small. Markings increasing slightly outwards from central rosette to about $\frac{3}{5}$ of radius, 2 to $2\frac{1}{2}$ in ·01 mm.—*C. asteromphalus*, Ehrb; Sch., *Atl.*, pl. lxiii. fig. 12.

Habitat.—Santa Monica deposit (Schmidt, Rae! Deby !). With type (Grunow).

Var. *omphalantha.* Grun, *ibid.*, 1884, p. 78; *C. omphalanthus*, Ehrb., *Mon. Ber. Ak.*, 1844, p. 266.—Diam. ·255 to ·45 mm. Central space sometimes absent, rosette conspicuous or less obvious. Markings subequal to about $\frac{3}{5}$ of radius, $2\frac{1}{2}$ in ·01 mm., thence decreasing gradually to the border, sometimes slightly smaller towards the central rosette. Border somewhat sharply constricted

at two somewhat distant points.—*C. omphalanthus*, Ehrb.; Ralfs in *Pritch. Inf.*, p. 828; Cleve and Möll., *Diat.*, Nos. 57, 215; Ehrb., *Diat. Spec. Typ.*, No. 6 (excl. Sch., *Atl.*, pl. lxiii. fig. 2).

The constriction at the border is similar to that of *C. biangulatus* (Sch., *Atl.*, pl. lxiii. fig. 13), but the latter is devoid of a rosette, and the dark band at the border is relatively much wider. This var. is frequently mistaken for *C. oculus-iridis*, and sometimes for *C. borealis*.

Habitat.—"Bermuda" (Eulenstein! * Greville!); Nottingham deposit (G. M. Brown! Firth! Cleve and Möller! O'Meara! Cleve! Hardman!);† Richmond, Va. (O'Meara! Cleve and Möller!); Maryland (O'Meara! Hardman!); "Virginia" (Hardman!); Calvert, county Maryland (Kinker! Weissflog!); Kékkö deposit (Grove!); Rappahannock (Greville!); Piscataway deposit (Cambridge!† Greville!); Delaware, U.S. (Cambridge!);† Patuxent earth (Rae!);† Holland's Cliff (Cleve!); San Benito deposit, California (Grove!).

Var. *brightwellioides*. Grun., *ibid.*, 1884, p. 78.—Diam. ·1125 to ·26 mm. Surface rising from the centre to the semiradius, thence descending to the border. Markings polygonal, gradually increasing outwards to the semiradius, where there is a distinct circlet of large areolæ, $2\frac{1}{2}$ in ·01 mm. at right angles to the radius, thence decreasing to the border; at the centre $3\frac{1}{2}$, at the semiradius 3, at the border 3 in ·01 mm.—Pant., *Fossil. Bacil. Ung.*, p. 71, pl. xvii. fig. 155.

The circlet of larger markings forms a transition to *C. bulliens*, Sch., and to the genus *Brightwellia*.

Habitat.—Santa Monica deposit (Grunow, Kinker!); Szakal and Szent Peter deposits (Pantocsek).

Var. *pulchra*. Grun., *ibid.*, 1884, p. 78.—Diam. ·36 mm. Central space smooth, ·01 mm. broad, surrounded by a distinct band of larger markings. Markings 2 to $2\frac{1}{3}$ in ·01 mm., decreasing quite close to the border.

* As to this locality, Eulenstein notes—"Locus dubius, verisimiliter ad pagum hujus nominis Americæ borealis non ad *insulas* referendus."

† In the collection of Dr Griffin.

This var. approaches var. *princeps* in the character of the central space, but is distinguished by its larger markings.

Habitat.—Santa Monica deposit (Grunow).

Var. *macrantha.* Grun., *ibid.*, 1884, p. 78.—Diam. ·45 mm. Central space still larger, ·026 mm. broad, round, smooth, or punctate. Markings subequal, 4 in ·01 mm.

Habitat.—Found on a mass of diatoms floating on the Elbe by Möller (*fide*, Grunow).

Var. *princeps.* Grun., *ibid.*, 1884, p. 78.—Diam. ·5 mm. Central space subcircular, ·025 mm. broad. Markings decreasing outwards to about semiradius thence increasing slightly towards the border, 5 to 6 in ·01 mm.—Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. figs. 1–3.

Separated from the preceding by its smaller markings.

Habitat.—With var. *macrantha* (Grunow, Van Heurck).

Var. *pabellanica.* Grun., *ibid.*, 1884, p. 79.—Diam. ·145 to ·16 mm. Central space ·0065 mm. broad, the surrounding band of markings evident, but less prominent than in vars. *princeps* and *macrantha.* Markings 5 in ·01 mm., somewhat smaller at the border, faintly punctate.—*C. asteromphalus*, var. *pabellana*, Grun. in Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 5.

Habitat.—Pabellan de Pico guano (Grunow); Peruvian guano (Hardman !).*

Var. *hybrida.* Grun., *ibid.*, 1884, p. 79, pl. iii. (C) fig. 9.—Diam. ·175 to ·35 mm. Surface convex toward the border. Central rosette evident, rarely indistinct. Markings 4 in ·01 mm., somewhat smaller close to the rosette than about the semiradius thence gradually decreasing towards the border to 6 in ·01 mm., sometimes indistinctly punctate, a circlet of minute apiculi sometimes present at the border.—Pant., *Fossil. Bacil. Ung.*, p. 71; Sch., *Atl.*, pl. cxiii. fig. 22; *C. centralis*, Sch., *Atl.*, pl. lxiii. fig. 1.

Habitat.—Felsö-, Esztergály, Kekkö and Szent Peter deposits (Pantocsek) Hvidingsoe, N. Sea, Cuxhaven, Marahon mouth, Davis

* In the collection of Julien Deby.

Straits, Franz Josef's Land (Grunow); "Yszee" (Kinker !); Melville Bay (Griffin !); Los Angelos deposit (O'Meara !).

C. bisinuatus. Sch., *Atl.*, pl. lxiii. figs. 14, 15.—Diam. from ·106 to ·145 mm. Surface showing 2 unsymmetrical depressions, convex inwards, close to the border. Central space absent; rosette distinct, composed of few (4 to 6) areolæ. Markings polygonal, 4 in ·01 mm., decreasing for a short distance from the rosette, thence subequal, and again decreasing near the border; close to the border 7 in ·01 mm.

Approaching *C. oculus-iridis* and *C. centralis.*

Habitat.—North Celebes (Grunow, Schmidt).

C. Weyprechtii. Grun., *Denk. Wien. Ak.*, 1884, p. 78, pl. iii. (C), fig. 8.—Diam. ·094 mm. Surface convex. Central space minute, rosette small. Markings gradually decreasing outwards; towards the centre $5\frac{1}{2}$, towards the border 7, in ·01 mm., each with a distinct central dot and minute puncta, 20 in .01 mm. Border broad, of two portions, the inner with delicate striæ 15 in ·01 mm., the outer hyaline.

This species is readily distinguished by its border.

Habitat.—Franz Josef's Land (Grunow).

C. undulans, Rattray, *C. undulatus*,* Cstr., *Diat. Chall. Exped.*, p. 159, pl. viii. fig. 3.—Diam. ·390 mm. Surface with two concentric elevations—one near the centre, the second close to the border. Central space irregular, about $\frac{1}{10}$ of diam. broad. Markings polygonal, largest on the elevations, increasing gradually from the central space to the inner edge of the inner elevation, decreasing suddenly at its outer edge, and again increasing to the inner edge of the outer elevation, smallest towards the border, about the semi-radius $1\frac{3}{4}$ to 2 in ·01 mm.; rows indistinct upon the elevations, secondary oblique decussating rows evident.

Habitat.—Pacific Ocean, H.M.S. Challenger (Castracane).

C. convexus. Sch., *Atl.*, pl. lx. fig. 15.—Diam. ·1 mm. Central space absent. Markings hexagonal, distinctly punctate, central

* Name preoccupied by Cleve (*Kongl. Sv. Vet.-Ak. Handl. Stockh.*, 1881, No. 5, p. 20).

dots minute; towards the centre 3, towards the border 5, and at the border 8, in ·01 mm.; secondary oblique decussating rows evident.

Habitat.—Springfield deposit, Barbados (Schmidt, Grunow, Hardman !).

Var. *bengalensis.* Grun., *Denk. Wien. Ak.*, 1884, p. 73.—Diam. ·08 to ·14 mm. Markings smaller; towards the centre 4, at the border 10, in ·01 mm.

Habitat.—Brackish water, coast of Bengal (Grunow).

Var. *diminuta.* Grun., *ibid.*, 1884, p. 73.—Markings still more minute; towards the centre 6 in ·01 mm., less distinctly punctate.

This var. approaches *C. radiosus*, Grun.

Habitat.—Brackish water, coast of Bengal (Grunow).

C. fimbriatus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 78.—Diam. ·1 to ·1125 mm. Central space and rosette absent. Markings polygonal; towards the centre 4, decreasing gradually outwards to 6 or 7, in ·01 mm.; radial rows most evident towards the border.—Ehrb., *Mikrog.*, pl. xxii. fig. 2; Ralfs in *Pritch. Inf.*, p. 829; Grun., *Denk. Wien. Ak.*, 1884, p. 74. *C. radiolatus*, Ehrb., Sch. *Atl.*, pl. lx. fig. 11.

Distinguished from *C. radiosus*, Grun., by its larger markings, which decrease less rapidly outwards in Grunow's species.

Habitat.—Caltanisetta (Ehrenberg); Sicilian marls (Grunow); Los Angelos deposit, Cal. (O'Meara !).

Var. *subradiata*, nov. *C. fimbriatus*, var., Van Heurck, *Syn. Diat. Belg.*, p. cxxxi. fig. 2.—Diam. ·063 mm. Markings 7, increasing gradually outwards to 6, in ·01 mm., about $\frac{2}{3}$ of radius from the centre, again diminishing more rapidly to the border.

Grunow, followed by Pantocsek, have united this to the species. The affinities to *C. radiatus* are striking.

Habitat.—Oran deposit (Van Heurck).

Var. *californica.* Grun., *ibid.*, 1884, p. 74.—Diam. ·07 to ·09 mm. Centre, often with small irregular puncta, one frequently larger than the others, and like the rudiment of a bristle. Markings

towards the centre 5 to 6 in ·01 mm., often smaller than nearer the border, at the border 8 to 9 in ·01 mm.

Habitat.—California deposits, San Diego (Grunow).

C. obversus, sp. n. Sch., *Atl.*, pl. lx. fig. 14 (no name).—Diam. ·075 to ·105 mm. Central space absent or subobsolete; rosette distinct. Markings polygonal, subequal or decreasing slightly for about $\frac{5}{6}$ of radius, thence decreasing more rapidly to the border; towards the centre 2 to $2\frac{1}{2}$, at the border 8, in ·01 mm.; central papillæ distinct; rows sometimes obscurely fasciculate; on the outer $\frac{1}{6}$ of the radius the rows are separated by clear radial lines. Border narrow, distinct.

From *C. fimbriatus* this is distinguished by the larger more robust markings and their more rapid decrease from the centre outwards.

Habitat.—Hong Kong (Hardman!); marsh ground, Wedel (Schmidt); Cambodia (Hardman!).

Var. *tenuior*, nov.—Diam. ·0625 mm. Central space and rosette absent. Markings subequal to about $\frac{2}{3}$ of radius, thence decreasing to the border; at the centre 6, near the border 10, in ·01 mm.

Habitat.—Rio Janeiro (Hardman!).

C. grandineus, sp. n. Sp. aff. *C. concinno*, Sch., *Atl.*, pl. lx. fig 16.—Diam. ·075 to ·19 mm. Surface moderately convex, flat on a narrow zone adjacent to the border. Central space absent; rosette distinct, sometimes inconspicuous. Markings polygonal, decreasing gradually from the centre outwards; towards the centre 4, towards the border 8, in ·01 mm.; rows straight; secondary oblique decussating rows manifest; minute apiculi at intervals of about ·006 mm., sometimes present at the border. Border with inner edge indistinct; striæ 4 to 6 in ·01 mm.

This species differs from *C. concinnus*, var. *jonesiana*, by the absence of apiculi and processes, and from *C. asteromphalus*, var. *hybrida*, by the absence of a zone of somewhat larger markings about the semiradius than nearer the centre.

Habitat.—Dredged off Heard Island, in 75 fathoms, by H.M.S. Challenger (Rae!); dredged in Royal Sound, Kerguelen, 28

fathoms, by H.M.S. Challenger (Rae !); Ballybrack (O'Meara !); Nottingham, Maryland, (O'Meara !); Cambridge deposit, Barbados (Johnson !).*

Var. *dentata*, nov.—Diam. ·1075 mm. Central rosette small; markings towards the centre 4, at the border 6, in ·01 mm.; apiculi large, at subequal intervals of about ·006 mm.

Habitat.— ? (Greville !); Barbados deposit (Johnson !).*

C. centralis, emend.; *C.? centralis.* Ehrb., *Abh. Ber. Ak.*, 1838, p. 129.—Diam. ·12 to ·255 mm. Central space absent or minute; a rosette obvious. Markings variable in size, subequal—4 to 4½ in ·01 mm. for about ⅝ of the radius, thence decreasing gradually outwards on the outer ⅙; sometimes 5 in ·01 mm., and decreasing gently from the semiradius; rows straight, sometimes subfasciculate towards the border, the fasciculi separated by narrow clear areas proceeding inwards from the apiculi; apiculi delicate, inserted at the border at intervals of about ·006 mm., 2 larger unsymmetrical, at an interval from each other somewhat greater than the semiradius. Border narrow; striæ 6 in ·01 mm.—Ehrb., *Mon. Ber. Ak.* 1844, p. 78; *Mikrog.*, pl. xviii. fig. 39; pl. xxii. fig. 1; Greg., *Trans. Roy. Soc. Edin.*, 1857, p. 501, pl. xi. fig. 49; Ralfs in *Pritch. Inf.*, p. 828; Grun., *Sitzungsb. naturw. Ges. Isis, Dresden*, 1878, p. 123; Van Heurck, *Syn. Diat. Belg.*, pl. ciii. fig. B; H. L. Smith, *Diat. Spec. Typ.*, Nos. 91, 92; Cleve and Möll., *Diat.*, Nos. 57, 164, 207, 215; *C. asteromphalus*, var. *centralis*, Grun., *Denk. Wien. Ak.*, 1884, p. 79; *C. centralis*, var. *micraster*, Grun.; Cleve and Möll., *Diat.*, No. 172; *C. centralis*, var., Cstr., *Diat. Chall. Exped.*, p. 155, pl. ii. fig. 3; *C. centralis forma minor*, Van Heurck., *Typ. Syn. Diat. Belg.*, No. 531 (excl. *C. centralis*,† Weisse, *Bull. Acad. Imp St Petersb.*, 1867, p. 122, pl. i. fig. 18; *C. centralis*,‡ O'Me., *Proc. Roy. Irish Ac.* 1875, p. 260, pl. xxvi. fig. 19; *C. centralis*,§ Sch., *Atl.*, pl. lxiii. fig. 1; *C. centralis*, Ehrb., *Mikrog.*, pl. xxi. fig. 3).

This species is sometimes confounded with *C. oculus-iridis*, *C.*

* In the collection of Dr Greville.

† This is *C. radiatus*, Ehrb.

‡ This is probablly *C. subtilis*, as stated by Grunow.

§ This is *C. asteromphalus*, var. *hybrida*, Grun.

radiatus, and *C. concinnus*. In balsam preparations the apiculi are hardly visible. It has been employed along with *Rhizosoleniæ* and *Denticellæ* by Max Schultze in 1859 in studying the movements of the granules in the interior of the frustule. In Ehrenberg's definition, the markings are given as about 6 in ·01 mm., as in some forms of *C. radiosus*, Grun. Castracane erects a var. solely on the presence of a striated border—a character clearly shown in Ehrenberg's figure (*Mikrog.*, pl. xviii. fig. 39). Some specimens named by Van Heurck *Coscinodiscus centralis forma minor* (*Typ. Syn. Diat. Belg.*, No. 531), from the Baltic, may belong rather to *C. oculus-iridis*, owing to the increase in the size of the markings outwards; *C. centralis* is readily distinguished from *C. asteromphalus* by the presence of the 2 unsymmetrical apiculi, and cannot be united with it in the same species as proposed by Grunow. The two processes were first observed by Mr E. Grove, F.R.M.S.

Habitat.—Oran and Caltanisetta deposits (Ehrenberg); Glenshira sand (Gregory!); Cherbourg (H. L. Smith!); grey mud dredged in 569 fathoms, lat. 63° 40′ N., long. 5° 28′ E. (Ehrenberg); Richmond (Ehrenberg, Cleve and Möller!); some guanos (Grunow); South Sea, Lagos, North Sea, Baltic (Grunow); Heligoland (Schultze!);* *Ascidia*, Hull (Gregory!* Greville!); Loch Fyne (Gregory! Greville!); Lamlash (Gregory! Greville! Dickie!); Firth of Forth at Granton (Rattray!); Melville Bay (Deby! Barnett!);† soundings, Gulf of California (H. L. Smith!); Ballybrack (O'Meara!); from *Laminaria saccharina*, Dalkey (O'Meara!); *Ascidia*, Dublin Bay (O'Meara!); Bermuda tripoli (Greville!); Fuur deposit, Jutland (O. N. Witt!); North Atlantic, lat. 51° 20′ N., long. 52° 25′ W., 232 fathoms (O'Meara!); Richmond Tunnel (O'Meara!); Davis Straits (O'Meara!); Lümfiord, Jutland (Hardman!); Patos Island guano (Hardman!);‡ Virginia (Hardman!);‡ Oran deposit (Deby!); Malahide (O'Meara!); Behring Sea, 1681 fathoms (H. L. Smith!); surface, Hong Kong Harbour, H.M.S. Challenger (O'Meara!); Monkstown, in tide pools (O'Meara!); *Ascidia*, Belfast (O'Meara!); Thames, at Sheerness (Grove!); Spezzia (Kinker!); Los Angelos

* In the collection of Dr Greville.
† In the collection of Dr Griffin.
‡ In the collection of Julien Deby.

deposit (O'Meara!); Baltic Sea (Van Heurck!); Teignmouth Grove!); Peruvian guano (Cleve! Macrae!); Sta Monica deposit (Cleve and Möller!) Sta Maria deposit (Grove!); Davis Straits, Patagonia; Nottingham deposit, Md. (Cleve and Möller!); Möller's Elbe material (Cleve!); Patagonia, 1375 fathoms, H.M.S. Challenger (Cleve!); Faroe Channel (Grove!); west coast Florida, U.S. Survey (Febiger!); Yeddo, Bohuslan, Kusu, Pabillan de Pico guano; Greenland; Successful Bay, Kerguelen (Cleve!).

C. floridulus. Sch., *Atl.*, pl. cxiii. fig. 16, *a*, *b*, *c*.—Diam. ·1135 mm. Surface flat. Central space irregular, small, about $\frac{1}{25}$ of diam. broad. Markings polygonal, largest round the central space, thence decreasing slightly outwards; towards the centre 4, towards the border 6, in ·01 mm.; minute puncta at the origin of the shorter rows; rows substraight; secondary oblique decussating rows evident; minute apiculi at angles of areolæ (seen in oblique aspects). Border narrow.

Distinguished from *C. obscurus* by the size of the markings and character of the border.

Habitat.—Sta Monica (Schmidt).

C. inæquisculptus, sp. n. Roundly elliptical; major axis ·175 mm., about $1\frac{1}{17}$ times minor. Surface slightly convex towards the centre. Central space absent; rosette distinct. Markings hexagonal, and increasing slightly outwards to about the semiradius, 4 to $4\frac{1}{2}$ in ·01 mm. in radial rows; beyond the semiradius large, unequal, and without order; central papillæ of the smaller faint, in the larger a faint elevated central ridge. Border indistinct; striæ 6 in ·01 mm.—(Pl. I. fig. 17.)

Habitat.—Moron deposit (Greville!).

C. megacentrum. Grove, MS.—Diam. ·12 to ·125 mm. Surface somewhat convex. Central space and rosette absent. Markings angular, $1\frac{1}{2}$ to 3 on the central area, which extends to $\frac{1}{4}$ or $\frac{1}{3}$ of the radius, beyond this $2\frac{1}{2}$ in ·01 mm., and increasing gradually outwards to about $\frac{2}{3}$ of radius, thence decreasing to the border; the central papillæ prominent; irregular on the central area, beyond this in straight rows, with secondary oblique decussating

rows manifest. Border narrow; striæ evident, 6 to 8 in ·01 mm. —(Pl. II. fig. 13.)

Habitat.—Oamaru deposit (Grove !).

C. secernendus. Sch., *Atl.*, pl. cxiv. fig. 1.—Diam. ·272 mm. Surface flat, towards the border convex. Central space absent; rosette distinct. Markings hexagonal, towards the border obtusely angular with long axes radial, pearly, decreasing gradually outwards; towards the centre $1\frac{1}{2}$ to 2, towards the border 2 to $2\frac{1}{2}$, in ·01 mm. Central papillæ prominent, those on the markings near the border placed towards their central side; secondary oblique decussating rows distinct, towards the border coarsely submoniliform, and separated by more distinct clear lines. Border narrow; striæ 8 to 10 in ·01 mm.

The appearance of the markings recall those of *Aulacodiscus margaritaceus*, var. *Kinkeri.*

Habitat.—Maryland (Thum).

C. moravicus, Grun. Sch., *Atl.*, pl. cxiv. fig. 2.—Diam. ·24 mm. Surface convex. Central space small; rosette distinct. Markings hexagonal, obscurely punctate, towards the border submoniliform, increasing slightly outwards for some distance from the central space, again decreasing towards the border; towards the centre $2\frac{1}{2}$, at the border 5, in. ·01 mm.; central papillæ faint, rows towards the border separated by evident clear lines; secondary oblique decussating rows evident. Border narrow, hyaline.

Distinguished from *C. asteromphalus* by the size and appearance of the markings.

Habitat.—Hungarian marl (Thum).

C. borealis. Bail., *Amer. Jour. Sci.*, 1856, p. 3.—Diam. ·15 to ·25 mm. Surface slightly depressed at the centre, somewhat convex towards the border. Central rosette usually distinct, of 6 to 8 large areolæ, sometimes subobsolete. Markings increasing regularly from the central rosette outwards; towards the rosette 3 to $3\frac{1}{2}$, near the border 2, in. ·01 mm.; central papillæ distinct, the division lines composed of rounded granules; secondary oblique rows distinct. Border distinct, dark, with close irregular coarse striæ.—Ralfs in

Pritch. Inf., p. 828; Sch., *Atl.*, pl. lxiii. fig. 11; H. L. Sm., *Diat. Spec. Typ.*, Nos. 90, 93, 95; *C. oculus-iridis*, var. *borealis*, Cleve, *Vega Exped., Vetensk. Jakttag. Stockh.*, Bd. iii. 1883, p. 488; Grun. *Denk. Wien. Ak.*, 1884, p. 77 (excl. *C. borealis*, Ehrb., *Mon. Ber. Ak.*, 1861, p. 294).

Distinguished from *C. oculus-iridis* by the coarser and more robust markings. Kitton's Holles cliff specimens sometimes show under low powers 4 minute elevations at the centre, indicating a transition to *C. excavatus* and *C. asteroides.*

Habitat.—Kamstchatka Sea, 1700 fathoms (Bailey !); Kurile Islands, 1329 fathoms (H. L. Smith !); Nottingham deposit (Kitton !); Sta Monica deposit (Weissflog, Type plate); California (Firth,* Grunow); Hong Kong (Weissflog !) Japan (Deby !); Naparima, Trinidad (Johnson !); Sta Barbara deposit (Kinker !); Cambodia (Hardman !);† Behring Sea, 1681 fathoms (H. L. Smith !); Holles cliff (Kitton !); Barbados (Johnson !); Cape Wankarema (Cleve).

C. megaporus. Ehrb., *Mon. Ber. Ak.*, 1861, pp. 280, 294.—Diam. about ·23 mm. Surface slightly convex. Central rosette absent. Markings subequal, $2\frac{1}{2}$ to 3 in ·01 mm.; near the border smaller, central papillæ small. Border broad.

Probably, as stated by Ehrenberg, a var. of *C. borealis.*

Habitat.—Lat. 60° 40′ N. long. 29° W. 1000 fathoms; lat. 62° 6′ N., long. 32° 21′ W., 1540 fathoms; lat. 59° 12′ N., long. 50° 38′ W., 1833 fathoms (Ehrenberg).

C. oculis-iridis. Ehrb., *Abh. Ber. Ak.*, 1839, p. 147. Diam. ·135 to ·3 mm. Central rosette distinct, sometimes small. Markings polygonal, non-punctate; towards the rosette usually smaller, from 3 to 4, thence increasing gradually outwards to $2\frac{1}{2}$ in ·01 mm., towards the border again decreasing to 5 or 6 in ·01 mm.; central papillæ sometimes prominent; secondary decussating rows well marked. Border narrow; striæ 6 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 42; pl. xix. fig. 2; Jan., *Abh. Schl. Ges. väter. Cult.*, 1862, Heft ii. p. 3, pl. i. B. fig. 6; pl. ii. A. fig. 4; *Gazelle Exped.*, taf. ii. fig. 2; Sch., *Atl.*, pl. lxiii. figs. 6, 7, 9; pl. cxiii. figs. 1, 3–5,

* In the collection of Dr F. W. Griffin.

† In the collection of Julien Deby.

20; Raben., *Alg. Europ.*, Nos. 2487, 2558; Cleve and Möller, *Diat.*, Nos. 3, 57, 162, 215, 258, 259, 276, 319; *C. oculus-iridis*, var. *genuina*, Grun., *Denk. Wien. Ak.*, 1884, p. 77; *C. centralis*, Ehrb., *Mikrog.*, pl. xxi. fig. 3; H. L. Sm., *Diat. Spec. Typ.*, No. 92; *C. omphalanthus*, Grun., in Sch., *Atl.*, pl. lxiii. fig. 2.

Grunow believes that *C. oculus iridis*, var.? *pacifica*, may belong to *C. asteromphalus*, the punctation of the markings having escaped Schmidt's observation. The specimen figured by Schmidt (*Atl.*, pl. cxiii. fig. 1) shows a less obvious increase of the markings outwards, and a more distinctly marked zone adjacent to the border; but gradations occur to such forms, as shown in fig. 3 of same plate.

Habitat.—Franz Josef's Land (Grunow!); Cherbourg (H. L. Smith!); Lümfiord, Jutland (Hardman!); mud from Glückstadt, Elbe above Cuxhaven (Rabenhorst and Schwarz!); Aegina (Schmidt); Bohuslan, and mud from Elbing, West Prussia (Cleve!); Java (Schmidt, Cleve and Möller! O'Meara!); Japan (Deby!) Inland Sea, Japan, H.M.S. Challenger (Rae!); Arafura Sea, H.M.S. Challenger! (Doeg!); Japan oysters (Doeg!); Hong Kong (Deby! Hardman! Greville! Grove! Firth!); surface, Hong Kong Harbour, H.M.S. Challenger (O'Meara!); Isle of Muntok, Indian Archipelago (Grove!); Sand Heads, Bay of Bengal (Macrae!); shell cleanings, Singapore (Hardman!); Ceylon (Macrae!); edible seaweeds, India (Macrae!); East Indies (Macrae!); Spitzbergen, Santos (Cleve); Cape Wankarema (Cleve and Möller!); Greenland (Cleve!); Penang Harbour (Rae!); Kusu (Cleve! O'Meara!); H.M.S. Challenger, lat. 32° 31′ N., long. 135° 39′ E., 1675 fathoms (Rae!); Atlantic Telegraph soundings (Roper!); North Atlantic, lat. 51° 20′ N., long. 52° 25′ W., 232 fathoms (O'Meara!); "Atlantic sounding" (Weissflog!); Sea of Kamstchatka, 1700 fathoms (Bailey!); Nancoori, Pensacola, Trinidad, California (Cleve and Möller!); Cambodia (Hardman!); marine deposit, Fiji Islands (Grove!); Patagonian and Ichaboe guanos (Janisch); Lobos di Afuera guano (Grove!); Peruvian guano (Hardman! Cleve! Janisch); Bolivian guano (Cleve! Greville!); "guano" (O'Meara!); Patos Island guano (Greville!); Californian guano (Greville!); Baytha, Elesd, Alsó-, Felsö-, Esztergály, Kékkö, Mogyorod, Szakal, Szent-Péter and Dolje deposits (Pantocsek!); Moron deposit (Greville!); Brünn Tegel (Cleve!); Mors (Cleve); Monterey (Stokes!); Santa Maria

deposit (Kinker!); Mejillones (Cleve! O'Meara!); Santa Monica (Schmidt, Grove! Kinker!); Nottingham deposit, Md. (Greville; Cleve and Möller!); "Maryland" (O'Meara!); Richmond tunnel (O'Meara!); Richmond, Va. (Cleveand Möller!); Bermuda tripoli (Greville!); "Barbados" (Johnson!); Cambridge deposit, Barbados (Johnson!); Holland's Cliff (Cleve!); Marstrand (Kinker!).

Var. *morsiana.* Grun., *ibid.*, 1884, p. 77.—Diam. ·23 mm. Surface slightly depressed towards the centre, the highest zone about $\frac{2}{3}$ of radius from centre. Central rosette distinct. Markings robust, increasing from the centre for $\frac{2}{3}$ of radius, thence diminishing to the border; towards centre 3, on highest zone 2 to $2\frac{1}{2}$, in ·01 mm.—Sch., *Atl.*, pl. lxiii. fig. 9 (no name); *C. asteromphalus*, Ehrb., (*fide* Grun., *ibid.*, p. 77); Sch., *Atl.*, pl. lx. fig. 7; *Cestodiscus radiatus*, Ehrb.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxix. fig. 5.

Distinguished by the wider zone at the border, upon which the markings decrease.

Habitat.—Mors deposit (Schmidt, Cleve!).

Var. *subspinosa.* Grun., *ibid.*, 1884, p. 67.—Diam. ·1665 mm. Central rosette small, often with a small central space. Markings more delicate, 4 to 5, at the border 7 to 8 in ·01 mm. Apiculi minute, numerous, at subregular intervals, inserted close to border. —Sch., *Atl.*, pl. lxiii. fig. 4 (no name).

Habitat.—Mejillones guano (Grunow, Grove, O'Meara!); Ichaboe guano (Schmidt).

Var. *tenui-striata.* Grun., *ibid.*, 1884, p. 77.—Diam. ·14 to ·15 mm. Surface very convex on outer portion. Markings at the centre 5, towards the border 6 or 7, in ·01 mm., non-apiculate.

A smaller form with smaller markings, 7 to 10 in ·01 mm., occurs in the Caspian.

Habitat.—Campeachy Bay (Grunow).

Var. *stelliger.* Sch., *Atl.*, pl. lxiii. fig. 8 (no name).—Diam. 2 mm. Central rosette distinct. Markings subequal to, but more irregular than those of the type; towards the border larger, and forming a circlet of distinct rosettes, placed at regular intervals about $\frac{1}{5}$ of radius from the border.

Habitat.—Java (Schmidt).

Var. *loculifera*, nov.—Diam. ·17 mm. Central space distinct, from ·0025 to ·005 mm. broad, subcircular, usually surrounded by a distinct band of large areolæ. Markings sometimes most evident on a narrow band around the border, the central papillæ faint.—(Pl. I. fig. 2.)

This var. is readily distinguished by the central space. Oamaru specimens have the band around the central space less evident, and the markings towards the centre 5, near the border $3\frac{1}{2}$ in ·01 mm.

Habitat.—Manilla Algæ (Grove!); Jackson's Paddock, Oamaru deposit (Grove!); Springfield deposit, Barbados (Doeg!); Mejillones (Grove!).

C. annulatus. Grun., *Denk. Wien. Ak.*, 1884, p. 74, pl. v. (E), fig. 57.—Diam. ·08 to ·165 mm. Surface with a distinct annular depression, about $\frac{1}{10}$ of radius broad, its inner edge about $\frac{1}{3}$ of radius from the centre, beyond this slightly convex to the border. Central space minute, subcircular; surrounded by a distinct rosette or band of large areolæ. Markings pent- or hex-agonal, gradually increasing outwards; near the centre 4, towards the border 3, in ·01 mm.; on the annular depression the areolæ replaced by isolated round granules with hyaline interspaces. Border narrow; striæ delicate, 8 to 10 in ·01 mm.

This species is readily distinguished by the annular depression.

Habitat.—Mors deposit (Grunow, Deby!).

C. groveanus,* sp. n.—Roundly elliptical, major axis ·21 mm. about $1\frac{1}{10}$ times minor. Surface flat from the centre for about $\frac{1}{3}$ of the radius, here rising abruptly to form a distinct elevated ring about ·01 mm. broad, thence descending more gradually with a slight outward concavity to the border. Central rosette distinct. Markings hexagonal, subequal, $3\frac{1}{2}$ in ·01 mm., but near the border enlarging to about $2\frac{1}{2}$; central papillæ faint; rows radial, straight; secondary oblique decussating rows most distinct near the border. Border striæ coarse, indistinct.—(Pl. I. fig. 11.)

Distinguished from all others by the prominent elevated ring. The transition to *C. excavatus*, var. *semilunaris*, Grun., is not abrupt.

Habitat.—Newcastle desposit, Barbados (Grove!).

* Dedicated to Edmund Grove, Esq., F.R.M.S., the well-known investigator of the Oamaru deposit, New Zealand.

C. suboculatus, sp. n. Sch., *Atl.*, pl. lxi. fig. 5 (no name)—Diam. ·1135 mm. Surface convex towards the border. Central space minute, rosette distinct. Markings polygonal, increasing from the rosette to the semiradius, thence gradually decreasing to the border; towards the centre 4, at semiradius 3, in ·01 mm. Central dots distinct. Border sharply defined, about $\frac{1}{14}$ of radius broad; striæ coarse, moniliform, 4 in ·01 mm.

According to Grunow, this is allied to *C. oculus-iridis*. It is, however, readily distinguished by its more robust markings and its broader border.

Habitat.—Springfield, Barbados (Schmidt).

C. pacificus, Rattray. *C. oculus-iridis*, var.? *pacifica*, Grun., *Denk. Wien. Ak.*, 1884, p. 77.—Diam. ·25 mm. Central space absent or small; rosette distinct. Markings polygonal, decreasing from the centre to the border; towards the centre 4, towards the border 6, in ·01 mm.; rows radial, straight, or subparallel; secondary oblique decussating rows evident. Border indistinct.—Sch., *Atl.*, pl. lx. fig. 13 (no name); *C. asteromphalus*, Ehrb.? *fide* Grun., *ibid.*, 1884, p. 77.

Habitat.—Monterey (Schmidt).

C. intermixtus, sp. n.—Diam. ·224 mm. Surface almost flat. Central space absent; rosette distinct. Markings hexagonal, increasing subregularly from the rosette outwards; towards the centre 4, near the border $2\frac{1}{2}$, in ·01 mm.; at $\frac{2}{3}$ of the radius from the centre a distinct zone about ·01 mm. broad, of much larger, unequal and irregularly arranged areolæ for the most part in 3 rows, those at the centre being largest; the central papillæ evident, rows straight. Border narrow, sharply defined; striæ 4 to 6 in ·01 mm., well marked.—(Pl. I. fig. 13.)

Habitat.—Santa Monica deposit (Weissflog!).

C. Monicæ, Rattray. *C. Janischii*, var.? *Monicæ*, Grun., *Denk. Wien. Ak.*, 1884, p. 76.—Diam. ·2275 mm. Central space rounded, about, $\frac{1}{30}$ of diam. broad. Markings around the central space large, free, circular, or elliptical, forming a distinct single band, beyond this obtusely angular, subpearly, with faint central dots, gradually increasing from the central space for about $\frac{4}{5}$ of the radius, again

diminishing slightly towards the border; towards the centre 5, towards the border 3½, in ·01 mm.—Sch., *Atl.*, pl. lxiii. fig. 10 (no name).

Habitat.—Santa Monica deposit (Grunow).

C. Kurzii, Grun. Sch., *Atl.*, pl. cxiii. fig. 17.—Diam. ·1135 mm. Central space small, surrounded by an inconspicuous band of areolæ. Markings sometimes rounded and free on one side, hexagonal and in contact on the opposite, increasing from the central space for about ⅔ of the radius, thence decreasing rapidly to the border; towards the centre 4, at ⅔ of the radius 2, at the border 6, in ·01 mm.; central papillæ evident; minute puncta at the origin of the shorter rows; secondary oblique decussating rows distinct. Border narrow.

Habitat.—Elephant Point (Grunow).

C. spinuligerus, sp. n. Sch., *Atl.*, pl. lxiii. fig. 3 (no name).—Diam. ·14 to ·24 mm. Central space subcircular, ·0065 mm. broad, surrounding band of large areolæ distinct. Markings polygonal, 3½ in ·01 mm., distinctly punctate, decreasing on outermost ¼ of radius to the border; small apiculi at the [illegible]ler obvious, at intervals of about ·0075 mm.—Sch., *Atl.*, pl [illegible]g. 19 (no name); *C. asteromphalus*, var. *spinuligera*, Grun. [illegible]*k. Wien. Ak.*, 1884, p. 79.

Sometimes united to *C. oculus iridis*. The apiculi resemble those of *C. concinnus*, W. Sm.

Habitat.—Monterey deposit (Schmidt, Grunow); Ichaboe guano (Schmidt; Sta Monica deposit (Firth !); Arica (Grüindler).

C. oamaruensis. Grove and Sturt, *Jour. Quek. Micr. Cl.*, 1887, p. 68.—Diam. ·185 to ·25 mm. Surface convex towards the centre. Central space obtusely angular, about ⅓ of diam. broad, surrounded by a distinct band of markings somewhat larger than those adjacent to them. Markings 4- to 6-angled, gradually decreasing from the centre outwards; towards the centre 4 to 4½, towards the border 6 to 7, in ·01 mm.; central papillæ small, indistinct; rows straight, or showing wide gentle curves; apiculi near the border minute, at unequal intervals of ·0075 to ·01 mm.; at the origin of the shorter rows minute puncta sometimes present. Border narrow,

hyaline.—Grun., *Bot. Centralbl.*, Bd. xxxiv. Nos. 2, 3, p. 35.—(Pl. I. fig. 1.)

The central space and surrounding band are similar to those of *C. asteromphalus*, var. *pabellanica*, Grun., but the markings of the latter are quite distinct. I agree with Mr Grove that this species cannot be assimilated to *C. perforatus*, var. *cellulosa*, as stated by Grunow.

Habitat.—Oamaru deposit (Grove!); Cambridge deposit Barbados (Johnson!).*

C. umbonatus. Greg., *Trans. Roy. Soc. Edin.*, 1857, p. 500, pl. x. fig. 48.—Diam. ·13 mm. Surface with central portion convex for about $\frac{3}{4}$ of radius, sharply defined, thence almost flat to the border. Central space absent; rosette distinct, ·0075 mm. broad. Mark areolate, subequal, 4 in. ·01 mm.; close to the border submoniliform rows, distinct; the secondary oblique rows less obvious.—Jan. *Abh. Schl. Ges. väter. Cult.*, 1861, pl. ii. fig. 5, 1862, p. 5.

The radiating lines between the markings noted by Janisch are not found in the type.

Habitat.—Lamlash (Gregory!);* Peruvian guano (Janisch).

C. Weissflogii. Sch., *Atl.*, specimen plate (Einladung zur Subscription), 1874, fig. 5.—Diam. ·0375 mm. Surface with 8 submarginal bullations having the peripheral portion most prominent. Central space circular, obscure, a distinct rosette of 3 areolæ meeting at its middle. Markings minute, 12 to 14 in ·01 mm., rows straight; no primary rays corresponding to the bullations, surface scabrous. —(Pl. I. fig. 25.)

This species is, in the opinion of Kitton, a *Melosira*. In the 8 submarginal bullations it shows affinity to the *Inflati* section of the genus *Aulacodiscus*.

Habitat.—New Zealand deposits (Kitton!); Samoa (Schmidt).

C. theskelos,† sp. n.—Diam. ·13 mm. Surface bearing a number of subradial curved or flexuous distinct lines, simple, or once dichotomous, and uniting around the centre to form a sharply curved

* In the collection of Dr Greville.

† θεσκελος, wonderful.

line. Central space and rosette absent. Markings hexagonal, largest and most unequal on a small central area, beyond this decreasing but slightly outwards, 8 to 9 in ·01 mm.; rows radial or subradial, the secondary oblique straight or slightly curved rows more evident. Border narrow; striæ delicate, 8 to 10 in ·01 mm. —(Pl. II. fig. 19.)

This genus forms the transition to the genus *Asteromphalus*.

Mr Kitton possesses an abnormal specimen of *Arachnoidiscus ornatus*, Ehrb., from Hong Kong, where the areolæ are irregularly disposed, and the surface is marked at irregular intervals by well-defined straight and curved, branching, or anastomosing lines, which seem to be homologues of the costate rays of that species, and correspond to the irregularly radial lines of *Coscinodiscus theskelos*. The radial costæ of *Stictodiscus Crozierii*, Kitton (*Month. Micr. Jour.*, vol. x. p. 275, pl. xxxviii. fig. 2), first found by Capt. Crozier in the Mauritius, but also occurring in the West Indies, have a similar character.

Habitat.—Santa Marta deposits, Cal. (Doeg!).

C. duriusculus, sp. n. Sch., *Atl.*, pl. lviii. fig. 8 (no name).—Diam. about ·0255 mm. Central space and rosette absent. Markings round, granular, about 6 to 7 in ·01 mm.; rows distinct, substraight, separated by wide hyaline cuneate interspaces. Border sharply defined, broad; striæ obvious, distant, 4 to 5 in ·01 mm.

The arrangement of the markings and the border distinguish this from all other species.

Habitat.—Patuxent River (Schmidt).

C. rotula, Grun. Sch., *Atl.*, pl. lvii. figs. 6, 7.—Diam. about ·0275 to ·03 mm. Central space circular, $\frac{1}{12}$ of diam. broad, sometimes excentric. Markings around the central space largest, consisting of free granules tapering outwards, elsewhere punctiform in straight or slightly curved widely separated radial rows; interspaces hyaline. Border sharply defined, with distinct granules opposite the ends of the radial rows.

Habitat.—Campeachy Bay (Schmidt, Weissflog!).

C. stelliger, Grun. Sch., *Atl.*, pl. lviii. fig. 10.—Diam. about ·035 mm. Central space circular, $\frac{1}{7}$ to $\frac{1}{8}$ of diam. broad, bearing a

few rounded isolated granules. Markings rounded, granular, most prominent near the margin of the central space, faint beyond the semiradius; rows widely separated by hyaline interspaces. Border distinct, sharply defined, with delicate markings only opposite the ends of the radial rows.

Distinguished from *C. rotula* by its larger markings; the central space does not become excentric.

Habitat.—Campeachy Bay (Schmidt).

C. perminutus, sp. n. *Melosira?* Sch., *Atl.*, pl. lix. fig. 7.—Diam. about ·03 mm. Central space distinct, circular, $\frac{1}{6}$ to $\frac{1}{7}$ of diam. broad. Markings punctiform, with small interspaces; rows straight, interspaces widest opposite origin of shorter rows; adjacent to border a distinct band of large areolæ 5 in ·01 mm.; a few scattered prominent round granules forming a circlet a short distance within the border, and one present at middle of central space. Border sharply defined, from $\frac{1}{4}$ to $\frac{1}{5}$ of radius broad, hyaline.

Habitat.—Campeachy Bank (Schmidt).

C. lunæ. Ehrb., *Mon. Ber. Ak.*, 1844, p. 201.—Diam. ·038 to ·057 mm. Central space rounded, about $\frac{1}{10}$ of diam. broad. Markings rounded, granular, subequal, 6 to 8 in ·01 mm.; rows separated by wide hyaline interspaces, only a few passing from the central space, adjacent to the border crowded on a narrow zone. Border narrow, hyaline.—Ehrb., *Mikrog.*, pl. xxxv. A. 21. fig. 7; Ralfs in *Pritch. Inf.*, p. 829; *C. cycloteres*, Cstr., *Diat. Chall. Exped.*, p. 161, pl. xxii. fig. 8.

Distinguished from *C. actinochilus* by the less crowded radial rows, and the less distinct narrower marginal zone.

Habitat.—Pancake ice, Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W.; also lat. 75° S., long. 170° W.; lat. 78° 10′ S., long. 162° W., 190 fathoms; from snow and ice taken from the sea, lat. 76° S., long. 165° W., near Victoria land; ex *Salpâ*, lat. 66° S., long. 157° W.; Gulf of Erebus and Terror, lat. 63° 40′ S., long. 55° W., 207 fathoms (Hooker); lat. 53° 55′ S., long. 108° 35′ E., 1950 fathoms; H.M.S. Challenger (Castracane, Rae!).

C. trochischos. Tru. and Witt, *Jeremie Diat.*, p. 14, pl. ii.

fig. 14.—Diam. ·05 mm. Surface flat. Centre indistinctly granular; radial subuniform plications running from the centre to the border, and producing a wheel-like appearance. Markings minute, angular, 8 to 10 (?) in ·01 mm.; radial rows between the plications evident; secondary oblique rows distinct. Border relatively broad, sharply defined; striæ delicate, but evident.

The appearance presented by the plications in the figure of Truan and Witt, are somewhat similar to the irregular clear areas found in *Aulacodiscus acutus*, Rattray (*Jour. Quek. Micr. Cl.*, vol. iv. ser. 2, p. 38, pl. iii. fig. 4.

Habitat.—Jeremie marl (Truan and Witt).

C. rhombicus, Grun. Van Heurk, *Syn. Diat. Belg.*, pl. cxxix. fig. 3.—Diamond-shaped, angles obtuse, those at extremities of major axis the more acute. Major axis ·0825, about $2\frac{1}{3}$ times minor. Surface showing a distinct elliptical central portion. Markings rounded, granular towards middle of central area, which elsewhere is hyaline, beyond this area rounded on a narrow zone; on a distinct zone adjacent to the border more minute subpunctiform, 10 in ·01 mm.; rows beyond central area straight, secondary oblique rows faint. Apiculi distinct, at subequal wide intervals.

Habitat.—Naparima deposit, Trinidad (Grove !).

C. rex. Wallich, *Jour. Roy. Micr. Soc. Lond.*, 1879, p. 688.—Diam. 1·25 to 1·4 mm. Central space and rosette absent, but an indistinctly defined central area, with round isolated granules and hyaline interspaces, as well as distant irregular, somewhat curved large apiculi, evident. Markings on the central area without order, beyond this area round, forming straight closely disposed radial rows, 3 in ·01 mm.; on the sides of the cylindrical valve the rows straight, and corresponding in position to those on flat surface of valve, but the markings smaller, 5 in ·01 mm.; central papillæ prominent; a circular smooth space sometimes present at junction of terminal and lateral portions of cylindrical valve.—Sch., *Atl.*, pl. cxiv. fig. 7; *C. regius*, Grun., *Jour. Roy. Micr. Soc. Lond.*, 1879, p. 688.

This species was originally distributed under the name of *C. rex.*

Habitat.—Bay of Bengal (Wallich, 1857; Kitton !); H.M.S. Challenger, Stat. 265, 2900 fathoms (Grunow); Nancoori (Cleve !).

C. biradiatus. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 42, pl. iv. fig. 7.—Sometimes elliptical. Diam. ·075 to ·085 mm. Surface slightly convex. Central space rounded, with isolated granules, sometimes absent. Markings smallest, rounded, and irregular at the centre; elsewhere obtusely angular, subpearly, enlarging to about $\frac{2}{3}$ of radius to 4 in ·01 mm., and again decreasing slightly outwards; rows regular, alternately longer and shorter; the spaces at the origin of the shorter rows smooth, hyaline. A narrow hyaline band just within the border. Border with a single band of minute granules, 4 in ·01.—Sch., *Atl.*, pl. lviii. fig. 2.

Habitat. — Barbados deposit (Greville! Cleve); Springfield deposit, Barbados (Schmidt, Doeg!);* Chalky Mt., Barbados (Firth!); Oamaru deposit (Grove!); Piscataway deposit (Greville!); Sta Monica deposit (Grove!).

C. elegantulus. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 42, pl. iv. fig. 8.—Diam. ·0425 to ·1 mm. Surface slightly convex. Central space absent. Markings polygonal, and in contact or rounded, free, and without order on a small slightly excentric area extending to about $\frac{1}{3}$ of radius or onwards to the semiradius, thence obtusely quadrangular, 4 to $4\frac{1}{2}$ in ·01 mm., subequal, or increasing slightly outwards; rows straight or curved, the shorter rows around the border inconspicuous, irregular. Border with a single row of minute granules, 8 in ·01 mm.—Sch., *Atl.*, pl. lviii. figs. 3–5.

Distinguished from *C. biradiatus*, Grev., by its more delicate markings and its larger central portion upon which these are irregularly placed. Its markings are smaller and less pearly than those of *C. patellæformis*, Grev., to which it is also allied.

Habitat.—Barbados deposit (Greville! Firth!); Cambridge deposit, Barbados (Rae!); Newcastle deposit (Firth!); Chalky Mount, Barbados (Weissflog!); Springfield deposit, Barbados (Doeg! Cole!).

C. aethes, sp. n.—Diam. ·05 mm. Surface almost flat. Central space quadrate, about $\frac{1}{10}$ of diam. broad. Markings round or obtusely angular, from the central space to about $\frac{7}{10}$ of the radius

* In the collection of Dr F. W. Griffin.

subequal 4, on the remaining subopaque sharply defined band adjacent to the border 6, in ·01 mm.; rows inconspicuous, on the outer portion of the subopaque band more evident, secondary irregular subconcentric rows faint. Border sharply defined, hoop-like; striæ delicate, 10 to 12 in ·01 mm.—(Pl. II. fig. 8.)

Habitat.—Sta Monica, California (Deby !).*

C. apiculatus. Ehrb., *Mon. Ber. Ak.*, 1884, p. 77.—Diam. ·0825 to ·1125 mm. Central space distinct, irregularly rounded, about $\frac{1}{16}$ of diam. broad. Markings rounded, often compressed in the direction of the radius; sometimes polygonal, non-punctate, increasing slightly from the centre to about the semi-radius, again decreasing gradually to the border; towards the centre 4, at the border 6 to 8, in ·01 mm. Border indistinctly defined on its inner side; striæ coarse, subregular, 6 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 43; Ralfs in *Pritch. Inf.*, p. 829; Grun., *Denk. Wien. Ak.*, 1884, p. 75; Sch., *Atl.*, pl. lxiv. figs. 5–8; Cleve and Möll., *Diat.*, Nos. 164, 215; Raben., *Alg. Europ.*, No. 2484; *C. apiculatis*, var., Sch., *ibid.*, pl. lxiv. fig. 9; *C. apiculatus?* Sch., *ibid.*, pl. lxiv. fig. 10.

This species, when its markings are polygonal and in contact, is distinguished from *C. radiatus*, Ehrb., by the presence of a central space. Ehrenberg compares it with *Pyxidicula gemmifera*, Ehrb. The specimen figured by Schmidt (*Atl.*, pl. lxiv. fig. 10) has an unusually prominent border.

Habitat.—Rappahannock, Va. (Bailey ! † Rogers !); † Richmond, Va. (Ehrenberg, Kinker !); "America" (Ralfs); Nottingham, U.S., (Cleve and Möller ! Hardman !); ‡ "Virginia" (Hardman !); Bermuda tripoli (Greville !); Delaware, Md. (O'Meara !); Mejillones (O'Meara !); Patos Island guano (Hardman !); ‡ Los Angelos (O'Meara !); Richmond tunnel (O'Meara !); Monkston tidepool (O'Meara !); Cambridge deposit, Barbados (Greville !); Upper Peruvian guano (Weissflog !); Piscataway (Weissflog ! Greville !); Moron deposit (Greville !); Monterey stone (Greville !); Sta Monica deposit (Cleve and Möller !); Maryland (Cleve !); Laguna Harbour, anchor ground (Rabenhorst and Schwarz !).

* In a *Coscinodiscus* Type-plate by Thum, in the collection of Julien Deby.
† In the collection of Dr Greville.
‡ In the collection of Julien Deby.

Var. *Woodwardii*, Rattray. *C. Woodwardii*, Eul., *Diat. Spec. Typ.*, No. 116.—Diam. ·1075 to ·15 mm. Central space small, sometimes absent; rosette usually distinct. Markings polygonal, subequal, or increasing slightly outwards, again decreasing near the border; towards the rosette 4, at about $\frac{5}{6}$ of radius 3, in ·01 mm.; at the border much more minute.—*C. Woodwardii*, Eul.; Sch., *Atl.*, pl. lxi. fig. 3 (not fig. 2); *C. apiculatus*, var. *ambigua*, Grun., *Denk. Wien. Ak.*, 1884, p. 75; *C. perforatus*, Cleve and Möller, *Diat.*, No. 57.

Specimens in Chincha guano are transitional to *C. Janischii.*

Habitat.—N. American tertiary deposits, mouth of the Maranhon, Cuxhaven (Grunow, Firth!); Virginia (Hardman!); Monterey (Möller!); Mejillones, Bolivia (Firth!); trawled in lat. 24° 42′ N., long. 152° 1′ W., by H.M.S. Challenger (Rae!); Piscataway deposit (Rogers!);* Rappahannock, Va. (Rogers!* Bailey!); Bermuda tripoli (Greville!); Delaware, Md. (O'Meara!); Richmond tunnel (O'Meara!); Richmond (Cleve and Möller!); Oamaru deposit (Grove!); Whampoa (Grove!); Chincha guano (Grove!); Lobos di Afuera guano (Grove!); Upper Peruvian guano (Weissflog!); Monterey (Weissflog! Cleve! Greville! Moller!); Moron deposit (O'Meara!); Nottingham deposit (Cleve and Möller!† Hardman! O'Meara!); Maryland (Cleve!); Pabillan de Pico guano, Holland's cliff (Cleve!).

Var. *maxima.* Grun., *ibid.*, 1884, p. 76.—Diam. ·315 mm. Central space hyaline, from $\frac{1}{22}$ to $\frac{1}{23}$ of diam. broad. Markings $3\frac{1}{2}$ to 4, at the border 7, in ·01 mm.

Habitat.—Richmond, Va. (Grunow).

C. perforatus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 78.—Diam. ·09 to ·1325 mm. Central space small but distinct, without a surrounding rosette. Markings rounded and granular or polygonal, on the same or on different valves, $3\frac{1}{2}$ to 4 in ·01 mm., somewhat smaller towards the border, the central dots obvious; minute puncta at the inner ends of the shorter radial rows. Border striæ, 6 in ·01 mm.—Ehrb., *Mikrog.*, pl. xviii. fig. 46; Sch., *Atl.*, pl. lx. fig. 12 (?); pl. lxiv. figs. 12–14; Smith, *Syn. Brit. Diat.*, ii. p. 85.

Specimens from Callao seaweeds sometimes exhibit at the

* In the collection of Dr Greville.

† *Diat.*, Nos. 215, 216.

border minute apiculi at intervals of about ·005 mm. Grunow (*Denk. Wien. Ak.*, 1884, p. 76), says that *C. Woodwardii*, Eul., is identical with the present species. In Kitton's specimens of Eulenstein's valves, however, which I have examined, there are no puncta at the inner ends of the shorter rows. These valves are, indeed, identical with *C. apiculatus*, var. *ambigua*, Grun., which must be abandoned. Kitton's slide was obtained from Herr E. Weissflog, who purchased Eulenstein's collection at his death, and therefore knew his *C. Woodwardii*.

Schmidt's figure (*Atl.*, pl. lx. fig. 12) resembles *C. centralis* much more than *C. perforatus*, in its more prominent central rosette, more crowded markings, and the more rapid diminution of their size towards the border. Its union to *C. perforatus* is doubtful; it is based only on the presence of minute puncta, opposite the origin of the shorter rows.

Habitat.—Richmond, Va.; Arica and Peruvian guano (Schmidt, Kinker! Cleve!); Thames mud (Roper); Medway (Dallas); New Nottingham (Firth!); Callao seaweeds (Firth!); Saldanha Bay guano (Cleve!).

Var. *cellulosa*. Grun., *Denk. Wien, Ak.*, 1884, p. 75.—Diam. ·135 to ·15 mm. Markings hexagonal, 3 to 5 in ·01 mm., smaller at the border; the puncta at the origin of the shorter rows indistinct; minute apiculi at intervals of about ·005 mm., sometimes present.—Sch., *Atl.*, pl. cxiv. fig. 5; in H. L. Sm., *Diat. Spec. Typ.*, No. 90; Janisch, *Gazelle Exped.*, taf. iv. figs. 6, 7.

Habitat.—With the species and on *Macrocystis* from the coast of Peru (Grunow); Callao seaweeds (Firth!); Japan (Kinker!); Maryland deposit (Rae!); Bermuda tripoli (Greville!); Nottingham, Maryland (O'Meara!); Oamaru deposit, Jackson's Paddock (Grove!); Chincha guano (Grove! Schmidt); Lobos di Afuera guano (Grove!); Japan (H. L. Smith!); Bolivian guano (Cleve!); San Benito deposit, California (Grove!).

Var. *delicatula*, nov. *C. perforatus*, var. *cellulosa*, Grun., *ibid.*, 1884, p. 75.—Central space minute. Markings towards the centre 5, towards the border 8, in ·01 mm.; puncta at the origin of the shorter rows small.

This var. approaches *C. fimbriatus*.

Habitat.—Moron deposit (Grunow).

*C. mossianus.** Grev., *Trans. Micr. Soc Lond.* 1865, p. 25, pl. iv. fig 22.—Diam. ·135 mm. Surface flat to about the semi-radius, thence sloping gently to the border. Central space minute. Markings pearly, obtusely quadrangular, those in the outermost band concavo-convex with large faint rounded central dots, increasing from the centre to about $\frac{1}{3}$ of the radius to 2 or $2\frac{1}{2}$ in ·01 mm., thence decreasing uniformly to the border to 3 in ·01 mm., irregular around the centre, the rows separated by clear lines.

Habitat.—Cambridge deposit, Barbados (Johnson !).†

C. gemmifer. Ehrb., *Mon. Ber. Ak.*, 1844, p. 201.—Diam. about ·06 mm. Central space circular, hyaline, about $\frac{1}{6}$ of diam. broad. Markings rounded, conspicuous, about 5 in ·01 mm., decreasing from the central space outwards; towards the border punctiform, interspaces hyaline; rows straight.—Ehrb., *Mikrog*, pl. xxxv. A. 22. fig. 3.

By Ehrenberg, compared with *Pyxidicula gemmifera*, Ehrb., from Virginia. Not to be united to *C. elegans*, Grev., as suggested by Pantocsek, since in the latter the radial rows are more crowded. Allied to *C. cycloteres*, Cstr., but distinguished by the less uniform markings, and to *C. actinochilus*, Ehrb., where, however, the radial rows are also more numerous.

Habitat.—Ice Barrier, Antarctic, lat. 78° 10′ S., long. 162° W.; Sounding, 190 fathoms, lat. 78° 10′ S., long. 162° W.; ex *Salpâ*, lat. 66° S., long. 157° W.; Sounding, 270 fathoms, lat. 63° 40′ S., long. 55° W. (Hooker); Monterey Stone (Cleve !).

Var. *campechiana*, nov. *C. gemmifer*, var., Grun. MS.—Diam. ·0375 mm. Central space round, $\frac{1}{7}$ to $\frac{1}{8}$ of radius broad, punctate. Markings most prominent near the central space; indistinct towards the border, the interspaces minutely punctate.

Habitat.—Campeachy Bay (Weissflog !).

C. flagrans, sp. n. Sch., *Atl.*, pl. lvii. fig. 46.—Outline hexagonal, with angles obtuse and sides slightly convex. Diam. about ·03 mm. Central space distinct. Markings rounded, granular, about 5 in

* Dedicated to Mr Moss of Lancaster, the friend and fellow worker of Mr Johnson.

† In the collection of Dr Greville.

·01 mm., closely disposed in the rows, interspaces hyaline; rows radial or somewhat oblique, straight; adjacent to the border a narrow hyaline band. Border striæ, evident, 6 in ·01 mm.

Habitat.—Springfield deposit, Barbados (Schmidt).

C. gemmatulus. Cstr., *Diat. Chall. Exped.*, p. 161, pl. xvii. fig. 9.—Diam. about ·04 mm. Central space indistinctly defined, about $\frac{1}{7}$ of diam. broad. Markings round, granular, subequal, minute with hyaline interspaces; a circlet of minute closely placed dots (apiculi?) at inner edge of border; rows irregular, radial. Border distinctly defined, about $\frac{1}{9}$ of radius broad; striæ delicate, 10 to 12 in ·01 mm.

Habitat.—Indian Ocean (Castracane).

C. actinochilus. Ehrb., *Mon. Ber. Ak.*, 1844, p. 200.—Diam. about ·035 mm. Central space circular, about $\frac{1}{6}$ of diam. broad, bearing isolated rounded granules. Markings granular, rounded, in straight, closely disposed rows, with interspaces at origin of shorter rows hyaline; just within the border punctiform, and arranged on a sharply defined band in still more crowded radial rows.—Ehrb., *Mikrog.*, pl. xxxv. A, 21. fig. 5; Ralfs in *Pritch. Inf.*, p. 829; *Craspedodiscus actinochilus*, Ehrb., *Abh. Ber. Ak.*, 1872, p 26.

Habitat.—Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W. (Hooker).*

C. galapagensis, Rattray. *C. griseus*, var. *gallopagensis*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 7; pl. cxxxii. fig. 1.—Diam. ·175 to ·19 mm. Surface flat from the centre to about semiradius, a slightly convex zone reaching inwards to the semiradius, about $\frac{2}{7}$ of the radius broad, and merging gradually into the almost flat portion adjacent to the border. Central space indistinct, rounded. Markings small, round or oval, granular, least crowded towards the centre, about 4 in ·01 mm.; interspaces hyaline, unequal; rows straight. Border narrow, hyaline.—(Pl. II. fig. 20.) *C. griseus*, Sch., *Atl.*, pl. lviii. fig. 1.

Distinguished from *C. nitidus* by the evident straight radial lines, sometimes most distinct on the outer half of the valve.

* *Fide* Ehrenberg.

Haitat.—Galapagos Islands (Grunow); Springfield deposit, Barbados (Schmidt); Oamaru deposit (Grove! Kinker!).

C. armatus. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 42, pl. iv. fig. 5.—Sometimes obtusely angular. Diam. ·0575 mm. Surface convex. Central space rounded, about $\frac{1}{2\,3}$ of diam. broad. Markings rounded, granular; towards the centre 5, towards the border 8, in ·01 mm.; rows straight or curved, radial or subradial, sometimes forming unequal obscure fasciculi, separated towards the border by delicate, clear, slightly elevated lines. Border indistinct. —*Cosmiodiscus armatus*, Sch. (not Grev.), *Atl.*, pl. lvii. fig. 4.

The delicate elevated lines represent, as noted by Greville, the more pronounced ridges of *Biddulphia Johnsoni*, Ralfs (*Trans. Micr. Soc. Lond.*, 1866, p. 4, pl. i. fig. 11). The union of *C. armatus* with the old genus *Cestodiscus*, as suggested by Habirshaw, is undesirable.

Habitat.—Barbados deposit (Greville!); Richmond, Va. (Schmidt).

C. obliquus, Rattray. *Cosmiodiscus obliquus*, Grev. MS.—Diam. ·0675 to ·075 mm. Surface slightly depressed towards the centre, and convex towards the border. Central space small, circular, bearing a few rounded granules. Markings obtusely angular or suboval, with long axis radial, $5\frac{1}{2}$ to 6 near the border, decreasing to 8 in ·01 mm.; rows radial, towards the border uniformly curved towards the same direction, at subregular intervals of about ·005 mm. short hyaline curved lines on the convex sides of the valve, and extending a short distance inwards as attenuating subulate spaces. Border narrow; striæ 8 to 10 in ·01 mm.—(Pl. I. fig. 14.)

Habitat.—Monterey deposit (Greville!); Santa Monica deposit (Grove!).

C. apages,* Rattray. *Cosmiodiscus normanianus*, Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 80, pl. viii. fig. 11.—Diam. ·045 to ·06 mm. Surface almost flat. Central space small, indefinite. Markings small, round, free granules disposed without order, with wide hyaline interspaces between the centre and semiradius; beyond this more punctiform, crowded, 8 to 10 in ·01 mm.; rows straight; distinct radial clear spaces extending from about semiradius to

* ἀπαγής, loose in texture.

border at intervals of ·01 to ·0125 mm., and expanding slightly outwards. Border sharply defined, hyaline.—*Coscinodiscus normanianus*, Grev., *ibid.*, Explan. pl. viii. fig. 11.

The name *normanianus* is undesirable, there being already a *C. Normanii*, which is distinct.

Habitat.—Barbados deposit (Norman, Grove !).

C. splendidulus, Rattray. *Cosmiodiscus normanianus*, Grove and Sturt (non Grev.), *Jour. Quek. Micr. Cl.*, 1887, p. 65, pl. vi. fig. 21.—Diam. ·055 to ·075 mm. Surface almost flat. Central space circular, about $\frac{1}{12}$ of diam. broad. Markings small, round, granular, with wide unequal interspaces, somewhat more crowded towards the border; rows straight, but inconspicuous, at regular intervals of about ·01 mm. narrow hyaline radial lines, most distinct towards the border, the outer ends of the intervening compartments convex outwards; a distinct but small apiculus at the outer ends of the hyaline lines. Border distinct, about $\frac{1}{15}$ of radius broad, hyaline.

Habitat.—Oamaru deposit (Grove !).

C. perikompsos,* Rattray. *Cosmiodiscus elegans*, Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 79, pl. viii. fig. 13.—Diam. ·0875 to ·095 mm. Surface slightly convex towards the border. Central space circular, $\frac{1}{12}$ to $\frac{1}{13}$ of diam. broad, faint. Markings obscure, punctiform, most distinct on the inner half of the valve, in faint radial rows; towards the border more crowded, 20 to 25 in ·01 mm., forming punctiform, evident, straight, radial striæ, 14 in ·01 mm., at subregular intervals of ·006 to ·0075 mm. distinct, straight, hyaline radial lines extending outwards from the central space, and most prominent towards their outer ends. Border narrow, hyaline. —(Pl. III. fig. 12.)

Habitat.—San Pedro (Grove !); Monterey (Hardman, Greville); Santa Monica deposit (Grove !).

Var. *curta*, nov.—Diam. ·04 to ·0625 mm. Central space distinct, rounded or irregular, small, rarely of considerable size. Markings 8 to 10 in ·01 mm., least crowded near the centre, near border more minute, forming closely disposed radial striæ 12 in ·01 mm.; rows separated at equal or subequal intervals by hyaline substraight lines,

* περίκομψος, very elegant. The specific name *elegans* is preoccupied (*q. v.*).

originating sharply at or near the semiradius, rarely nearer the central space, and terminating at inner edge of the marginal striæ. Border sharply defined, hyaline.

Habitat.—Elesd deposit (Hardman !).*

C. tenuis, Rattray. *Cosmiodiscus tenuis*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxv. fig. 13.—Diam. ·055 to ·0675 mm. Surface slightly convex towards the border. Central space circular, obscure or subobsolete. Markings faint, minute rounded granules most evident and most laxly disposed towards the centre; about the semiradius 6 in ·01 mm., towards the border punctiform, arranged in straight radial striæ, 8 to 10 in ·01 mm.; between the striæ, at subregular intervals, narrow, short, indistinct, hyaline spaces. Border distinct, from $\frac{1}{10}$ to $\frac{1}{13}$ of radius broad; striæ delicate, obscure, 16 in ·01 mm.

Habitat.—Monterey deposit (Van Heurck); San Pedro (Grove !).

C. evadens, sp. n. Sch., *Atl.*, pl. lvii. fig. 44.—Diam. ·0625 mm. Central space and rosette absent. Markings rounded, granular, about 5 in ·01 mm., towards the border smaller; interspaces hyaline, unequal, largest towards the centre; rows straight; apiculi evident at intervals of about ·015 mm., inserted at inner edge of border. Border sharply defined, narrow; striæ radial, 6 to 8 in ·01 mm.

Habitat.—Springfield deposit, Barbados (Schmidt).

Var. *parvula.* Sch., *Atl.*, pl. lvii. fig. 45 (no name).—Diam. ·0325 mm. Markings subequal to border, non-apiculate. Border less sharply defined; striæ short, but distinct, 6 in ·01 mm.

Habitat.—Springfield deposit, Barbados (Schmidt).

C. undatus, Grun. Pant., *Fossil. Bacil. Ung.*, p. 74, pl. x. fig. 89; pl. xxvii. fig. 252. *Actinocyclus* (*alienus*, Grun. var.?) *undatus*, Cleve, *Jour. Queh. Micr. Cl.*, 1885, p. 174, pl. xiii. fig. 14.—Diam. ·07 to ·1 mm. Surface with two concentric elevations and depressions. Central space indistinct, circular, about $\frac{1}{23}$ of diam. broad, with rounded isolated granules. Markings rounded, granular, 5 to 6 in ·01 mm., near the border punctiform; rows separated by narrow

* In the collection of Mr Julien Deby.

clear lines, small subulate spaces opposite the origin of the shorter rows; apiculi sometimes many, minute, inserted close to the border, at wide subequal intervals. Border narrow.

Pantocsek's figure shows no apiculi, but in their stead small clear spaces surrounded by a faint circlet of markings. In the presence of these spaces it differs from *C. intumescens*, Pant.

Habitat.—Brünn Tegel (Cleve !); Alsó-, Felsö-Esztergály, Kékkö, Szakal, Szent Peter deposits (Pantocsek !).

C. agapetos,* sp. n. *C. nitidulus*, Grun., var.? Sch., *Atl.*, pl. cxiii. fig. 18.—Diam. ·05 mm. Central space and rosette absent. Markings small, rounded, granular, largest towards the centre, decreasing outwards, on a zone adjacent to the border punctiform; interspaces wide, hyaline, smaller towards the border; rows obscurely radial towards the centre, becoming obviously radial near the border. Border narrow.

Distinguished from *C. nitidulus* by the greater reduction in size of the markings outwards and the evident zone adjacent to the border.

Habitat.—Aegina (Schmidt).

C. exiguus, sp. n. Sch., *Atl.*, pl. lviii. fig. 30 (no name).—Diam. about ·04 mm. Central space distinct, rounded; no rosette. Markings minute, granular; beyond the semiradius punctiform, disposed at unequal intervals in evident radial straight rows; interspaces largest towards the centre, hyaline; non-apiculate. Border sharply defined, narrow; striæ, 6 to 8 in ·01 mm.

Habitat.—Campeachy Bay (Schmidt).

Var. *aequalis*, nov. Sch., *Atl.*, pl. lviii. fig. 31 (no name).—Diam. about ·056 mm. Central space angular, from $\frac{1}{10}$ to $\frac{1}{12}$ of diam. broad. Markings minute, granular or subpunctiform, 5 to 6 in ·01 mm., somewhat more prominent around central space; interspaces narrow, hyaline. Border sharply defined, at its middle a distinct concentric line, hyaline.

Habitat.—Mors deposit (Schmidt).

C. apollinis. Ehrb., *Mon. Ber. Ak.*, 1844, p. 200.—Diam. ·0815 mm. Central space rounded, about $\frac{1}{14}$ of diam. broad, bearing a

* ἀγαπητὸς, lovely.

few isolated rounded granules. Markings rounded, granular, 5 in ·01 mm., somewhat more crowded around the central space; rows straight, shorter irregular rows around the border; interspaces hyaline. Border distinct, striæ delicate.—Ehrb., *Mikrog.*, pl. xxxv. A. 22. fig. 4; Ralfs. in *Pritch. Inf.*, p. 829; *C. scintillans*, Grev., *Quart. Jour. Micr. Sci.*, 1863, p. 230, pl. ix. fig. 6 (excl. *C. scintillans*, Grev., H. L. Smith, *Diat. Sp. Typ.*, No. 99.

Habitat.—Cambridge deposit, Barbados (Browne); pancake ice, Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W.; lat. 78° 10′ S., long. 162° W., in 190 fathoms; ex *Salpis*, lat. 66° S., long. 157° W.; lat. 63° 40′ S., long. 55° W., in 207 fathoms (Hooker); Oamaru deposit (Grove! Doeg!); Norway (Deby!); Barbados (Cleve).

Var. *compacta*, nov. *C. scintillans*, Grev. (?) Sch., *Jahresb. d. Kom. z. Untersuch. d. deutsch. Meer, Kiel*, 1874, p. 94, pl. iii. fig. 33.—Diam. ·04 mm. Central space more inconspicuous, rounded. Markings decreasing towards the border, the radial rows more crowded, the shorter rows larger than in type.

Habitat.—Sölsvig (Schmidt).

C. diplostictus, Grun. Van Heurck., *Syn. Diat.* Belg., pl. cxxxii. fig. 3.—Diam. ·068 mm. Central space indistinct. Markings of two kinds: large rounded granules at wide irregular intervals, arranged in inconspicuous radial rows, and becoming somewhat smaller towards the border; and minute puncta, least crowded towards the centre, also arranged in numerous radial rows, with hyaline interspaces opposite the ends of the shorter rows. Border sharply defined; striæ distinct, 6 to 8 in ·01 mm.

Habitat.—Balearic Islands (Van Heurck).

C. decussatus, Grove and Sturt MS.—Diam. ·01 mm. Surface flat, slightly convex near the border. Central space circular, about $\frac{1}{20}$ of diam. broad. Markings minute, round, granular, with unequal hyaline interspaces, most crowded towards the border; about the semiradius 6, at the border 8 or 9, in ·01 mm.; rows straight, secondary oblique decussating rows distinct beyond the semiradius. Border narrow, distinct; striæ faint, 10 in ·01 mm. —(Pl. I. fig. 7.)

Distinguished from *C. apollinis* by the smaller size and evident decussate arrangement of the markings.

Habitat.—Bain's Farm upper stratum, Oamaru deposit (Grove !).

C. biplicatus, Grun. Van. Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 6.—Diam. ·0715 mm. Surface with an elongated somewhat curved depression on each side of the valve at about $\frac{2}{3}$ of radius from the centre. Central space angular, about $\frac{1}{22}$ of diam. broad. Markings punctiform, most evident about semiradius, around the border on a band about $\frac{1}{8}$ of radius broad, minute in crowded radial rows; rows substraight. Border narrow, hyaline.

Distinguished from *C. pellucidus* by the elongated depressions, the central space, more prominent markings, and distinct narrow band adjacent to the border.

Habitat.—Samoa Islands, Cuxhaven (Van Heurck).

C. bengalensis, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 9.—Diam. ·1065 mm. Surface with a faint undulation about $\frac{2}{3}$ of radius from centre. Central space indistinct, rounded, bearing isolated granules. Markings punctiform, somewhat laxly disposed around the centre 7 to 8 in ·01 mm., increasing slightly to about $\frac{2}{3}$ of radius, again decreasing to the border; secondary oblique decussating rows faint; apiculi small, at intervals of about ·01 mm. Border sharply defined; striæ delicate, regular.

Distinguished from *C. pellucidus* by its somewhat more prominent markings, its apiculi, and striated border.

Habitat.—Elephant Point, Bengal (Van Heurck).

C. pellucidus, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxxii. fig. 8.—Diam. ·0325 to ·045 mm. Surface sometimes with a faint undulation about the semiradius. Central space absent. Markings punctiform, recognised with diffiulty, least crowded towards the centre; interspaces clear; rows straight or substraight. Border hyaline.—Cleve and Möll., *Diat.*, No. 172; *Odontodiscus pellucidus*, Grun., *Vega Exped., Vetensk. Jakttag. Stockh.*, Bd. iii. 1883, p. 488.

Habitat.—Davis Straits (Grunow, Cleve and Möller !); Greenland (Cleve !); Maghellan Straits (Cleve !).

C. lacustris. Grun., *Kongl. Sv. Vet.-Ak. Handl. Stockh.*, 1880., No. 2, p. 114.—Circular or elliptical. Diam. ·035 to ·075 mm. Surface with a slight unilateral undulation close to the centre. Central space and rosette absent. Markings minute, 6 to 10 in ·01 mm.; rows radial, non-fasciculate, most distinct about semiradius; a circlet of minute apiculi, about 6 in ·01 mm., close to the border, within these a few scattered irregularly placed, but similar apiculi forming an indistinct inner circlet.—Grun., *Denk. Wien. Ak.*, 1884, p. 85, pl. iv. (D), fig. 30; Van Heurck, *Syn. Diat. Belg.*, pl. c. fig. 42; Cleve and Möller, *Diat.*, No. 172; Van Heurck, *Typ. Syn. Diat. Belg.*, No. 535; Kitton, *Norfolk Diat.*, Nos. 3, 21; *Cyclotella punctata*, W. Sm., *Syn. Brit. Diat.*, ii. p. 87; *Stephanodiscus punctatus*, Grun.

The name *punctatus* of Smith cannot be retained, because of *Coscinodiscus punctatus*, Ehrb., which is a distinct species. Distinguished from *C. capensis* by the absence of a central space and the irregularity of the inner circlet of smaller apiculi, and from *C. plicatulus* by the more delicate markings and the nearer approximation of the circlet of apiculi to the border.

Habitat.—Kara Sea, Jamal (Cleve! Grunow); Market Weighton, England (Grunow); Locality (?) (Kinker!); Yorkshire (Van Heurck!); Wisbeach (S. Smith!);* Breydon (Kitton!).

Var. *septentrionalis.* *C.* (*lacustris*, var.?) *septentrionalis*, Grun., *ibid.*, 1884, p. 85, pl. iv. (D), figs. 28, 33.—Diam. ·0365 to ·055 mm. Surface with a short central transverse plication, most pronounced at the centre. Markings larger, 8 to 9 in ·01 mm., decreasing somewhat near the border; apiculi closely placed, arranged in a single circlet near the border.

Grunow has found specimens at Cuxhaven and in the Gulf of Bothnia, intermediate between this var. and the type.

Habitat.—Franz Josef's Land, Kara Sea, Jamal, Cape Wankarema (Grunow); Canton River, Whampoa (Grove!); Balearic Islands (Cleve!).

Var. *hyperborea.* *C.* (*lacustris*, var.?) *hyperboreus*, Grun., *ibid.*, 1884, p. 85, pl. iv. (D), fig. 26.—Diam. ·037 mm. Undulation near the centre faint. Markings angular, 10 in ·01 mm.; subequal

* In the collection of W. Smith.

to the circlet of apiculi, thence more minute to the border; apiculi more distant, forming a single circlet.—Cleve and Möll., *Diat.*, No. 319.

Habitat.—Franz Josef's Land, Kara Sea, Cape Wankarema (Grunow; Cleve and Möller!).

Var. *marina*, Grun. Cleve and Möll., *Diat.*, No. 172.—Diam. ·025 to ·03 mm. Markings 10 in ·01 mm., most evident towards the centre, near border 12 to 14 in ·01 mm.; radial rows faint, secondary oblique rows more evident slightly concave outwards; apiculi numerous, minute, about 4 in ·01 mm.

Habitat.—Davis Straits (Cleve and Moller!).

Var. *australiensis*. *C.* (*lacustris*, var. ?) *australiensis*, Grun., *ibid.*, 1884, p. 86, pl. iv. (D) fig. 31 *a*, *b*; fig. 32.—Sometimes roundly elliptical. Diam. ·02 to ·073 mm. Surface convex on one half of valve, concave on the opposite; a short transverse central band. Markings punctiform, 10 to 12 in ·01 mm.; apiculi distinct, forming a single circlet.

The apiculi are less crowded than in var. *septentrionalis*, but more so than in var. *hyperborea*.

Habitat.—Brackish water, Australia (Grunow); Whampoa mud, China (Kitton!); Yarra Yarra, Australia (Kitton!).

C. plicatulus, Grun., *Denk. Wien. Ak.*, 1884, p. 86, pl. iv. (D), fig. 27.—Diam. ·04 mm. Central space absent. Markings small, rounded, granular, about 6 in ·01 mm., smaller and subpunctiform towards the border; interspaces hyaline, irregular, subequal to the zone of the processes, thence smaller to the border; secondary irregularly concentric rows distinct; apiculi numerous, large, at subregular intervals of ·005 to ·006 mm., inserted some distance from the border.

Habitat.—Monterey tripoli, Cal. (Grunow).

C. pulcherrimus, sp. n.—Diam. ·035 to ·045 mm. Surface flat. Central space absent. Markings rounded, granular, 5½ to 6 in ·01 mm.; from the centre 5 to 8 narrow hyaline lines passing outwards to the semiradius, and terminating in funnel-shaped expansions; between the latter and the centre the markings

irregular, but beyond the semiradius in distinct, somewhat bent, subradial non-fasciculate rows. Border narrow; striæ 6 to 8 in ·01 mm. distinct.—(Pl. II. fig. 1.)

Habitat.—Galapagos Islands (Weissflog!).

C. tabularis, Grun., *Denk. Wien. Ak.*, 1884, p. 86.—Diam. ·0605 mm. Central space subcircular, distinct, about $\frac{1}{8}$ of diam. broad, bearing a few isolated round granules. Markings round, granular, subequal or somewhat smaller near the border, about 6 in ·01 mm. interspaces hyaline; a narrow clear band about $\frac{1}{13}$ of radius broad adjacent to the border; at its outer edge distinct apiculi at intervals of about ·01 mm. Border narrow, hyaline.—Sp. n.? Sch., *Atl.*, pl. lvii. fig. 43.

Habitat.—Table Bay (Schmidt).

C. Thumii. Cleve, *Jour. Quek. Micr. Cl.*, 1885, p. 175, pl. xiii. fig. 17.—Obtusely triangular, or with a unilateral compression, or circular. Diam. ·075 to ·125 mm. Central space subcircular, about $\frac{1}{6}$ of diam. broad, hyaline, with diverticula extending outwards between the longer rows of markings. Markings small, rounded, granular, 6 in ·01 mm., subequal to but most crowded around the border; the rows straight, with wide hyaline interspaces towards the centre.—Sch., *Atl.*, pl. cxiv. fig. 10.

Specimens with two central spaces, separated by a single row of markings, sometimes occur.

Habitat.—Brünn Tegel (Weissflog!); Mähren deposit (Deby!).

C. comptus. Cstr., *Diat. Chall. Exped.*, p. 157, pl. xiii. fig. 9.—Diam. ·125 mm. Central space circular, hyaline, about $\frac{1}{10}$ of diam. broad, without a distinct limiting band of markings. Markings punctiform; rows radial, only a few reaching the central space, the majority terminating about $\frac{3}{5}$ of radius from the centre, on the other $\frac{2}{5}$ of radius crowded, interspaces hyaline. Border distinct, narrow, hyaline.

Distinguished from *C. Thumii* by the smaller markings, fewer of which pass to the central space, and by their more crowded arrangement on the outer $\frac{2}{5}$ of the radius. The relationship to *C. dimorphus*, noted by Castracane, is remote.

Habitat.—Antarctic Ocean, H.M.S. Challenger (Castracane).

C. confertus, sp. n. Sch., *Atl.*, pl. lviii. fig 22 (no name).—Diam. about ·04 mm. Central space and rosette absent. Markings punctiform, interspaces large, unequal, but markings not crowded either towards centre or border; rows faint. Border indistinctly defined; striæ 6 in ·01 mm.

Distinguished from *C. marginulatus* var. *sparsa*, Grun., from Campeachy Bay, by the radial arrangement of the markings and the less prominent border.

Habitat.—Cape of Good Hope (Schmidt).

C. polygonus. Cstr., *Diat. Chall. Exped.*, p. 161, pl. xxii. fig. 6.—Polygonal. Diam. about ·1 mm. Central space subcircular, about $\frac{1}{15}$ of diam. broad, bearing a few isolated granules. Markings punctiform, interspaces hyaline, unequal, largest opposite the origin of the shorter rows; rows often interrupted; a narrow hyaline band adjacent to the border.

The polygonal outline of the valve is probably the result of fracture. This species is close to *C. Thumii*, but is distinguished by the absence of crowding in the arrangement of the markings towards the border, and by the hyaline zone adjacent to the latter.

Habitat.—Antarctic Ocean, H.M.S. Challenger (Castracane).

C. elongatus, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxv. fig. 14.—Elongately elliptical. Major axis ·083 mm., about $5\frac{1}{2}$ times minor. Central space absent. Markings punctiform, irregular on a small central area; rows straight along the minor axis, elsewhere convex towards the extremities of the major axis and radial, most crowded on a narrow band around the border. A single interrupted row running along the major axis; apiculi 2, minute, one at each end of the major axis; interspaces hyaline.

The valve figured by Van Heurck (*ibid.*, pl. cxxv. fig. 15) is similar, but possesses an evident pseudonodule, and so belongs to *Actinocyclus*.

Habitat.—Mejillones guano (Van Heurck).

C. pauper. Tru. and Witt, *Jeremie Diat.*, 1888, p. 13, pl. ii. fig. 11.—Diam. ·06 mm. Surface flat. Central space circular, $\frac{1}{12}$ of diam. broad. Markings round or obtusely angular, subpearly; towards the centre $3\frac{1}{2}$, towards the border $4\frac{1}{2}$, in ·01 mm.; smaller

round punctiform granules placed irregularly amongst the larger; rows inconspicuous, most crowded, and submoniliform near the border; interspaces narrow, hyaline. Border with inner edge indistinct; striæ, 4 to 6 in ·01 mm., evident.

Distinguished from *C. elegans* by the more irregular markings, narrower interspaces, and less evident radial rows.

Habitat.—Jeremie deposit, Hayti (Weissflog !).

C. elegans. Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 3, pl. i. fig. 6.—Diam. ·045 to ·08 mm. Central space irregularly round, about $\frac{1}{11}$ of diam. broad. Markings round, 4 to $4\frac{1}{2}$ in ·01 mm.; subequal to the narrow band at the border; upon this band minute granular interspaces, hyaline, large opposite the origin of the shorter rows; rows upon the band at border crowded; apiculi minute, widely placed, sometimes absent. Border striæ obvious, 6 to 8 in ·01 mm.—Sch., *Atl.*, pl. lviii. fig. 7; Pant., *Fossil. Bacil. Ung.*, p. 73, pl. xvi. fig. 141; pl. xxiv. fig. 216; Janisch, *Gazelle Exped.*, taf. iv. fig. 6; Cleve and Möll. *Diat.*, No. 164; *C. margaritaceus*, Cstr., *Diat. Chall. Exped.*, p. 164, pl. xviii. fig. 3. In H. L. Smith, *Diat. Spec. Typ.*, No. 99.

Not to be united to *C. gemmifer*, as suggested by Pantocsek.

Habitat.—Monterey deposit (Hardman! Greville! Firth! Kinker!); Bajtha, Elesd, Kékkö, Szent Peter and Dolje deposits (Pantocsek); Piscataway (Weissflog!); Los Angelos (O'Meara! Hardman!);* Sta Monica deposit (Cleve and Möller! Cleve! Firth!); Springfield, Barbados (Doeg!); Japan (H. L. Smith!); Newcastle deposit, Barbados (Doeg!); S. California, Pacific coast (Cleve).

Var. *parvipunctata.* Tru. and Witt, *Jeremie Diat.*, p. 14, pl. ii. fig. 22.—Diam. ·08 mm. Markings more minute; towards the centre 6, towards the border 8, in ·01 mm.; rows radial, more numerous; interspaces less evident, small.

Habitat.—Jeremie deposit (Truan and Witt).

C. spiniferus, Grove and Sturt. Grun., in *Bot. Centralbl.*, Bd. xxxiv. Nos. 2, 3, 1888, p. 35.—Diam. ·0875 to ·1125 mm. Surface

* In the collection of Julien Deby.

rising from the centre for about $\frac{1}{4}$ of radius to the highest zone; this zone convex, most sharply defined on its inner side, its outer edge passing gradually into the outer portion, which slopes downwards to the border. Central space subcircular, about $\frac{1}{14}$ of diam. broad, with a few isolated granules at its middle. Markings angular, towards the centre 8, on the highest zone largest, subpearly 5, beyond this subequal to the border and 6, in ·01 mm.; secondary subconcentric bands evident on the highest zone, distinct subulate areas opposite origin of shorter rows about the semiradius; apiculi at the border evident, rarely obscure, at intervals of about ·01 mm. Border narrow, hyaline.—*C. elegans*, var. *spinifera*, Grove and Sturt, *Jour. Quek. Micr. Cl.*, vol. iii. ser. 2, 1887, p, 69, pl. v. fig. 9.

Grunow justly separates this from *C. elegans*, Grev., but it is quite distinct from *Cestodiscus pulchellus*, Grev., to which he is inclined to assimilate it.

Habitat.—Oamaru deposit (Doeg! Grove!).

C. griseus. Grev., *Quart. Jour. Micr. Sci.*, 1863, p. 230, pl. ix. fig. 7.—Diam. ·065 to ·0825 mm. Surface rising slightly from centre to about semiradius, thence sloping gradually to the border. Central space rounded, $\frac{1}{15}$ to $\frac{1}{17}$ of diam. broad, inconspicuous, bearing several rounded granules. Markings rounded, granular, 6 in ·01 mm., decreasing slightly near the border; the rows straight or slightly bent, the shorter rows irregularly placed; interspaces narrow, minute; apiculi at intervals of about ·01 mm., sometimes inserted at the border.—Sch., *Atl.*, pl. lviii. figs. 13, 14 (excl. fig. 1).

Small hyaline irregular spaces sometimes occur here and there on the surface, and there may be a narrow clear band within the border. Sometimes confounded with *C. apollinis.*

Habitat.—Cambridge deposit, Barbados (Greville! Johnson!); Barbados (Cleve! Firth!); Monterey (Weissflog!); Los Angelos (O'Meara!); Santa Maria deposit (Grove!); Santa Monica deposit (Cleve!).

Var. *apiculata*, nov.—Diam. ·0875 mm. Similar to the type, but the markings increasing from the central space outwards; towards the centre 8, towards the border 6, in ·01 mm.; the rows more crowded; a circlet of minute apiculi at the border.

Habitat.—"Barbados, 1865" (Greville!).

C. cribrosus. Tru. and Witt, *Jeremie Diat.*, p. 14, pl. ii. fig. 25. —Diam. ·06 to ·08 mm. Surface flat. Central space irregularly angular, small. Markings rounded, increasing slightly from the central space for about $\frac{1}{4}$ of the radius, thence subequal to the distinct band adjacent to the border, upon this band punctiform; towards the centre $3\frac{1}{2}$, at the semiradius 3, on the marginal band 6, in ·01 mm.; rows straight, secondary oblique rows inconspicuous. —Sch., *Atl.*, pl. lxiv. fig. 11 (no name).

Habitat.—Jeremie deposit (Truan and Witt); Moron deposit (Schmidt).

C. subdivicus. Tru. and Witt, *Jeremie, Diat.*, p. 14, pl. ii. fig. 13.—Diam. ·05 mm. Surface almost flat. Central space and rosette absent. Markings rounded or obtusely angular, decreasing gradually from the centre to the border; towards the centre 4 to $4\frac{1}{2}$ (?) in ·01 mm.; at the border punctiform; rows reaching the centre few, those intervening shorter and less prominent, on a narrow band adjacent to the border, numerous.

Schmidt has assigned this species to *Actinocyclus*, but Truan and Witt have failed to find a pseudonodule, and so justly unite it to *Coscinodiscus.*

Habitat.—Jeremie deposit (Truan and Witt).

C. undulatus. Cleve, *Kongl. Sv. Vet.-Ak. Handl. Stockh.*, 1881, No. 5, p. 20, pl. v. figs. 60*a*, 60*b*.—Diam. ·0575 to ·096 mm. Central space elongately elliptical, major axis $\frac{1}{7}$ to $\frac{1}{9}$ of diam., 2 to 3 times minor, sometimes subcircular. Surface slightly convex or subplain at the centre; a distinct broad elevated zone about the semiradius, with the inner edge less abrupt than the outer. Markings pearly, round, granular, increasing slightly from the central space to the elevated zone; towards the centre 4, on the highest zone 4; somewhat larger but more crowded on a narrow but distinct band adjacent to the border; again smaller, punctiform, about 6 in ·01 mm., interspaces hyaline. Border with its inner edge sometimes distinct, striæ moniliform, 6 to 7 in ·01 mm.

Habitat.—Galapagos Islands (Weissflog! Cleve!).

C. bathyomphalus. Cleve, *Vega Exped., Vetensk. Jakttag. Stockh.*, Bd iii., 1883, p. 489, pl. xxxviii. figs. 81*a*, 81*b*.—Diam.

·02 to ·03 mm. Surface with central portion funnel-shaped, round, and sharply defined in valve aspect. Central space and rosette absent. Markings small, rounded, granular, decreasing from the edge of the central portion to the border; rows straight.

In Spitzbergen specimens the central portion extends to about $\frac{2}{3}$ of the radius, and the rows of markings are traceable to the centre. Cape Wankarema valves show the central portion extending to about $\frac{1}{4}$ of the radius, and surrounded by a distinct irregular clear band, bearing faint markings. The markings outside of this band increase gradually outwards to the border (Cleve, *ibid.*, pl. xxxviii. fig. 81*b*). The latter may be distinguished as var. *wankaremensis.*

Habitat.—Spitzbergen (Cleve !).

*C. grayianus,** sp. n.—Diam. ·105 mm. Surface rising slightly from the edges of the central space to the elevated ring, the latter sharply defined, ·0075 mm. broad, placed near the semiradius, from its outer edge, a short gentle slope passing gradually into the flat band reaching the border. Central space subcircular, $\frac{1}{8}$ to $\frac{1}{9}$ of diam. broad, bearing several round isolated granules. Markings around the central space round, granular, with narrow hyaline interspaces; beyond this to the elevated ring in moniliform rows, 6 in ·01 mm., upon this ring angular 4 in ·01 mm., with distinct central papillæ and irregular, beyond this again in moniliform rows to the inner edge of the flat band adjacent to the border, upon this band round, free, granular, with narrow hyaline interspaces, 6 in ·01 mm.; apiculi minute at subregular intervals. Border ·005 mm. broad, with distinct striæ 8 in ·01 mm.—(Pl. II. fig. 12.)

Habitat.—Antarctic ooze 1950 fathoms (Rae !).

C. notabilis, sp. n.—Diam. ·1 mm. Surface almost flat. Central space circular, sharply defined, about $\frac{1}{20}$ of diam. broad. Markings subpearly, obtusely angular, increasing from the centre to about the semiradius, again decreasing to the border; towards the centre 5, at the semiradius $4\frac{1}{2}$, at the border 8, in ·01 mm.; on a narrow indistinct zone, adjacent to the border, punctiform; rows with slight bendings, separated by narrow clear lines, most evident

* Named in honour of W. J. Gray, Esq., M.D.

opposite the origin of the shorter rows. Secondary subconcentric rows evident on central $\frac{2}{3}$ of the valve. Border narrow, hyaline.—(Pl. II. fig. 6.)

Habitat.—Gazelle Expedition (Weissflog!); lat. 53° 55′S., long. 108° 35′ E., 1950 fathoms, H.M.S. Challenger (Kinker!).

C. subnotalilis, sp. n.—Diam. ·05 to ·075 mm. Surface flat. Central space and rosette absent. Markings round, granular, subequal 7 to 8 in ·01 mm., irregular, on a small indistinctly defined central, sometimes slightly excentric area, elsewhere in radial or subparallel, irregularly subfasciculate straight or slightly flexuous rows; the rows in each fasciculus parallel to those at its middle or side on the same valve; secondary concentric bands faint; interspaces between the rows linear, small and subulate at the origin of the shorter rows.—(Pl. I. fig. 8; Pl. II. fig. 18.)

Distinguished from *C. notabilis* by the absence of a central space and the smaller size of the markings.

Habitat.—Chalky Mount, Barbados (Firth!).

Var. *marina*, nov.—Diam. ·055 mm. Central space small, rounded, with a small central rounded granule. Markings obtusely angular, 6 to 7 in ·01 mm. Border striæ evident, 6 in ·01 mm.

Habitat.—Gazelle Expedition (Weissflog!).

*C. Kochii.** Pant., *Fossil. Bacil. Ung.*, p. 71, pl. xxii. fig. 197.—Diam. ·11 to ·15 mm. Surface flat to semiradius, two broad low undulations on outer half of valve. Central area minute, surrounded by a narrow dark line. Markings towards the centre angular, beyond the semiradius more rounded, subequal, 7½ to 8, on a distinct band adjacent to the border more crowded, 10 in ·01 mm.; rows separated by evident clear lines, distinct subulate spaces opposite the origin of the shorter rows, and most evident towards the centre; at wide subregular intervals minute clear spaces (apiculi?) close to the border. Border narrow, hyaline.

Pantocsek's original specimen is non-fasciculate. Distinguished from *C. dubiosus* by the undulate surface, the more distinctly separated radial rows, and the more evident subulate interspaces.

Habitat.—Szent Peter deposit (Pantocsek!).

* Dedicated to Prof. A. Koch of Klausenburg.

C. biharensis. Pant., *Fossil. Bacil. Ung.*, p. 71, pl. xiv. fig. 119; pl. xvi. fig. 139.—Diam. ·11 to ·18 mm. Surface slightly depressed at centre; somewhat convex about the semiradius. Central space absent; a rosette sometimes present. Markings hexagonal, compressed in direction of radius, on a small, sharply-defined, rounded central area, extending to $\frac{1}{6}$ or $\frac{1}{7}$ of radius obtusely angular, large, unequal; beyond this much smaller, 4 to $4\frac{1}{2}$ in ·01 mm., increasing slightly to about the semiradius, again decreasing gradually to the border; broadish in a direction at right angles to the radius, the central papillæ prominent; rows straight or slightly curved; secondary subconcentric rows faint. Border narrow, hyaline.

Habitat.—Elesd marl, Hungary (Pantocsek !).

C. neogradensis. Pant., *Fossil. Bacil. Ung.*, p. 74, pl. ii. fig. 18. —Diam. ·06 to ·13 mm. Surface with two concentric undulations about the semiradius, the inner the more evident. Central space and rosette absent. Markings towards the centre angular 8, beyond the inner undulation rounded 6, between the hyaline spaces at the border somewhat larger, 5 to $5\frac{1}{2}$ in ·01 mm., these spaces 15 to 27, narrow, subregular, with their long axes radial, each formed by the interruption of a row of markings; rows straight, between the undulations moniliform. Border sharply defined; striæ delicate, 16 in ·01 mm.

Distinguished from *C. intumescens* by the undulations, markings, and hyaline marginal spaces. With this *Cosmiodiscus barbadensis* (Grev., *Trans. Mic. Soc. Lond.*, 1866, p. 80, pl. viii. fig. 12) may be identical.

Habitat.—Alsö-, Felsö-, Esztergály, Kékkö, Mogyorod, Szakal and Szent Peter deposits (Pantocsek !).

C. intumescens. Pant., *Fossil. Bacil. Ung.*, p. 74, pl. ii. fig. 17. —Diam. ·1 to ·155 mm. Surface with a wide shallow undulation extending from a short distance within the semiradius almost to the border. Central space and rosette absent. Markings towards the centre angular $6\frac{1}{2}$, soon becoming small rounded granules, $5\frac{1}{2}$ in ·01 mm., at the border on an evident band again more minute; rows straight, small subulate clear spaces opposite the origin of the shorter rows; on the band adjacent to the border minute

hyaline spaces at subregular wide intervals. Border narrow; striæ obscure, 12 to 14 in ·01 mm.

Pantocsek includes this species in his section *Cestodisci.* The absence of distinct apiculi and the more lax arrangement of the markings towards the border appear to me to bring it closer to the *Radiati.*

Habitat.—Bajtha, Alsö-, Felsö-, Esztergály, Kékkö, Mogyorod, Szakal and Szent Peter deposits (Pantocsek !).

C. hungaricus. Pant., *Fossil. Bacil. Ung.*, p. 73, pl. ix. fig. 73. —Diam. ·072 to ·0825 mm. Surface slightly convex, depressed at the centre; colour brown, light blue and grey in concentric zones. Central space small, but distinct, rounded. Markings round or obtusely angular, closely disposed 6, towards the border more crowded, punctiform, 8 to 9 in ·01 mm.; rows straight; secondary oblique decussating rows most evident near the border; apiculi distinct, 10 to 15, at subequal intervals. Border sharply defined, about $\frac{1}{8}$ to $\frac{1}{6}$ of radius broad; striæ delicate, 14 to 16 in ·01 mm.

Habitat.—Kékkö, Szakal, Szent Peter and Felsö-Esztergály deposits (Pantocsek !).

Var. *Szabói*, Rattray. *C. Szabói*, Pant., *Fossil. Bacil. Ung.*, p. 74, pl. xviii. fig. 167.—Diam. ·064 mm. Central space small, smooth. Markings punctiform, 10 in ·01 mm.; near the border a circlet of minute, rounded, distant, smooth spaces.

Habitat.—Szent Peter deposit (Pantocsek !).

C. apiculiferus,* Rattray. *C. armatus*, Pant., *Fossil. Bacil. Ung.*, p. 74, pl. x. fig. 90.—Diam. ·034 to ·049 mm. Surface almost flat or slightly convex, somewhat depressed at the centre. Central space and rosette absent. Markings obtusely angular; at the centre 8, soon increasing to 6 in ·01 mm., and thence subequal to the border; minute, subulate, hyaline spaces opposite the origin of the shorter rows; rows straight; apiculi 7 to 14, prominent, inserted close to the border. Border narrow, hyaline.

Habitat.—Alsó-, Felsö-, Esztergály deposits (Pantocsek !).

C. Mártonfii.† Pant., *Fossil. Bacil. Ung.*, p. 72, pl. xv. fig. 132.

* The name *C. armatus* has been preoccupied by Greville (*Trans. Micr. Soc. Lond.*, 1861, p. 42, pl. iv. fig. 5) for a distinct form, and must be abandoned.

† Dedicated to Prof. L. Mártonfi of Számos-Ujvár.

—Diam. ·048 to ·0625 mm. Surface almost flat. Central space small, bearing a few isolated granules. Markings punctiform, 15 in ·01 mm.; rows straight, most evident on outer $\frac{2}{3}$ of valve; apiculi distinct, at intervals of about ·005 mm., forming a circlet a short distance from border. Border narrow, hyaline.

Pantocsek places this species amongst the *Fasciculati*, but in his type the rows are radial and straight, so that it belongs rather to the *Radiati*.

Habitat.—Elesd marl (Pantocsek !).

C. patera. Cstr., *Diat. Chall. Exped.*, p. 155, pl. ii. fig. 6.—Diam. ·0575 to ·089 mm. Surface hat-shaped, centre slightly depressed, thence rising slightly for about $\frac{1}{3}$ of radius to form a distinct circular ridge, whence it descends more rapidly, becoming almost flat near the border. Central space irregularly rounded, $\frac{1}{12}$ to $\frac{1}{16}$ of diam. broad, bearing a few isolated granules. Markings punctiform, subequal, but becoming more crowded and sometimes slightly smaller towards the border; towards the centre 8 to 10, towards the border 10, in ·01 mm.; rows irregularly and faintly fasciculate, sometimes at unequal intervals, stopping short of the border; interspaces hyaline. Border narrow, hyaline.

Distinguished from *C. umbonatus*, Cstr. (non Greg.) by the arrangement of the markings and the absence of apiculi.

Habitat.—Pacific Ocean, 2900 fathoms, H.M.S. Challenger (Castracane); Gazelle Expedition (Weissflog !); Szent Peter deposit (Pantocsek); Barbados deposit (Johnson !).

C. densus. Grove and Sturt MS.—Diam. ·0575 mm. Central space distinct, irregularly rounded, about $\frac{1}{12}$ of diam. broad. Markings closely disposed, rounded, granular, 6 in ·01 mm., subequal for about $\frac{4}{5}$ of radius, somewhat smaller towards the border; radial rows obscure; short, secondary, transverse or slightly oblique, somewhat flexuous rows more evident, non-decussating for about $\frac{4}{5}$ of radius, beyond this to the border decussating, but more faint. —(Pl. II. fig. 9.)

Readily distinguished by the subopaque appearance and short secondary rows.

Habitat.—Oamaru deposit (Kinker !).

C. subsalsus. Juh.-Dannf., *Bih. Sv. Vet.-Ak. Handl. Stockh.*, 1882, p. 47, pl. iii. fig. 33.—Diam. ·035 to ·045 mm. Surface slightly convex. Central space and rosette absent. Markings pearly, 12 in ·01 mm., somewhat less dense at the centre than about the semiradius, near the border more minute, and forming a somewhat broad distinct zone of irregular width; rows radial or subparallel, obscurely fasciculate; secondary irregular concentric bands visible.

Habitat.—Subfossil at Sunda, in Blidö Upland; Upland, Lidö, Norrtelje, Karlshamn (Juhlin-Dannfelt).

C. Trinitatis, Rattray. *Cestodiscus* (*pulchellus*, var.) *Trinitatis*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxvi. fig. 4.—Diam. ·04 mm. Surface with the central portion extending to about $\frac{5}{8}$ of radius, its outer edge subcrenulate, distinct. Central space circular, about $\frac{1}{12}$ of diam. broad. Markings rounded, granular, decreasing gradually towards the border, least crowded near the central space; towards the centre 6, towards the border 12, in ·01 mm.; apiculi distinct, large at somewhat unequal wide intervals, inserted near the outer edge of the central portion.—*Cestodiscus pulchellus*, Habirsh., *Cat. Diat.*, ed. 2, 1885, § *Cestodiscus.*

Habitat.—Naparima deposit (Van Heurck, Grove!).

C. disciger. Ehrb., *Mon. Ber. Ak.*, 1843, p. 271.—Diam. ·056 mm. Central space distinctly defined, large, irregularly circular, not smooth. Markings minute, equal, in contact, hardly conspicuous, 15 in ·01 mm.—Kütz., *Sp. Alg.*, p. 123.

Kützing, followed by Ralfs, contrasts this species with *C. perforatus*, hence the markings were probably also in radial, straight, or almost straight rows.

Habitat.—Virginian deposit (Kützing); Ems, near Wiener; marine mud, from Norderney; $2\frac{1}{2}$ fathoms, Crildmar; marshy ground, Wöhrden (Ehrenberg).

C. cervinus. Ralfs in *Pritch. Inf.*, p. 831.—Diam. ·135 to ·2125 mm. Surface convex towards the centre. Colour fawn. Central space and rosette absent. Markings minute; rows straight. —*Hyalodiscus cervinus.* Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 95, pl. v. fig. 9.

This approaches *C. granulatus*, but is distinguished by its convex surface and more minute markings. From *C. dubiosus* it is distinguished by the absence of subulate spaces and of clear scattered puncta. According to Mr E. Grove probably *C. radiosus*, Grun.

Habitat.—Arctic regions (Sutherland); shell cleanings, West Indies (Brightwell); ex *Ascidiis*, Roundstone Bay, Co. Galway; ex *Ascidiis*, Co. Clare (O'Meara).

C. granulatus. Ehrb., *Mon. Ber. Ak.*, 1845, p. 75.—Diam. ·049 mm. Markings minute, granular, equal, 9 to 10 in ·01 mm.; rows crowded.—Ralfs in *Pritch. Inf.*, p. 830.

The appearance of the centre of this unfigured species is unknown.

Habitat.—Stratford Cliff and Holles Cliff, Va. (Ehrenberg).

C. punctulatus. Greg., *Trans. Roy. Soc. Edin.*, 1857, p. 500 pl. x. fig. 46.—Diam. ·045 to ·085 mm. Central space and rosette absent. Markings faint, minute, punctiform, forming delicate radiating lines, less distinct towards the centre than the oblique decussating lines; at wide irregular intervals scattered clear but faint round dots. Border sharply defined; striæ faint, 20 to 22 in ·01 mm.—Greg., *Trans. Micr. Soc. Lond.*, 1857, p. 83, pl. i. fig. 48. Ralfs in *Pritch. Inf.*, p. 831; H. L. Smith, *Diat. Sp. Typ.*, No. 96.

The fine radiating lines recall those of *C. stellaris*, Roper. Gregory regarded the valves as probably belonging to *Melosira* or *Orthosira*. O'Meara (*Proc. Roy. Ir. Ac.*, 1875, p. 265) has probably not seen the true species.

Habitat.—Jamaica (Greville!); Cumberland inlet, lat. 66° N. (Arnott!); Lamlash Bay and Loch Fyne (Gregory!); locality (?) (Rae!); *Ascidia*, Co. Clare (O'Meara).

C. radiopunctatus. Harting, *Verh. Kon. Ak. Wetensch. Amsterdam*, 1864, No. 2, p. 8, pl. i. fig. 3.—Diam.? At the centre a minute stellette of small dark puncta. Markings small, round, granular, subequal; rows radial, inconspicuous, secondary irregularly concentric bands manifest. Border with short delicate striæ.

Harting compares the markings to those of *C. profundus*, Ehrb. (*Mikrog.*, pl. xxxv. B. fig. 8), but they are smaller. In the latter,

too, there is no central stellette, the markings decrease distinctly outwards, and there are no striæ at the border.

Habitat.— Banda Sea, 1200 fathoms; Tamelijk, 2050 fathoms (Harting).

C. clivosus. Pant., *Fossil. Bacil. Ung.*, p. 72, pl. ii. fig. 16.— Diam. ·13 to ·15 mm. Colour bluish at the centre, outside of this with concentric zones of brown, green, or dark grey. Surface slightly depressed at the centre for about $\frac{2}{5}$ of radius, outside of this showing 4 concentric zones, alternately elevated and depressed, the second narrowest and most sharply defined, the fourth similar, but merging gradually into the outermost portion. Central space indistinct, with scattered isolated granules sometimes absent Markings punctiform, towards the centre more distinct and irregular 8, towards the border 10, in ·01 mm.; rows closely disposed; a broad hyaline band within the border. Border striæ faint, 8 to 10 in ·01 mm.

The colour of this species recalls that of some *Actinocycli, e.g., A. Ehrenbergii*, Ralfs (*Pritch. Inf.*, p. 834).

Habitat.—Kékkö, Szakal, and Szent Peter deposits (Pantocsek !); Alsó-, Felsö-, Esztergály, Mogyorod (Pantocsek !).

Var. *latefasciata*, Grun. Pant., *ibid.*, p. 72, pl. xxvii. fig. 253.— Diam. ·11 mm. Central space subcircular, evident, about $\frac{1}{18}$ of diam. broad. The hyaline band within the border much wider, $\frac{1}{6}$ to $\frac{1}{7}$ of radius.

Habitat.—Alsó-, Felsö-, Esztergály deposits (Pantocsek !).

C. depressus. Gregory MS.—Diam. ·03 mm. Surface with a distinct central depression extending to about $\frac{1}{3}$ of radius. Central space and rosette absent. Markings minute, resolved with difficulty, most evident around outer edge of the depression; rows straight. Border sharply defined, with delicate striæ, 16 to 18 in ·01 mm.

Habitat.—Arran Island (Greville !); Patos Island guano (Norman !); * Maghellan Straits (Cleve !); Cape Wankarema (Cleve !); Greenland (Cleve !).

* In the collection of Dr Greville.

C. ludovicianus,* Rattray. *Janischia? antiqua*, Grun. Van Heurck., *Syn. Diat. Belg.*, pl. xcv. *bis*, figs. 10, 11.—Diam. ·335 mm. Surface slightly convex towards the centre. Central space and rosette absent. Markings minute, 16 in ·01 mm.; rows radial, straight, but faint; oblique decussating rows more evident; apiculi prominent, at intervals of ·01 to ·015 mm., forming a circlet at a distance of ·03 mm. from the border; at opposite sides of the valve the apiculi, replaced by a narrow curved hyaline band, bearing well-marked radial striæ, these bands at a somewhat greater distance from the border, and sometimes interrupted; close to the border a circlet of minute apiculi at intervals of ·005 to ·0075 mm., and recognised only with difficulty.

The specific name *antiquus* cannot be adopted, being preoccupied for a distinct form.

Habitat.—Jutland, Cementstein (Deby !).

C. polurrhaptos,† sp. n.—Roundly elliptical. Major axis ·1125 mm., about $1\frac{2}{7}$ times the minor. Surface with two obtusely conical elevations, most evident and meeting at, the centre thence diverging and becoming more faint in outline towards the border, symmetrical with respect to the minor axis; their outer ends embracing about $\frac{1}{4}$ of the circumference. Central space and rosette absent. Markings hexagonal, most evident at the centre; towards the centre 10 to 12, towards the border 14 to 16, in ·01 mm.; rows straight; secondary oblique rows obscure; minute subulate spaces opposite the origin of the shorter rows towards the centre; apiculi evident, scattered irregularly over the surface. Border sharply defined, hyaline.—(Pl. III. fig. 4.)

Cyclotella Castracanei, Eul. MS., as noted by Kitton, shows some affinity to this species in the lobes about the centre, but in the former these lobes do not meet at the centre. The valve marking is also distinct.

Habitat.—Santa Marta deposit, Cal. (Doeg !).

Grunow refers, in a note to Mr E. Grove, dated May 8, 1888, to a species which he names *C. florescens*, Grun. This, according to

* Named in honour of Mr Louis Deby.

† πολυρραπτος, embroidered.

Mr Grove, resembles *C. dubiosus*, but the delicate hexagonal markings are rather larger; the surface is dotted over with small spots (apiculi?), each standing in the middle of a rosette of 5 to 6 areolæ; there is a submarginal circlet of apiculi; the rows are straight, radial and non-fasciculate; there are small subulate interspaces at the origin of the shorter rows, and the border is striated.

§ VII. Elaborati.

Valves elongately elliptical. Markings rounded or granular; the rows chiefly disposed with relation to the major axis.

C. naviculoides. C.? naviculoides, Tru. and Witt, *Jeremie Diat.*, p. 14, pl. ii. fig. 10.—Rhomboidal. Major axis ·0875 to ·11 mm., 3 to 3½ times minor. Surface flat. Central space and rosette absent. Markings pearly, decreasing somewhat from the centre outwards about 4 in ·01 mm.; round and irregular at the centre, obtusely angular, and in faint rows subparallel to the major axis towards the more acute extremities; irregularly transverse or oblique decussating secondary rows sometimes evident, on a sharply defined narrow band adjacent to the border punctiform, and in more evident oblique decussating rows. Border narrow; striæ obvious, 8 to 10 in ·01 mm.

Distinguished from *C. lewisianus* by the more rhomboidal outline, and the closer arrangement of the markings, which are disposed in less evident rows.

Habitat.—Jeremie deposit (Truan and Witt); Monte Gubbio (Grove!).

C. paleaceus, Rattray. *Stoschia? paleacea*, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 6.—Elongately and irregularly elliptical, frequently with shallow lateral undulations. Major axis ·0425 to ·1 mm., from 4 to 8 times the longest transverse axis. Surface almost flat; central space and rosette absent. Markings polygonal, 6 to 7 in ·01 mm. Subequal, without order or in obscure rows; around the border a distinct band of areolæ.

Habitat.—Nancoori (Hardman!);* Naparima (Kitton).

* In the collection of Julien Deby.

C. lewisianus. Grev., *Trans. Micr. Soc. Lond.*, 1866, p. 78, pl. viii. figs. 8–10.—Valve elliptical, the sides slightly protuberant at ends of minor axis. Major axis ·0875 mm., about 2 to $2\frac{1}{3}$ times minor. Central space absent. Markings rounded, about 4 in ·01 mm., largest and irregular around the centre; the rows straight and slightly curved along and parallel to the major axis, diverging and bent away from the minor; on a narrow band at the border crowded, and forming oblique decussating rows. Border indistinct; striæ 8 in ·01 mm.—Sch., *Atl.*, pl. lxvi. fig. 12; Pant., *Fossil. Bacil. Ung.*, p. 70, pl. xxv. fig. 232; Cleve and Möll., *Diat.*, No. 162.

Habitat.—Nancoori (Cleve and Möller! Cleve! Kinker! Hardman!); Maryland (Cleve); Rappahannock, U.S. (Greville!); Richmond, Va. (Greville!); Szent Peter deposit (Pantocsek); Naparima, Trinidad (Firth! Kitton); Jones Cliff, Maryland (Marshall!);* Trinidad deposit (Greville!); Artesian well, Cambridge, Maryland (Doeg!);† Monroe Fortress, North America (Weissflog!); Monte Gubbio (Grove!); Nottingham deposit (Greville!); Los Angelos, Cal. (Cambridge!);‡ Calvert County, Maryland (Grove!).

Var. *moronensis*, nov. Major axis ·0825 mm. about $2\frac{1}{5}$ times minor. Central space absent. Markings polygonal, in contact 3 to $3\frac{1}{2}$ in ·01 mm., subpearly, the rows arranged as in the type, the band at the border sharply defined on its inner side, with markings decreasing slightly outwards from 6 to 8 in ·01 mm. Striated border absent.

Habitat.—Moron deposit (Hardman!). §

Var. *similis*, nov.—Rhombic. Major axis ·05 mm., about $1\frac{2}{3}$ times minor. Markings round, granular, almost symmetrical with respect to the major and minor axes; the rows concave towards the former on each half of valve; at the centre about $3\frac{1}{2}$ in ·01 mm., near the border crowded, subpunctiform.—(Pl. III. fig. 10.)

Habitat.—South Naparima, Trinidad (Hardman!). ‖

* In the collection of Dr F. W. Griffin.

† Forwarded by Professor H. L. Smith. According to Dr Griffin, the boring also contained *Coscinodiscus excavatus* var. *genuina*, and *C. excavatus* var. *quadriocellata*.

‡ In the collection of Dr F. W. Griffin.

§ In the collection of Dr Greville.

‖ In the collection of Mr Julien Dely.

C. gracilentus, sp. n.—Elongately elliptical, the extremities bluntly cuneate. Major axis ·095 to ·125 mm., about 5½ times minor. Central space and rosette absent; a circular central area about ·006 mm. broad, with irregular angular granules, and bounded by a narrow hyaline line. Markings hexagonal, 4½ to 5 in ·01 mm., somewhat smaller around the border; rows curving outwards from the central area, and continued almost parallel to the major axis almost to its extremities, near the latter diverging outwards; irregular along the minor axis. Border narrow, hyaline.—(Pl. I. fig. 9.)

Habitat.—Naparima, Trinidad (Weissflog! Firth!).

§ VIII. Cocconeiformes.

Valves roundly elliptical. Markings cocconeoid.

C. cocconeiformis. Sch., *Atl.*, pl. lviii. figs. 25, 26, 28.—Diam. ·023 to ·035 mm. A narrow hyaline band, sometimes tapering towards the extremities running from the centre towards the border. Markings small, round, granular, crowded on a narrow, sometimes distinctly defined band adjacent to the border; rows straight from the centre at right angles to the hyaline band, elsewhere diverging and slightly curved towards the extremities of this band; interspaces hyaline.

Distinguished by the arrangement of the markings.

Habitat.—Monterey (Schmidt, Weissflog!); Sta Marta deposit (Doeg!).

Var. *latior*, nov. Sch., *Atl.*, pl. lviii. fig. 23 (no name).—Subcircular. Diam. ·0165 mm. Central hyaline band extending close to the border, its sides convex with a slight median constriction. Markings more minute, irregular, rows undifferentiated, the band adjacent to the border absent.

Habitat.—Campeachy Bay (Schmidt).

Var. *brevior*, nov. Sch. *Atl.*, pl. lviii. fig. 24 (no name).—Diam. ·038 mm. Central space small, roundly elliptical. Markings subequal, or decreasing slightly towards the border, in more evident diverging rows. Border narrow, formed by two concentric bands of granules.

Habitat.—Cape of Good Hope (Schmidt).

Var. *tenuior*, nov. *C. cocconeiformis*, var. Sch., *Atl.*, pl. lviii. fig. 27.—Diam. ·0425 mm. Hyaline central band narrow, extending to the border. Markings in parallel rows on a narrow but distinct median area, about the minor axis; at border more minute, forming delicate striæ.

Habitat.—Leton Bank (Schmidt).

It should be noted that specimens described by Grove and Sturt, from the Oamaru deposit, under the designation of *Triceratium coscinoides*, are rather triangular forms of *Coscinodiscus*, and thus form a connecting link to the great but heterogeneous groups of forms at present classed in the genus *Triceratium*.

SPECIES EXCLUSÆ VEL INQUIRENDÆ.

C.? polystigma. Ehrb. (*Mon. Ber. Ak.*, 1843, p. 271), belongs to *Auliscus* (Rattray, *Jour. Roy. Micr. Soc. Lond.*, 1888, p. 897).

C.? bifrons, Cstr. (*Diat. Chall. Exped.*, p. 156, pl. ii. fig. 1) and *C.? janus*, Cstr. (*ibid.*, p. 157, pl. ii. fig. 2), are inadequately defined species, which, with more probability belong to *Actinocyclus*.

C. craspedodiscus, Kütz. (*Sp. Alg.*, p. 126), is *Craspedodiscus elegans*, Ehrb., but distinct from *Coscinodiscus craspedodiscus*, O'Me., and from *Craspedodiscus Coscinodiscus*, Ehrb. (*Mon. Ber. Ak.*, 1844, p. 266; *Mikrog.*, pl. xxxiii. 15. fig. 8, &c.).

C. Auliscus, Kütz. (*Sp. Alg.*, p. 126), belongs to *Auliscus* (Rattray, *Jour. Roy. Micr. Soc. Lond.*, 1888, p. 896).

C. Barklyi, Coates (*Quart. Jour. Micr. Sci.*, 1861, p. 138), from Yarra Yarra, has been correctly named by Grunow *Actinocyclus Barklyi*. The species was dedicated to Sir H. Barkly, formerly President, Royal Society, Melbourne.

C.? rudis, Cstr. (*Diat. Chall. Exped.*, p. 162, pl. xxii. fig. 4), from the Philippine Islands, belongs to *Stephanopyxis*.

C. minimus, Schum. (*Schrift. Phys. Oek. Ges. Konigsb.*, 1867, p. 62, pl. iii. fig. 72). This Baltic specimen is a *Cyclotella*, and approaches *Cy. striata*, var. *baltica*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xcii. fig. 13), also from the Baltic.

C. minor, Weisse (*Bull. Ac. Imp. Sci. St Petersb.*, 1855, p. 276, pl. i. fig. 4). The Simbirsk valves, so named by Weisse, were designated *C. polycora* by Ehrenberg, and belong to *Pyxidicula*.

C. parma, Bail. According to Prof. H. L. Smith (*Lens*, 1872, p. 232), who has examined the Bailey Collection bequeathed to the Boston Society of Natural History, this is *Stictodiscus californicus*, Grev. (*Trans. Micr. Soc. Lond.*, 1861, p. 79, pl. x. fig. 1), of which Bailey's *Triceratium parma* is a triangular form.

C. pyxidicula, Kütz. (*Sp. Alg.*, p. 126) is *Pyxidicula Coscinodiscus*, Ehrb. (*Mon. Ber. Ak.*, 1844, p. 85), and belongs to *Craspedodiscus*, approaching *Cr. elegans*, Ehrb.

C. cruciatus, Kütz. (*Bacil.*, p. 132, pl. xxviii. fig. 10), belongs to *Pyxidicula*. Specimens have been procured from Vera Cruz and Richmond, Va.

C. pyxis, Ehrb. (*Mikrog.*, pl. xxxiii. 13. fig. 3*), is perhaps a *Melosira*. It cannot be united to *Coscinodiscus*.

C. quindenarius. This name is erroneously quoted in Habirshaw's *Cat. Diat.*, 2nd ed., § *Coscinodiscus*, for *Actinocyclus quindenarius*, Ehrb. (*Mikrog.*, pl. xxi. fig. 17).

C. fuscus, Norman (*Trans. Micr. Soc. Lond.*, 1861, p. 7, pl. ii. fig. 3) is *Actinocyclus Ralfsii*, not *A. Barklyi*, nor *Actinoptychus Barklyi*, as stated in Habirshaw's *Cat. Diat.*, 2nd ed., § *Actinocyclus*.

C. græcus, Kütz. (*Bacil.*, p. 132), was named *Discoplea græca* by Ehrenherg (*Mon. Ber. Ak.*, 1840, p. 208; *Mikrog.*, pl. vi. 2. figs. 1 *a–c*). It is a *Melosira* or perhaps a *Cyclotella*. Specimens were found in Grecian marl.

C. flavicans, Ehrb. (*Abh. Ber. Ak.*, 1841, p. 412, pl. i. 3. fig. 17), has been defined as small, with delicate, non-radiate markings, being yellow by transmitted, and white by reflected light. Ralfs accepted this species (*Pritch. Inf.*, p. 831) as belonging to *Coscinodiscus;* to me it seems rather to be a *Melosira*. Simbirsk specimens, named *C. flavicans* by Weisse (*Bull. Ac. Imp. Sci. St Petersb.*, 1855, p. 276, pl. figs. 5 *a*, *b*), have coarse markings, and must be excluded. *C. flavicans?* Ehrb. (*Mikrog.*, pl. xxxix. 2. figs. 19, 20), are also distinct. Ehrenberg's original specimens were from Peru and St Domingo.

C. arafusœnis. O'Me., *Quart. Jour. Micr. Sci.*, 1877, p. 463.—Diam. ·375 mm. Central space small. Markings areolate, at the margin subhexagonal, diminishing in size towards the ends; shorter, broader, and more robust than in *C. craspedodiscus*, O'Me., radial rows terminating some distance from the centre, but of more equal

length than in *C. craspedodiscus*. Kitton believes that this "species," founded on specimens procured by H.M.S. Challenger in the Arafura Sea, is identical with *C. nobilis*. The comparison of the markings with those of *C. Janischii*, var. *arafurensis*, Grun., appears to me to preclude this conclusion. In a letter to Mr E. Grove, dated May 8, 1888, Grunow states that *C. arafusœnis*, O'Me., is probably identical with *C. Janischii*, var. *arafurensis*, and this seems more likely. The name may be abandoned without inconvenience. No figure of the original was published.

The specimen from Jeremie deposit, figured by Truan and Witt (*Jeremie Diat.*, p. 14, pl. ii. fig. 23), and referred to as *Coscinodiscus ?* spec., is pointed out as differing from *Stephanophyxis turris* by the absence of a circlet of apiculi. Grunow, however, enrolls some Franz Josef's Land valves without apiculi as vars. of the last named species, *e.g.*, *S. turris*, var. *cylindrus forma inermis* (*Denk. Wien. Ak.*, 1884, p. 87, pl. v. (E), figs. 10, 11), and *S. turris*, var. *arctica forma macropora*, *ibid.*, pl. v. (E), fig. 20. With this last, from the large size of its markings, the Jeremie valve seems to be identical. It also approaches *Pyxidicula Weyprechtii*, Grun., but differs in not showing a gradual diminution in size of the markings outwards.

The union of *Actinocyclus alienus*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. cxxv. figs. 10, 11), from Santa Monica; *A. incertus*, Grun. (Van Heurck, *ibid.*, pl. cxxv. fig. 4), from Santa Monica and Monterey; *A. subtilis*, Ralfs (*Pritch. Inf.*, p. 835; Van Heurck, *ibid.*, pl. cxxiv. fig. 7; pl. cxxv. figs. 9, 11; *Typ. Syn. Diat. Belg.*, Nos. 519, 520; Cleve and Möll., *Diat.*, No. 171), from Ilfracombe, Plymouth, &c.; and *A. Roperii*, Grun. (Van Heurck, *ibid.*, pl. cxxv. figs. 5, 6), from Carteret, &c., to *Coscinodiscus*, as proposed by Grun. (*Denk. Wien. Ak.*, 1884, p. 83), is undesirable since in all a pseudonodule is recognisable.

C.? ovalis. Roper (*Quart. Jour. Micr. Sci.*, 1858, p. 22, pl. iii. fig. 4), is *Actinocyclus Roperii*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. cxxv. figs. 5, 6), and synonymous with *Eupodiscus Roperii*, Breb. (*Jour. Quek. Micr. Cl.*, 1870, p. 41). The name *ovalis* (*Actinocyclus ovalis*), as being the older specific designation, should stand. Specimens were obtained by H.M.S. Challenger in a sounding made near Yedo, Japan. There is a true *Coscinodiscus ovalis* distinct from Roper's form, and devoid of a pseudonodule.

C. minutus. Kütz. (*Bacil.*, p. 131, pl. i. fig. 14).—Diam. ·0125 mm. Central space and rosette absent. Markings minute, punctiform, without order. Border striæ punctate. This species, which Ralfs admits (*Pritch. Inf.*, p. 831), and which was founded on specimens procured in mud from the river Elbe at Cuxhaven by Sonder, belongs to *Cyclotella*, and is probably *Cy. salina*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xcii. fig. 11), as found in the Thames at Greenwich. Schumann's *C. minutus* (*Schrift. Phys. Oek. Ges. Königsb.*, 1867, p. 62, pl. iii. fig. 71) is distinct, though it too belongs to *Cyclotella*.

C. sinensis. O'Me., *Quart. Jour. Micr. Sci.*, 1879, p. 122.—Diam. ·3575 mm. Central space stellate, because of the different lengths of the rows of markings. Markings close, distinctly larger towards the border.

This species, procured by H.M.S. Challenger in Hong Kong Harbour, is problematical. O'Meara has so named, with a query, fossil valves from Los Angelos and Mejillones now in the British Museum, but the specimens to which his "finder" numbers refer belong to *C. gigas*, Ehrb. The original may have been *C. diorama* Sch. (*Atl.*, pl. lxiv. fig. 2), or *C. mirificus*, Cstr. (*Diat. Chall. Exped.*, p. 154, pl. iii. fig. 6), also from Hong Kong Harbour.

C. Smithii. O'Me., *Proc. Roy. Ir. Acad.*, 1875, p. 262.—This is *C. minor*, W. Sm., not Ehrb. (*Syn. Brit. Diat.*, i. p. 23, pl. iii. fig. 36), and is identical with *Melosira nivalis*, W. Sm. (*ibid.*, ii. p. 58, pl. liii. fig. 336), with which it must be united, though approaching *M. distans*, Kütz., from Bilin. O'Meara, whilst pointing out the inaccuracy of Smith's specific name *minor*, and replacing it by that of *Smithii*, continues the error with respect to the genus.

C. striatus, Kütz. (*Bacil.*, p. 131, pl. i. fig. 8), is *Cyclotella striata*, var. *intermedia*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xcii. fig. 10), not *Cy. striata*, as stated by Van Heurck (*ibid.*, pl. xcii. fig. 6). *Cy. dallasiana*, W. Sm., now preserved in Smith's collection in the British Museum, shows no clear band between the central areolate portion and the radially striated band at the border, it is thus also identical with *C. striata*, var. *intermedia*, Grun., with which also agrees *Cosc. striatus*, Ehrb. (Jan. and Raben., in

Rabenh., *Beitr. z. näh. Kennt. u. Verbreit. d. Algen*, Leipzig, 1863, Heft i. p. 8, pl. iv. fig. 4), from Honduras and from Peruvian guano (Jan., *Abh. Schl. Ges. väter. Cult.*, 1862, Heft ii. p. 4, pl. i. A. fig. 5). In Cleve's collection, at present preserved in the Royal Botanical Museum, Stockholm, *C. striatus* is recorded from Kiel Harbour, and specimens from this locality are not unfrequent in Rabenh., *Alg. Europ.*, No. 1697.

C. varius. Schum. (*Schrift. Phys. Oek. Ges. Königsb.*, 1867, p. 62, pl. iii. fig. 76), is inadequately diagnosed—the appearances described being mostly those resulting from differences in focussing. There are about 6½ rows of markings in ·01 mm. The name may be abandoned without inconvenience.

C.? heterostigma. Ehrb., *Mon. Ber. Ak.*, 1872, p. 297.—Specimens so named were recorded by Ehrenberg, from a depth of 3 fathoms, in the Greenland Sea, near Sabini Island. Ehrenberg regarded them as probably belonging to *Gallionella.* They may be united to *Melosira.* They reached ·0475 mm. in diam., had punctiform irregular markings, smaller ones being disposed among the larger.

C. tenellus. Ehrb., *Mon. Ber. Ak.*, 1854, p. 238.—Diam. ·075 mm. Markings 8½ to 9 in ·01 mm., equal; rows radiating. Specimens are recorded by Ehrenberg from the Atlantic Ocean. Ralfs has admitted the species, but notes that the characters given are insufficient to distinguish it from *C. radiolatus* and *C. subtilis.* The species may be abandoned without inconvenience.

Cosmiodiscus imperfectus. Grun., *Denk. Wien. Ak.*, 1884, p. 69. —Grunow refers to this species as figured in Sch., *Atl.*, pl. iii. figs. 17, 18. For pl. iii. he means pl. xxxvi. The forms are quite distinct from *Coscinodiscus punctulatus*, though distributed, according to Schmidt, as a var. of this species by Möller. I have followed Schmidt in naming them *Aulacodiscus suspectus* (Rattray, *Jour. Roy. Micr. Soc. Lond.*, 1888, p. 339), which I regard as the simplest species of the genus *Aulacodiscus.* Grunow, overlooking Schmidt's earlier name, proposed in 1884 a new one, namely, *Aulacodiscus imperfectus*, but in the same sentence he notes that the absence of processes is opposed to this determination, and so gives another name still, *Cosmiodiscus imperfectus.* The absence of processes, however, is not of itself sufficient to exclude the species from *Aulacodiscus*, since these are also entirely absent from *A.*

apedicellatus, Rattray, and abnormal valves of *A. Kittoni*. They are also sometimes absent (hence inconstant) in otherwise normal valves of *A. formosus* and *A. excavatus*. For these reasons the species seems to me to have more claim to rank with *Aulacodiscus* than with Greville's later genus *Cosmiodiscus*.

C. vulgaris. Schum., *Schrift. Phys. Oek. Ges. Königsb.*, 1867, p. 62, pl. iii. figs. 74 *a–c*.—Surface moderately convex. Centre mostly with markings similar, sometimes larger than the others, more rarely without markings. Markings equal, angular, about 7½ in ·01 mm.

Schumann explains that, with the object glass raised, the markings appear round, when depressed angular and resolved into 4 smaller markings. To this he joins *C. radiatus*, Ehrb. (*Mikrog.*, pl. xxxv. A. 17. fig. 6); *C. perforatus*, Ehrb. (*Mikrog.*, pl. xviii. fig. 46); *C. intermedius*, Ehrb. (*Mikrog.*, pl. xxxiii. 13. fig. 3); and *C. radiolatus*, Ehrb. (*Mikrog.*, pl. xxxix. 2. fig. 18). The species is insufficiently diagnosed, and by the union of the above-named species of Ehrenberg it becomes too extensive. It may be abandoned, being probably identical with *C. radiosus*, Grun.

Specimens were observed by Schumann from the Baltic.

C. intermedius, Ehrb., *Mikrog.*, pl. xxxiii. 13. fig. 3.—Diam.? Central space and rosette absent. Markings increasing gradually from centre outwards; rows radial, non-fasciculate.

This species found in San Francisco tripoli cannot be determined with certainty. The markings are similar to those of *C. argus*, Ehrb. (*Mikrog.*, pl. xxi. fig. 2), though smaller. *C. radiolatus* Ehrb. (*Mikrog.*, pl. xxxix. 2. fig. 18) is identical, but distinct from *C. radiolatus* (*Mikrog.*, pl. xxii. fig. 4), which recalls *C. radiatus*, var. *subæqualis*, Grun.

C. radiolatus. Ehrb., *Abh. Ber. Ak.*, 1841, p. 412, pl. i. 3. fig. 19; pl. ii. 6. fig. 16.—Markings small, equal, 9 in ·01 mm.; rows radial. The specimens figured by Ehrenberg (*Mikrog.*, pl. xviii. fig. 36; pl. xxii. fig. 4; pl. xxxix. 2. fig. 18), from Peru, Cuba, and Richmond, Va, disagree to a considerable extent, and may not represent the same species. Ralfs distinguished *C. radiolatus* from *C. apollinis* by the absence of a central space. Janisch, on the other hand (*Abh. Schl. Ges. väter. Cult.*, 1862, Heft ii. p. 4; pl. ii. B, fig. 17), describes the markings as strong, round, and decreasing

towards the centre and border. *C. radiolatus*, Weisse (*Bull. Ac. Imp. Sci. St Petersb.*, 1855, p. 276, pl. i. fig. 7), in which the fasciculi recall those of *C. symmetricus*, Grev., and *C. radiolatus*, Weisse (*ibid.*, 1868, p. 122, pl. i. fig. 26),—which is identical with *C. radiatus*, Ehrb.—are thus distinct. In Prof. Cleve's collection, specimens named *C. radiolatus* from Ichaboe guano occur. Compare *C. radiosus*, Grun.

Cosmiodiscus carconensis. Möll., *Typ. Pl.*, No. 100 (*fide* Habirsh., *Cat. Diat.*, § *Coscinodiscus.*—I am unacquainted with this form. In the opinion of Mr E. Grove, F.R.M.S., it is probably an early name for *Stephanodiscus carconensis*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xcv. figs. 1–4), which is stated by Habirshaw (*Cat. Diat.*, § *Stephanodiscus*) to be synonymous with *Coscinodiscus carconensis.*

Aulacodiscus apedicellatus, Rattray (*Jour. Roy. Micr. Soc. Lond.*, 1888, p. 349), has been sometimes associated with *Coscinodiscus.*

Nomina nuda.

C. actinocyclus, Ehrb., *Mikrog.*, p. 130.—Specimens procured by Dr Philippi from the Bramahputra, near Burrisal, in 17 fathoms, December and January 1842.—Ehrb., *Abh. Ber. Ak.*, 1872, p. 261.

C. amplius. Ehrb., *Abh. Ber. Ak.*, 1872, p. 202.—Specimens collected by Capt. Rogers in the China Sea, lat. 22° 36′ N., long. 116° 38′ E., in 17 fathoms.

C. asymmetricus, Grun., Cleve and Möll., *Diat.*, Nos. 154, 155.—Specimens procured from the Balearic Islands. This name occurs in *Denk. Wien. Ak.*, 1884, p. 86, where Grunow draws attention to its occurrence in the seas of the Southern Hemisphere up to the Antarctic Regions.

C. centranthus. Ehrb., *Mikrog.*, p. 139.—Specimens procured from the river Tenasserim, Further India.

C. delawarensis, Grun. Cleve and Möll., *Diat.*, No. 211. Specimens recorded from Delaware.

C. discoplea. Ehrb., *Mikrog.*, p. 131.—Specimens are recorded from the Ganges, near Calcutta, in March (?), April, May, and June 1842, and from the Bramahputra, near Burrisal, in June, July, August, and December 1842.

C. fasciatus. Ehrb., *Mon. Ber. Ak.*, 1855, p. 301.—Specimens recorded from Simbirsk. This may probably be synonymous with *C. simbirskianus.*

C. fenestratus. Ehrb., *Mikrog.*, p. 130.—Specimens procured by Dr Philippi, in February 1842, from the Ganges, near Calcutta. Brackish and marine. By Ehrenberg stated to be allied to *C. minor*, Ehrb.

C. indicus. Ehrb., *Mikrog.*, p. 131.—Specimens procured in December 1845, from the Ganges. Brackish or marine.

C. javanensis, Grun. Cleve and Möll., *Diat.*, No. 145.—Specimens from the Balearic Islands (?).

C. lineolatus. Ehrb., *Abh. Ber. Ak.*, 1872, p. 148.—Specimens procured in the Indian Ocean, off Zanzibar, by Capt. Pullen, in 2200 fathoms.

C. longispinus, Grun. Cleve and Möll., *Diat.*, No. 276.—Specimens procured from California.

C.? mesacmæus. Ehrb., *Mikrog.*, p. 142.—Doubtfully united to *Coscinodiscus* by Ehrenberg. Specimens were procured in Blumen-Erde, Canton, in 1847, and from a bottom deposit at the mouth of the Si-Kiang. Marine.

C. mesodictyon. Ehrb., *Mikrog.*, p. 131.—Specimens were probably procured from the mouth of the Ganges in December 1845. Fresh water. This may have been a *Melosira* or *Cyclotella.*

C. microcentrum. Ehrb., *Abh. Ber. Ak.*, 1872, p. 213.—Specimens collected by Capt. Niejahr, near Mél Island, Paranagua Bay, coast of Brazil, in phosphorescent water, August 16 to 17, 1870.

C. obliquus? This name is recorded with a query in Habirshaw's *Cat. Diat.*, 2nd ed., § *Coscinodiscus.*

C. pumilo. Ehrb., *Abh. Ber. Ak.*, 1872, p. 167.—Specimens procured by Weisse from the following localities:—Taganrog Roads, Kimburgs Kaja Kossa, south of Berdjanskaja Kossa, off Bertsnaskaja Kossa, south-east of Birtjutskaja Kossa, in the Sea of Azof (Ehrenberg).

C. subtilissimus. Ehrb., *Mon. Ber. Ak.*, 1861, p. 281.—Specimens recorded from lat. 62° 6′ N., long. 32° 21′ W., in 1540 fathoms, and from lat. 59° 12′ N., long. 50° 38′ W., in 1833 fathoms.

C. japonicus. Ehrb., *Abh. Ber. Ak.*, 1872, p. 198.—Specimens were procured in the Sea of Japan, in 24, 60, and 65 (?) fathoms.

C. japonicus, Cleve,? recorded in Habirshaw's *Cat. Diat.*, 2nd ed., § *Cestodiscus*, was most probably distinct.

C. tenerrimus. Ehrb., *Abh. Ber. Ak.*, 1872, p. 208.—Specimens were procured by Capt. Gerder in the White Sea, in 1857.

C. wallichianus, Grun. Cleve and Möll., *Diat.*, No. 183.—Specimens procured from the Southern Ocean.

C. complexus, Stodder, has been recorded in Habirsh., *Cat. Diat.*, § *Coscinodiscus.* The photograph there referred to has not been published.

C. Febigerii, H. L. Sm., is also recorded in Habirsh., *Cat. Diat.*, § *Coscinodiscus*, where it is stated with some doubt to be a var. of *C. excavatus.**

C. Challengeri, Janisch † ("fragments not rare"); *C. marylandicus*, Grun.; and *C. pumilus*, Grun., are names that have been applied to forms said to occur in Cleve and Möll., *Diat.*, No. 216, which is a preparation from the Nottingham deposit, U.S.

A very distinct and interesting species has recently been discovered by Mr Edmund Grove, and named by him *Coscinodiscus lacunosus*, of which the following is a diagnosis:—Diam. ·0575 mm. Surface and central portion extending to about ⅓ of radius, slightly elevated, the elevation at its outer edge passing into a faint depression, which in turn rises gently to the marginal zone. Central space distinct, angular with a single well-marked circular granule, elsewhere hyaline. Markings round, granular, closely disposed on the central elevation and near border, least crowded on the depressed zone, subequal 5½ to 6 in ·01 mm.; rows radial, straight, at subregular intervals between the rows large hyaline interspaces almost as conspicuous as the intervening rows; apiculi minute but distinct, inserted at inner edge of border opposite the outer ends of the hyaline areas. Border with inner edge well defined, ·002 mm. wide; striæ delicate but distinct, 8 to 10 in ·01 mm.

Habitat.—Totara, New Zealand fossil (Grove!).

* With respect to *C. Febigerii*, Mr E. Grove informs me that there is no doubt as to its identity with *C. excavatus*, and that Prof. W. H. Smith himself has admitted this.

† According to Mr Grove, *C. Challengeri* is probably identical with *C. Gazellæ.*

The clear radial areas in this species recall those of *Aulacodiscus acutus*, Rattray (*Jour. Roy. Micr. Soc.*, 1888, p. 368; *Jour. Quek. Micr. Cl.*, vol. iv. ser. 2. (1889), p. 38, pl. iii. fig. 4).

Artificial Key.

1.	Surface with 8 submarginal bullations and a large central rosette of 3 areolæ,	*Weissflogii.*
	No such bullations,	2.
2.	Irregular radial distinct bent costæ passing from centre to border, sometimes branching,	*theskelos.*
	No such costate lines,	3.
3.	Markings crowded on one half of valve, on other half in distant curved rows, converging to a small excentric area,	*sphæroidalis.*
	Markings otherwise,	4.
4.	Surface with 2 lateral obtusely conical plications meeting at centre,	*polurrhaptos.*
	No such folds,	5.
5.	A narrow hyaline band, sometimes tapering outwards, running from centre towards border. Markings towards ends of bands in curves, convex towards centre, and symmetrical with respect to the band,	*cocconeiformis.*
	No such cocconeoid structure,	6.
6.	Surface without low undulations,	7.
	Surface with low undulations,	8.
7.	Central space absent or minute,	9.
	Central space present and more evident,	10.
9.	Markings non-fasciculate,	11.
	Markings obviously fasciculate,	12.
	Markings obscurely fasciculate,	13.
11.	Markings areolate, round, granular on a distinct unilateral lunate depression. A rosette usually present,	*lunatus.*
	Markings angular, at centre $3\frac{1}{2}$, at border 4, in ·01 mm.; rows oblique, decussating, forming many distinct areas on surface. Border broad, distinct,	*implicatus.*
	Markings of 2 kinds, the larger areolate unequal, 2 to $2\frac{1}{2}$ in ·01 mm., prominent, within these the smaller more faint,	*bisculptus.*
	No such areas or markings,	14.
14.	Valves reniform,	*reniformis.*
	Valves diamond-shaped,	15.
	Valves elliptical,	16.
	Valves circular,	17.
	Valves obtusely angular. Markings towards centre 3, at border 4 to $4\frac{1}{2}$, in ·01 mm.	*subangulatus.*
	Valves subcircular. Markings angular, towards centre 8, gradually increasing outwards for $\frac{3}{4}$ radius to 6, on a distinct band adjacent to border punctiform 10 to 12, in ·01 mm.; subulate spaces evident near marginal band,	*moronensis.*

15.	Markings areolate, at centre 3, at border 6, in ·01 mm., irregular,	*lanceolatus.*
	Markings rounded at centre, on a distinct band at border punctiform, oblique rows evident,	*naviculoides.*
16.	Around border a circlet of prominent apiculi. Markings angular, 6 in ·01 mm.,	*spinulosus.*
	Apiculi two, one at each end of minor axis. Markings minute,	*sarmaticus.*
	Apiculi prominent at intervals of ·01 to ·0125 mm. Markings towards centre rounded 6, at $\frac{2}{3}$ radius angular 8, on a sharp band at border punctiform 12, in ·01 mm.,	*ovalis.*
	Non-apiculate,	18.
18.	Markings rounded, granular,	19.
	Markings angular,	20.
19.	An elliptical central area distinct,	*oblongus.*
	An elongate, irregular, central hyaline area. Markings sometimes punctiform,	*humilis.*
	Appearance otherwise,	21.
21.	Markings without order,	22.
	Markings in more definite order,	23.
22.	Markings small, brilliant, granular, with wide unequal hyaline interspaces. Valves elongately elliptical,	*tenuisculptus.*
	Markings 4 to 5 in ·01 mm. A broad hyaline band adjacent to border,	*inexpectatus.*
23.	Markings on central portion subradial or irregular, on a well-defined band at border minute in radiating striæ,	*ellipticus.*
	Markings at centre irregular, elsewhere in rows slightly curved along, and parallel to major axis, diverging and bent away from minor axis,	*lewisianus.*
	Markings punctiform, rows curved and diverging away from major axis,	*elongatus.*
20.	An elliptical central area with markings in rows parallel to major axis, elsewhere rows radial,	*obovatus.*
	No such central area,	24.
24.	Valve elongately and irregularly elliptical. Markings 6 to 7 in ·01 mm.,	*paleaceus.*
	Valve regularly elliptical. Markings 4½ to 5 in ·01 mm., smaller at border; rows curving outwards from central area, thence parallel to major axis, and again diverging outwards,	*gracilentus.*
17.	Markings towards centre without order, with wide interspaces, towards border in straight radial rows, 3 in ·01 mm.; apiculi curved, placed towards centre,	*rex.*
	Appearance otherwise,	25.
25.	Markings areolate, delicate, central area distinct, bounded by a ring of minute apiculi; rows radial. Valve readily overlooked,	*prætor.*
	No such central area,	26.

26.	Markings minute, rows radial. Surface convex,	*cervinus.*
	Markings punctiform, interspaces large; rows radial, faint, ...	*confertus.*
	Markings minute, granular, equal, 9 to 10 in ·01 mm.; rows radial, crowded, ...	*granulatus.*
	Markings punctiform, irregular, and with hyaline interspaces for $\frac{2}{3}$ of radius, on outer third more minute, forming radiating striæ. Non-apiculate, ...	*Hauckii.*
	Markings rounded, granular, irregular for $\frac{2}{3}$ of radius, on outer third forming evident striæ. Apiculi 8, large, at outer edge of central portion, ...	*hirtulus.*
	No such arrangement of markings, . . .	27.
27.	On a small central area the markings large, unequal, beyond this much smaller, $2\frac{1}{2}$ in ·01 mm., increasing gradually outwards to about $\frac{2}{3}$ of radius, thence decreasing; irregular on the central area, beyond it in radial rows, . .	*megacentrum.*
	No such central area, ...	28.
28.	Five to 8 narrow hyaline lines radiating around centre, expanding outwards, and terminating at semiradius, ...	*pulcherrimus.*
	No such hyaline areas, ...	29.
29.	Markings irregular, ...	30.
	Markings in more definite order, . . .	31.
30.	Markings punctiform, ...	32.
	Markings rounded, granular, ...	33.
	Markings angular, ...	34.
32.	Interspaces wide, a circlet of long, narrow, curved apiculi inserted some distance from border, .	*insutus.*
	Interspaces unequal, largest either towards centre or towards border. Valves dissimilar, . .	*dimorphus.*
	Interspaces subobsolete, markings crowded. Apiculi minute, scattered over surface at wide intervals, ...	*impolitus.*
	Interspaces minute; markings largest, most crowded and most evident at centre; apiculi at border only, minute. Border broad, sharply defined, striæ 17 to 18 in ·01 mm., . .	*granulosus.*
33.	Markings somewhat smaller towards border; interspaces largest towards centre. Border distinctly hyaline, ...	*exasperans.*
	Markings small, more crowded towards centre. Border prominent, ...	*cinctus.*
	Markings subpearly, rounded, robust. Interspaces wide, ...	*nitidus.*
	Markings large, round, still more robust. Border broad, not sharply defined, with coarse striæ, .	*subnitidus.*
34.	Markings 4 to 5 in ·01 mm., smaller towards border; a circlet of irregular apiculi at border,	*antediluvianus.*
	Markings large and irregular, on a band adjacent to border, and at irregular intervals elsewhere. A circlet of large radial subregular areolæ forming the border, ...	*luxuriosus.*
	No such markings, ...	35.

35.	Markings 1¼ to 1¾ in ·01 mm., at border a single band of subquadrate areolæ 3 in ·01 mm.,	*megacoccus.*
	Markings 2½ to 3 in ·01 mm., smaller towards border, not robust. Border about ⅓ of radius, broad, with distant evident striæ,	*irregularis.*
	Markings otherwise,	36.
36.	Markings areolate to about semiradius, on outer portion distinct radial flexuous lines,	*subareolatus.*
	No such lines,	37.
37.	Markings towards centre 3, decreasing rapidly outwards, at border 6, in ·01 mm. Border striæ coarse,	*turgidus.*
	Markings 3½ in ·01 mm.; apiculi long, delicate, inserted at inner edge of border, and extending beyond circumference,	*nottinghamensis.*
31.	Markings rounded, granular; towards centre 5, towards border 8, in ·01 mm.; at intervals near border delicate, clear, elevated lines,	*armatus.*
	Markings largest and irregular beyond semiradius, smaller towards centre and border; rows radial. A rosette,	*inæquisculptus.*
	Appearance otherwise,	38.
38.	At centre a large prominent nodule,	*nodulifer.*
	No such nodule,	39.
39.	Markings in conspicuous, regular, concentric circles,	*patina.*
	Markings robust, 2½ in ·01 mm., punctate, subregularly, but more obscurely concentric; radial rows obscure,	*velatus.*
	Markings small, round, granular, subequal; radial rows inconspicuous, secondary irregularly concentric bands evident. A minute central rosette,	*radiopunctatus.*
	No such concentric arrangement,	40.
40.	Rows inconspicuously radial, on outer portion obscurely fasciculate secondary oblique outwardly curved decussating rows distinct towards border,	*decrescens.*
	Rows radial,	41.
	Rows oblique and decussating,	42.
41.	Surface rising for about ¾ of radius, here descending abruptly, thence flat to border; markings areolate 3, increasing outwards to 2½ in ·01 mm. on highest zone, here decreasing suddenly, and from this to border subequal. A rosette,	*epiphanes.*
	Surface with central portion convex and sharply defined. Markings areolate. A rosette,	*umbonatus.*
	Surface with central portion funnel-shaped. Markings granular, decreasing from edge of central portion to border,	*bathyomphalus.*
	Surface regularly convex between centre and border. Markings subequal, areolate, 3½ in ·01 mm. Border striæ 5 to 6 in ·01 mm.,	*luctuosus.*
	No such surfaces,	43.
43.	Markings somewhat excentric, 4 to 4½ in ·01 mm.; rows curved,	*elegantulus.*
	No such excentricity or curvature,	44.

44.	Elongated subulate spaces at origin of shorter rows. Markings polygonal, without order to ⅓ of radius, thence obtusely angular, 4 in ·01 mm., and in radial rows, ...	*patellæformis.*
	Subulate spaces opposite shorter rows. Markings 12 in ·01 mm. Apiculi some distance from border. Inner layer with a zone of costæ round central space, ...	*Baileyi.*
	Subulate spaces most evident towards centre. Surface undulate beyond semiradius. Markings 7½ to 8, at border 10, in ·01 mm., at wide intervals minute clear spaces close to border, .	*Kochii.*
	No such appearance, ...	45.
45.	Near centre a sharply defined band of 2 rows of large areolæ. Markings towards centre 4, at border 2½, in ·01 mm. A distinct rosette, .	*intermixtus.*
	No such band, ...	46.
46.	Markings resolved with difficulty, . . .	47.
	Markings larger, easily resolved, . . .	48.
47.	Markings 12 in ·01 mm.; apiculi scattered at wide intervals over surface, but most crowded towards border, ...	*fragilissimus.*
	Appearance otherwise, ...	49.
49.	Markings more delicate, rows radial. Surface with scattered clear dots, ...	*punctulatus.*
	A distinct central depression. Markings most evident at its edge, ...	*depressus.*
48.	Markings areolate, ...	50.
	Markings round, granular or punctiform, . .	51.
50.	Apiculate, ...	52.
	Non-apiculate, ...	53.
52.	Apiculi 7 to 14, prominent. Markings at centre 8, increasing outwards to 6, in ·01 mm., then subequal, ...	*apiculiferus.*
	Apiculi 6, large, with slight median constriction. Markings 6 to 8 in ·01 mm., decreasing near border. Rosette minute, ...	*subaulacodiscoidalis.*
	Apiculi 2, asymmetrical at border. Markings 4 to 5 in ·01 mm., decreasing gradually outwards. A rosette, ...	*centralis.*
	Apiculi prominent, long, acicular, inserted at inner edge of border, and reaching its outer edge. Markings towards centre 4½ to 5, at semiradius 3½, near border 4, in ·01 mm., .	*pectinatus.*
	Apiculi prominent, truncate. Markings towards centre 4, at semiradius 3, towards border 3½, in ·01 mm., radial rows inconspicuous, . .	*egregius.*
	Apiculi scattered irregularly over surface. Markings towards centre 8 to 10, towards border 14 to 16, in ·01 mm. Subulate clear lines opposite shorter rows, ...	*dubiosus.*
	Apiculi scattered over surface at irregular intervals. Markings 8 in ·01 mm., subequal, central dots distinct, ...	*denticulatus.*
	Apiculi minute, in a circlet at border. Markings towards centre 4, towards border 8, in ·01 mm.,	*grandineus.*

53.	Two asymmetrical distant curved depressions at border. A rosette, ...	*bisinuatus.*
	No such depressions, ...	54.
54.	Border broad, its inner edge with 2 asymmetrical constrictions. No rosette, ...	*biangulatus.*
	No such constrictions, ...	55.
55.	Largest markings forming a conspicuous, not sharply defined zone towards semiradius, .	*bulliens.*
	No such zone, ...	56.
56.	Markings towards centre 6 in ·01 mm., decreasing slightly outwards, secondary rows obscure. Border sharp, $\frac{1}{7}$ of radius broad, striæ evident, 4 in ·01 mm., ...	*compositus.*
	Markings $3\frac{1}{2}$, towards border $2\frac{1}{2}$, in ·01 mm. Surface with an elevated ring about $\frac{1}{3}$ of radius from centre, ...	*groveanus.*
	Markings $1\frac{1}{2}$ to 2, at border 2 to $2\frac{1}{2}$, in ·01 mm., pearly; central papillæ prominent, directed centrally towards border, ...	*secernendus.*
	Markings 5 to 7 in ·01 mm., slightly smaller towards centre, papillæ evident, minute subulate areas at origin of shorter rows. Border opaque, ...	*debilis.*
	Markings increasing gradually outwards, at centre 8, at border 6, in ·01 mm.; irregular on a small central area, secondary oblique rows distinct, ...	*traducens.*
	Markings $2\frac{1}{2}$ to 3 in ·01 mm., subequal, central papillæ small. Border broad, ...	*megaporus.*
	Markings $6\frac{1}{2}$ to 7 in ·01 mm., decreasing rapidly outwards, interspaces evident opposite shorter rows, ...	*profundus.*
	Markings towards centre 4, towards border 6, in ·01 mm., decrease outwards gradual. A rosette,	*pacificus.*
	Markings towards centre 6 to 9, near border 9 to 10, in ·01 mm.; minute subulate lines at origin of shorter rows, ...	*radiosus.*
	Markings towards centre 4, decreasing outwards to 6 or 7, in ·01 mm.; radial rows most evident towards border, fimbriate, ...	*fimbriatus.*
	Markings towards centre 2 to $2\frac{1}{2}$, at border 8, in ·01 mm., decrease outwards rapid; radial rows separated by distinct lines around border, central papillæ distinct, ...	*obversus.*
	Markings towards centre 2 to $2\frac{1}{2}$, near border 6, in ·01 mm. Border narrow, ...	*radiatus.*
	Markings towards centre 2 to $2\frac{1}{2}$, robust. Border broad, sharply defined, striæ coarse, . .	*marginatus.*
	Markings towards centre $1\frac{1}{2}$ to $2\frac{1}{2}$, central papillæ prominent. Border conspicuous, raised, striæ coarse, ...	*robustus.*
	Markings towards centre 3 to 4, increasing to $2\frac{1}{2}$ or 3, at border 5 or 6, in ·01 mm. Central rosette large, ...	*oculus-iridis.*
	Markings towards centre 8, at semiradius 4, towards border 8, in ·01 mm., on outer half of valve in inconspicuous, subconcentric bands, .	*antarcticus.*
	Markings at centre large, unequal, suddenly decreasing to 4 or $4\frac{1}{2}$ in ·01 mm., again increasing to about semiradius, thence decreasing to border. Border distinct, smooth, . . .	*biharensis.*

	Markings at centre 6 to 7, at semiradius 5 to 5½, on a sharply defined marginal band (⅛ of radius broad) 10 in ·01 mm.; rows on marginal band oblique, decussating, ...	*exutus.*
	Markings towards centre 4, increasing outwards to 2 or 3, at border 4 or 5, in ·01 mm., secondary oblique rows indistinct, ...	*argus.*
	Markings 3½, subequal to semiradius, at ⅔ radius 3 in ·01 mm., thence decreasing to border. Border broad, striæ 4 to 5 in ·01 mm , . .	*glaberrimus.*
	Markings towards centre 3, decreasing outwards to 5, at border 8, in ·01 mm.; secondary oblique rows evident, ...	*convexus.*
	Markings towards centre 3 to 3½, towards border 4, in ·01 mm. Surface convex, ...	*asperulus.*
51.	Apiculate, ...	57.
	Non-apiculate, ...	58.
	Apiculi robust, spine-like, with blunt free ends inserted at middle of small round hyaline spaces. Markings round, granular, 6 in ·01 mm., subpunctiform towards border. Central space small, ...	*johnsonianus.*
	Apiculi robust, 6, symmetrical, free ends blunt. Markings round, granular, 4 in ·01 mm., subequal. Central space absent, .	*stokesianus.*
	Apiculi distinct. Valves dissimilar—the one with markings round, granular, 3 in ·01 mm., on a submarginal band 8 in ·01 mm., and in oblique decussating rows, interspaces distinct —the other with markings angular, without interspaces, ...	*superbus.*
57.	Apiculi evident, about ·015 mm. apart. Markings round, granular, about 5 in ·01 mm., toward border smaller; interspaces hyaline, largest towards centre, ...	*evadens.*
	Apiculi conspicuous in a circlet some distance from border. Markings granular, secondary rows irregularly concentric, interspaces hyaline, distinct, ...	*plicatulus.*
	Apiculi distinct, a circlet close to border, within this a few scattered apiculi forming an irregular inner circlet. Markings 6 to 10 in ·01 mm., .	*lacustris.*
	Apiculi in two circlets, those of the inner prominent, distant, and at opposite sides of valve replaced by a narrow curved hyaline band, those of the outer minute, close. Markings 16 in ·01 mm., ...	*ludovicianus.*
	Markings in radial rows separated by wide cuneate interspaces. Border sharp, striæ obvious, ...	*duriusculus.*
	Markings small, on a zone adjacent to border punctiform, interspaces wide, smaller towards border, rows obscurely radial towards centre, obviously radial near border, ...	*agapetos.*
58.	Markings decreasing gradually outwards, towards centre 4 to 4½ in ·01 mm., at border punctiform; rows radial, those reaching centre few, on a narrow band at border many, . . .	*subdivicus.*
	Markings round for ½ to ⅔ of radius, on outer portion polygonal 2½ to 3, at border 6, in ·01 mm., ...	*diversus.*

12.	A distinct elevated ring at ⅔ of radius from centre. Apiculi minute,	*whampoensis.*
	No such ring,	59.
59.	Around centre three or more large lanceolate dark distant spaces, sometimes meeting at centre to form a stellette. Marking towards centre 6, border 8 to 9 in ·01 mm.,	*symbolophorus.*
	Around centre, five or six large areolæ. Markings 16 to 20 in ·01 mm.,	*stellaris.*
	No such markings,	60.
60.	An apiculus at middle of outer ends of each fasciculus; no octagonal figure,	*Rothii.*
	Appearance otherwise,	61.
61.	An apiculus at middle of outer ends of each fasciculus. Surface with an octagonal figure. Apiculi placed at the angles of the octagon,	*angulatus.*
	Apiculi not confined to middle of outer ends of fasciculi,	62.
62.	Apiculi many upon and between the fasciculi. Markings 15 to 16 in ·01 mm.,	*polyacanthus.*
	Apiculi absent or minute, the rows in the fasciculi converging towards border, about 6 in each fasciculus at border. Markings 8 in ·01 mm., decreasing a little outwards,	*Normani.*
	Apiculi interfasciculate,	63.
63.	Apiculi 3 to 5, spine-like, inserted some distance from border. Markings towards centre 4½ to 5, towards border 6; rows in each fasciculus parallel to central row,	*echinatus.*
	Apiculi distinct. Markings cestodiscoid, angular, 8 in ·01 mm., on a broad band at border punctiform 10 in ·01 mm.; rows fasciculate, those in each fasciculus parallel to central row,	*pusillus.*
	Apiculi minute. A narrow hyaline band around central area. Markings 6 to 7 in ·01 mm., rows subparallel,	*odontodiscus.*
	Appearance otherwise,	64.
64.	Fasciculi evident only beyond semiradius. Markings 6 in ·01 mm. Border with crowded oblique decussating rows, non-apiculate,	*Kützingii.*
	At border a circlet of small round clear spaces, non-apiculate,	*Grunowii.*
	No such markings and spaces,	65.
65.	Rows in each fasciculus parallel to central row,	66.
	Rows in each fasciculus parallel to corresponding side rows,	67.
	Nine asymmetrical prominent rows from centre to border; intervening rows subradial,	*barbadensis.*
66.	A single minute apiculus close to border. Markings towards centre 6, at border 8, in ·01 mm.,	*lentiginosus.*
	No such apiculus,	68.
68.	Prominent interfasciculate radial rows,	69.
	No such rows present,	70.

69.	A distinct marginal band. Markings granular, decreasing slightly outwards, secondary oblique rows on marginal band. Apiculi large, interfasciculate,	*proteus.*
	No such marginal band,	71.
71.	Markings actinocycloid, towards centre 8, towards border 10, in ·01 mm., decussating rows most evident towards border. Non-apiculate,	*actinosus.*
	Markings sub-actinocycloid, towards centre 6, towards border subpunctiform 8, in ·01 mm. Apiculi interfasciculate, distinct, some distance from border,	*partitus.*
	Markings equal, 4 in ·01 mm., secondary oblique rows straight. Non-apiculate,	*senarius.*
	Markings equal, 8 to 10 in ·01 mm., secondary oblique rows straight. Apiculi interfasciculate,	*interlineatus.*
70.	Markings 3½ to 4 in ·01 mm.; secondary rows straight, obvious. Non-apiculate,	*denarius.*
	Markings robust, increasing to semiradius, thence decreasing, at centre 4, at semiradius 3 to 3½, at border 6, in ·01 mm. Central papillæ evident,	*simbirskianus.*
	Markings punctiform,	72.
72.	Markings 15 to 16 in ·01 mm., irregular on a small round central area, elsewhere in 8 broad fasciculi. Apiculi interfasciculate,	*glacialis.*
	Markings punctiform on a distinct band at border, elsewhere 6 in ·01 mm. Apiculi prominent, interfasciculate,	*odontophorus.*
	Markings arranged in striæ, at intervals not reaching centre. Apiculi distinct in a circlet near border. Border broad,	*marginulatus.*
67.	Border crenulate,	*crenulatus.*
	Border uniformly curved,	73.
73.	Fasciculi and rows curved,	74.
	Fasciculi and rows straight,	75.
74.	Markings areolate, 6 in ·01 mm., sometimes non-apiculate,	*curvatulus.*
	Markings rounded, decreasing rapidly outwards. Non-apiculate,	*semipennatus.*
75.	Markings 6 to 10 in ·01 mm. Apiculi interfasciculate,	*subtilis.*
	Markings 10 in ·01 mm. Apiculi prominent. Valve minute,	*minutellus.*
13.	Two asymmetrical distant processes. Minute apiculi at outer ends of radial clear lines. Markings 7 to 8, decreasing outwards to 12, in ·01 mm.,	*concinnus.*
	No such processes and apiculi,	76.
76.	Markings excentric,	77.
	Markings non-excentric,	78.
77.	Markings irregular on a small excentric area. Border prominent,	*africanus.*
	Markings similar. A distinct small central nodule. Border simple,	*vetustissimus.*

78.	Markings polygonal, 7 in ·01 mm., decreasing outwards; rows fasciculate beyond semiradius, oblique rows concave outwards, . .	*suspectus.*
	Markings 8 in ·01 mm., decreasing slightly towards border; rows radial, on outermost portion of valve parallel, subfasciculate secondary rows obscure, slightly concave outwards, ...	*subglobosus.*
	Markings otherwise, ...	79.
79.	A distinct irregularly broad band adjacent to border. Markings 12 in ·01 mm., irregular on a small central area, ...	*subsalsus.*
	No such irregular marginal band, ...	80.
80.	Markings round, granular, and without order, with hyaline interspaces to about semiradius, thence more crowded in rows and subequal, 10 in ·01 mm.; rows subfasciculate to border,	*atlanticus.*
	Markings not so disposed, ...	81.
81.	Markings round, granular, subequal, 7 to 8 in ·01 mm., irregular on a small indistinctly defined central area, elsewhere in radial subparallel and irregularly subfasciculate rows; secondary concentric bands faint, . . .	*subnotabilis.*
	Markings 3½ to 4 in ·01 mm., angular; radial rows faint, secondary concentric bands evident, ...	*isoporus.*
	No concentric arrangement visible, . . .	82.
82.	Markings punctiform, 12 to 15 in ·01 mm. Apiculi large, distant. A distinct band at border, with evident oblique decussating rows,	*doljensis.*
	Markings round, granular about 4 in ·01 mm., beyond semiradius subfasciculate, interspaces wide, ...	*nitidulus.*
	Markings polygonal, 8 to 10 in ·01 mm. A circlet of numerous apiculi a short distance within border. Border distinct, striæ evident,	*corolla.*
10.	Markings consisting of a few wavy lines diverging outwards from central space, and confined to central half of valve, ...	*venulosus.*
	No such markings, ...	83.
83.	Markings round, granular, 6 in ·01 mm., smaller towards border, radial rows obscure, short transverse somewhat flexuous rows more evident, non-decussating except near border, .	*densus.*
	Markings small, round, granular, interspaces unequal, radial rows obscure, radial hyaline lines at regular intervals extending from centre, a distinct apiculus at outer end of each, ...	*splendidulus.*
	Markings round, granular, without order to semiradius, beyond this punctiform in radial rows; radial clear spaces at subregular intervals, extending outwards from semiradius, ...	*apages.*
	Markings obtusely angular or oval, 5½ to 6, at border 8, in ·01 mm.; rows radial, towards border uniformly curved towards same direction, with hyaline curved lines extending inwards a short distance at subregular intervals, ...	*obliquus.*

	Markings punctiform, on inner half of valve in faint radial rows on outer half forming striæ 14 in ·01 mm., at subregular intervals straight hyaline radial lines, most prominent at their outer ends, ...	*perikompsos.*
	Markings faint, minute, granular, most loosely disposed towards centre, at semiradius 6 in ·01 mm., towards border punctiform in radial striæ, 8 to 10 in ·01 mm., between the striæ at subregular intervals indistinct short hyaline spaces, ...	*tenuis.*
	Appearance otherwise, ...	84.
84.	Outline hexagonal. Markings round, granular, about 15 in ·01 mm., rows radial or somewhat oblique, ...	*flagrans.*
	Outline polygonal, ...	*polygonus.*
	Outline diamond-shaped. Markings rounded and granular towards middle of large central area; on a distinct zone adjacent to border subpunctiform, 10 in ·01 mm.; rows radial, apiculi distinct, ...	*rhombicus.*
	Outline elliptical or diamond-shaped. Markings rounded. 6 to 8 in ·01 mm.; rows radial, oblique decussating rows at border; interspaces hyaline,	*punctatus.*
	Outline circular, ...	85.
85.	Central space excentric. Markings punctiform in distant rows, on a band around central space larger, ...	*rotula.*
	Central space slightly excentric. Markings round, granular, 5 in ·01 mm., rows fasciculate, those in each fasciculus parallel to central row,	*inclusus.*
	Central space non-excentric, ...	86.
86.	Central space extending to $\frac{2}{3}$ radius, hyaline. Markings punctiform; rows close, radial, .	*vacuus.*
	Central space extending almost to semiradius. Markings polygonal, delicate, increasing slightly outwards; rows radial, ...	*liocentrum.*
	Central space extending to $\frac{2}{3}$ radius, sharply defined. Markings punctiform, 28 in ·01 mm., forming radial striæ, ...	*mesoleius.*
	Central space large, not smooth. Markings 15 in ·01 mm., ...	*disciger.*
	Central space large (·0375 mm. broad), at its centre a group of irregular apiculi, at its edge a circlet of similar apiculi at wide intervals. Markings 6 to 7 in ·01 mm., ...	*Gazellæ.*
	Central space otherwise, ...	87.
87.	Radial plications from centre to border, producing a wheel-like aspect, ...	*trochiscos.*
	No such plications, ...	88.
88.	Markings non-fasciculate, ...	89.
	Markings obscurely fasciculate, ...	90.
	Markings obviously fasciculate, ...	91.
89.	Surface with a distinct central portion extending to about $\frac{2}{3}$ radius, and with its outer edge crenulate, sharp, ...	*Trinitatis.*
	No such crenulate portion, ...	92.

92. { Delicate puncta at origin of shorter rows, . . 93.
No such puncta, ... 94.

93. { Puncta at origin of shorter rows sometimes visible. A distinct band of large markings round central space. Markings towards centre 4 to 4½, towards border 6 to 7, in ·01 mm. Apiculi near border in a circlet, minute, . *oamaruensis.*
No such band nor apiculi, ... 95.

95. { Distinct apiculi at angles of areolæ, . . . *floridulus.*
No such apiculi, ... 96.

96. { An evident rosette. Markings towards centre 2½, increasing outwards for a short distance, at border 5 in ·01 mm., submoniliform, obscurely punctate; central papillæ faint. Rows radial, ... *moravicus.*
No rosette, ... 97.

97. { A distinct band of large round markings around central space. Markings towards centre 5, increasing slightly for ⅕ of radius, towards border 3½ in ·01 mm., subpearly. Central papillæ faint, ... *Monicæ.*
No such distinct band, ... 98.

98. { Markings rounded or angular, 3½ to 4 in ·01 mm. Central dots distinct; central space small, . *perforatus.*
Markings robust, rounded on one side of valve, hexagonal on the other; towards the centre 4, at ⅔ of radius 2, at border 6, in ·01 mm.; decrease to border rapid. Central papillæ prominent; rows radial, ... *Kurzii.*
Markings towards centre 4½, at semiradius 3½, at border 5 or 6, in ·01 mm. Central space rounded; no distinct central papillæ or dots, . *conformis.*
Markings towards centre 2½, at ⅕ of radius 2, at border 4 to 5, in ·01 mm., subpearly. Central papillæ faint. Central space minute or subobsolete, ... *obscurus.*

94. { Markings unequal and irregular. Central space irregular, ... *anastomosans.*
No such irregular markings, ... 99.

99. { An elevated band at semiradius; markings inside this and towards border moniliform 6, on the elevated band 4, in ·01 mm., . . *grayianus.*
Highest zone reaching inwards to about ¼ of radius, most abrupt on inner edge. Markings towards centre 8, on highest zone 4, beyond this 6, in ·01 mm. Subequal subulate spaces opposite shorter rows, ... *spiniferus.*
No such sharp elevated band, ... 100.

100. { Markings of 2 kinds—the larger distant, rounded, in inconspicuous radial rows — the smaller closer, punctiform, in evident radial rows; hyaline spaces opposite shorter rows, . . *diplostictus.*
Markings not of such distinct kinds—rounded, . 101.
Markings not of such distinct kinds—angular, . 102.

101.	Markings 6 in ·01 mm.; rows radial, separated by wide hyaline intervals; adjacent to border a distinct band, with markings similar to those on rest of valve,	*lunæ.*
	Markings minute, granular, faint beyond semiradius, radial rows distinct,	*stelliger.*
	Marking minute, granular, most distinct towards centre, beyond semiradius punctiform; rows radial. Border sharply defined,	*exiguus.*
	Markings punctiform; rows radial, interspaces widest opposite shorter rows. Adjacent to border a band of large faint areolæ. A few scattered prominent round granules on a circlet some distance from border and one at centre,	*perminutus.*
	Markings otherwise,	103.
103.	Adjacent to border a distinct band, with smaller markings than on rest of valve,	104.
	No such band,	105.
104.	The marginal band narrow,	106.
	The marginal band broad,	107.
	Central space elongately elliptical. An undulation at semiradius, with inner edge less abrupt than outer,	*undulatus.*
106.	Markings rounded, granular, 4 to 4½ in ·01 mm. Interspaces hyaline, large opposite shorter rows,	*elegans.*
	Markings 6 in ·01 mm. A marginal hyaline band. Apiculi distinct,	*tabularis.*
107.	Rows radial, less crowded, and prominent within a distinct marginal band,	*actinochilus.*
	No such appearance,	108.
108.	Markings towards centre 3½, at semiradius 3, at border 6, in ·01 mm. Central space irregular, small; rows radial, secondary rows indistinct,	*cribrosus.*
	Markings obtusely angular; central space subquadrate. Border hoop-like, striæ evident,	*aethes.*
105.	Apiculate,	109.
	Non-apiculate,	110.
109.	Apiculi large, distant, inserted some distance within border. Markings rounded or obtusely angular; towards centre 6, towards border 8 to 9, in ·01 mm., and punctiform; secondary rows most evident near border,	*hungaricus.*
	Apiculi minute. Markings 6 in ·01 mm., decreasing slightly near border; rows radial, interspaces narrow,	*griseus.*
110.	Markings rounded, granular, about 5 in ·01 mm., conspicuous, decreasing rapidly from centre outwards, towards border punctiform, more faint; rows radial,	*gemmifer.*
	Markings rounded, granular, near border punctiform; without order,	*confusus.*
	Markings rounded, granular; interspaces hyaline, unequal; rows radial,	111.

111.	Markings about 4 in ·01 mm., least crowded towards centre. Border narrow,	*galapagensis.*
	Markings about 4 in ·01 mm. Border broad, sharply defined, striated,	*gemmatulus.*
	Markings towards centre 3½, towards border 4½, in ·01 mm.; smaller round granules at intervals among the larger; rows inconspicuous, moniliform at border, interspaces narrow,	*pauper.*
	Markings otherwise,	112.
112.	Markings granular; about semiradius 6, at border 8 to 9, in ·01 mm.; rows radial, secondary rows distinct only beyond semiradius. Central space circular. No rosette,	*decussatus.*
	No such secondary rows,	113.
113.	Surface with an elongated somewhat curved depression on each side of valve, about ⅔ of radius from centre. Rows radial,	*biplicatus.*
	No such depressions,	114.
114.	Markings obtusely angular, robust, subpearly, 4 in ·01 mm.; rows radial, alternately longer and shorter, spaces at origin of shorter rows hyaline,	*biradiatus.*
	Markings delicate, granular, rounded, 5 in ·01 mm.; rows radial, interspaces wide,	*apollinis.*
	No such wide interspaces,	115.
115.	Markings obviously punctate,	116.
	Markings not obviously punctate,	117.
116.	Border broad, of two parts, inner striated, outer hyaline,	*Weyprechtii.*
	Border simple. Markings robust, 3½ to 4, increasing outwards to 2½ or 3, in ·01 mm.; central dots prominent. A rosette,	*asteromphalus.*
117.	Markings in contact and angular throughout,	118.
	Markings not in contact and angular throughout,	119.
118.	Markings towards centre 3, increasing outwards to 2 at border, 5 in ·01 mm.; secondary oblique rows not conspicuous,	*crassus.*
	Markings more delicate, towards centre 3½, increasing outwards to 2½ on an angular somewhat elevated zone, at border 6 or 7 in ·01 mm.; radial rows indistinct,	*heteroporus.*
	Markings towards centre 4, at ⅔ radius 3 in ·01 mm., thence rapidly decreasing to border; radial rows distinct, secondary rows inconspicuous,	*boliviensis.*
119.	Markings rounded, sometimes angular, towards centre 6, increasing slightly outwards, at border 6 to 8 in ·01 mm.; central papillæ prominent, rows radial,	*apiculatus.*
	Markings towards central space rounded 4, soon becoming hexagonal and gradually increasing outwards to 1¾ or 2 in ·01 mm. near border,	*gigas.*
102.	Adjacent to border a distinct band, with markings well-defined, elsewhere outlines of markings faint. Markings 3½ to 4 in ·01 mm., increasing but little outwards,	*Janischii.*
	No such band adjacent to border,	120.

120.	Markings minute, delicate 8, towards border recognised with difficulty, 10 to 12 in ·01 mm.; rows radial, oblique rows faint; no hyaline band adjacent to border, Border narrow, hyaline,	*imperator.*
	Markings punctiform, 15 in ·01 mm.; rows radial. Apiculi distinct in a circlet near border,	*Martonfii.*
	Appearance otherwise,	121.
121.	No rosette,	122.
	A rosette distinct,	123.
122.	A distinct band of large areolæ around large central space. Markings punctate $3\frac{1}{2}$ in ·01 mm., decreasing on outer $\frac{1}{7}$ of radius. Apiculi at border obvious,	*spinuligerus.*
	No such band,	124.
124.	Markings punctiform,	125.
	Markings larger,	126.
125.	Markings 13 in ·01 mm.; rows radial,	*cingulatus.*
	Rows radial, only a few reaching large central space, the others mostly ending about $\frac{2}{5}$ of radius from border; interspaces large, hyaline,	*comptus.*
126.	Markings rounded, granular, 6 in ·01 mm.; rows radial; interspaces wide towards centre,	*Thumii.*
	Markings obtusely quadrangular, pearly, increasing to $\frac{1}{3}$ of radius to 2 or $2\frac{1}{2}$, thence decreasing to border to 3 in ·01 mm.; irregular around centre; rows radial,	*mossianus.*
	Markings obtusely angular, subpearly; towards centre 5, at semiradius $4\frac{1}{2}$, at border 8, in ·01 mm.; radial rows separated by narrow clear lines. Secondary subconcentric rows evident,	*notabilis.*
	Markings hexagonal,	127.
127.	Markings increasing outwards to $\frac{6}{7}$ of radius, towards centre 5, at $\frac{6}{7}$ of radius $3\frac{1}{2}$ in ·01 mm.; rows radial, at intervals the markings in isolated oblique rows. Apiculi minute, forming a circlet at border,	*lutescens.*
	Markings not oblique at intervals,	128.
128.	Markings towards centre 8, beyond semiradius 4, in ·01 mm.; rows towards border subfasciculate. Apiculi 2, at an interval equal to about $\frac{1}{3}$ circumference,	*modestus.*
	Non-apiculate,	129.
129.	Markings $2\frac{1}{2}$ to 3 in ·01 mm., subpearly, with central papillæ obscure,	*dubius.*
	Markings towards centre 3 to $3\frac{1}{2}$, increasing outwards almost to border to 2 or $2\frac{1}{2}$ in ·01 mm.; rows radial, with secondary rows evident,	*entoleion.*
	Markings towards centre $4\frac{1}{2}$, at semiradius 4, in ·01 mm., smaller at border,	*flexilis.*
	Markings 7 to 8 in ·01 mm., decreasing but little towards border. Border striæ delicate,	*josefinus.*
	Markings towards centre 6, towards border 8, in ·01 mm. Central space broad ($\frac{1}{11}$ diam.)	*mirificus.*

123. A distinct band of smaller markings adjacent to border. Apiculi numerous, minute, inserted at inner edge of border, ... *blandus.*
No such band nor apiculi, ... 130.

130. Markings towards centre 3 to 3½, near border 2, in ·01 mm.; robust, central papillæ prominent, *borealis.*
Markings towards centre 4, at semiradius 3, in ·01 mm. Border $\frac{1}{14}$ radius broad, striæ coarse, *suboculatus.*

90. Central space bearing 2 large conspicuous round granules, ... *bioculatus.*
No such granules, ... 131.

131. Apiculi numerous, forming a double circlet; undulation about semiradius, slight, . . *capensis.*
No such arrangement of apiculi, ... 132.

132. Apiculi many in one circlet. Markings 24 in ·01 mm. Border broad, hyaline, . . . *hyalinus.*
Apiculi large. Markings towards centre rounded 4½, outwards polygonal 6, in ·01 mm. Border striæ delicate, ... *tuberculatus.*
No such apiculi and markings, ... 133.

133. Distinct subulate spaces at origin of shorter rows. Markings 7 in ·01 mm., the fasciculi separated by inconspicuous radial lines. Valve large, . *nobilis.*
No such distinct subulate spaces, . . . 134.

134. Markings towards centre 4½, at semiradius 3½, in ·01 mm., thence decreasing outwards. Centre with one large irregular marking, . . . *æginensis.*
Markings towards centre 5 or 6, at border 9, in ·01 mm.; around central space rounded, elsewhere often quadrilateral, ... *Payeri.*

91. Fasciculi unequally curved, rows parallel to side row of each. Markings rounded, granular 10, in an indistinct band at border 15 to 16, in ·01 mm. Apiculi interfasciculate, . . *divisus.*
Fasciculi not so curved. Rows parallel to central row in each fasciculus, ... 135.

135. A single prominent spine-like apiculus near border, outside of this a circlet of minute apiculi, ... *kryophilus.*
No such apiculus, ... 136.

136. Markings oval, with long axis radial or a little oblique. Border broad, striæ coarse, . . *planiusculus.*
No such markings, ... 137.

137. Distinct radial interfasciculate rows, . . 138.
No radial interfasciculate rows, . . . 139.

138. Apiculi interfasciculate, robust; a distinct zone adjacent to border, ... *extravagans.*
Markings sometimes subradial, decreasing outwards, ... *Gregorii.*

139. Each adjacent pair of rows in each fasciculus originating farther and farther from central row; interspaces hyaline. Non-apiculate, . *fasciculatus.*
Markings subpearly, 4 in ·01 mm.; around border a narrow clear space, ... *symmetricus.*

8. Central space absent or minute, . . . 140.
Central space present larger, ... 141.

140.	Non-apiculate,	142.
	Apiculate,	143.
142.	Surface with several concentric zones of different and brilliant hues. A broad hyaline band within border. Central space indistinct,	*clivosus.*
	No such zones,	144.
144.	Near border a circlet of radially elongate clear spaces. Surface with two concentric undulations. Central space absent,	*neogradensis.*
	No such clear spaces,	145.
145.	An annular depression about $\frac{1}{3}$ of radius from centre. Central space minute. Rosette distinct,	*annulatus.*
	A wide undulation extending from a short distance within the semiradius almost to border. Markings towards centre $6\frac{1}{2}$, angular, soon becoming rounded, and $5\frac{1}{2}$ in ·01 mm. No hyaline band at border. Central space absent. Minute hyaline spaces at subregular wide intervals near border,	*intumescens.*
	Undulation faint near semiradius. Markings punctiform, delicate, least crowded towards centre; rows substraight, radial,	*pellucidus.*
	Appearance otherwise,	146.
146.	Alternate or opposite rounded or cuneate elevations and depressions around centre. Central space absent. A rosette,	*excavatus.*
	An elongated unilateral depression near centre. Central space minute,	*impressus.*
	Surface with 6 to 12 shallow depressions near centre. Markings at centre 2 to $2\frac{1}{2}$, at $\frac{1}{3}$ radius 3 to $3\frac{1}{2}$, thence increasing to $1\frac{1}{2}$ in ·01 mm.; smaller at border,	*asteroides.*
143.	A short transverse central plication. Central space absent,	*plicatus.*
	Two shallow concentric undulations. Central space indistinct, granular,	*undatus.*
141.	Surface hat-shaped,	147.
	Surface with faint undulations,	148.
147.	Centre much depressed. Markings punctiform, fasciculate. Apiculi prominent, inserted at border,	*obnubilus.*
	Centre less depressed. Markings punctiform, faintly fasciculate, the rows at unequal interspaces stopping short of border, and leaving small hyaline areas,	*patera.*
148.	Undulation single, about $\frac{2}{3}$ of radius from centre. Central space indistinct. Markings 7 to 8 in ·01 mm. Apiculi small,	*bengalensis.*
	Two undulations, one near centre, the other close to border. Markings increasing outwards between these, largest on the elevations,	*undulans.*

42. Adjacent to border a distinct zone of large areolæ, with inner edge wanting, and outer convex outwards, ... *zonulatus.*
Adjacent to border a single zone of still larger areolæ, perfect, with inner and outer edges convex, ... *heteromorphus.*
No such zone, ... 149.

149. Outer portion of valve scarcely siliceous from $\frac{1}{3}$ to $\frac{1}{2}$ of radius broad, with distant subuniform costæ, ... *sol.*
Four subgelatinous, cuneate, symmetrical protuberances. A distinct band adjacent to border; within this markings in oblique rows $2\frac{1}{2}$, upon it 6, in ·01 mm., and in radial rows, *bipartitus.*
Markings round, robust, pearly, smooth, decreasing rapidly at border; at centre 2, at border 4, in ·01 mm. Border sharp, of 1 to 2 bands of round granules, ... *vigilans.*
Markings angular, 10 in ·01 mm. A narrow hyaline band adjacent to border. Apiculi many, outer ends obtuse, ... *cristatus.*
No such outer portion or markings, . . . 150.

150. Inner $\frac{2}{3}$ of border hyaline between distant radial lines, outer $\frac{1}{3}$ with 8 to 10 striæ in ·01 mm., . *circumdatus.*
Border prominent, hoop-like, striæ $1\frac{1}{2}$ in ·01 mm. Markings $1\frac{1}{2}$ in ·01 mm., a few more minute at wide intervals; papillæ obscure, . . *aphrastos.*
Border subopaque, prominent, with coarse striæ. Markings 2 to $2\frac{1}{2}$ in ·01 mm., radial rows inconspicuous, ... *concavus.*
Border otherwise, ... 151.

151. A single large apiculus near border, outside it a circlet of more minute ones. Markings on a narrow zone adjacent to border, granular, . *leptopus.*
No such isolated large apiculus, . . . 152.

152. Surface convex towards centre. Markings towards centre 4 to $4\frac{1}{2}$, towards border 3, in ·01 mm., ... *tumidus.*
Surface convex; markings at centre 2, at border 3, in ·01 mm.; at border a single band of quadrate, equal areolæ, ... *Mölleri.*
Surface almost or quite flat, ... 153.

153. Markings hexagonal, 2 in ·01 mm.; at intervals more minute, more distinct areolæ. A distinct band of larger areolæ adjacent to border, . *pulchellus.*
No interspersed minute areolæ, ... 154.

154. Markings hexagonal, $2\frac{1}{2}$ to 3 in ·01 mm. Apiculi clavate at inner edge of hyaline border, . . *macraeanus.*
No such apiculi, ... 155.

155. Markings $1\frac{1}{2}$, decreasing outwards to 2 in ·01 mm.; a ring of very large areolæ adjacent to border. Non-apiculate, ... *splendidus.*
No such distinct ring, ... 156.

156.	Markings 2½ to 4 in ·01 mm., oblique rows straight. Apiculi minute or absent, . .	*lineatus.*
	Markings 3½ to 4 in ·01 mm., oblique rows substraight. Border ⅐ of radius broad, with distinct striæ; inner and outer parts separated by a narrow line, ...	*marginato-lineatus.*
	Markings 6 in ·01 mm.; rows straight. Apiculi minute. Border narrow, ...	*anguste-lineatus.*
	Markings 9 in ·01 mm., decreasing but slightly outwards, on a narrow band at border 14 to 16 in ·01 mm., oblique rows substraight, . .	*sublineatus.*
	Oblique rows more curved, ...	157.
157.	Markings at centre 5 to 6, towards border 10, in ·01 mm.; robust, decrease outwards rapid. Apiculi many, at unequal intervals, . .	*decipiens.*
	Markings towards centre 2, at border 4 to 6, in ·01 mm.; not robust, decrease outwards rapid. Apiculi many, prominent in a circle at border,	*minuens.*
	Decrease in size of markings outwards more gradual, ...	158.
158.	Markings obtusely angular, subpearly, towards centre 2½, at border 3, in ·01 mm. Non-apiculate, ...	*antimimos.*
	Markings at centre 4, towards border 8, in ·01 mm.; oblique rows concave outwards. Apiculate, ...	*excentricus.*
	Markings 2½ in ·01 mm. Border narrow. (Diam. ·02), ...	*subconcavus.*
	Markings towards centre 5, at border 6, in ·01 mm. Apiculi numerous, distinct. Border broad, hyaline, ...	*peruanus.*
	Markings at centre 6, at border 9 to 10, in ·01 mm.; oblique rows curved outwards. Apiculi minute. Border narrow, ...	*minor.*
	Markings otherwise, ...	159.
159.	Markings towards centre 4, at border 6, in ·01 mm., secondary larger hexagonal areolæ distinct, ...	*labyrinthus.*
	No such secondary areolæ, ...	160.
160.	Markings 8 to 9 in ·01 mm., decreasing somewhat outwards. Centre occupied by a single, evident circular areola. Apiculi in a circlet at border,	*pseudo-lineatus.*
	Markings towards centre 3, towards border 4, in ·01 mm. No such central areola. Border striæ 3 to 4 in ·01 mm., ...	*antiquus.*

ACTINOGONIUM.

Ehrb. emend., *Mon. Ber. Ak.*, 1847, p. 54.—Circular or obtusely angular. Surface with a central area, sometimes distinct. Colour pale grey, the rays more pearly. Markings on central area areolate, obtusely angular or rounded; rays broad, distinct, straight, curved or flexuous, sometimes subclavate, regular or irregular, confined to a zone about the semiradius, or extending nearer to the border, radiating to the angles of the valve, more

rarely at unequal intervals opposite its sides; on the interradial areas round or angular irregularly disposed unequal granules; interspaces unequal, hyaline. Border hyaline, sometimes broad and angular.

Van Heurck has recorded his belief that *Actinogonia* are but the "valves interieures" of *Asterolampræ*. Of this I have observed no evidence, but the possibility of their being so is to be borne in mind.

A. multiradiatum, sp. n.—Diam. ·05 mm. Surface with central area, extending to about $\frac{2}{5}$ of radius from centre. Colour pale grey, somewhat darker towards border. Markings on central area subangular, unequal areolæ about 4 in ·01 mm., most crowded at the centre; rays short, occupying middle third of valve, broad, subclavate, or with lateral irregular lobes; the interradial areas hyaline, on outer third bearing rounded or irregularly angular pearly granules, without order, and decreasing outwards, 4 to $4\frac{1}{2}$ in ·01 mm.; adjacent to the border a hyaline band of unequal breadth. Border narrow, hyaline.—(Pl. III. fig. 15.)

Habitat.—Barbados deposit (Hardman !).*

A. septenarium. Ehrb., *Mon. Ber. Ak.*, 1847, p. 54.—Obtusely angular. Diam. ·055 to ·07 mm. Surface with central area indistinctly defined, about ·01 mm. broad. Markings on central area small, round or obtusely angular subequal granules, with hyaline unequal interspaces; rays distinct, extending between central area and angles of valve; short similar rays sometimes intercalated towards outer portion of interradial areas, rarely two rays proceeding side by side towards the same angle, straight or gently curved, rarely flexuous; on the interradial areas round unequal irregularly disposed granules, with hyaline unequal interspaces, few close to the outer sides of those areas. Border angular, about $\frac{1}{10}$ of radius broad, widest opposite the middle of the interradial areas.—Ehrb., *Mikrog.*, pl. xxxvi. fig. 39; Ralfs in *Pritch. Inf.*, p. 813, pl. v. fig. 55; Ehrb., *Abh. Ber. Ak.*, 1875, p. 38, pl. i. fig. 4; Van Heurck, *Syn. Diat. Belg.*, pl. cxxvii. fig. 8; *A. quinarium*, Habirsh., *Cat. Diat.* § *Actinogonium.*

Habitat.—Cambridge deposit, Barbados (Van Heurck); Springfield deposit, Barbados (Hardman !).

* In the collection of Mr Julien Deby.

Artificial Key.

1.	Circular. Rays confined to a narrow area about semiradius. Central area sharply circumscribed. Border narrow, ...	*multiradiatum.*
	Angular. Rays passing between central area and angles of valve. Central area not sharply circumscribed. Border angular, broad, . .	*septenarium.*

BRIGHTWELLIA.

Ralfs, *Pritch. Inf.*, p. 940.—Circular. Surface flat from the centre outwards to the circle of large areolæ, beyond this slightly convex, and sloping downwards to the border. Colour pale smoky grey, when dry sometimes purplish or brown. Central space circular or obtusely angular, distinct, hyaline; more rarely minute or absent; a rosette rarely differentiated. Markings areolate, rarely subcircular towards the centre; a distinct circlet formed by a single row of large areolæ, and situated between $\frac{1}{3}$ and $\frac{2}{3}$ of radius from centre; within this circlet the rows evident, substraight, or oblique, curved, and decussating, beyond the circle radial; evident costæ and primary rays sometimes radiating at regular intervals between the circlet of large areolæ and the border; minute apiculi sometimes present. Border narrow, hyaline, with delicate radial striæ.—*Craspedodiscus, pro parte*, Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 95; *Heterodictyon*, Grev., *Trans. Mic. Soc. Lond.*, 1863, p. 67.

This genus approaches *Coscinodiscus* through *C. bulliens.*

§ 1. Acostatæ.

Non-costate.

B. splendida, Rattray.—*Heterodictyon splendidum*, Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 67, pl. iv. fig. 7.—Diam. ·045 mm. Central space small, indistinct. Markings toward the centre subcircular, soon becoming obtusely angular; the circlet of large areolæ at about $\frac{3}{5}$ of radius from centre, within this circlet the markings about 4 in ·01 mm., in obscure slightly curved radial rows; the secondary decussating rows undifferentiated; the hyaline interspaces largest towards the centre; beyond the circlet the rows moniliform, with markings decreasing gradually outwards from 5 to 8 in ·01 mm.,

and separated around the border by narrow hyaline lines; non-apiculate. Border hyaline.

Habitat.—Cambridge deposit, Barbados (Johnson !).

B. excellens, sp. n.—Diam. ·07 mm. Central space obtusely triangular, about $\frac{1}{14}$ of diameter broad, not sharply defined. Markings towards the central space subcircular, soon becoming areolate; the circlet of large areolæ about $\frac{5}{7}$ of radius from centre; within this circlet the markings subequal, 4 in ·01 mm., in evident curved oblique decussating rows; beyond the circlet decreasing gradually outwards from 5 or $5\frac{1}{2}$ to 8 in ·01 mm.; minute apiculi at intervals of ·0075 mm., inserted close to the border. Border striæ delicate, 14 in ·01 mm.—(Pl. III. fig. 16.)

This species was labelled by Greville *B. splendida*, but his specific name cannot be adopted, since the distinct *Heterodictyon splendidum* now becomes *B. splendida.*

Habitat.—Barbados deposit (Greville !).

B. hyperborea, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 8.—Diam. ·065 mm. Central space and rosette absent. Markings areolate; the circlet of large areolæ about $\frac{1}{2}$ of radius from centre, within this circlet the markings subequal, 3 to $3\frac{1}{2}$ in ·01 mm., in substraight decussating rows, beyond the circle subequal, or decreasing but slightly near the border, 5 in ·01 mm. Border hyaline.—Grun., *Denk. Wien. Ak.*, 1884, p. 70, pl. ii. (B), fig. 64.

Habitat.—Dredged by U.S.S. Gettysburg, lat. 35° 25′ N., long. 69° 42′ E., in 2924 fathoms; marine deposit, Franz Josef's Land (Grunow).

B. elaborata. Grev., *Trans. Micr. Soc. Lond.*, 1861, p. 73, pl. ix. fig. 1.—Diam. ·08 mm. Central space absent; an inconspicuous rosette. Markings areolate; the circlet of large areolæ about $\frac{5}{8}$ of radius from the centre, within this circlet the markings increasing slightly outwards from the rosette 3 to $3\frac{1}{2}$ in ·01 mm., in inconspicuous radial, and secondary oblique curved decussating rows; beyond the circlet increasing in breadth uniformly, but somewhat rapidly outwards, but subequal radially, 4 in ·01 mm. Border hyaline.

Habitat.—Barbados (Johnson !).

B. coronata. Ralfs in *Pritch. Inf.*, p. 940; *Craspedodiscus*

coronatus, Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 95, pl. v. fig. 6.—Diam. ·12 to ·185 mm. Central space subcircular, $\frac{1}{16}$ to $\frac{1}{23}$ of diam. broad, sometimes surrounded by a band of evident areolæ. Markings areolate; the circlet of large areolæ from $\frac{2}{3}$ to $\frac{7}{10}$ of radius from centre, within this circlet the markings subequal, 4 in ·01 mm., in regular, oblique, greatly curved, decussating rows, beyond the circlet decreasing and slightly curved outwards from 5 to 8 in ·01 mm.; minute apiculi at intervals of about ·0075 mm. sometimes visible, inserted close to the border at the outer ends of faint subhyaline lines. Border striæ 12 to 14 in ·01 mm.—*B. pulchra*, Grun.; Van Heurck, *Syn. Diat. Belg.*, pl. cxxviii. fig. 9; *Bot. Centralbl.*, Bd. xxxiv. 1888, Nos. 2, 3, p. 35; Grove and Sturt, *Jour. Quek. Micr. Cl.*, 1887, p. 67; *B. Murrayi*, Cstr., *Diat. Chall. Exped.*, 1886, p. 138, pl. x. fig. 2.

Habitat.—Cambridge deposit, Barbados (Greville! Johnson!); Oamaru deposit (Grove!); Bridgewater deposit, Barbados (Johnson!); "Barbados" (Greville! Johnson!).

Var. *radians*, nov.—Diam. ·15 mm. Central space obtusely triangular, with outwardly convex sides. Markings areolate; the circlet of large areolæ about $\frac{1}{3}$ of radius from centre, beyond this circlet the markings decreasing more distinctly outwards from 4 to 8 or 9 in ·01 mm.; primary rays evident at intervals of ·0075 mm., non-costate.—(Pl. III. fig. 14.)

Habitat.—Barbados deposit (Greville!).

§ 2. Costatæ.

Costæ distinct.

B. Johnsonii. Ralfs, *Trans. Micr. Soc. Lond.*, 1866, p. 4, pl. i. fig. 11.—Diam. ·07 to ·1075 mm. Central space minute, round or angular, a rosette sometimes evident. Markings areolate, sometimes subcircular towards the centre, with narrow hyaline interspaces, the circlet of large areolæ from $\frac{2}{7}$ to $\frac{3}{7}$ of radius from centre, within this circlet the markings subequal or somewhat smaller towards the centre, 3 in ·01 mm., in obscure radial and more evident, sometimes irregularly curved or flexuous decussating secondary rows, beyond the circlet decreasing gradually outwards from 4 to 6 or 6½ in ·01 mm. at border, with evident primary rays at

subregular intervals, and narrow costate ridges towards the outer ends of these rays. Border with delicate striæ, 16 to 18 in ·01 mm. —Walker and Chase, *New and Rare Diat.*, ii. p. 2, pl. v. fig. 10.

Habitat.—Springfield deposit, Barbados (Hardman!); "Barbados" (Johnson! Greville!); Cambridge deposit, Barbados (O'Meara!).

Artificial Key.

1.	Primary rays obvious outside of circlet of large areolæ, narrow prominent costæ at their outer ends,	*Johnsonii.*
	No such costæ,	2.
2.	Central space subcircular, surrounded by a band of large markings. Markings within the circlet of large areolæ in uniform, curved, decussating rows, beyond the circlet decreasing gradually outwards. Apiculi faint,	*coronata.*
	Central space obtusely triangular, without a distinct limiting band of markings. Markings subcircular towards central space. Apiculi minute, but more evident,	*excellens.*
	Central space minute, indistinct. Markings within the circlet of large areolæ rounded, in obscure curved radial rows, beyond the circlet the rows moniliform. Non-apiculate,	*splendida.*
	No central space,	3.
3.	An inconspicuous rosette. Markings outside circlet of large areolæ increasing gradually in breadth towards border,	*elaborata.*
	No rosette. Markings within the circlet of large areolæ in substraight decussating rows without order, beyond the circlet subequal or slightly smaller near border,	*hyberborea.*

STELLADICUS, GEN. N.

Circular. Central space and rosette absent. Markings: rays clavate, their inner ends rounded, meeting at centre, attenuating towards border, at middle of the interradial areas similar, but narrow faint rays extending to middle of inner ends of compartments, and continued thence as straight narrow lines to the border; the compartments each consisting of three subequal parts separated by narrow straight lines reaching the border, the inner end of each part convex towards the centre; areolate, the areolæ forming oblique, straight, decussating rows. Border narrow, hyaline.—*Asterolampra, pro parte* Norman, *Trans. Micr. Soc. Lond.*, 1861, p. 6.

S. stella, Rattray. *Asterolampra stella.* Norman, *Trans. Micr. Soc. Lond.*, 1861, p. 6, pl. ii. fig. 1.—Diam. ·09 mm. Markings:

clavate rays 6, about ·005 mm. broad at their widest part, the faint interradial rays about ·0025 mm. broad, the compartments reaching about $\frac{5}{9}$ of radius from circumference, the areolæ subequal, 8 (?) in ·01 mm.

Norman provisionally united this species to *Asterolampra*. The remarkable appearance of the compartments and rays are sufficient to justify its separation as the type of a new genus.

Habitat.—Sierra Leone (Norman).*

ASTEROLAMPRA.

Ehrb. emend., *Mon. Ber. Ak.*, 1844, p. 73.—Circular, rarely obtusely and regularly angular. Surface subplain, a central or subcentral portion hyaline with distinct rays, the outer portion with evident compartments separated by distinct, rarely subobsolete, hyaline intervals. Colour subhyaline to subpearly or pale grey. Central space sometimes distinct, hyaline, granular or subareolate; a rosette rarely differentiated, a central areolate area frequent. Markings on central area areolate, rarely arranged in concentric zones; rays diverging from centre or from outer edge of areolate area, rarely from an excentric point, straight, arcuate of sharply geniculate at or near their middle, frequently dichotomous, narrow, more rarely broad or subobsolete, sometimes confined to a relatively narrow, submarginal zone; interradial areas hyaline, or with faint diffuse lines passing to centro-lateral angles of compartments; compartments with inner ends convex or concave towards the centre, sometimes transversely or obliquely truncate, rarely asymmetrical with respect to the rays; granular, with irregular, wide, hyaline interspaces or areolate, rarely subhyaline or minutely punctate; the areolæ in oblique, decussating, substraight rows parallel to the edges of the inter-compartmental intervals; frequently a single row of larger areolæ fringing the compartments; those bounding its inner edge most evident, rarely large and conspicuous; at each centro-lateral angle a single areola, frequently larger and sometimes protuberant into the interradial spaces; single or double granules rarely present at outer ends of intervals. Border narrow, hyaline, rarely broad, obtusely angular and subpearly.—

* Communicated by Mr Frederick Kitton.

Asterolampra, Grev., *pro parte*, *Trans. Micr. Soc. Lond.*, 1860, p. 162; *Craspedodiscus, pro parte*, Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 95.

§ 1. MARGINATÆ.

A large central areolate area. Rays many, short, confined to a submarginal zone. Compartments minute.

A. marginata. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 50, pl. viii. fig. 30.—Diam. ·0625 to ·1125 mm. Central area extending outwards for $\frac{3}{4}$ to $\frac{5}{6}$ of radius, its outer edge sinuate, subregular; a rosette distinct. Markings on central area areolate, the areolæ decreasing slightly from the rosette outwards from 4 to 5 in ·01 mm.; rows radial, straight, non-fasciculate; secondary oblique decussating rows faintly marked; rays short, straight, radiating from the distal angles of the sinuations about $\frac{1}{5}$ of radius long; the compartments restricted to a narrow (about ·0025 mm. broad) zone adjacent to the border, their inner ends convex towards the centre; the intervals represented by shallow indentations.—Eulenst., *Diat. Spec. Typ.*, No. 16; *Craspedodiscus marginatus*, Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 95, pl. v. fig. 7; Ralfs in *Pritch. Inf.*, p. 832; *A. marginata*, var. *minor*, Walker and Chase, *New and Rare Diat.*, ii. p. 7, pl. v. fig. 8.

This species forms the connecting link between *Coscinodiscus*, *Brightwellia*, and *Asterolampra*. In the first named genus it approaches *C. bulliens*, the markings on its central area also recall those of *C. concinnus*, var. *jonesiana*. The interspaces between the rays are larger, more peripherally placed, and less sharply defined than in any *Brightwellia*, whilst its regularly areolate central portion is homologous to the irregular inconstant areolate areas of *Asterolampra decora*, *A. affinis*, *A vulgaris*, &c.

Habitat.—Barbados deposit (Johnson! Greville! Grove! Eulenstein!); Springfield deposit, Barbados (Walker and Chase); Cambridge deposit, Barbados (Deby!); Chalky Cliff, Barbados (Deby!).

§ 2. DUCTILES.

A central areolate area. Compartments conical in outline with distant subradial branching, but evident lines.

A. ralfsiana. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 50, pl. viii. fig. 31.—Diam. ·07 to ·08 mm. Central space sub-circular, ·0025 to ·003 mm. broad, sometimes slightly excentric; an evident subcentral areolate area reaching between $\frac{1}{4}$ and $\frac{1}{5}$ of radius from centre. Markings on subcentral area unequal, $2\frac{1}{2}$ to 4 in ·01 mm., the smaller sometimes confined to a band contiguous to the central space; rays many, straight, those on one side of valve sometimes somewhat longer than those on opposite side; the compartments reaching from $\frac{1}{3}$ to $\frac{5}{17}$ of radius inwards, outline obtusely conical, sides subuniformly convex, non-areolate but with distinct subradial lines at intervals of ·0025 to ·003 mm., frequently dichotomising towards the border; intervals wide, reaching close to the border; adjacent to the border a circlet of delicate subregular striæ, 4 to $4\frac{1}{2}$ in ·01 mm.—Eul., *Diat. Spec. Typ.*, No. 16.

Habitat.—Barbados deposit (Greville! Grove, Eulenstein!); Cambridge deposit, Barbados (Deby!).

§ 3. Submargaritaceæ.

Aspect subpearly. A central space usually distinct. Rays broad or undifferentiated, the inner ends of the compartments extending to edge of central space.

A. ambigua. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 54, pl. viii. figs. 42–45.—Diam. ·0175 to ·0525 mm. Central space angular, distinct, hyaline, up to about ·01 mm. broad. Markings: rays straight, sometimes slightly constricted at the middle or expanding regularly outwards and merging into the compartments; interradial spaces expanding outwards, at outer end trilobate, the small central lobe homologous to the interval between the compartments; two straight or slightly curved lines (concave towards each other) passing from the inner end of this lobe, and meeting at or near the inner ends of the interradial spaces; the compartments reaching about $\frac{1}{6}$ of radius inwards, sometimes indistinctly defined, subhyaline; small isolated round granules sometimes present close to the sides of the interradial spaces.

Rarely the outline of the valve is obtusely angular.

Habitat.—Cambridge deposit, Barbados (Johnson!); Barbados (Greville!); locality? (Deby!).

A. dubia. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 54, pl. vii. fig. 41.—Diam. ·0375 mm. Central space quadrate, with sides slightly concave, hyaline. Markings: rays hardly differentiated, interradial spaces 4, cruciform ovate, their inner ends rounded, the outer subacute, homologous to the intervals between the compartments; two distinct subparallel radial lines extending from their inner ends to the sides of the protuberant outer extremities, an evident round clear granule adjacent to the border and opposite the outer ends of each interradial space; the compartments extending to edges of central space, subhyaline. Border distinct, its inner edge round.

Habitat.—Barbados deposit (Greville !).

A. aliena. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 55, pl. viii. fig. 46.—Diam. ·0525 mm. Central space circular, about ·01 mm. broad, not sharply defined. Markings: a minute clear round granule at middle of central area; rays broad; interradial spaces expanding uniformly outwards, the sides straight, their outer ends placed obliquely to the direction of the intervals, two faint subparallel lines extending between their inner ends and the edges of the intervals; the compartments reaching almost to the semiradius, subhyaline; the intervals narrow, of uniform breadth, a small round granule at their outer extremities. Border about $\frac{1}{7}$ of radius broad, subhyaline, its inner edge slightly angular, the angles corresponding in position to the outer ends of the intervals.

Habitat.—Barbados deposit (Greville !).

§ 4. Traducentes.

Sometimes regularly polygonal. Central space distinct, granular or subareolate, rarely hyaline. Rays narrow. Compartments subhyaline, or their inner edge with a distinct band of larger areolæ, their outer portion subareolate or granular.

A. stellulata. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 54, pl. vii. fig. 40.—Sometimes regularly and obtusely polygonal. Diam. ·045 mm. Central space irregularly concavo-convex, about ·0075 mm. long by ·005 mm. broad. Markings: rays 7 to 9, straight, regular; the compartments reaching about $\frac{3}{8}$ of radius inwards, their inner ends sigmoid on each side of, and protuberant

towards the centre opposite the rays; the puncta obscure, isolated towards their inner, more crowded around their outer extremities, 10 to 12 in ·01 mm.; the intervals narrow, of uniform width, their outer ends convex outwards not reaching the border, a small round clear granule intervening between them and the border. Border narrow, indistinct.

Habitat.—Barbados deposit (Greville !).

A. kittoniana. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 53, pl. viii. fig. 39.—Diam. ·04 to ·05 mm. Central space heptangular, about $\frac{1}{6}$ of diam. broad, its sides deeply concave outwards. Markings: central space punctato-areolate; rays 7, straight; the compartments reaching from $\frac{1}{3}$ to $\frac{1}{4}$ of radius inwards, their inner ends concave inwards on each side of, but protuberant opposite, the rays, sometimes bluntly conical, their centro-lateral angles rounded, the markings obscure; the intervals alternating rapidly outwards, not reaching the border, their outer ends acute, with a minute rounded granule at each side sometimes visible.

Habitat.—Barbados deposit (Kitton, Deby !).

A. traducens, sp. n.—Diam. ·0875 mm. Central area circular, ·025 mm. broad. Markings on central area isolated, rounded or irregular and minute granules; rays 16, narrow; at middle of interradial spaces a narrow, sharply defined, sublinear area extending from their inner ends to the intervals; the compartments with inner ends faintly defined, an outwardly concave row of minute puncta stretching between the extremities of the adjacent intervals, those contiguous to the intervals somewhat more prominent, elsewhere the puncta more minute and obscure; intervals tapering rapidly outwards, their outer ends acute, not reaching the border. Border obtusely and subregularly polygonal.—(Pl. III. fig. 22.)

Habitat.—Barbados deposit (Deby !).

A. pulchra. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 53, pl. viii. figs. 37, 38.—Obtusely and regularly octagonal. Diam. ·0625 mm. Central space distinct, $\frac{1}{5}$ to $\frac{1}{6}$ of diam. broad, its sides deeply concave outwards between the radii. Markings: on central space a few irregular, obtusely angular or subareolate granules;

rays 8, straight; at middle of interradial spaces 2, somewhat faint subparallel lines radiating to the inner edges of the intervals; the compartments reaching about $\frac{1}{3}$ of radius inwards, the inner ends concave towards the centre on each side of, but somewhat protuberant opposite, the rays, subhyaline or obscurely and minutely punctate, sometimes with a band of more prominent oblong markings at their inner ends; the intervals alternating gradually outwards, their outer ends convex, and not reaching the border with a distinct oval, obliquely placed granule at one or both sides, those belonging to one interval sometimes meeting each other at the extremity of the interval.

Habitat.—Barbados deposit (Greville).

A. scutula. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 52, pl. viii. fig. 47.—Diam. ·04 mm. Central space regularly hexagonal, about ·005 mm. broad, its sides convex, towards the centre hyaline. Markings: rays 6, straight; the compartments reaching about $\frac{3}{8}$ of radius inwards, their inner ends flat or somewhat concave, protuberant at outer ends of rays; a single large areola at their centro-lateral angles, tapering slightly outwards, its inner end convex but hardly protuberant, elsewhere the puncta obscure or unresolved; the intervals evident, expanding somewhat rapidly outwards, their outer ends rounded, reaching the border. Border narrow, hyaline.

Habitat.—Barbados deposit (Johnson!).

A. simulans. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 52, pl. viii. fig. 36.—Circular or regularly and obtusely polygonal. Diam. ·0375 to ·06 mm. Central space polygonal, ·01 mm. broad, its sides convex towards the centre, hyaline. Markings: rays 9, straight; at middle of interradial spaces somewhat broader, more faint lines radiating to the edges of the intervals; the compartments reaching about $\frac{1}{3}$ of radius inwards, their inner ends concave, concentric, with circumference sometimes slightly protuberant at ends of rays; a distinct band of narrow radially elongate areolæ fringing their inner ends, those at the centro-lateral angles most distinct and largest; contiguous to the outer ends of the areolæ a narrow, hyaline, indistinct band concave towards the border, beyond this the puncta obscure, about 5 in ·01 mm., sometimes

subareolate; intervals narrow, unequal, sometimes expanding slightly at their outer ends. Border narrow, its inner edge with minute irregularities, hyaline.

This species is sometimes confounded with *A. punctata*, from which it differs in the appearance of its central space and the sculpturing of its compartments.

Habitat.—Barbados deposit (Greville!); Springfield deposit, Barbados (Hardman!).*

A. æmulans. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 52, pl. viii. figs. 34, 35.—Diam. ·05 to ·075 mm. Colour pale brownish-grey at centre and along the rays, elsewhere pale smoky grey. Central portion distinct, extending outwards for $\frac{1}{6}$ to $\frac{4}{15}$ of radius. Markings on central portion areolate, the areolæ $3\frac{1}{2}$ to 4 in ·01 mm.; rays straight or but slightly arcuate; the compartments reaching from $\frac{2}{5}$ to $\frac{8}{15}$ of radius inwards, their inner ends convex, with short distinct radial lines about 4 in ·01 mm., and extending outwards for $\frac{1}{3}$ to $\frac{1}{4}$ of their length, their outer portion with delicate areolæ decreasing rapidly outwards from 5 or $5\frac{1}{2}$ to 8 in ·01 mm.; the intervals of equal breadth, their outer ends rounded, not reaching the border. Border distinct, hyaline.

Both of the typical specimens in Greville's collection show a distinctly areolate central portion, and there is no indication of its ever being solid, as stated by Greville. In neither are the markings on the compartments isolated puncta.

Habitat.—Barbados deposit (Greville!).

§ 5. Eximiæ.

Central areolate area sometimes present. Rays rarely unequal, and meeting at an excentric point or absent. Compartments areolate, their inner ends convex, concave, or obliquely truncate towards the centre, sometimes bounded by a prominent areolate fringe; intervals rarely obsolete or occluded at their inner ends, reaching the border.

A. nicobarica, Grun. Van Heurck, *Syn. Diat. Belg.*, pl. cxxvii. fig. 7.—Diam. ·055 mm. Central space angular, about ·01 mm.

* In the collection of Mr Julien Dely.

broad. Markings: a single rounded granule at middle of central space; rays between central space and middle of inner ends of compartments absent, but well-defined, subpearly, subclavate areas, tapering gradually outwards, sometimes slightly flexed, and each with a narrow delicate flexuous line close to their edges, extending between central space and outer ends of intervals, the lines from the sides of the adjacent areas meeting one another at their inner ends; the compartments reaching about $\frac{1}{2}$ of radius inwards, their inner edge convex towards the centre, the areolæ distinct, 6 to 8 in ·01 mm.; at irregular wide intervals a few more prominent rounded dots; the intervals not reaching the border, tapering outwards, their outer ends convex.

Habitat.—Nancoori deposit (Van Heurck).

A. punctata. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 51, pl. viii. fig. 32.—Diam. ·0625 mm. Central areolate area subelliptical to round, about ·0075 mm. broad. Markings on central area unequal, 4 to 5 in ·01 mm.; rays 6 to 7, straight; the compartments reaching about $\frac{1}{4}$ of radius inwards, their inner ends transversely truncate or slightly concave inwards opposite the radii; the areolæ few, granular towards their inner ends, towards the border 6 to 7 in ·01 mm., with hyaline interspaces; the intervals wide, sometimes expanding slightly outwards, their outer ends transversely truncate, sometimes reaching the border. Border, narrow, indistinct, subhyaline.

The smaller areolæ on the central area may be so disposed as to form an arc round a somewhat larger unilateral areola.

Habitat.—Barbados deposit (Greville !).

A. balearica. Cleve, *Kongl. Sv. Vet.-Akad. Handl. Stockh.*, 1881, Bt. xviii. No. 5, p. 20, pl. v. fig. 59.—Diam. ·0715 mm. Central space circular, hyaline, about $\frac{1}{12}$ of diam. broad. Markings: rays straight, symmetrical; the compartments reaching about $\frac{7}{9}$ of radius inwards, their inner ends uniformly and considerably convex, areolate; the areolæ subequal or decreasing but slightly outwards, 9 in ·01 mm.; the intervals of uniform breadth, their outer ends convex, reaching about $\frac{5}{6}$ of radius from centre.

From *A. Grevillei* this differs not only in the coarser character

of the areolæ, but also in the appearance of the rays and length of the intervals between the compartments. In *Asterom. centraster*, Johnson, the intervals between the compartments are continued to the centre, and their outer ends are swollen and knob-like. From *A. brebissoniana* it is readily distinguished by the straight instead of geniculate rays.

Habitat.—Balearic Islands (F. Söderlund).

A. lævis. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 51, pl. viii. fig. 33.—Diam. ·0275 mm. Central rosette and areolate area absent. Markings: rays 6, straight; the compartments reaching to about $\frac{3}{8}$ of radius inwards, their inner ends transversely truncate, sometimes slightly flexuous or concave towards the centre; the areolæ obscure, least evident towards the border; the intervals extending to about $\frac{1}{2}$ of distance between their inner ends and border, tapering gradually outwards with the inner ends convex. Border narrow, hyaline, indistinct.

Habitat.—Barbados deposit (Greville !).

A. marylandica. Ehrb., *Mon. Ber. Ak.*, 1844, p. 76, fig. 10 (June 1844).—Diam. ·0375 to ·15 mm. Central areolate area absent. Markings: the rays 4 to 12, straight, or but slightly arcuate, diverging from the centre, sometimes dichotomous or dividing into three equal rami; the compartments reaching from $\frac{3}{8}$ to $\frac{1}{2}$ of radius inwards, their inner ends uniformly convex towards centre, sometimes truncate or slightly emarginate, rarely unsymmetrical; the areolæ distinct, subequal on inner $\frac{2}{3}$ of compartments, or decreasing more regularly outwards from 6 to 10 in ·01 mm.; a band of more prominent submuriform areolæ around their edges; the intervals expanding gradually outwards, their outer ends reaching close to the border.—Müller, *Abh. Ber. Ak.*, 1841, p, 232, pl. vi. fig. 4 (no name); Bail., *Amer. Jour. Sci.*, 1845, vol. xlviii. pl. iv. fig. B; Brightw., *Quart. Jour. Micr. Sci.*, 1860, p. 94, pl. v. fig. 3; Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 47, pl. ii. figs. 14, 15; Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 108, pl. iii. figs. 1–4; Ralfs in *Pritch. Inf.*, p. 836, pl. xi. fig. 33; Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 44, pl. vii. figs. 1–3; *A. septenaria*, Johnson, *Amer. Jour. Sci.*, 1852, ser. 2, vol. xiii. p. 33; *A. impar*, Shadb., *Trans. Micr. Soc.*

Lond., 1854, p. 17, pl. i. fig. 14; *A. pelagica*, Ehrb., *Mon. Ber. Ak.*, 1854, p. 238; *A. hexactis*, Ehrb., *Abh. Ber. Ak.*, 1872, pl. ix. figs. 1, 2; *A. marylandica*, var. *ausonia*, Cstr., *Atti Accad. pontif. nuov. Lincei, Roma*, 1875, p. 393, pl. vi. fig. 4.

Dr Wallich distinguished as var. β (Wallich, *ibid.*, 1860, p. 48, pl. ii. fig. 14), forms having compartments with rounded inner ends and simple rays, and as var. γ (*ibid.*, pl. ii. fig. 15), those having the inner ends of the compartments truncate, and the rays conspicuously ramose near their central ends, the branching of the rays being symmetrical with respect to a diameter of the valve. Péragallo (*Diat. Baie d. Villefranche*, 1888, p. 74) still adheres to Wallich's var. β, naming it *A. marylandica*, var. *major*, but it appears to me hardly sufficiently distinct from the type to merit varietal recognition. Wallich's var. γ may be named *A. marylandica*, var. *ramosa*. Forms named by Thum *A. adriatica*, Grun., and now in the collection of Mr Julien Deby, do not differ from *A. marylandica*. One of the specimens named *A. adriatica* has a delicate round granule (probably an apiculus) at the outer extremity of each interval, thus recalling *A. marylandica*, var. *ausonia*, Cstr.

Mr Edmund Grove has proposed to establish a var. *appropinquans*, for specimens in which one of the compartments is smaller than the others, two of the intercompartmental intervals being in consequence unusually approximated.

Habitat.—Maryland (Ehrenberg); Barbados deposit (Greville !); Springfield deposit, Barbados (Hardman !); Cambridge deposit, Barbados (Greville ! Hardman !); Newcastle deposit, Barbados (Grove !); Piscataway deposit (Johnson, Greville ! Roper !); * Monterey stone (Greville ! Walker-Arnott); Richmond deposit, Va. (Johnson); Rappahannock (Greville); Bermuda tripoli (Dallas); Oamaru deposit (Grove !); Indian Ocean soundings, lat. 5° 37′ S., long. 61° 33′ E., 2200 fathoms, Captain Pullen (Greville !); Indian Ocean, off Zanzibar (Ehrenberg); from *Salpæ*, Bay of Bengal (Wallich); Alexandria (Hardman !); Port Natal (Shadbolt); from *Comatulæ*, Mediterranean Sea (Müller); *Holothuria*, China (Thum); South Naparima, Trinidad (Greville); Rembang Bay (Deby !); † Tegel von Mähren, Austria (Deby !).

* In the collection of Dr R. K. Greville.

† In a type slide of material from this locality prepared by Herr Thum.

A. rotula. Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 111, pl. iii. fig. 5.—Diam. ·11 mm. Central areolate area absent. Markings: rays diverging from a central point, simple, or rarely dichotomous, straight or slightly arcuate; on the interradial spaces faint, indefinite, narrow dark bands extending for some distance inwards from the centro-lateral angles of the compartments; the compartments reaching about $\frac{2}{3}$ of radius inwards, their inner edges somewhat obliquely truncate on each side of the rays; areolæ faint; the intervals of uniform breadth, their outer ends convex close to the border.—Cstr., *Atti Accad. pontif. d. nuov. Lincei*, 1875, p. 393, pl. vi. fig. 3*a*; *A. Grevillii Wallich*, var. *adriatica*, Grun. in Van Heurck, *Syn. Diat. Belg.*, pl. cxxvii. fig. 12.

This species is distinguished from those forms of *Asteromphalus variabilis* having a faintly differentiated subobsolete ray by the more straight rays and the less obliquely truncate inner ends of the compartments. Péragallo (*Diat. Baie d. Villefranche, Paris*, 1888, p. 74) has already pointed out the identity of Grunow's *A. Grevillii*, var. *adriatica* with *A. rotula.*

Habitat.—Monterey stone (Walker-Arnott); Adriatic Sea, Balearic Islands (Van Heurck).

A. dallasiana. Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 115, pl. iv. fig. 10.—Diam. ·075 mm. Central areolate area absent. Markings: rays 7, slightly and uniformly arcuate towards the same direction; the compartments reaching to $\frac{2}{3}$ of radius inwards, their inner ends transversely truncate, areolate; the areolæ subequal 10, near border 12, in ·01 mm.; the intervals attenuating slightly outwards, and again somewhat swollen at the extremities, reaching close to the border.—*Asteromphalus dallasianus*, Ralfs in *Pritch. Inf.*, p. 836.

Although a centro-lateral area is present there is no such subobsolete interval as would be found in *Asteromphalus.* Compare also in this respect some forms of *Asteromphalus variabilis.*

Habitat.—Nottingham deposit, U.S. (Greville !).

A. brebissoniana. Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 114, pl. iii. fig. 9.—Diam. ·075 mm. Central areolate area absent. Markings: rays sharply bigeniculate about $\frac{3}{5}$ of their length from the centre, two or three adjacent rays meeting

one another a short distance from centre whither they are continued as a simple line; the compartments reaching from $\frac{5}{8}$ to $\frac{5}{9}$ of radius inwards, their inner ends transversely truncate or but slightly convex, areolate; the intervals attenuating gradually outwards, their outer ends slightly swollen and knob-like.

Greville has pointed out that the geniculate flexure of the rays found also in *Asteromphalus imbricatus*, *Asterom. Darwinii*, *Asterom. elegans*, *Asterom. Brookei*, and in a less degree in *Asterom. shadboltianus*, goes to establish the validity of the union of *Asteromphalus* and *Asterolampra* into a single genus, but this ignores the apparently more constant characters in the latter connected with the subobsolete interval between two of the compartments.

Habitat.—Monterey stone (Walker-Arnott).

A. Grevillei. Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 113, pl. iv. fig. 21.—Diam. ·075 to ·085 mm. Central areolate area absent, sometimes represented by two plano-convex areolæ. Markings: rays straight, or gently arcuate, diverging from an angular arched central line rarely from a central point; the compartments reaching about $\frac{1}{2}$ of radius inwards, their inner ends curved or flattened on opposite sides of rays convex towards the centre, their margin formed by a narrow subhyaline band, elsewhere minute subpunctiform granules, least evident towards the border, sometimes unresolved; the intervals of uniform breadth; their outer ends transversely truncate, sometimes terminating a considerable distance from the border. Border narrow, indistinct.—*Asteromphalus Grevillei*, Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 47, pl. ii. fig. 15.

The radii vary in number from 7 to 17, and exhibit considerable variation in their mode of origin, an adjacent pair frequently uniting and being connected by a common short central stalk with the central portion.

Habitat.—Moron deposit (Greville! Hardman!); Indian Ocean 2200 fathoms, Captain Pullen (Wallich); Rappahannock deposit, U.S. (E. W. Dallas); Monterey stone (Walker-Arnott).

A. princeps,* nov. *A. Grevillei*, var. *eximia.* Cstr., *Diat. Chall.*

* The name *eximia* cannot be adopted, as it has been preoccupied by Greville (*Trans. Micr. Soc. Lond.*, 1865, p. 99) for a distinct form (see *infra*).

Exped., 1886, p. 136, pl. v. fig. 5.—Diam. ·18 mm. Central areolate area absent. Markings: rays regularly arcuate, rarely substraight, 2 or 3 short lines diverging from the centre and branching somewhat irregularly 2 to 4 times before reaching half the distance to the inner ends of the compartments; the compartments reaching about $\frac{2}{3}$ of radius inwards, their inner ends transversely truncate or slightly concave towards the centre, the areolæ forming a distinct band along their outer edge and 6, elsewhere in straight oblique decussating rows and about 8, in ·01 mm., decreasing but slightly towards the border; the intervals attenuating regularly outwards and reaching the border. Border distinct, hyaline.

Habitat.—Equatorial Atlantic, H.M.S. Challenger (Castracane).

A. brightwelliana. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 48, pl. viii. figs. 26, 27.—Diam. ·075 to ·095 mm. Surface markedly convex. Central areolate area absent. Markings: rays straight or subuniformly curved, springing from a somewhat excentric point, the compartments reaching about $\frac{1}{3}$ of radius inwards, their inner ends concave towards the centre, unequal, areolate; a single band of areolæ adjacent to their inner ends large, 3 to $3\frac{1}{2}$ in ·01 mm., the others much smaller 6, decreasing gradually outwards to 8 or 9 in ·01 mm., the intervals of equal breadth (about ·0035 to ·004 mm.), their outer ends reaching the border.

Habitat.—Springfield deposit, Barbados (Hardman!); "Barbados" deposit (Greville!); Cambridge deposit, Barbados (Deby!).

A crenata. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 47, pl. viii. figs. 4–16.—Diam. ·05 to ·075 mm. Central areolate area absent. Markings: rays straight; the compartments reaching about $\frac{1}{3}$ of radius inwards, their inner ends concave towards the centre and concentric with the border, areolate, a single band of large unequal areolæ at their inner ends 4 in ·01 mm., and from 3 to $3\frac{1}{2}$ times as long as broad, beyond this band 6, decreasing rapidly outwards to 10 in ·01 mm.; the intervals expanding slightly outwards and of uniform breadth, reaching the border.

This species is distinguished from *A. concinna* by the shape of the inner ends of the compartments and from *A. vulgaris* by their regularity.

Habitat.—Barbados deposit (Greville! Hardman!).

A. eximia. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 99, pl. viii. fig. 10.—Diam. ·0875 to ·15 mm. Central rosette distinct, a central areolate area about ·036 mm. broad. Markings: on central area unequal areolæ 2 to 2½ in ·01 mm.; rays 22, straight; the compartments reaching about ⅔ of radius inwards, their inner ends convex, bounded by a single band of large areolæ about 3 in ·01 mm., and gradually decreasing in length from the radii outwards, elsewhere the areolæ quadrate, decreasing gradually from 3 to 4 in ·01 mm.; intervals of equal breadth throughout, their outer ends convex, reaching close to the border.

Habitat.—Cambridge deposit, Barbados (Hardman); Chalky Cliff, Barbados (Deby !).

A. concinna. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 46, pl. vii. figs. 10–12.—Diam. ·075 to ·1 mm. Central areolate area absent. Markings: rays straight, springing from centre; the compartments reaching from ⅓ to ½ of radius inwards, their inner ends transversely truncate or slightly convex, bounded by a single band of large areolæ 4 in ·01 mm. at their inner ends, sometimes a single large areola at each of the centro-lateral angles of the compartments; elsewhere the markings 6, decreasing gradually outwards to 8 in ·01 mm.; the intervals expanding slightly outwards, their outer ends convex, reaching close to the border.

Habitat.—Barbados deposit (Greville !).

A. vulgaris. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 47, pl. vii. figs. 17–20.—Diam. ·0375 to ·055 mm. A central areolate area sometimes present. Markings on central area unequal, about 4 in ·01 mm., rarely a single round areola only present; rays straight; the compartments reaching from ⅓ to ¼ of radius inwards, their inner ends concave at the middle, bounded by a distinct band of large areolæ about 4 in ·01 mm., those at the centro-lateral angles largest, and protruding into the interradial spaces with their central ends rounded, sometimes those of the adjacent compartments expanding towards one another at their inner ends so as almost to exclude the intervals; elsewhere the areolæ 8 to 10 in ·01 mm.; the intervals expanding for some distance outwards, their outer ends sometimes reaching close to the border, convex outwards.—

A. vulgaris, var. *a*, Grev., *ibid*, 1862, p. 47; *A. vulgaris*, var. *b*, Grev., *ibid.*, p. 47, pl. vii. fig. 21; Eul., *Diat. Spec. Typ.*, No. 16.

Rarely in valves from Cambridge deposit, Barbados, the areolæ on the central area are unequal and much larger, from $1\frac{1}{4}$ to $1\frac{1}{2}$ in ·01 mm., and are disposed subconcentrically towards its outer margin.

Habitat.—Barbados deposit (Greville! Hardman! Eulenstein!) locality? (Deby!); Springfield deposit, Barbados (Hardman!); Cambridge deposit, Barbados (Hardman!); Oamaru deposit (Grove! Deby!).

Var. *planior*. *A. vulgaris*, var. *c*. Grev., *ibid.*, 1862, p. 47, pl. vii. fig. 22.—Diam. ·07 mm. Central area distinct, subcircular, from $\frac{1}{4}$ to $\frac{1}{5}$ of diam. broad. Markings on central area $2\frac{1}{2}$ to 3 in ·01 mm., unequal; the compartments reaching about $\frac{5}{17}$ of radius inwards, the largest centro-lateral areolæ but slightly protuberant, sometimes bulging laterally towards their inner ends; the intervals clavate, their outer ends convex, close to the border.

Habitat.—Barbados deposit (Greville! Grove!).

Var. *cellulosa*. *A. vulgaris*, var. *d*. Grev., *ibid*, 1862, p. 47, pl. vii. fig. 23; pl. viii. fig. 24.—Diam. ·055 to ·1 mm., central area subcircular, from $\frac{1}{3}$ to $\frac{3}{8}$ of diam. broad, a central rosette sometimes distinct. Markings on central area subequal, 4 in ·01 mm., without order or in obscure oblique decussating rows; the compartments reaching from $\frac{1}{4}$ to $\frac{2}{5}$ of radius inwards, the band of areolæ at their inner ends extending outwards almost to border, about 4 in ·01 mm. across; the marginal areolæ protruding slightly at their central inwardly convex ends.

Habitat.—Barbados deposits (Greville! Deby! Grove!).

A. decorata. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 46, pl. vii. fig. 13.—Diam. ·08 mm. Central areolate area absent. Markings: rays straight, the compartments reaching about $\frac{1}{4}$ of radius inwards, their inner ends transversely truncate, bounded by a single band of oblong areolæ 2 to $2\frac{1}{4}$ in ·01 mm. broad and about ·01 mm. long, those opposite the rays with long axis radial, the others becoming more and more oblique, those at the angles of the compartments ·0175 mm. long, with outer ends attenuated; the

remaining areolæ decreasing gradually outwards from 6 to 9 in ·01 mm.; the intervals expanding gradually outwards, their outer ends swollen, knob-like, reaching close to the border.

Habitat.—Barbados deposit (Greville !).

A. splendida. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 48, pl. viii. fig. 25.—Diam. ·09 mm. Central areolate area distinct. Markings on central area unequal, about $2\frac{1}{2}$ in ·01 mm.; rays straight, or but slightly arcuate, many; the compartments reaching about $\frac{1}{3}$ of radius inwards, their inner ends concave, bounded by a band of large areolæ $3\frac{1}{2}$ in ·01 mm. broad, and extending outwards almost to the border, those at the edges of the compartments with inner ends protuberant and rounded; close to the border the areolæ obscure, subpunctiform, 12 in ·01 mm.; the intervals of uniform (·0025 mm.) width, their outer ends close to the border, convex outwards.—*A. vulgaris*, var. *e*, Grev., *ibid*, 1862, p. 47.

The great length of the areolæ at the inner ends of the compartments at once distinguishes this species from *A. vulgaris*, to which Greville united it with some hesitation.

Habitat.—Barbados deposit (Greville !).

A. uraster. Grove and Sturt, *Jour. Quek. Micr. Cl.*, 1887, p. 143, pl. xiii. fig. 42.—Diam. ·06 mm. Central areolate area distinct. Markings on central area large, 2 to $2\frac{1}{2}$ in ·01 mm.; rays straight, two passing to the apex of one of the compartments; the compartments reaching from $\frac{2}{3}$ to $\frac{3}{5}$ of radius inwards, conical, their inner ends obtusely angular, the areolæ evident, decreasing but slightly outwards, from $5\frac{1}{2}$ to 6 in ·01 mm.; the intervals broad, attenuating somewhat outwards, not reaching the border; a minute round granule (apiculus?) at the outer end of each.

The apex of the compartment that receives the two radii is somewhat obliquely truncate, and slightly concave towards the centre.

Habitat.—Oamaru deposit (Gray).

A. rylandsiana. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 49, pl. viii. figs. 28, 29.—Diam. ·04 to ·05 mm. Central areolate area about ·01 mm. broad. Markings on central area subequal, about 3 in ·01 mm.; rays 7 to 12, straight, or but slightly flexed; the

compartments reaching about $\frac{3}{10}$ of radius inwards, their inner ends transversely truncate, or slightly convex towards the centre, their adjacent sides formed by a large cuneate areola, distinctly protruding by an inwardly convex inner end into the interradial space and attenuating towards the border, elsewhere the areolæ evident, decreasing rapidly outwards from 4 to 8 in ·01 mm.; the intervals obsolete. Border narrow, indistinct.

Habitat.—Barbados deposit (Greville! T. G. Rylands, de Brébisson, Grove!); Springfield deposit, Barbados (Hardman!).

*A. tenerrima,** sp. n.—Diam.? Central areolate area absent. Markings: rays 4 to 7, straight; the compartments reaching from $\frac{3}{8}$ to $\frac{1}{2}$ of radius inwards, their inner ends concave towards centre or transversely truncate; a single band of large areolæ bounding the inner ends, outside of this a single large lanceolate areola extending close to the border bounding the interval; elsewhere the areolæ small, evident, in distinct radial and less manifest subregular concentric zones; intervals extending to border of uniform width.—(Pl. III. figs. 18, 20.)

Habitat.—?

A. affinis. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 45, pl. vii. figs. 7–9.—Diam. ·0675 to ·1175 mm. Colour pale grey, the rays more opaque. Central areolate area from ·0125 to ·0175 mm. broad, sometimes absent; a small central space rare. Markings on central portion 2 in ·01 mm., hyaline; rays straight or slightly flexuous towards the inner ends; the compartments reaching from

* This species is established on two specimens occurring in a photograph now in the possession of Mr Julien Deby, the history of which is given in a letter addressed to him by Herr E. Weissflog dated 27th July 1878. Herr Weissflog says:—"I have received a letter from Mr F. Habirshaw of New York, in which he says—'The late John E. Gavit . . . engraved the fine plate in Bailey's "New Species," &c. (Smith's *Contrib.*). He also some years ago made a plate which suddenly disappeared—neither plate nor impressions could be found. In overhauling the effects of Judge Johnson (of *Asterodiscus* in *Silliman's Jour.*) two impressions were found, and it is believed that they are the only two extant. A few days since we photographed the one sent us, and we hope that you will be pleased with the result. If there are more wanted, I would like MM. Deby and Delogne to have copies.' Herewith you will find two proofs, and you will oblige much by remitting one to M. Delogne." Nothing further is known of the specimens. They seem, however, to come from the Barbados deposit.

$\frac{1}{4}$ to $\frac{1}{5}$ of radius inwards, their inner ends slightly angular towards centre or transversely truncate, the areolæ decreasing but slightly towards the border, the outermost row more conspicuous; the intervals of equal breadth throughout, their outer ends reaching the border. Border narrow, hyaline.

Habitat.—Barbados deposit (Johnson! Deby!); Oamaru deposit (Grove!); Newcastle estate, Barbados (Grove!).

A. decora. Grev., *Trans. Micr. Soc. Lond.*, 1862, p. 45, pl. vii. figs. 4–6.—Diam. ·04 to ·1375 mm. Central space subtriangular or absent, central areolate area distinct, up to ·02 mm. broad. Markings on the central area unequal, 2 to 3 in ·01 mm., those around its edges largest; rays 5 to 19, straight, sometimes meeting at the centre; the compartments reaching about $\frac{1}{4}$ of radius inwards, their inner ends slightly concave or convex towards the centre, the areolæ at their inner ends largest, 4 in ·01 mm., the others decreasing rapidly outwards from 6 to 10 in ·01 mm.; the intervals of uniform breadth, reaching close to the border.—*A. decora*, var. Cstr., *Diat. Chall. Exped.*, 1886, p. 136, pl. xvi. fig. 9.

In the variation in size of the areolate central area this species approaches the forms of *A. vulgaris* and *A. rylandsiana.* The size of the markings on the compartments is less, according as the specimens decrease in size.

Habitat.—Cambridge deposit, Barbados (Greville! Hardman!); "Barbados" deposit (Greville!); Springfield deposit, Barbados (Hardman!); Oamaru deposit (Deby! Grove! Hardman!).

Var. *concentrica*, nov.—Diam. ·095 to ·125 mm. Central areolate area from ·0375 to ·065 mm. broad, a rosette sometimes evident. Markings on central area $1\frac{1}{2}$ to 2 in ·01 mm., with central dots large, and in evident concentric, less evident radial rows; rays many; at middle of interradial space a distinct hyaline area, with inner end close to border of central area; the compartments reaching from $\frac{1}{3}$ to $\frac{3}{7}$ of radius, areolæ obvious, 4 to 8 in ·01 mm.; the intervals gradually increasing in breadth outwards, their outer ends transversely truncate.

Habitat.—Oamaru deposit (Grove! Hardman! Deby!); Cambridge deposit, Barbados (Johnson!).

A. Weissflogii. Van Heurck, *Syn. Diat. Belg.*, pl. cxxvii. fig. 9.—Diam. ·06 mm. Central space minute, irregular. Markings: rays 6, straight, robust; at middle of interradial spaces faint subhyaline areas about ·0025 mm. broad, extending between central space and intervals; the compartments distinct, extending from $\frac{1}{3}$ to $\frac{5}{12}$ of radius inwards, their inner ends concave inwards on each side of, but protuberant opposite the rays; delicate, radial, subindefinite, faint striæ, about ·0025 mm. long fringing their inner edge; a somewhat larger more prominent areola at the centro-lateral angles; elsewhere the markings punctiform, obscure; the intervals sharply defined, tapering slightly towards, but not reaching the border with outer ends convex outwards; a delicate, irregularly sinuate, continuous zone of unequal breadth contiguous to the inner ends of the compartments. Border about $\frac{1}{12}$ to $\frac{1}{15}$ of radius broad, its inner edge sharply defined, hyaline.—Pelletan, *Les Diat. Nat. Hist.*, tom. i. p. 206, fig. 112; tom. ii. fasc. i. p. 170, fig. 427; *A.* (*pulchra*, var.?) *Weissflogii*, Grun.; Van Heurck, *loc. cit.*

In some specimens the outer ends of the intervals between the compartments are not swollen, as shown in Professor Van Heurck's figure.

Habitat.—Barbados deposit (Deby!); Cambridge deposit, Barbados (Van Heurck).

Artificial Key.

1.	Rays straight or subuniformly curved, springing from a somewhat excentric point,	*brightwelliana.*
	No such excentricity,	2.
2.	A large central areolate area; rays short, numerous, compartments convex inwards; intervals subobsolete,	*marginata.*
	No such structure,	3.
3.	Compartments obtusely conical, ornamented with distinct subradial dichotomising lines,	*ralfsiana.*
	No such markings on the compartments,	4.
4.	Rays obsolete or subobsolete or broad; interradial spaces small,	5.
	Rays narrow, sublinear; valve subpearly,	6.
	Rays linear; valve more hyaline,	7.
	Rays broad, subclavate, passing between central space and intervals between compartments, with inner ends convex to centre,	*nicobarica.*
5.	Interradial spaces tapering towards centre,	8.
	Interradial spaces tapering towards border; a distinct round granule adjacent to border opposite outer end of each,	*dubia.*

8.	Interradial spaces with outer ends trilobate; the median lobes homologous to subobsolete intervals between the compartments; a central hyaline area sometimes present; radii sometimes broad, . .	*ambigua.*
	Interradial spaces with inner ends acute; a small round central granule and one at outer end of each interval. Border broad, inner edge angular, .	*aliena.*
6.	Central area granular or subareolate, . . .	9.
	Central area hyaline, without markings, . . .	10.
9.	Inner ends of compartments with short distinct radial lines, ...	11.
	No such radial lines. Markings on compartments obscure, at each side of the outer ends of the intervals a minute granule, ...	*kittoniana.*
	No such radial lines. Compartments with inner ends faint, an outwardly concave row of minute puncta stretching between outer extremities of adjacent intervals, ...	*traducens.*
11.	One portion of compartments areolate; no granules at outer ends of intervals, ...	*aemulans.*
	Outer portion of compartments subhyaline or obscurely punctate, a distinct oval granule at one or both sides of outer ends of intervals, . .	*pulchra.*
10.	Markings on compartments obscure, a small round granule opposite the outer end of each interval, .	*stellulata.*
	A distinct areola at centro-lateral angle of each compartment, hardly protuberant into interradial space, ...	*scutula.*
7.	A band of large areolæ at inner ends of compartments,	12.
	No such areolate band, ...	13.
12.	The band of large areolæ prominent, 2 to 2½ in ·01 mm., the lateral areolæ of each compartment not protuberant; inner ends of compartments convex inwards; the intervals expanding gradually outwards. No central areolate area, ...	*decorata.*
	The areolæ still longer and narrower (3½ in ·01 mm. across), the lateral areolæ protuberant; inner ends of compartments concave towards centre, intervals not expanding outwards. A central areolate area,	*splendida.*
	The limiting band of areolæ less prominent but distinct, ...	14.
14.	Inner ends of compartments transversely truncate or convex towards centre, ...	15.
	Inner ends of compartments distinctly concave towards centre, ...	16.
	Inner ends of compartments transversely truncate or concave towards centre. A single large lanceolate areola bounding each side of the intervals between compartments. No central areolate area, .	*tenerrima.*
	Inner ends of compartments concave inwards on each side of rays; a delicate irregularly sinuate zone contiguous to inner ends of compartments, a minute irregular central space, ...	*Weissflogii.*

15.	Lateral areolæ larger, non-protuberant; intervals expanding slightly outwards. No areolate central area,	*concinna.*
	Lateral areolæ not larger; intervals of uniform breadth; a central areolate area,	*decora.*
	Lateral areolæ not larger, but larger areolæ at inner ends of compartments, gradually decreasing away from the rays; a central rosette; compartments with inner ends convex; intervals of equal breadth,	*eximia.*
	Lateral areolæ not larger. Compartments with inner ends mostly convex. No areolate central area. Intervals expanding somewhat outwards,	*marylandica.*
16.	Lateral areolæ protuberant,	17.
	Lateral areolæ non-protuberant; intervals expanding slightly outwards. No central areolate area,	*crenata.*
17.	Intervals narrow, frequently almost occluded at the inner ends. A central areolate area,	*vulgaris.*
	Intervals narrow, sometimes expanding outwards; contiguous to the large areolæ, a narrow hyaline band, concave towards border,	*simulans.*
13.	Intervals between compartments obsolete, a large subcuneate areola at sides of each compartment, and contiguous to that of adjacent compartments,	*rylandsiana.*
	Intervals present. No such subcuneate areolæ,	18.
18.	Markings on compartments subgranular, few towards their inner ends,	*punctata.*
	Markings on compartments areolate throughout,	19.
19.	No central areolate area,	20.
	A central areolate area present,	21.
20.	A subasteromphaloid centro-lateral area differentiated; inner ends of compartments transversely truncate; intervals attenuating outwards,	*dallasiana.*
	No such subasteromphaloid area,	22.
22.	Rays sharply bigeniculate beyond their middle; compartments with inner ends slightly swollen, faint lines passing inwards a short distance from their inner ends,	*brébissoniana.*
	Rays straight,	23.
23.	A circular hyaline central space; compartments with inner ends convex centrally, the intervals of uniform, breadth, reaching $\frac{5}{8}$ of radius from centre,	*balearica.*
	Compartments with inner ends transversely truncate to slightly concave inwards; intervals short, extending to half length of compartments,	*lævis.*
	Compartments with inner ends somewhat obliquely truncate; intervals of equal breadth, outer ends close to border. Rays sometimes dichotomous,	*rotula.*
	Compartments with inner ends transversely truncate, to concave; rays straight or curved, frequently branching 2 to 4 times before reaching half distance to compartments,	*princeps.*
21.	Two rays passing to apex of one of the compartments, an apiculus at outer end of each interval,	*uraster.*
	No such radii,	24.

24.	Central areolate area inconspicuous—represented by two areolæ; rays sometimes dichotomous; compartments with inner ends curved or flattened on opposite sides of rays, the areolæ obscure or unresolved, ...	*Grevillei.*
	Central areolate area sometimes large; compartments with inner ends concave to truncate, areolæ obvious, ...	*affinis.*

ASTEROMPHALUS.

Ehrb. Emend., *Mon. Ber. Ak.*, 1844, p. 198.—Circular, more rarely flabelliform, oval, or suboblong. Colour pale grey or subhyaline, inconspicuous. A centro-lateral area distinct, extending to or slightly beyond the middle of the clear area, rarely quite across the latter—ovate, clavate, or with sides somewhat deeply constricted. Markings: rays distinct, simple or dichotomous, extending from the apex or also from the sides of the centro-lateral area, straight, arcuate, flexuous or sharply bigeniculate, sometimes with short lateral rami passing obliquely outwards from the geniculations; interradial spaces hyaline, or with a subdistinct median area, continuous with the intervals between the compartments; the compartments equal or unequal, their inner ends convex, obliquely to transversely truncate or concave; areolæ distinct or inconspicuous, the outermost row most manifest; intervals between the compartments tapering slightly outwards, or of uniform width, their outer ends rarely expanded, sometimes not reaching border; a subobsolete interval distinct, straight, rarely distinctly arcuate; sometimes an obscure granule (apiculus?) at outer ends of intervals. Border narrow, hyaline.—*Spatangidium*, de Bréb., *Bull. Soc. Linn. Normand.*, 1857, p. 296; *Asterolampra*, Grev., *pro parte*, *Trans. Micr. Soc. Lond.*, 1860, p. 102; *Mesasterias*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 392; *Actinogramma*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 254.

§ 1. Obscuri.

The outer ends of the rays penetrating a short distance into the apices of the compartments; the intercompartmental intervals prolonged as definitely marked areas to the centre.

Astreom. centraster. Johnston, *Quart. Jour. Micr. Sci.*, 1860, p. 12, pl. i. fig. 10.—Diam. about ·072 mm. Colour of compart-

ments pale buff. Markings: rays straight, narrow, meeting near centre the inner ends of the intercompartmental areas, their outer ends knob-like, and reaching about $\frac{3}{7}$ of radius from centre; the compartments extending to about semiradius, their inner ends convex, inwards; the intervals attenuating slightly outwards, their outer ends swollen knob-like, reaching close to the border.—Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 124; Ralfs in *Pritch. Inf.*, p. 838.

One of the intercompartmental areas is more developed than the others at its inner end, but the outer portion of the same area is not subobsolete, as is usual in *Asteromphalus.*

Habitat.—Elide guano (Johnson).

§ 2. Centrales.

Clear, median portion of valve not markedly excentric. Rays straight, arcuate or geniculate; compartments with inner ends concave or convex towards centre, sometimes transversely or obliquely truncate; intervals rarely markedly expanded towards their outer ends.

Asterom. wallichianus. Ralfs in *Pritch. Inf.*, p. 837.—Rarely roundly elliptical. Diam. ·0375 to ·055 mm. Centro-lateral area V-shaped. Markings: rays 5 or 6, straight, diverging from centre; the compartments reaching to about semiradius, their inner ends transversely truncate or slightly concave towards centre; the concavity subconcentric with the circumference; the areolæ distinct; the intervals of uniform width, extending close to border; the subobsolete interval attenuating rapidly outwards.—Cleve, *Bih. k. Sv. Vet.-Ak. Handl. Stockh.*, 1873, Bd. 1, No. 11, p. 5, pl. i. fig. 1; *Asterolampra wallichiana*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 115, pl. iv. fig. 11.

Faint lines pass towards the centre on the interradial spaces from the inner angles of the compartments.

Habitat.—"Bermuda tripoli" (E. W. Dallas); Nottingham deposit, Maryland (Greville!); Santa Monica deposit (Hardman!).*

Astreom. variabilis, nov. *Asterolampra variabilis.* Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 111, pl. iii. figs. 6–8.—Diam. ·07 to ·125

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mm. Centro-lateral area V-shaped, with inner end obtuse or subacute, sometimes reaching the centre. Markings: rays substraight or somewhat curved, well-defined, frequently dichotomous, sometimes meeting in a small semicircular line curving round the centre; the compartments reaching from $\frac{2}{3}$ to $\frac{3}{5}$ of radius inwards, their inner ends obliquely truncate, on each side of rays straight or slightly concave towards the centre; areolæ distinct, decreasing gradually outwards from 5 to 8 in ·01 mm.; the intervals straight or slightly arcuate, their outer ends convex outwards, sometimes hardly reaching the border.

Habitat.—Monterey stone (Greville! Arnott! Kitton!);* Santa Monica deposit (Grove!).

Asterom. Hookerii. Ehrb., *Mon. Ber. Ak.*, 1844, p. 200, pl. (June) fig. 3.—Diam. ·064 mm. Centro-lateral area with sides straight, parallel or slightly concave outwards, sometimes suddenly contracting near the subobsolete interval, its inner end conical. Markings: rays straight; the compartments reaching from $\frac{1}{2}$ to $\frac{8}{13}$ of radius inwards, their inner ends rounded; areolæ delicate; the intervals attenuating slightly outwards, reaching the border.—Ehrb., *Mikrog.*, pl. xxxv. A. 21. fig. 2; Ralfs in *Pritch. Inf.*, p. 836, pl. xi. fig. 34; *A. Buchii*, Ehrb., *ibid.*, 1844, p. 200, pl. (June) fig. 4,—7 rays; *A. Cuvierii*, Ehrb., *ibid.*, 1844, p. 200, pl. (June) fig. 7,—9 rays; *Mikrog.*, pl. xxxv. A. 21. fig. 1; Janisch, *Abh. Schl. Ges. väter. Cult.*, 1861, p. 160; *A. Humboldtii*, Ehrb., *ibid.*, 1844, p. 200, pl. (June) fig. 6,—8 rays; *Mikrog.*, pl. xxxv. A. 21, fig. 3; Janisch and Rabenh., *Beitr.*, p. 4, pl. iii. fig. 11; Sch., *Atl.*, pl. xxxviii. figs. 18–20; *Asterolampra Hookerii*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 114.

Habitat.—Pancake ice, Antarctic Ice Barrier, lat. 78° 10′ S., long. 162° W.; lat. 75° S., long 170° W.; lat. 64° S., long. 160° W. (J. D. Hooker); Peruvian guano (Janisch); H.M.S. Challenger, lat. 53° 55′ S., long. 108° 35′ E., 1950 fathoms (Grove! Rae!).

Asterom. shadboltianus. Ralfs in *Pritch. Inf.*, p. 838.—Diam. ·0775 mm. Centro-lateral area ovate, attenuating rapidly towards its outer end, angular at the centre. Markings: rays 5, straight, or with a small geniculation near their middle; the compartments reaching about $\frac{3}{4}$ of radius inwards, their inner ends transversely

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truncate, those bordering the subobsolete interval more oblique and concave inwards at the middle; areolæ delicate, decreasing outwards from 9 to 14 in ·01 mm.; the intervals attenuating slightly outwards, their outer ends expanded and knob-like, not reaching the border.—*Asterolampra shadboltiana*, Grev., *Trans. Micr. Soc. Lond.*, 1860 p. 121, pl. iv. fig. 19.

Distinguished from *A. Brookei* by the outline of the centro-lateral space, the less marked geniculation of the radii, and the relatively shorter intervals between the compartments.

Habitat.—Indian Ocean soundings, 2200 fathoms, Captain Pullen (Greville !); Mejillones (Grove !).

Asterom. roperianus. Ralfs in *Pritch. Inf.*, p. 838.—Diam. ·07 to ·165 mm. Centro-lateral area with inner end angularly rounded, the sides sharply constricted and expanding thence to their outer ends. Markings: rays bigeniculate at their middle; the flexions more pronounced on the rays proceeding from the sides than on those passing from the extremity of the centro-lateral area; the compartments reaching to $\frac{2}{3}$ of radius inwards, their inner ends transversely truncate, those adjacent to the subobsolete interval more oblique; areolæ 12 to 16 in ·01 mm., obscure; the intervals broad, their edges parallel, the outer ends sometimes slightly expanded, not reaching the border.—Sch., *Atl.*, pl. xxxviii. fig. 15; *Asterolampra roperiana*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 120, pl. iv. fig. 14; *Mesasterias abyssi*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 392, pl. ix. fig. 7.

Habitat.—Indian Ocean soundings, 2200 fathoms, Captain Pullen (Roper ! * Greville !); Mejillones guano (Deby ! Grove !).

Asterom. Brookei.† Bail., *Amer. Jour. Sci.*, vol. xxii. ser. 2, 1856, p. 2, pl. i. fig. 1.—Diam. ·0725 to ·075 mm. Centro-lateral area with a median constriction, its central end sometimes transversely truncate or subrotund. Markings: rays straight or slightly flexuous, sometimes sharply bigeniculate towards their outer extremities, with short lateral rami proceeding from the angles; the compartments reaching from $\frac{3}{5}$ to $\frac{1}{2}$ of radius inwards, their inner ends tranversely truncate or slightly concave inwards, some-

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† Named in honour of Lieut. Brooke of the U.S. Navy.

times obtusely rounded; those bordering the subobsolete interval with the inner ends more oblique and slightly concave inwards; the areolæ evident, decreasing gradually outwards, from 8 to 10 in ·01 mm.; the intervals tapering gradually outwards, their outer ends convex, reaching close to the border.—Ralfs in *Pritch. Inf.*, p. 837, pl. v. fig. 79; Cleve, *Bih. k. Sv. Vet.-Ak. Handl. Stockh.*, 1873, Bd. i. No. 13, p. 10; Sch., *Atl.*, pl. xxxviii. figs. 21–23; *Asterolampra Brookei*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 119, pl. iv. fig. 18; *Actinogramma Brookei*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 254.

Habitat.—Sea of Kamtschatka, 1700 fathoms (Bailey!); Behring Sea, 1158 fathoms (H. L. Smith!); Atlantic soundings (Roper); Santa Monica deposit; (Deby!) loc.? (Grove!).

Var. *robusta*, nov. *A. robustus*, Cstr., *Atti Accad. Pontif. nuov. Lincei*, 1875, p. 393, pl. vi. fig. 5.—Rotundato-obovate. Markings: rays sharply bigeniculate at their middle; the compartments reaching from $\frac{2}{3}$ to $\frac{1}{2}$ of radius inwards, the inner ends of those adjacent to subobsolete interval transversely truncate, of the others somewhat concave inwards; intervals broad, with sides parallel reaching the border.—*A.* (*Brookei*, var.?) *robustus*, Péragallo, *Diat. Baie Villefranche*, p. 75, pl. ii. fig. 15.

Habitat.—Mediterranean Sea (Castracane).

Asterom. Beaumontii, Ehrb., *Mon. Ber. Ak.*, 1844, p. 200, fig. 5.—Diam. ·04 mm. Centro-lateral area with sides parallel, and inner ends conical. Markings: rays sharply bigeniculate at their middle, the compartments reaching about $\frac{2}{3}$ of radius from circumference, conical, with inner ends obtusely angular; areolæ distinct, from 6 to 8 (?) in ·01 mm.; intervals between compartments attenuating gradually outwards, reaching the border.

From this species I exclude *Spatangidium heptactis*, de Bréb. (*Bull. Soc. Linn. Normand.*, 1857, p. 292, pl. iii. fig. 2); *S. ralfsianum*, Norman (not Grev.), *Quart. Jour. Micr. Sci.*, 1859, p. 161, pl. vii. figs. 7, 8); and *Asterolampra heptactis*, Grev. (*Trans. Micr. Soc. Lond.*, 1860, p. 122), which Janisch (*Abh. Schl. Ges. väter. Cult*, 1861, p. 160) has proposed to unite with it. Janisch also proposes to unite to *Asterom. Beaumontii* the forms figured by

Schmidt (*Atl.*, pl. xxxviii. figs. 6, 7), which belong rather to *Asterom. heptactis.*

Habitat.—Pancake ice (Pfankuchen Eise), Ice Barrier, Antarctic Ocean, lat. 78° 10′ S., long. 162° W. (J. D. Hooker); H.M.S. Challenger (Deby !).

Asterom. moronensis, Rattray. *Asterolampra moronensis*, Grev., *Quart. Jour. Micr. Sci.*, 1863, p. 230, pl. ix. fig. 8.—Diam. ·06 to ·075 mm. Centro-lateral area with sides at first almost parallel, and then converging rapidly so as to meet about half way between centre and apices of compartments, beyond the point of union a simple line passing to the centre. Markings: rays sharply geniculate at or slightly beyond their middle, short lateral rami passing obliquely outwards from the geniculations; the compartments reaching from $\frac{2}{3}$ to $\frac{5}{9}$ of radius inwards, their inner ends obliquely truncate, straight or slightly concave towards the centre, those adjacent to the subobsolete interval with one side much longer than the other; the areolæ obvious, decreasing outwards from 6 to 10 in ·01 mm., the oblique decussating rows straight or slightly curved at their inner ends; the intervals narrow, expanding gradually towards their outer ends, which reach close to the border, at the middle of the expanded portion a distinct radial dark subconical area.—Sch., *Atl.*, pl. xxxviii. fig. 24.

Habitat.—Moron deposits near Seville (Greville ! Hardman ! Norman !);* Santa Monica deposit (Hardman ! Deby !).

§ 3. Excentrici.

Sometimes elliptical, rarely suboblong. Clear median portion of valve sometimes markedly excentric; the centro-lateral area extending beyond the centre often subclavate, sometimes malleiform. The subobsolete ray rarely arcuate, the others straight or curved; a lunate ridge sometimes visible at outer ends of rays.

Asterom. wyville-thomsonianus. O'Me., *Jour. Lin. Soc.* (Botany), vol. xv. p. 57, pl. i. fig. 5.—Diam. ·06 mm. Central areolate area absent. Markings: rays 6, straight; the compartments 6, five equal smaller, reaching about $\frac{2}{5}$, the sixth larger reaching about $\frac{3}{5}$ of radius inwards, their inner ends uniformly convex towards the

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centre; areolæ distinct; intervals attenuating gradually outwards, reaching the border.

Habitat.—Kerguelen Island, H.M.S. Challenger (O'Meara).

Asterom. stellatus. Ralfs in *Pritch. Inf.*, p. 838.—Diam. ·045 to ·07 mm. Centro-lateral area elongate, faintly subclavate with slight median constriction. Markings: rays straight or slightly curved, springing from apex and side of centro-lateral area, sometimes dichotomous; the compartments reaching about $\frac{3}{8}$ of radius inwards, their inner ends conical, with sides slightly convex; areolæ obscure, decreasing outwards from 14 to 20 in ·01 mm., most evident near the apices of the compartments; the intervals rapidly attenuating outwards, reaching close to the border.—*Asterolampra stellata*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 124, pl. iv. fig. 20.

This species approaches *A. hiltonianus*, but is distinguished by the more straight radii, which are never geniculate; in the appearance of the centro-lateral area it comes near to *A. elegans*.

Habitat.—Indian Ocean, soundings 2200 fathoms, Captain Pullen (Greville !); *Holothuria*, China (Deby !).

Asterom. elegans. Grev., *Quart. Jour. Micr. Sci.*, 1859, p. 161, pl. vii. fig. 6.—Diam. ·075 to ·14 mm. Centro-lateral area elongate, its inner end rounded, straight or somewhat bent near the subsolete ray. Markings: rays simple, dichotomous, rarely branching, more frequently with sharp, rarely obtuse geniculations near their middle or somewhat closer to the central area; the compartments conical, their inner ends subacute; the areolæ most evident at the inner ends, elsewhere obscure, decreasing outwards from 12 to 16 in ·01 mm.; intervals narrow, attenuating outwards, their outer ends close to the border.—Ralfs in *Pritch. Inf.*, p. 837, pl. v. fig. 87.; Sch., *Atl.*, pl. xxxviii. figs. 1, 2; *Asterolampra elgans*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 118, pl. iv. fig. 16; *Actinogramma Jupiter*, Ehrb., *Abh. Ber. Ak.*, 1872, p. 392, pl. ix. fig. 3; *Ac. Venus*, Ehrb., *ibid.*, pl. ix. fig. 4; *Ac. Saturnus*, Ehrb., *ibid.*, pl. ix. fig. 5; *Ac. Sol*, *ibid.*, pl. ix. fig. 6.

Habitat.—Indian Ocean soundings, 2200 fathoms, Captain Pullen (Greville !); Californian guano (Norman !);* dredged by

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H.M.S. Challenger, lat. 5° 54′ N., long. 147° 2′ W., 2550 fathoms (Hardman!);* Indian Ocean (Ralfs); Gazelle Expedition (Janisch); S.S. Buccaneer, off Ascension Island (Grove!).

Asterom. imbricatus. Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 46, pl. ii. fig. 9.—Roundly elliptical to subcircular. Diam. ·06 to ·085 mm. Centro-lateral area clavate, widest at its exremity or somewhat nearer to the narrow end, frequently extending across and beyond the centre of the valve. Markings: rays sharply bigeniculate at their middle, the geniculations regular, forming a distinct roundly elliptical figure around the centro-lateral area; the compartments reaching about $\frac{1}{3}$ of radius inwards, their inner ends conical, with sides convex, those adjacent to the subobsolete ray with ends obliquely truncate; areolæ obscure, subpunctiform; intervals narrow, at first attenuating, then subequal in breadth, their outer ends convex outwards, reaching close to border.—Ralfs in *Pritch. Inf.*, p. 837; *Asterolampra imbricata*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 119, pl. iv. fig. 17.

Wallich distinguishes as var. β, forms with "plani sutures," *i.e.*, non-geniculate rays, and as var. γ, forms with "the capitate extremity of the basal ray," *i.e.*, of the centro-lateral area emarginate. To his former var. the name *rectiradiata* may be given; the latter is unimportant.

Habitat.—Indian Ocean soundings, 2200 fathoms, Captain Pullen (Greville!); Bay of Bengal (Wallich); Natal (Roper).

Asterom. hiltonianus. Ralfs in *Pritch. Inf.*, p. 837.—Diam. ·075 to ·135 mm. Centro-lateral area slightly constricted at its outer $\frac{1}{3}$, sometimes attenuating gradually towards its outer end. Markings: rays springing from apex and sides of centro-lateral area, straight or subuniformly arcuate, sometimes geniculate about their middle, and concave towards the subobsolete interval, rarely dichotomous opposite central end of centro-lateral area; the compartments reaching about $\frac{2}{3}$ of radius inwards, their inner ends sharply conical, those adjacent to the subobsolete interval somewhat more obtuse, with sides somewhat convex; areolæ obscure, 20 to 24 in ·01 mm., towards border resolved with difficulty; the

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intervals attenuating rapidly outwards, reaching close to the border. —*Asterolampra hiltoniana*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 117, pl. iv. fig. 15; H. L. Smith, *Diat. Spec. Typ.*, No. 49.

Habitat.—Indian Ocean soundings, 2200 fathoms, Captain Pullen (Greville ! Roper !); Algoa Bay guano (Greville !); South Pacific, 2900 fathoms (H. L. Smith !).

Asterom. flabellatus. Grev., *Quart. Jour. Micr. Sci.*, 1859, p. 160, pl. vii. figs. 4, 5.—Flabelliform, subtriangular or subcircular. Diam. ·0425 to ·06 mm., the minor axis from ·0375 to ·05 mm. Centro-lateral area subclavate, the sides more rarely almost parallel towards the central end, inner end rounded. Markings: rays straight or slightly curved; the compartments longer towards the subobsolete interval, reaching from $\frac{2}{5}$ to $\frac{2}{3}$ of radius inwards, their inner ends conical, sometimes transversely truncate; areolæ obscure; the intervals tapering slighty outwards, extending to border.—Janisch, *Abh. Schl. Ges. väter. Cult.*, 1861, p. 160; Ralfs in *Pritch. Inf.*, p. 837; Sch., *Atl.*, pl. xxxviii. figs. 10, 12; *A. flabellatus*, var. *tergestina*, Grun., Van. Heurck, *Syn. Diat. Belg.*, pl. cxxvii. figs. 5, 6; *Asterolampra flabellata*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 116; *Spatangidium flabellatum*, de Bréb., *Bull. Soc. Linn. Normand*, 1857, p. 297, pl. iii. fig. 3; *S. peltatum*, de Bréb., *ibid.*, p. 298, pl. iii. fig. 4.

Habitat.—Rembang Bay (Deby !); Peruvian guano (de Brébisson, Janisch); Campeachy Bay, Yokohama and Hong Kong (Schmidt); California guano (Greville !); Corsican algæ (de Brébisson); Teignmouth Ascidia (Grove !).

Asterom. cleveanus, Grun. Sch., *Atl.*, pl. xxxviii. figs. 13, 14.—Roundly elliptical to oval. Major axis ·045 to ·075 mm.; minor ·04 to ·0625 mm. Centro-lateral area tapering towards outer ends, the inner end angular, sometimes with sides slightly concave outwards. Markings: rays springing from apex and sides of centro-lateral area, straight or concave towards subobsolete ray, sometimes dichotomous; the compartments longest towards extremities of major axis, shortest towards the minor, reaching from $\frac{1}{2}$ to $\frac{2}{3}$ of radius inwards; their inner ends rounded or somewhat obliquely truncate; areolæ delicate, 12 to 14 in ·01 mm., the

intervals straight or slightly arcuate, of uniform breadth, their outer ends rounded close to the border, sometimes prolonged inwards as subdistinct areas on the interradial spaces.—*Asterom. wallichianus*, Cleve (not Grev.), *Bih. k. Sv. Vet.-Akad. Handl. Stockh.*, 1873, Bd. i. No. 11, p. 5, fig. 1; Cleve and Möller, *Diat.*, Nos. 145, 146.

Habitat.—Surface of Java Sea (Cleve, Schmidt); Manilla mud (Grove!); Muntok, East Indian Archipelago (Grove!).

Asterom. reticulatus. Cleve, *Bih. k. Sv. Vet.-Akad. Handl. Stockh.*, 1873, Bd. 1, No. 11, p. 5, pl. i. fig. 2.—Diam. ·051 mm. Centro-lateral area with sides uniformly concave and inner end rounded, a sharp angular bend at outer extremity of one of its sides. Markings: rays arcuate, flexuous or sharply bigeniculate at their middle; the compartments reaching about $\frac{9}{14}$ of radius inwards, their inner ends transversely truncate, that on one side of subobsolete interval convex towards centre; areolæ distinct, 7 in ·01 mm.; the intervals broad, expanding gradually outwards to their middle, and again contracting uniformly towards their outer subacute ends, not reaching the border, the subobsolete interval arcuate, concave towards that compartment, having the inner end convex.

Habitat.—Surface of Java Sea (Cleve).

Asterom. Darwinii. Ehrb., *Mon. Ber. Ak.*, 1844, p. 200, pl. (June), fig. 1.—Diam. ·0625 to ·0875 mm. Centro-lateral area short and broad, sometimes subconical, or with sides almost parallel and converging suddenly to the centre. Markings: rays sharply geniculate about their middle, with short lateral rami proceeding from the angles; the compartments few, 5, reaching from $\frac{1}{2}$ to $\frac{3}{4}$ of radius inwards, of unequal length, their inner ends transversely truncate, those bordering the subobsolete interval with the inner ends more oblique; the areolæ decreasing gradually outwards from 8 to 12 in ·01 mm.; the intervals tapering ontwards, their outer ends rounded, reaching close to the border.—Ralfs in *Pritch. Inf.*, p. 837, pl. v. fig. 86; Sch., *Atl.*, pl. xxxviii. fig. 16; *A. Rossii*, Ehrb., *Mon. Ber. Ak.*, 1844, p. 200, pl. (June), fig. 2; *Mikrog.*, pl. xxxv. A. 21. fig. 4; *A. Brookei*, Grun. (not Bail.), Sch., *Atl.*, pl. xxxviii. fig. 9; *Asterolampra Darwinii*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 116, pl. iv. figs. 12, 13.

Habitat.—Monterey stone (Arnott !* Kitton !);* Antarctic Ocean, lat. 78° 10′ S., long. 162° W. (Ehrenberg, Ralfs); Santa Monica deposit (Hardman !)†.

Asterom. rarus, Rattray. *A. elegans*, Grev. var. Wallich., *Trans. Micr. Soc. Lond.*, 1860, p. 46, pl. ii. fig. 10.—Diam. ·0525 mm. Centro-lateral area distinct, extending over $\frac{5}{7}$ of disc, its extremity conical, the sides parallel, with a wide deep lateral conical indentation. Markings: rays of two kinds—one straight opposite the centro-lateral area, the others sharply bigeniculate at their middle; the compartments symmetrical with respect to the diameter corresponding to the subobsolete interval, of unequal length, their inner ends obtusely rounded, that opposite the subobsolete interval most obtuse; areolæ distinct, 6 in ·01 mm.; the intervals tapering outwards, reaching the border.

Habitat.—Salpœ, Indian Ocean (Wallich).

Asterom. heptactis. Ralfs. in *Pritch. Inf.*, p. 838, pl. viii. fig. 21. —Diam. ·0425 to ·175 mm. Centro-lateral area subclavate, the sides slightly sinuate, sometimes almost parallel, the inner end conical. Markings: rays sharply, 1- or 2- geniculate at or slightly beyond their middle; delicate lines traceable from the geniculations to the angles of the compartments; the compartments sometimes of unequal lengths, but symmetrical with respect to the diameter corresponding to centro-lateral area, reaching from $\frac{1}{2}$ to $\frac{2}{3}$ of radius inwards, their inner ends transversely truncate or slightly concave towards centre; areolæ delicate, 6 in ·01 mm.; the rows bounding the compartments obvious; intervals broad, a distinct lunate ridge at their outer ends.—*Spatangidium heptactis*, de Bréb., *Bull. Soc. Linn. Normand.*, 1857, p. 296, pl. iii. fig. 2; *S. ralfsianum*, Norman, *Quart. Jour. Micr. Sci.*, 1859, p. 161, pl. vii. figs. 7, 8; *Asterolampra heptactis*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 122; *Asterom. ralfsianus*, Grun., Sch., *Atl.*, pl. xxxviii. figs. 5–8 (excl. *Asterom. Beaumontii*, Ehrb., *Mon. Ber. Ak.*, 1844, p. 200).

Habitat. — Californian guano (Greville !); Peruvian guano (Grove ! de Brébisson, Schmidt, Greville !); Ichaboe guano (J.

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† In the collection of Mr Julien Deby.

T. Norman !); Atlantic soundings (Ralfs); Gazelle Expedition, Yokohama (Schmidt); Pabellan di Pico guano (Deby !); *Holothuria*, China (Deby !); Færoe Islands, H.M.S. Knight Errant (Grove !); loc.? (Grove !).

Asterom. arachne. *Spatangidium arachne*, de Bréb., *Bull. Soc. Linn. Normand.*, 1857, p. 296, pl. iii. fig. 1.—Broadly ovate to subcircular. Diam. ·045 to ·06 mm. Markings: rays 5, the central subobsolete longest, its proximal end expanded, malleiform, reaching between proximal ends of lateral rays, excentric; the lateral rays in two unsymmetrical pairs, the lower pair substraight, or slightly convex towards the central ray, the upper pair more curved in their proximal portions and more convex towards the lower; their inner ends expanded but more rounded than that of central ray; their outer ends sometimes slightly swollen and not reaching the border; compartments of unequal length, their inner ends convex, that opposite the subobsolete ray concave inwards, the areolæ decreasing but slightly outwards, 6 to 7 in ·01 mm.; rows evident, those adjacent to the rays somewhat more prominent. Border narrow, hyaline.—Ralfs in *Pritch. Inf.*, p. 837; Sch., *Atl.*, pl. xxxviii. figs. 3, 4; *Asterom. malleus*, Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 47, pl. ii. fig. 11; *Asterom. malleiformis*, Wallich, *ibid.*, Explan. pl. ii. fig. 11; *Asterolampra arachne*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 123; *Excentron cancroides*, Ralfs, *ibid.*, p. 837.

Habitat.—Peruvian guano (Greville ! Grove !); Ichaboe guano (Norman !); Indian Ocean soundings, 2200 fathoms, by Capt. Pullen (Greville !); locality ? (Dickie !); from *Salpæ*, Indian Ocean (Wallich); Guanape guano (Deby !); Arica, and Gazelle Expedition (Schmidt); S.S. Buccaneer, off Ascension Island (Grove !).

Asterom. nankoorensis. Grun., *Reise d. Novara*, 1870, p. 104, pl. i. A. fig. 22.—Oval to subcircular. Length, ·065 to ·075 mm.; breadth, ·0625 to ·065 mm. Centro-lateral area with sides concave outwards, the inner end conical. Markings excentrically disposed; rays—one more robust, arcuate, proceeding from apex of centro-lateral area, a few others more delicate; the compartments of unequal lengths, their inner ends concave towards centre; areolæ

delicate; intervals expanding gradually outwards, and reaching border; their inner ends uniformly curved away from one another, closed, and continued almost to the angles of the centro-lateral area; opposite this area and at middle of largest compartments 5 short rays, the two lateral longest, attenuating outwards, and with the inner ends slightly swollen and knob-like, the subobsolete interval nearer one of the larger intervals than the other, not reaching the border.

Allied to *A. arachne.*

Habitat.—Nancoori deposit (Grunow).

Asterom. sarcophagus. Wallich, *Trans. Micr. Soc. Lond.*, 1860, p. 47, pl. ii. fig. 12.—Subregularly oblong, the sides slightly concave about their middle, the concavities of those adjacent to the subobsolete interval greater than the others. Length ·045, breadth ·0225 mm. Centro-lateral area expanding gradually outwards. Markings: rays 6, subuniformly arcuate, the three from each side uniting in two excentric points, which are connected by a short transverse line at right angles to the major axis; the compartments reaching from $\frac{1}{2}$ to $\frac{2}{3}$ of radius inwards, unequal, but symmetrical with respect to the major axis, their inner ends convex; areolæ distinct, $4\frac{1}{2}$ (?) in ·01 mm., in obscure radial rows; the intervals attenuating outwards, their outer ends convex outwards, close to the border.—*Asterolampra sarcophagus*, Grev., *Trans. Micr. Soc. Lond.*, 1860, p. 124.

Distinguished from *A. arachne*, its nearest ally, by its outline, and the different character of its rays and compartments.

Habitat.—Indian Ocean (Wallich).

The following species from Peruvian guano have been founded on the number of the rays on the central portion of the valve, and cannot be retained:—

A. denarius. Janisch. (*Abh. Schl. Ges. väter. Cult.*, 1860, p. 160, pl. ii. B. fig. 22—unpublished). Oval. Diam. ·045 mm. Rays 10, straight. Areolæ small.

A. Brébisonii. Janisch (*ibid.*, 1861, p. 160, pl. ii. B. fig. 28—unpublished). Rays 12, straight. Areolæ small, on compartments adjacent to subobsolete interval.

A. Pringsheimii. Janisch (*ibid.*, 1861, p. 160, pl. ii. B. fig. 25

—unpublished). Diam. ·07 mm. Rays 14, zig-zag. Areolæ small.

A. Cohnii. Janisch (*ibid.*, 1860, p. 160, pl. ii. B. fig. 26—unpublished). Diam. ·0805 mm. Rays 15, zig-zag. Areolæ small. *A. cleveanus*, Janisch, is erroneously mentioned in Habirshaw's *Cat. Diat.*, § *Asteromphalus*, as found in *Abh. Schl. Ges. väter. Cult.*, 1860, p. 160, pl. ii. B. fig. 26. The name given in the paper quoted being *A. Cohnii.*

A. Ehrenbergii. Janisch. (*ibid.*, 1861, p. 161, pl. ii. B. fig. 27—unpublished). Almost circular. Diam. ·095 mm. Rays 16, zig-zag-shaped. Areolæ small.

A. Braunii. Janisch. (*ibid.*, 1861, p. 161, pl. ii. B. fig. 28—unpublished). Diam. ·105 mm. Rays 17, zig-zag. Areolæ small.

Artificial Key.

1.	Outline flabelliform to subtriangular, oval or elliptical,	2.
	Outline circular or subcircular,	3.
	Outline subregularly oblong, with sides slightly concave at their middle,	*sarcophagus.*
2.	Compartments longest adjacent to subobsolete interval, decreasing away from this; rays simple; areolæ obscure,	*flabellatus.*
	Compartments shortest at ends of minor axis; rays sometimes dichotomous; the areolæ delicate, 12 to 14 in ·01 mm.,	*cleveanus.*
3.	Rays simple,	4.
	Rays sharply bigeniculate,	5.
4.	Intervals between compartments prolonged to centre; outer ends of rays knob-like, penetrating a short distance into compartments,	*centraster.*
	No such structure,	6.
6.	Inner ends of compartments concave towards centre; centro-lateral area acutely V-shaped; subobsolete interval rapidly attenuating outwards,	*wallichianus.*
	Inner ends of compartments obliquely truncate on each side of the rays; centro-lateral area sometimes with inner ends obtuse,	*variabilis.*
	Inner ends of compartments rounded,	7.
	Inner ends of compartments more conical,	8.
	Inner ends of compartments concave towards centre; centro-lateral area with sides concave outwards; one ray opposite this area more robust than the others; intervals expanding outwards,	*nankoorensis.*
7.	Structure markedly excentric; central area opposite subobsolete interval, maleiform, extending across the hyaline portion of valve,	*arachne.*
	Structure not markedly excentric,	9.

9.	Compartments 6, 5 equal smaller reaching to $\frac{2}{5}$ of radius, the sixth larger, reaching to $\frac{3}{5}$ of radius,	*wyville-thomsonianus.*
	Compartments otherwise, ...	10.
10.	Centro-lateral area ovate; rays straight or slightly flexed; intervals not reaching border, . . .	*shadboltianus.*
	Centro-lateral area with sides more parallel, slightly concave outwards; intervals attenuating slightly reaching border, ...	*Hookerii.*
8.	Compartments reaching about $\frac{3}{5}$ of radius inwards, longest at sides of subobsolete intervals; rays straight; intervals rapidly attenuating outwards, .	*stellatus.*
	Compartments reaching $\frac{2}{3}$ of radius inwards; rays towards subobsolete interval simply flexed, . .	*hiltonianus.*
5.	Centro-lateral area sharply constricted at middle, thence expanding markedly outwards, compartments reaching to $\frac{2}{3}$ of radius inwards; the inner ends transversely truncate, ...	*roperianus.*
	Centro-lateral area otherwise, ...	11.
11.	Outer ends of intervals expanded, a distinct dark area at middle of wider portion, ...	*moronensis.*
	No such intervals, ...	12.
12.	Structure not markedly excentric, ...	13.
	Structure markedly excentric, ...	14.
13.	Compartments with inner ends conical, . . .	15.
	Compartments with inner ends obtusely angular; intervals attenuating outwards, ...	*Beaumontii.*
	Compartments with inner ends obtusely rounded or truncate; rays sometimes simple, sometimes wavy; geniculations not regular, ...	*Brookei.*
15.	Rays often dichotomous, bigeniculate at or within their middle; centro-lateral area elongate, . .	*elegans.*
	Rays simple; the geniculations regular, forming an elliptical figure round the centro-lateral area. Centro-lateral area clavate, ...	*imbricatus.*
14.	Subobsolete ray distinctly curved, ...	*reticulatus.*
	Subobsolete ray straight, ...	16.
16.	Centro-lateral area with sides parallel, and showing a deep regular median indentation, ...	*rarus.*
	No such indentation, ...	17.
17.	Centro-lateral area short, broad, or subconical; compartments few, large; rays geniculate about their middle. No lunate ridge, ...	*Darwinii.*
	Centro-lateral area subclavate or with sides slightly sinuate to almost parallel; rays geniculate at or beyond their middle; a lunate ridge frequently present at outer ends of intervals, ...	*heptactis.*

LIRADISCUS.

Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 4.—Circular, subcircular, or elliptical. Surface slightly convex or dome-shaped, flatter towards the border. Colour pale grey. Central space absent. Markings consisting of evident or more delicate lines, anastomosing irregularly or forming subregular or unequal areolæ, on a more or less irregular

band adjacent to the border subradial, with few anastomoses or lateral rami, and sometimes dichotomous; apiculi at the angles of the meshes sometimes distinct. Border narrow, hyaline, more rarely broad, with evident striæ.

§ 1. Circulares.

Outline circular.

L. furcatus. Grove, MS.—Circular. Diam. ·0875 to ·095 mm. Surface slightly convex. Markings prominent, areolæ at centre few, large, unequal, sometimes triramose, and reaching ·06 mm. in length, mostly 1 to 1½ in ·01 mm.; adjacent to border the lines straight or curved, radial, or subradial, frequently dichotomous, but without anastomoses; the areolæ hyaline, or with minute rounded granules at their centre. Border delicate, but distinct, about ·0025 mm. broad.—(Pl. III fig. 23.)

Habitat.—Marine deposit, Fiji Islands (Grove!).

L. capensis. Cleve, *Kongl. Sv. Vet.-Akad. Handl. Stockh.*, 1881, Bt. xviii. No. 5, p. 22, pl. v. fig. 61.—Circular. Diam. ·04 mm. Markings irregularly radiating or oblique, sometimes ramose, but not anastomosing lines, with large hyaline interspaces; at intervals a few rounded, elongate or irregular dots distinct. Border sharply defined; striæ obvious, 15 in ·01 mm.

Cleve places this species with some hesitation in the present genus, believing that it might be better to range it in *Cyclotella.* The relationships, however, which he points out with *Cy. striata,* Kütz. (Van Heurck, *Syn. Diat. Belg.*, pl. xcii. figs. 6–8), and *Cy. dallasiana,* W. Sm., are remote, whilst the general aspect of the lines on the surface is liradiscoid.

Habitat.—Cape of Good Hope (F. Hauck).

L. barbadensis. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 5, pl. i. fig. 14.—Diam. ·05 to ·105 mm. Surface slightly convex for ½ to ⅚ of radius, beyond this almost flat to the border. Markings evident areolæ, from 1 to 3 in ·01 mm., sometimes obtusely angular, the band adjacent to the border subregular.

Habitat.—Cambridge deposit, Barbados (Johnson!); "Barbados" (Greville! Johnson!).

§ 2. ELLIPTICI.

Outline roundly or elongately elliptical.

(α) Elongately elliptical.

L. ellipticus. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 99, pl. viii. fig. 6.—Major axis ·0775 to ·105 mm., from 2 to $2\frac{5}{8}$ times minor, the extremities of the major axis acute. Surface but slightly convex. Markings delicate; areolæ 2 to $3\frac{1}{2}$ in ·01 mm.; the band adjacent to the border narrow indistinct, with the subradial lines 4 to $4\frac{1}{2}$ in ·01 mm.

Habitat.—Cambridge deposit, Barbados (Johnson!); "Barbados" (Greville! Johnson!).

(β) Roundly elliptical.

L. oblongus, Grun. Cleve and Möller, *Diat.*, No. 276.—Major axis ·04 to ·05 mm., about twice the minor; the extremities obtuse. Surface slightly convex. Markings delicate; areolæ 4 in ·01 mm.; subequal or slightly smaller adjacent to the border, without order. Border narrow, sharply defined.

Habitat.—California (Cleve and Möller!).

L. ovalis. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 5, pl. i. figs. 15, 16.—Major axis ·04 to ·06 mm., from $1\frac{5}{7}$ to $1\frac{5}{11}$ times minor. Surface markedly convex. Markings prominent; the areolæ towards the centre sometimes imperfect; the band adjacent to the border irregular, narrow; apiculi irregular, inserted at the angles of the areolæ.

Greville represents the girdle as a narrow hyaline band extending for a short distance beyond the convex portion of the valve.

Habitat.—Cambridge deposit, Barbados (Johnson!); Oamaru deposit (Grove!).

L. marginatus. Grove MS.—Major axis ·0475 mm., about $1\frac{1}{4}$ times minor; extremities obtuse. Surface slightly convex. Markings robust, areolate; the areolæ irregular and unequal, 2 to 3 in ·01 mm., largest at the centre bearing a few faint rounded granules; marginal band distinct, its outer edge crenate. Border narrow, hyaline.—(Pl. III. fig. 13.)

Habitat.—Oamaru deposit (Grove!).

L. minutus. Grev. (*Trans. Micr. Soc. Lond.*, 1865, p. 47, pl. v. fig. 6), belongs to *Cresswellia.* The surface of the type is dome-shaped, and the markings on the central portion regular, and areolæ 3½ to 4 in ·01 mm.

Artificial Key.

1.	Outline circular,	2.
	Outline elliptical,	3.
2.	Lines not anastomosing. Border broad, striæ 15 in ·01 mm.,	*capensis.*
	Lines anastomosing,	4.
4.	Areolæ unequal, 1 to 3 in ·01 mm. Band adjacent to border subregular,	*barbadensis.*
	Areolæ unequal, few; around border the lines radial or subradial, often dichotomous,	*furcatus.*
3.	Extremities of major axis acute. Surface slightly convex. Markings delicate, areolæ 2 to 3½ in ·01 mm. Band adjacent to border narrow,	*ellipticus.*
	Extremities of major axis not acute,	5.
5.	Surface slightly convex,	6.
	Surface markedly convex. Markings prominent, areolæ often imperfect at centre; apiculi acicular, irregular, inserted at angles of areolæ,	*ovalis.*
6.	Markings delicate, 4 in ·01 mm., subequal; non-apiculate. Border narrow, sharply defined,	*oblongus.*
	Markings evident; marginal band of areolæ distinct, its outer edge crenate; areolæ unequal, irregular, 2 to 3 in ·01 mm., largest at the centre,	*marginatus.*

PORODISCUS.

Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 63.—Valves elliptical, circular or rhombic; sometimes the opposite valves of a frustule of unequal sizes. Surface slightly convex, dome-shaped or conical, with transversely truncated ends. Colour pale smoky grey. Central space circular to roundly elliptical, faintly punctate or hyaline, its outline smooth or finely crenulate. Markings small, round, granular, papilliform, or areolate; rows radial, more rarely inconspicuous or undifferentiated, secondary oblique rows sometimes evident; fasciculi frequently distinct; interspaces largest near the central space, sometimes absent; spines long, acicular or hour-glass-shaped, frequent; a sharply defined marginal band rare. Border inconspicuous.—*Craspedodiscus, pro parte*, Grun.; Sch., *Atl.*, pl. lxvi. figs. 7–9; *Craspedoporus, pro parte*, Grove and Sturt, *Jour. Quek. Micr. Cl.*, 1887, p. 67.

P. splendidus. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 46,

pl. v. fig. 5.—Circular, sometimes roundly elliptical. Diam. ·075 mm. Surface convex, forming a low dome. Central space circular, about ·015 mm. broad, sharply defined, hyaline. Markings large, areolate, increasing slightly to about semiradius, thence decreasing similarly to the border; around central space $4\frac{1}{2}$, at semiradius 3 to $3\frac{1}{2}$ in ·01 mm.; rows radial, straight; secondary oblique rows inconspicuous. Border inconspicuous.

Habitat.—Springfield deposit, Barbados (Hardman).

Var. *marginata*, nov. *Craspedodiscus ovalis*, Grun., in Sch., *Atl.*, pl. lxvi. fig. 6.—Roundly elliptical. Major axis ·065 mm., about $1\frac{1}{4}$ times minor. Central space with small round, free granules. Markings areolate and subequal, 4 in ·01 mm. for about $\frac{4}{5}$ of radius, on a distinct band adjacent to the border, round, granular, 8 in ·01 mm.; interspaces between radial rows evident only on a band adjacent to border; secondary oblique decussating rows more evident.—*Porodiscus splendidus*, var.? Sch., *ibid.*

Habitat.—Sprinfield deposit (Schmidt).

P. nitidus. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 65, pl. iv. fig. 4.—Circular or subcircular. Diam. ·05 to ·07 mm. Surface uniformly and moderately convex. Central space circular or roundly elliptical, hyaline, ·0075 mm. broad. Markings areolate, rarely obtusely angular towards the central space, increasing for a short distance outwards from this space, thence decreasing gradually to the border; towards the central space $4\frac{1}{2}$, near the border 8, in ·01 mm.; rows radial, straight, non-fasciculate. Border narrow.

The markings being areolate, there are no such hyaline interspaces as are shown in Greville's figure. Sometimes faint fasciculi are observed on one valve of a frustule, the opposite valve being non-fasciculate.

Habitat.—Cambridge deposit, Barbados (Johnson !).

Var. *armata*, nov.—Diam. ·0525 to ·095 mm. Central space circular, ·0075 mm. broad. Markings sometimes forming coarse moniliform striæ towards the border; spines acicular, about ·01 mm. long, sometimes shorter, inserted about $\frac{3}{4}$ of radius from centre; interfasciculate rarely a few at irregular intervals nearer the border. Girdle ·0125 mm. broad in a valve ·0525 mm. in diam., the

hyaline striæ at right angles to its edge undifferentiated.—(Pl. III. fig. 17.)

A specimen occurs in Dr Greville's collection in the British Museum, labelled *P. conicus*, and another labelled *P. major*. From both of these the present var. is quite distinct.

Habitat.—"Barbados" (Johnson! Greville!); Cambridge deposit, Barbados (Johnson!).

P. major. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 64, pl. iv. fig. 2.—Fragmentary. Diam. ? Surface slightly convex. Central space subcircular, ·0175 mm. broad, bearing almost invisible minute puncta, and a large round more distinct slightly excentric granule, its outer edge minutely crenate. Markings small, round, granular; towards the central space 8, nearer the border 10, in ·01 mm.; rows radial, straight, in faint fasciculi, most originating at about ·015 mm. from the central space, the others proceeding from the edge of this space; interspaces at origin of shorter rows hyaline.

Habitat.—Cambridge deposit, Barbados (Greville!).

Var. *densa*, nov. *P. major*, Grev., *ibid.*, 1865, p. 46.—Diam. ? Central space circular to oval, ·0075 to ·01 mm. broad, its outer edge smooth. Markings subareolate, near the central space $5\frac{1}{2}$ to 6 in ·01 mm.; interspaces around the central space more minute. —(Pl. III. fig. 21.)

The central space and markings at once distinguish this var.

Habitat.—Cambridge deposit, Barbados (Johnson!); "Barbados" (Greville!).

P. elegans. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 65, pl. iv. fig. 1.—Circular. Diam. ·0625 to ·095 mm. Surface rounded and dome-shaped. Central space circular, ·0075 mm. broad, sharply defined, hyaline. Markings obtusely angular or subareolate, decreasing gradually from the central space outwards, around the central space 6, near the border 10 to 12, in ·01 mm.; rows radial, straight; fasciculi distinct; interspaces minute, largest around the central space. Girdle cylindrical, ·03 mm. broad, in a frustule, ·06 mm. in diam.; a narrow hyaline band at each extremity;

the interval minutely punctate; at subregular intervals narrow hyaline straight lines at right angles to the edges of the valve.

The fasciculi are bounded by two adjacent radial rows, somewhat more conspicuous than the intervening rows. In one of the valves in Greville's collection it is possible to trace downwards from the central space a cylindrical siliceous tube which is of sufficient length to have passed to a plane corresponding in position to the edges of the valve.

Habitat.—Cambridge deposit, Barbados (Greville! Johnson!); "Barbados" (Greville! Johnson!).

P. spiniferus, sp. n.—Circular. Diam. ·0875 mm. Surface dome-shaped. Central space circular, ·0075 mm. broad. Markings areolate, subequal, 7 in ·01 mm.; rows radial, straight; fasciculi evident; bounded by two rows of more prominent submuriform areolæ; spines robust, conical, about ·03 mm. long, interfasciculate, forming a circlet at about $\frac{1}{3}$ of distance between central space and edge of valve. Girdle cylindrical, ·0375 mm. broad; a narrow hyaline band at each extremity; the clear substraight lines at right angles to its edges distinct.—(Pl. III. fig. 19.)

Habitat.—Cambridge deposit, Barbados (Johnson!).

P. oblongus. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 63, pl. iv. fig. 5.—Subacutely elliptical. Major axis ·05 mm. long, about $2\frac{1}{2}$ times minor. Surface sloping gradually downwards from edge of central space. Central space roundly elliptical, with major axis corresponding in direction to minor axis of valve. Markings angular, decreasing regularly and somewhat rapidly from central space to border; around the central space $4\frac{1}{2}$, at border 10, in ·01 mm.; rows radial, substraight. Border narrow, hyaline.—*P. ovalis*, Grev., *ibid.*; Explan. pl. iv. fig. 5; *Craspedodiscus oblongus*, Grun.; Sch., *Atl.*, pl. lxvi. figs. 7–9.

This species approaches in appearance *Coscinodiscus oblongus*, Grev. (*Trans. Micr. Soc. Lond.*, 1866, p. 4, pl. i. figs. 9, 10).

Habitat.—Barbados deposit (Johnson!).

P. Stolterfothii. Cstr., *Diat. Chall. Exped.*, 1886, p. 139, pl. xii. fig. 8.—Rhombic, with angles obtuse. Major axis ·077 mm. long, about $1\frac{7}{10}$ times minor. Surface slightly convex towards

centre, towards the border subplain. Central space roundly elliptical, with major axis about ·01 mm. long, and corresponding in direction to minor axis of valve, delicately punctate. Markings areolate, gradually increasing from central space outwards; towards the central space 6 or 6½, towards the border 4 to 4½, in ·01 mm.; rows radial, straight; secondary oblique decussating rows indistinct. Border narrow, hyaline.

Habitat.—Pacific Ocean, from a sounding made at the equator by H.M.S. Challenger (Castracane).

P. conicus. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 65, pl. iv. fig. 3.—Diam. ·025 to ·0525 mm. Major axis of frustule from ·0625 to ·0875 mm. Surface a more or less elongate regular cone, transversely truncate at the extremities, the opposite valves of a frustule of unequal height. Central space? Markings obtusely angular or subareolate, 6 in ·01 mm., subequal; rows radial, straight, non-fasciculate; secondary oblique decussating rows evident, from the truncated ends of the cone a few short tapering clear lines, distinct. Girdle cylindrical, from ·025 to ·0325 mm. broad; a narrow band at each extremity, hyaline, the intervening portion clouded with diffuse parallel lines.

Habitat. — Cambridge deposit, Barbados (Johnson!); Barbados (Johnson! Greville!); Bridgewater deposit, Barbados (Johnson!!).

P. hirsutus. Grove and Sturt, *Jour. Quek. Micr. Cl.*, 1887, p. 143, pl. xiv. fig. 54.—Circular. Diam. ·075 to ·0875 mm. Surface flat from central space to the sharply-defined marginal band, the latter sloping gently to the border. Central space circular, sharply defined, ¼ to ⅕ of diam. broad, surrounded by a narrow, more hyaline, sometimes interrupted band, with irregular outer edge. Markings rounded, prominent papillæ, with hyaline interspaces and without order; processes hour-glass-shaped, at subregular intervals, inserted on inner edge of marginal band, between these processes delicate radial striæ, 6 to 8 in ·01 mm., extending outwards to about middle of band; adjacent to border a circlet of evident papillæ at intervals of ·0075 to ·01 mm. Border narrow, bearing minute granules, 6 in ·01 mm.

This species approaches *Melosira sulcata forma coronata*, Grun. (Van Heurck, *Syn. Diat. Belg.*, pl. xci. fig. 24).

Habitat.—Oamaru deposit (Grove and Sturt!).

P. interruptus. Grove and Sturt (*Jour. Quek. Micr. Cl.*, 1887, p. 67, pl. v. fig. 8) has been found by Mr H. Morland (*Jour. Quek. Micr. Cl.*, 1887, p. 167) to be the opposite valve of *Craspedoporus elegans*, Grove and Sturt (*ibid.*, 1887, p. 64, pl. v. fig. 6; Rattray, *Jour. Roy. Micr. Soc.*, 1888, p. 919).

Artificial Key.

1.	Valves irregularly conical, with transversely truncated extremities, the opposite valves of a frustule of unequal heights,	*conicus.*
	Valves elliptical. Markings decreasing rapidly outwards, around central space 4½, at border 10, in ·01 mm.,	*oblongus.*
	Valves rhombic. Markings increasing gradually outwards, towards central space 6 to 6½, towards border 4 to 4½ in ·01 mm.,	*Stolterfothii.*
	Valves circular or subcircular,	2.
2.	Surface slightly convex; interspaces large around central space,	*major.*
	Surface almost flat to marginal band. Markings papilliform. Processes hour-glass-shaped,	*hirsutus.*
	Surface dome-shaped,	3.
3.	Markings distinctly fasciculate,	4.
	Markings non-fasciculate,	5.
4.	Spines robust, conical, interfasciculate, forming a circlet at about ⅓ of distance between central space and edge of valve. Markings areolate, 7 in ·01 mm.,	*spiniferus.*
	Spines absent. Markings obtusely angular or subareolate; around central space 6, near border 10 to 12, in ·01 mm.,	*elegans.*
5.	Markings areolate, increasing slightly to about semiradius, thence decreasing similarly to border; around central space 4½, at semiradius 3 to 3½, in ·01 mm. Spines absent,	*splendidus.*
	Markings towards central space 4½, near border 8, in ·01 mm., sometimes with long spines,	*nitidus.*

THAUMATONEMA.

Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 76.—Concatenate, discoid. Surface flat, or rising but slightly from centre for ½ to ⅔ of radius, thence sloping steeply downwards to edge of girdle and slightly concave at middle of outer portion. Colour

pale grey, the processes subhyaline. Markings punctiform or areolate, forming evident radial rows or striæ; radial costæ at subregular intervals, sometimes distinct. Process single, springing from centre of valve, proximal portion nodular or elongated and columnar, distal portion biramose, the rami equal, diverging symmetrically, their outer ends swollen and knob-like, rounded or elliptical.

This genus forms the transition between the circular forms of the Diatomaceæ and the armed *Chætocerotidæ*. Apart from the process, the valves approach *Coscinodiscus*, and the nodular proximal portion of that of *Thaumatonema costatum* is but a greater development of, and so homologous with, the nodule of *Coscinodiscus nodulifer*, this development being still more marked in *T. barbadense*.

T. barbadense. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 76, pl. v. fig. 26.—Circular. Diam. ·03 to ·04 mm.; height of central portion of valve above edge of girdle ·075 to ·1 mm. Surface flat to about $\frac{2}{3}$ of radius. Markings punctiform, closely arranged in evident striæ, the striæ 6 to $6\frac{1}{2}$ in ·01 mm. Process ·0185 to ·0315 mm. long, with proximal portion columnar, the rami of the upper portion stout, distance between the extremities of the rami ·0135 to ·0185 mm.

Habitat.—Cambridge deposit, Barbados (Johnson !); "Barbados" (Johnson !).

T. costatum. Grev., *Trans. Micr. Soc. Lond.*, 1865, p. 97, pl. viii. fig. 3.—Fragmentary. Roundly elliptical. Major axis ·055 mm., about $1\frac{1}{5}$ times minor. Surface flat to about semiradius. Markings areolate, increasing slightly to about semiradius, thence decreasing gradually and becoming more faint to border; at semiradius 5, at border 8 to 9, in ·01 mm., at subregular intervals of about ·0125 mm., radiating evident costæ. Process evident, the proximal median portion nodular, the diverging rami more delicate, straight, their outer ends elliptical, knob-like; length of rami including terminal knob ·0175 mm., major axis of knob ·01 mm., about $2\frac{1}{2}$ times minor.

Habitat.—Cambridge deposit, Barbados (Johnson !).

Artificial Key.

Markings punctiform. No costæ. Proximal portion of process columuar, ...	*barbadense.*
Markings areolate. Evident radial costæ. Proximal portion of process nodular, ...	*costatum.*

PEPONIA.

Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 75.—Central portion roundly elliptical, rarely subquadrate, with obtuse angles and convex sides, opposite the extremities of the minor axis a small regular cone, with free end rounded. Surface subplain. Colour pale grey. Central space absent. Markings areolate, sometimes increasing slightly from centre to semiradius, and again decreasing to border; between the lateral cones and the central portion a narrow hyaline band, at the extremities of the cones a small round, hyaline area. Border narrow, hyaline.

P. barbadensis. Grev., *Trans. Micr. Soc. Lond.*, 1863, p. 76, pl. v. fig. 25.—Central portion with major axis from ·0375 to ·075 mm.; distance between apices of lateral cones ·0475 to ·0925 mm. Markings towards the centre 4, at the semiradius $3\frac{1}{2}$, at the border 8, in ·01 mm., without order or in inconspicuous radial and short oblique rows; on the lateral cones decreasing towards their apices, sometimes absent.

Habitat.—Bridgewater deposit, Barbados (Johnson !); Cambridge deposit, Barbados (Johnson ! Greville); "Barbados" (Johnson ! Greville !).

EXPLANATION OF PLATES.

Plate I.

Fig. 1. *Coscinodiscus oamaruensis*, Grove and Sturt. × 660.
Fig. 2. ,, *oculus-iridis*, var. *loculifera*, nov. × 660.
Fig. 3. ,, *modestus*, sp. n. × 660.
Fig. 4. ,, *debilis*, sp. n. × 460.
Fig. 5. ,, *imperator*, Janisch. × 250.
Fig. 6. ,, *interlineatus*, sp. n. × 660.
Fig. 7. ,, *decussatus*, Grove and Sturt MS. × 660.
Fig. 8. ,, *subnotabilis*, sp. n. × 660.
Fig. 9. ,, *gracilentus*, sp. n. × 660.
Fig. 10. ,, *subareolatus*, sp. n. × 660 (worn specimen).
Fig. 11. ,, *groveanus*, sp. n. × 460.
Fig. 12. ,, *antediluvianus*, sp. n. × 660.
Fig. 13. ,, *intermixtus*, sp. n. × 660.
Fig. 14. ,, *obliquus*, Rattray. × 660.
Fig. 15. ,, *sphæroidalis*, sp. n. × 660.
Fig. 16. ,, *subtilis*, var. *lineolata*, nov. × 660.
Fig. 17. ,, *inæquisculptus*, sp. n. × 460.
Fig. 18. ,, *luxuriosus*, sp. n. × 660.
Fig. 19. ,, *glaberrimus*, sp. n. × 660.
Fig. 20. ,, *argus*, var. *subtraducens*, nov. × 460.
Fig. 21. ,, *nitidus*, var. *moronensis*, Grun. MS. × 660.
Fig. 22. ,, *planiusculus*, sp. n. × 660.
Fig. 23. ,, *granulosus*, Grun. × 460.
Fig. 24. ,, *whampoensis*, Grove MS. × 660.
Fig. 25. ,, *Weissflogii*, Sch. × 660.

Plate II.

Fig. 1. *Coscinodiscus pulcherrimus*, sp. n. × 660.
Fig. 2. ,, *lutescens*, sp. n. × 660.
Fig. 3. ,, *leptopus*, var. *discrepans*, nov. × 660.
Fig. 4. ,, *africanus*, var. *wallichiana*, Grun. × 660.
Fig. 5. ,, *minutellus*, sp. n. × 660.
Fig. 6. ,, *notabilis*, sp. n. × 660.
Fig. 7. ,, *actinosus*, Grove MS. × 660.
Fig. 8. ,, *aethes*, sp. n. × 660.
Fig. 9. ,, *densus*, Grove and Sturt MS. × 660.
Fig. 10. ,, *pusillus*, Grove MS. × 660.
Fig. 11. ,, *antimimos*, sp. n. × 660.
Fig. 12. ,, *grayianus*, sp. n. × 660.
Fig. 13. ,, *megacentrum*, Grove MS. × 660.
Fig. 14. ,, *epiphanes*, sp. n. × 660.
Fig. 15. ,, *superbus*, var. *nova-zealandica*, Grove MS. × 660.
Fig. 16. ,, *moronensis*, Johnson. × 660.
Fig. 17. ,, *vetustissimus*, Pant. × 660.
Fig. 18. ,, *subnotabilis*, sp. n. × 660.
Fig. 19. ,, *theskelos*, sp. n. × 660.
Fig. 20. ,, *galapagensis*, Rattray. × 660.

PLATE III.

Fig. 1. *Coscinodiscus implicatus*, sp. n. × 330.
Figs. 2. ,, *prætor*, Grove MS. × 660 (central portion)
Fig. 3. ,, *prætor*, Grove MS. × 660 (periphery).
Fig. 4. ,, *polurrhaptos*, sp. n. × 660.
Fig. 5. ,, *partitus*, Grove and Sturt MS. × 660.
Fig. 6. ,, *subtilis*, var. *scabra*, nov. × 660.
Fig. 7. ,, *excavatus*, var. *deliquescens*, nov. × 660.
Figs. 8. ,, *luctuosus*, Grove MS. × 660.
Fig. 9. ,, *luctuosus*, Grove MS. × 660 (zonal aspect).
Fig. 10. ,, *lewisianus*, var. *similis*, nov. × 660.
Fig. 11. ,, *implicatus*, var. *picturata*, nov. × 460.
Fig. 12. ,, *perikompsos*, var. *curta*, nov. × 660.
Fig. 13. *Liradiscus marginatus*, Grove MS. × 660.
Fig. 14. *Brightwellia coronata*, var. *radians*, nov. × 460.
Fig. 15. *Actinogonium multiradiatum*, sp. n. × 660.
Fig. 16. *Brightwellia excellens*, sp. n. × 660.
Fig. 17. *Porodiscus nitidus*, var. *armata*, nov. × 660.
Fig. 18. *Asterolampra tenerrima*, nov. × ?
Fig. 19. *Porodiscus spiniferus*, sp. n. × 330.
Fig. 20. *Asterolampra tenerrima*, nov. (small form). × ?
Fig. 21. *Porodiscus major*, var. *densa*, nov. × 660.
Fig. 22. *Asterolampra traducens*, sp. n. × 660.
Fig. 23. *Liradiscus furcatus*, Grove MS. × 660.

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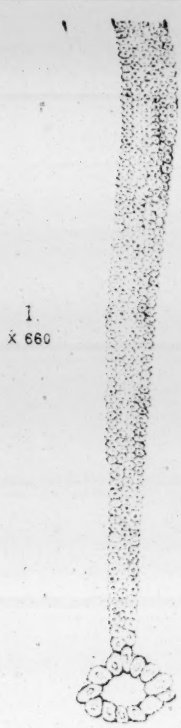
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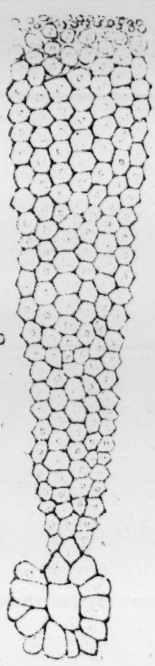
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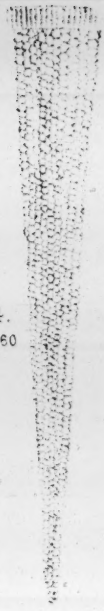
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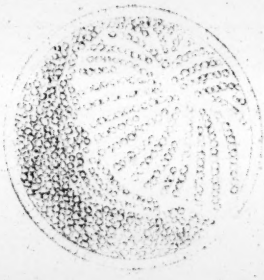
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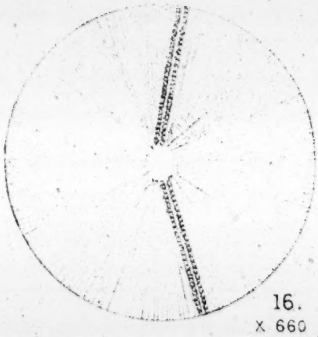
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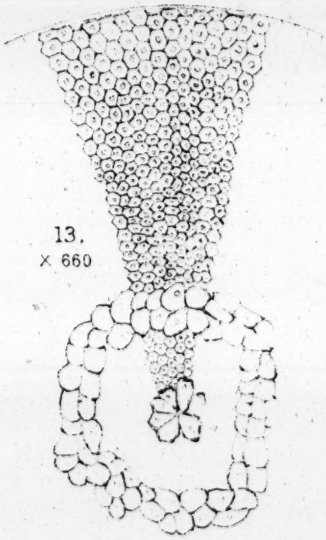
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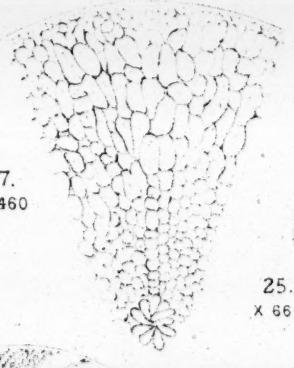
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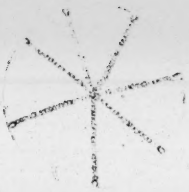


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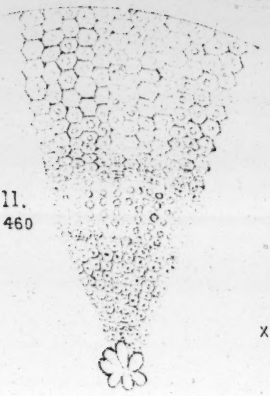
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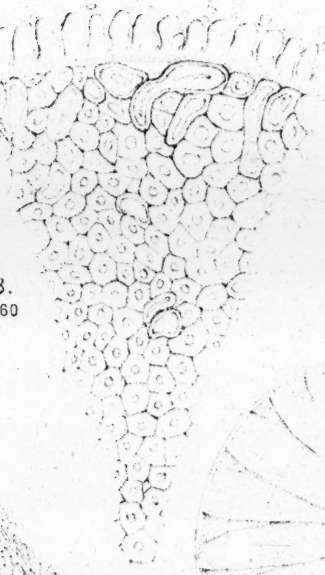
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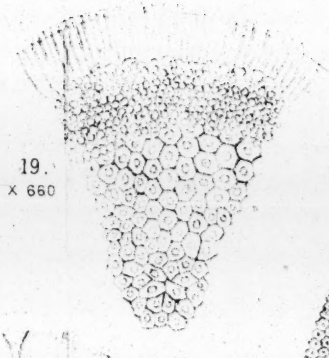


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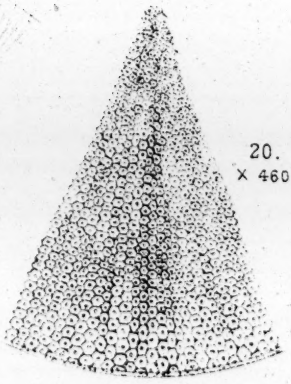


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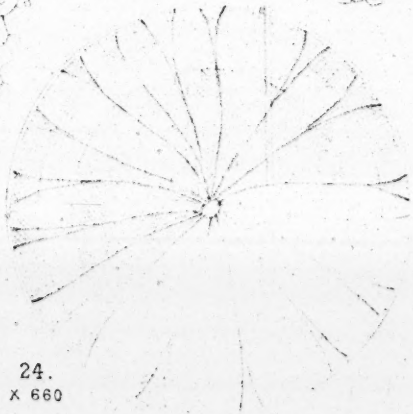




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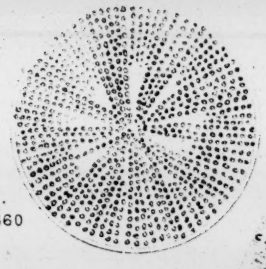
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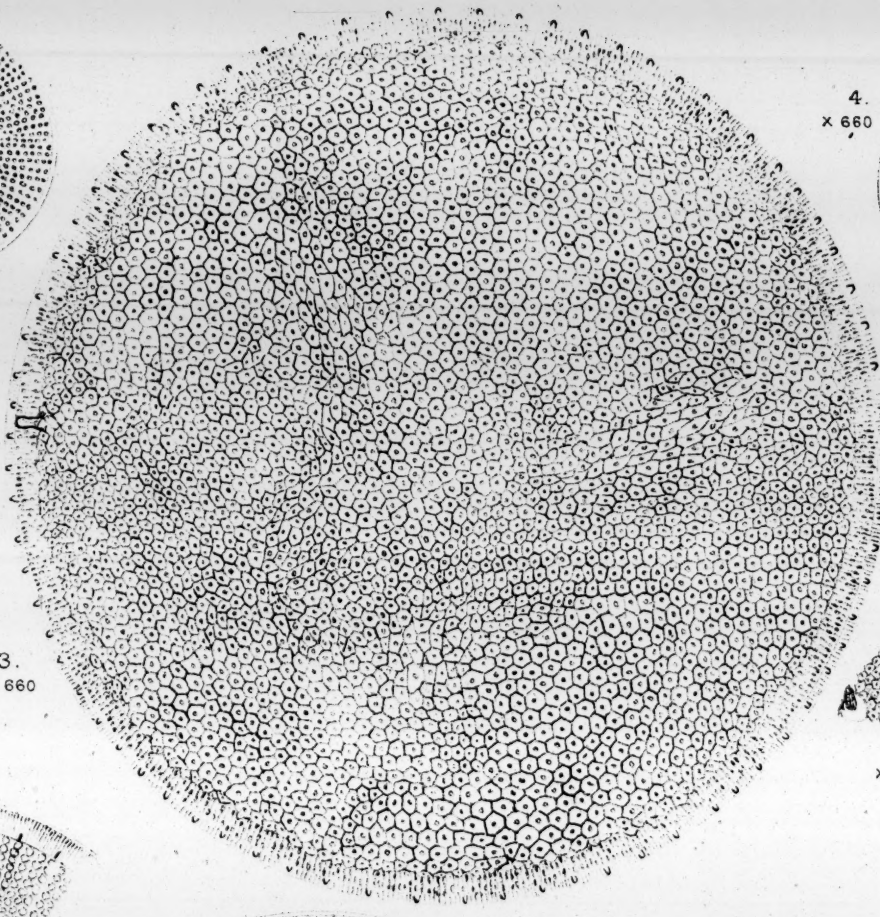
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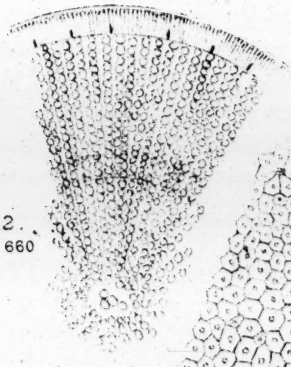


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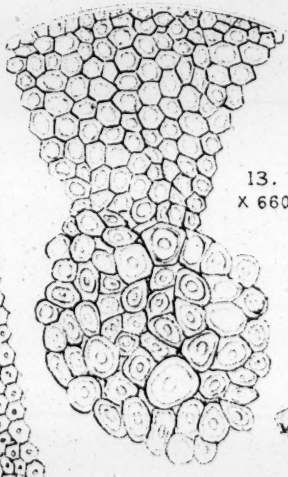


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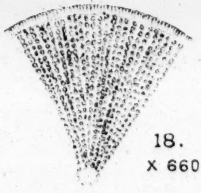
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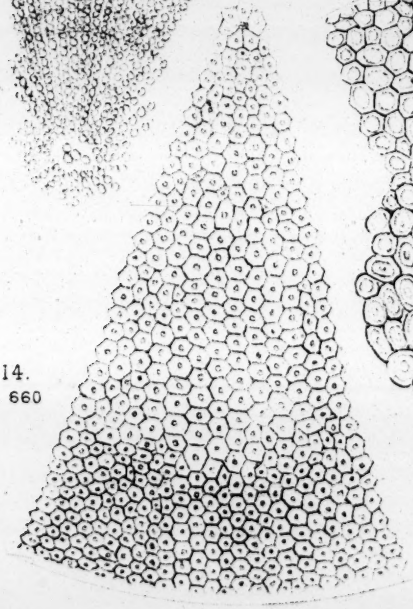
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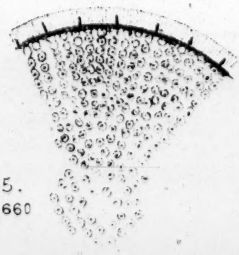
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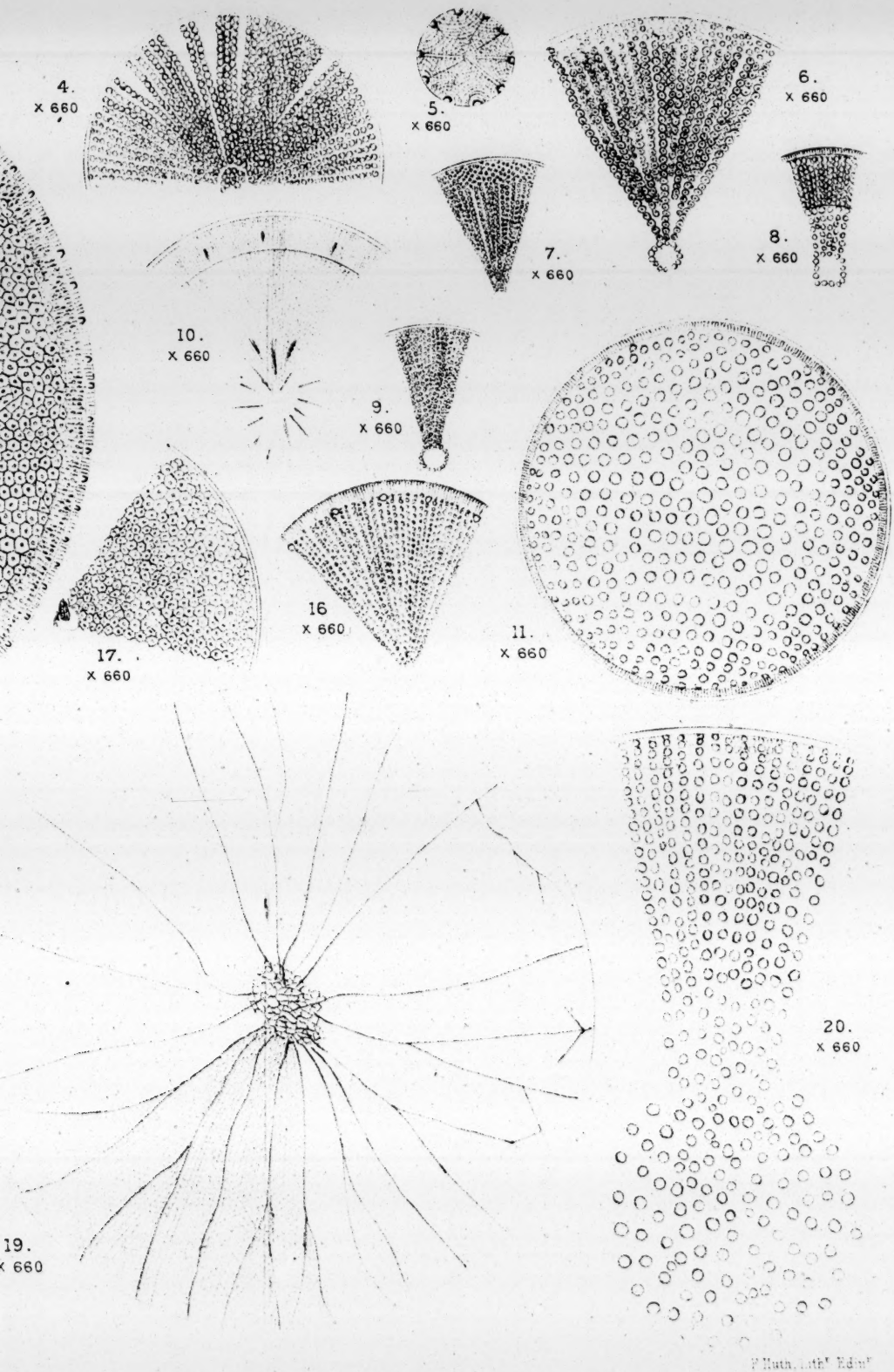


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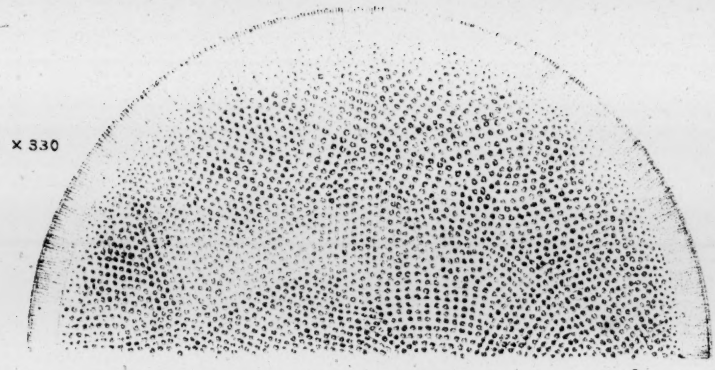
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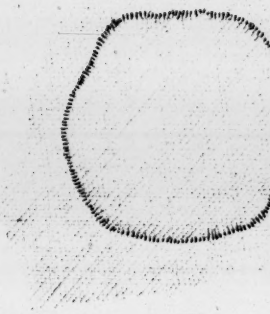
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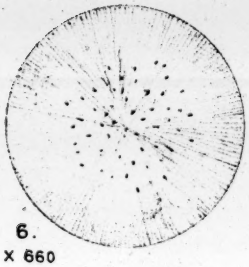


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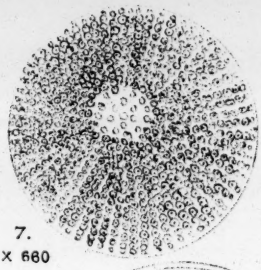


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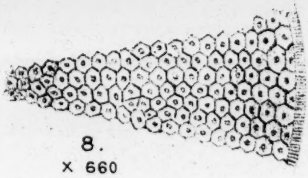




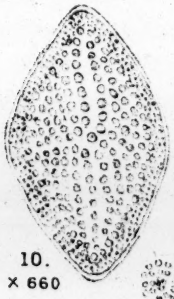
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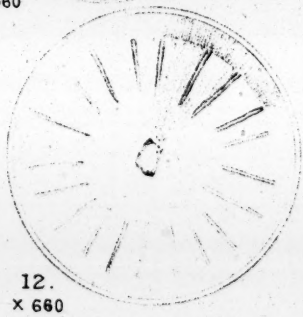
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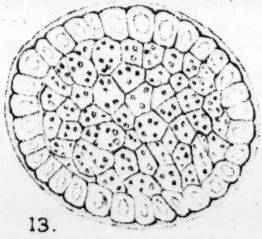
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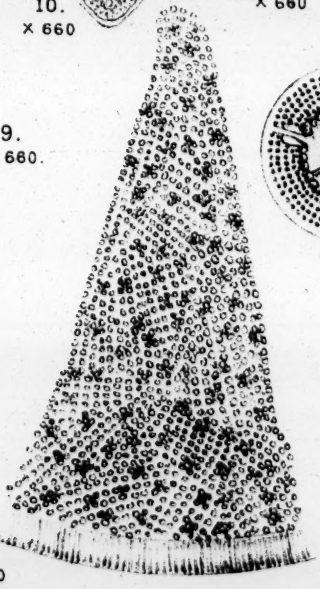
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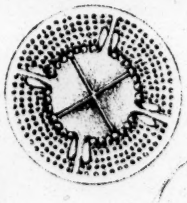
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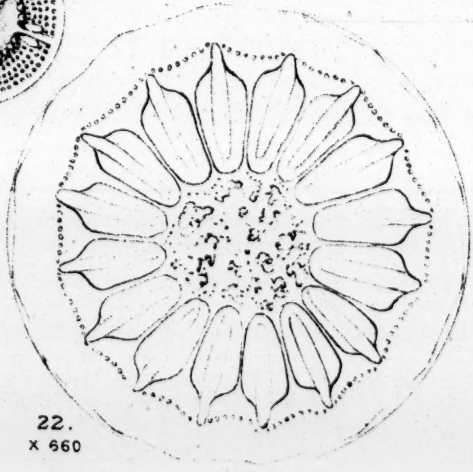


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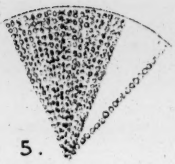
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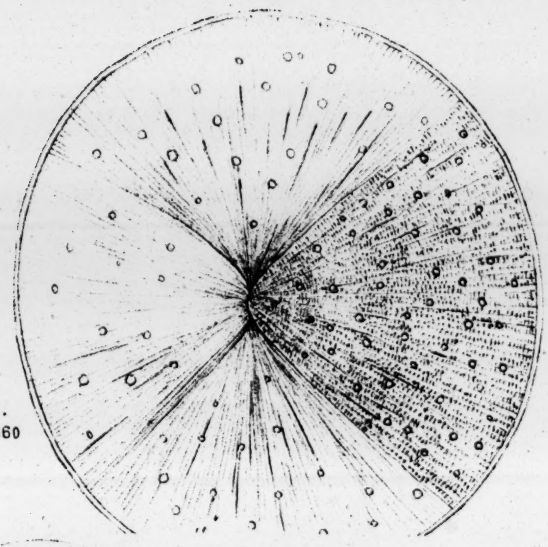


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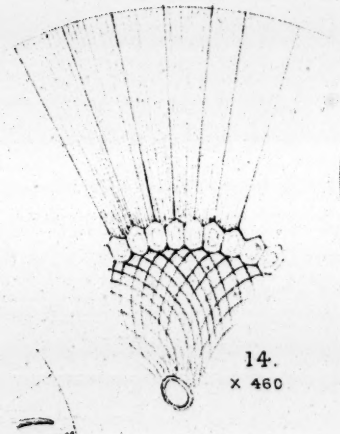
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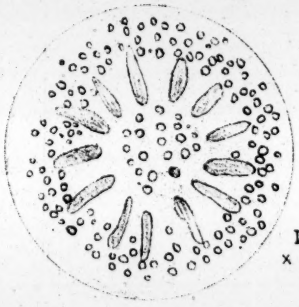
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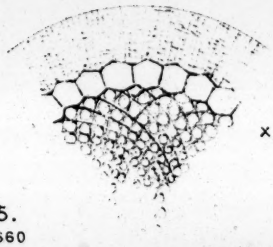
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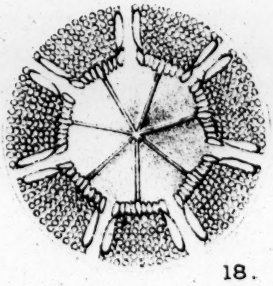


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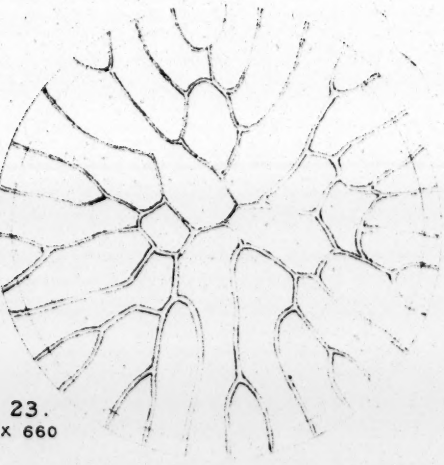


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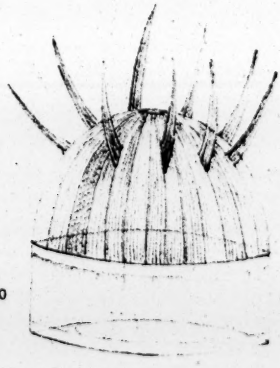


18.



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F. Huth, Lith.r Edin.r

US AND ALLIED GENERA. Pl. III.

Molecular Constitution of Matter. By Sir William Thomson.

(Read July 1, 1889.)

§ 1. The scientific world is practically unanimous in believing that all tangible or palpable matter, molar matter as we may call it, consists of groups of mutually interacting atoms or molecules. This molecular constitution of matter is essentially a deviation from homogeneousness of substance, and apparent homogeneousness of molar matter can only be homogeneousness in the aggregate. "A body is called homogeneous when any two equal and similar parts of it, with corresponding lines parallel and turned towards the same parts, are undistinguishable from one another by any difference in quality."* I now add that unless the "part" of the body referred to consists of an enormously great number of molecules, this statement is essentially the definition of crystalline structure. It is, indeed, very difficult to imagine equilibrium, static or kinetic, in an irregular random crowd of molecules. Such a crowd might be a liquid,—I can scarcely see how it could be a solid. It seems, therefore, that a homogeneous isotropic solid is but an isotropically macled crystal; that is to say, a solid composed of crystalline portions having their crystalline axes or lines of symmetry distributed with random equality in all directions. The proved highly perfect optical isotropy of the glass of object-glasses of great refracting telescopes, and of good glass prisms, seems to demonstrate that the ultimate molecular structure is fine-grained enough to let there be homogeneous crystalline portions, which contain very large numbers of molecules while their extent throughout space is very small in comparison with the wave-length of light.

§ 2. An ideal skeleton or framework for a homogeneous assemblage of bodies, or of material systems of any kind, or of qualities or properties of any kind, distributed periodically throughout space, is defined and explained in § 45 (*a*) to (*j*) below, substantially

* Thomson and Tait's *Treatise on Natural Philosophy*, new edition, vol. i. part ii. §§ 675–678; or *Elements of Natural Philosophy*, §§ 646–649.

taken from Bravais' doctrine of homogeneous assemblages, which we may look upon as the grammar of molecular construction.

SPACE-PERIODIC PARTITIONING (§§ 3–13).

§ 3. Given a homogeneous assemblage of points: let it be required to partition all space accordingly. The thing to be done is concisely defined in the second sentence of § 6 below.

§ 4. The problem is clearly indeterminate. Here is a solution which has obvious relation to Brewster's kaleidoscope and the corresponding doctrine of electric images, and which may be important in respect to Vortex Theory for a crystal or ether. From P, a point of the given assemblage, draw a line, PN, of any length in any direction, provided only that N is not a point of the assemblage of P's. Do the same relatively to every other of the P-assemblage. We thus have a homogeneous assemblage of double points, PN. Let Q be any point in space, and let Σ denote summation for all the PN's. Let $\phi(D)$ be a function which decreases as D increases from 0 to ∞. The equation

$$\Sigma[\phi(QP) - \phi(QN)] = 0,$$

expresses a locus for Q which partitions space periodically, and divides each periodic portion into two cells containing respectively an N and a P. Every cell containing an N is a parallel pervert (footnote on § 45*a* below) of every cell containing a P. That this is true we see by drawing any straight line to equal distances in opposite directions through the point midway between N and P. Its ends are similarly related, one of them to all the N's; the other to all the P's.

§ 5. Here is a perfectly general solution. Around any one of the points P describe a closed surface S, of which the greatest distance from P is less than that of P's nearest neighbour. Describe an equal, homochirally similar, and same-ways oriented surface around every other point P. None of these surfaces cuts or touches any other. Expand all of them simultaneously, equally, and without altering shape or orientation, till one of them touches another. All corresponding pairs of the surfaces touch simultaneously at corresponding points. Continue the expansion, annulling in each case the mutually enclosed portions of the expanding surfaces, and sub-

stituting the portion of fixed surface traced, or left behind, by the expanding line of mutual intersection. This portion of surface we shall call (after my brother, Professor James Thomson) an interface. Follow the same rule when another, another, and another contact takes place. When the borders of two of the growing interfaces thus traced meet and begin to intersect, annul their projecting portions, so that the intersection and what is left of the expansion of its previous border now constitute the boundary of the interface. Continue the process until fresh growing intersections of interfaces are formed, and the ends of these growing intersections meet, and at last nothing is left of the expanded original surfaces, and therefore nothing of space is left unenclosed by the cells—polyhedrons of interfaces—thus constructed.

§ 6. The interfaces formed in § 5 are generally curved, but, as we shall see (§ 7), may be plane, and are so in particular cases of special interest. In every case each cell contains one, and only one, of the P's; there is no interstitial space between them; they are all equal, homochirally similar, and con-orientational.

§ 7. If the initiating surface, S, of § 5 is a polyhedron of plane facets, the periodic partition to which it leads is in polyhedrons of plane facets. So it is also if the initiating surface is any ellipsoid with P for centre.

§ 8. Let S be a sphere. The partitional polyhedron, to which it leads, is the dodekahedron obtained by drawing planes through the middle points of the lines between P and its twelve next-neighbours, perpendicular to these lines.

§ 9. If S is an ellipsoid similar to and con-orientational with that determined in § 47 below, the partitional polyhedron to which it leads is the rhomboidal dodekahedron to which the rhombic dodekahedron of § 21 below is converted by the homogeneous strain of § 46. In this case the whole number of contacts of the expanding surfaces (§ 5) is twelve, and they all take place simultaneously.

§ 10. If the assemblage becomes equilateral, the partitional dodekahedrons of §§ 8, 9 become, each of them, the rhombic dodekahedron of § 21.

§ 11. If S is an ellipsoid, having conjugate diameters along lines from P to other three points of the assemblage, and of magnitudes proportional to the distances from P to the nearest points in these

lines, the partitional polyhedron to which it leads is a parallelepiped.

§ 12. If the three points chosen are nearest neighbours of P (§ 45*i* below), we are led to the *best conditioned* (or least oblique) of all the infinity of parallelepipedal partitions possible. This is the most obvious and the best known of the periodic partitions of space.

§ 13. Taking the parallelepipedal partitioning of § 11, let P′ be the farthest corner from P, so that PP′ is the longest diagonal of the parallelepiped. Let PA, PB, PC be conterminous edges and A′P′, B′P′, C′P′ their opposites conterminous in P′. Draw the planes ABC, A′B′C′. We thus divide the parallelepiped into three parts—an octohedron ABCA′B′C′; and two tetrahedrons, PABC, P′A′B′C′, which are parallel mutual perverts (footnote on § 45*a* below). This grouping of eight points of a homogeneous assemblage is, as we shall see later, important in the dynamics of molecular structure, or at all events in Boscovich's theory.*

On Boscovich's Theory (§§ 14–44 and §§ 62–71).

§ 14. Without accepting Boscovich's fundamental doctrine that the ultimate atoms of matter are points endowed each with inertia and with mutual attractions or repulsions dependent on mutual distances, and that all the properties of matter are due to equilibrium of these forces, and to motions, or changes of motion, produced by them when they are not balanced; we can learn something towards an understanding of the real molecular structure of matter, and of some of its thermodynamic properties, by consideration of the static and kinetic problems which it suggests. Hooke's exhibition of the forms of crystals by piles of globes, Navier's and Poisson's theory of the elasticity of solids, Maxwell's and Clausius' work in the kinetic theory of gases, and Tait's more recent work on the same subject—all developments of Boscovich's theory pure and simple—amply justify this statement.

§ 15. Boscovich made it an essential in his theory that at the

* Theoria Philosophiæ Naturalis redacta ad unicam legem virium in natura existentium, auctore P. Rogerio Josepho Boscovich, Societatis Jesu, nunc ab ipso perpolita, et aucta, ac a plurimis præcendentium editionum mendis expurgata. Editio Veneta prima ipso auctore præsente, et corrigente. Venetiis, MDCCLXIII. Ex Typographia Remondiniana superiorum permissu, ac privilegio.

smallest distances there is repulsion, and at greater distances attraction; ending with infinite repulsion at infinitely small distance, and with attraction according to Newtonian law for all distances for which this law has been proved. He suggested numerous transitions from attraction to repulsion, which he illustrated graphically by a curve,—the celebrated Boscovich curve,—to explain cohesion, mutual pressure between bodies in contact, chemical affinity, and all possible properties of matter—except heat, which he regarded as a sulphureous essence or virtue. It seems now wonderful that, after so clearly stating his fundamental postulate which included inertia, he did not see inter-molecular motion as a necessary consequence of it, and so discover the kinetic theory of heat for solids, liquids, and gases; and that he only *used* his inertia of the atoms to explain the known phenomena of the inertia of palpable masses, or assemblages of very large numbers of atoms.

§ 16. It is also wonderful how much towards explaining the crystallography and elasticity of solids, and the thermo-elastic properties of solids, liquids, and gases, we find without assuming more than one transition from attraction to repulsion. Suppose, for instance, the mutual force between two atoms to be zero for all distances exceeding a certain distance, I, which we shall call the radius of the sphere of influence; to be repulsive when the distance between them is $<\zeta$; zero when it is $=\zeta$; and attractive when it is $>\zeta$: and consider the equilibrium of groups of atoms under these conditions.

A group of two would be in equilibrium at distance ζ; and only at this distance. This equilibrium is stable.

A group of three would be in stable equilibrium at the corners of an equilateral triangle of sides ζ; and only in this configuration. There is no other configuration of equilibrium except with the three in one line. There is one, and there may be more than one, configuration of unstable equilibrium, of the three atoms in one line.

§ 17. The only configuration of stable equilibrium of four atoms is at the corners of an equilateral tetrahedron of edges ζ. There is one, and there may be more than one, configuration of unstable equilibrium of each of the following descriptions:—

(1) Three atoms at the corners of an equilateral triangle, and one at its centre.

(2) The four atoms at the corners of a square.

(3) The four atoms in one line.

There is no other configuration of equilibrium of four atoms, subject to the conditions stated above as to mutual force.

Important questions as to the equilibrium of groups of five, six, or greater finite numbers, of atoms occur, but must be deferred. The Boscovichian foundation for the elasticity of solids with no inter-molecular vibrations is the subject of §§ 62–71 below. A few preliminary remarks here may be useful.

§ 18. Every infinite homogeneous assemblage * of Boscovich atoms is in equilibrium. So, therefore, is every finite homogeneous assemblage, provided that extraneous forces be applied to all within influential distance of the frontier, equal to the forces which a homogeneous continuation of the assemblage through influential distance beyond the frontier, would exert on them. The investigation of these extraneous forces for any given homogeneous assemblage of single atoms—or of groups of atoms as explained below—constitutes the Boscovich equilibrium-theory of elastic solids.

§ 19. To investigate the equilibrium of a homogeneous assemblage of two or more atoms, imagine, in a homogeneous assemblage of groups of i atoms, all the atoms except one held fixed. This one experiences zero resultant force from all the points corresponding to itself in the whole assemblage, since it and they constitute a homogeneous assemblage of single points. Hence it must experience zero resultant force also from all the other $i-1$ assemblages of single points. This condition, fulfilled for each one of the atoms of the compound molecule, clearly suffices for the equilibrium of the assemblage, whether the constituent atoms of the compound molecule are similar or dissimilar.

§ 20. When all the atoms are similar—that is to say, when the mutual force is the same for the same distance between every pair—it might be supposed that a homogeneous assemblage, to be in equilibrium, must be of single points; but this is not true, as we see synthetically, without reference to the question of stability, by the following

* "*Homogeneous assemblage of points*, or *of groups of points*, or *of bodies*, or *of systems of bodies*," is an expression which needs no definition, because it speaks for itself unambiguously. The geometrical subject of homogeneous assemblages is treated with perfect simplicity and generality by Bravais, in the *Journal de l'École Polytechnique*, cahier xxxiii. pp. 1–128 (Paris, 1850).

examples of homogeneous assemblages of symmetrical groups of points, with the condition of equilibrium for each when the mutual forces act.

§ 21. *Preliminary.*—Consider an equilateral* homogeneous assemblage of single points, O, O′, &c. Bisect every line between nearest neighbours by a plane perpendicular to it. These planes divide space into rhombic dodekahedrons. Let A_1OA_5, A_2OA_6, A_3OA_7, A_4OA_8, be the diagonals through the eight trihedral angles of the dodekahedron inclosing O, and let $2a$ be the length of each. Place atoms Q_1, Q_5, Q_2, Q_6, Q_3, Q_7, Q_4, Q_8, on these lines, at equal distances, r, from O; and do likewise for every other point, O′, O″, &c., of the infinite homogeneous assemblage. We thus have, around each point A, four atoms, Q, Q′, Q″, Q‴, contributed by the four dodekahedrons of which trihedral angles are contiguous in A, and fill the space around A. The distance of each of these atoms from A is $a-r$.

§ 22. Suppose, now, r to be very small. Mutual repulsions of the atoms of the groups of eight around the points O will preponderate. But suppose $a-r$ to be very small; mutual repulsions of the atoms of the groups of four around the points A will preponderate. Hence for some value of r between zero and a, there will be equilibrium. There may, according to the law of force, be more than one value of r between zero and a giving equilibrium; but whatever be the law of force, there is one value of r giving *stable* equilibrium, supposing the atoms to be constrained to the lines OA, and the distances r to be constrainedly equal. It is clear from the symmetries around O and around A, that neither of these constraints is necessary for mere equilibrium; but without them the equilibrium might be unstable. Thus we have found a homogeneous equilateral distribution of 8-atom groups in equilibrium. Similarly, by placing atoms on the three diagonals, B_1OB_4, B_2OB_5, B_3OB_6, through the six tetrahedral angles of the dodekahedron around O, we find a homogeneous equilateral distribution of 6-atom groups in equilibrium.

§ 23. Place, now, an atom at each point O. The equilibrium will be disturbed in each case, but there will be equilibrium with a

* This means such an assemblage as that of the centres of equal globes piled homogeneously, as in the ordinary triangular-based, or square-based, or oblong-rectangle-based, pyramids of round shot, or of billiard-balls.

different value of r (still between zero and a). Thus we have 9-atom groups and 7-atom groups.

§ 24. Thus, in all, we have found homogeneous distributions of 6-atom, of 7-atom, of 8-atom, and of 9-atom groups, each in equilibrium. Without stopping to look for more complex groups, or for 5-atom or 4-atom groups, we find a homogeneous distribution of 3-atom* groups in equilibrium by placing an atom at every point O, and at each of the eight points A_1, A_5, A_2, A_6, A_3, A_7, A_4, A_8. There are four obvious ways of seeing this, found by choosing one or other of the four diagonals through trihedral angles referred to in § 21. Take, for example, A_1OA_5, and its congeners for all the dodekahedrons. These triplets include all the A's. (Compare § 25 below.)

§ 25. Lastly, choosing A_2, A_3, A_4, so that the angles A_1OA_2, A_1OA_3, A_1OA_4, are each obtuse,† we make a homogeneous assemblage of 2-atom‡ groups in equilibrium by placing atoms at O, A_1, A_2, A_3, A_4. There are four obvious ways (compare § 24 above) of seeing this as an assemblage of di-atomic groups, one of which is as follows:—Choose A_1 and O as one pair. Through A_2, A_3, A_4, draw lines same-wards parallel to A_1O, and each equal to A_1O. Their ends lie at the centres of neighbouring dodekahedrons, which pair with A_2, A_3, A_4 respectively.

§ 26. For the Boscovich theory of the elasticity of solids, the consideration of this homogeneous assemblage of double atoms is very important. Remark that every O is at the centre of an equilateral tetrahedron of four A's; and every A is at the centre of an equal and similar, but contrary-ways oriented, tetrahedron of O's. The corners of each of these tetrahedrons are respectively A, and three of its twelve nearest A-neighbours; and O and three of its twelve nearest O-neighbours. By aid of an illustrative model showing four of the one set of tetrahedrons with their corner atoms painted blue, and one tetrahedron of atoms in their centres painted red, the mathematical theory which had been communicated to the Royal Society of Edinburgh, was illustrated to Section A of the British Association at its recent meeting in Newcastle.

* This is the assemblage described in the footnote on § 71 below.

† This also makes A_2OA_3, A_2OA_4, and A_3OA_4 each obtuse. Each of these six obtuse angles is equal to $180° - \cos^{-1}(1/3)$.

‡ This is the assemblage described in § 69 below, and used in §§ 67, 68, 70.

§ 27. In this theory* it is shown that in an elastic solid constituted by a single homogeneous assemblage of Boscovich atoms, there are in general two different rigidities, n, n_1, and one bulk-modulus, k; between which there is essentially the relation

$$3k = 3n + 2n_1,$$

whatever be the law of force. Here n_1 denotes what are called the diagonal rigidities, and n the facial rigidities relative to the primitive cube of § 53 below. By facial and diagonal rigidities relative to any given cube I mean rigidities defined in the usual manner,† one of them according to shearing parallel to any face of the cube, the other according to shearing in planes parallel to any plane-diagonal of the cube.

§ 28. A remarkable result of my mathematical investigation is, that the facial rigidity, relatively to the primitive cube of § 52, is double the diagonal rigidity in the case in which each atom experiences force only from its twelve nearest neighbours. The law of force may be so adjusted as to make $n_1 = n$; and in this case we have $3k = 5n$, which is Poisson's relation. But no such relation is obligatory when the elastic solid consists of a homogeneous assemblage of double, or triple, or multiple Boscovich atoms. On the contrary, any arbitrarily chosen values may be given to the bulk-modulus and to the rigidity, by proper adjustment of the law of force, even though we take nothing more complex than the homogeneous assemblage of double Boscovich atoms above described.

Boscovichian Kinetic Theory of Crystals, Liquids, and Gases.

§ 29. The most interesting and important part of the subject, the kinetic, must, for want of time, be but slightly touched in the present communication. I hope to enter on it more fully in a future communication to the Royal Society of Edinburgh.

§ 30. To avoid circumlocutions, I shall call any velocity *moderate*, which is comparable with the maximum velocity acquired by two atoms attracting one another from rest, at distance I. It is the velocity that in the circumstances each would have when their

* See §§ 62–71 below.

† Thomson and Tait's *Natural Philosophy*, 2nd ed., vol. i. part 2, § 680; also reprint of *Mathematical and Physical Papers*, vol. iii. art. xcii. part 1.

distance becomes diminished to ζ. When I speak of atoms or groups moving "rapidly," I mean that the velocities are moderate as thus defined.

§ 31. Let us consider what would follow if we had given at any time, scattered randomly but equably all through space, simple Boscovich atoms moving with velocities randomly equal in all directions. As we are supposing the masses of all the atoms equal, we may call the mass of each unity: thus $\Sigma\frac{1}{2}v^2$ for all the atoms in any part of space at any time, is the total of their kinetic energy. Both the number of atoms and their total energy we shall suppose to be equal in all very large equal volumes.

§ 32. The result of a collision between two atoms is essentially the same as that of the collision of two equal balls supposed simply repellent at contact, as in the elementary kinetic theory of gases as worked out by Maxwell and Tait;* but the size of the balls that would give the same result depends, for each collision, very complexly on the law of force, and on the velocities and lines of motion of the atoms before the collision. As long as there is no case of collision between more than two atoms, the average energy of the free atoms at any time, and the law of the distribution of energy among the multitude in their free paths between collisions, is not affected by this complication, and is the same as if the atoms were equal hard globes merely repellent at contact. It is only when the results of unequal distributions of density, of energy, or of components of momentum, are to be traced, and the laws of the relation of pressure to density, or of thermal conduction, or of viscosity are to be investigated, that we can take into account the law of force, and can find differences from what the results would be if we had merely the hard equal balls to deal with.

§ 33. But now suppose, while two atoms are in collision, a third to come within their influential distance, so that three shall be in collision at the same time. All three *may* go clear, or two of them may remain in collision, or in other words, fall into combination, and *one* go free. It is scarcely possible that all three can remain in

* Maxwell, *Philosophical Magazine*, 1860, and *Philosophical Transactions*, 1867 and 1878; Tait, "On the Foundations of the Kinetic Theory of Gases," *Trans. Roy. Soc. Edin.*, vol. xxxiii., read May 14 and December 6, 1886, and January 7, 1887.

collision—that is to say, can combine. It will certainly be a very rare incident that they remain for any considerable time in collision; but I cannot prove that the case may not occur in which none will go free, and the three will remain in combination.

§ 34. If the initially-given velocities are very great, the general result, even of triple collisions, will be to leave the individual atoms free. The comparatively rare double atoms resulting from triple collisions, and the still rarer triplets, will be liable to be separated again into single atoms by all fresh collisions. This is the case of a perfect monatomic gas, at a temperature much higher than the Andrews' critical point.

§ 35. But if the originally-given velocity be exceedingly small, the result of exceedingly nearly every triple collision will be to form a combination of at least two of the three colliding atoms. Immediately after the collision by which it was formed, each doublet will generally have considerable relative motion of its two atoms; that is to say, the two will describe orbits round their common centre of inertia: or, in the extreme case of no moment of momentum round this point, they will oscillate relatively to their centre of inertia to and fro in a straight line; the centre of inertia itself generally having a considerable velocity. Still supposing the average velocities of the free atoms to be very small, and their number to be very great in comparison with that of the double-atoms, we now see that the general effect of the collisions between double and single atoms must be to diminish the energies of the relative and absolute motions of the constituents of the doublets, and so reduce the doublets more and more nearly to the condition of pairs of atoms in relative equilibrium (§ 16 above), at distance ζ asunder, with centre of inertia of each pair moving very slowly through space.

§ 36. But now consider the effect of a collision between two doublets each with little or no intestine commotion before the collision, and with its centre of inertia moving very slowly through space. The case in which the same description would be applicable to the four atoms after the collision, whether in the same pairs or in interchanged pairs, would be exceedingly rare. So also would be the case of the four atoms remaining combined. The result in exceedingly nearly every case would be a triplet with considerable intestine commotion, and its centre of inertia moving rapidly

through space, and a single atom moving rapidly through space. The general tendency of subsequent collisions between these rapidly-moving triplets and single atoms, with the multitude of slowly-moving single atoms throughout space, would be to diminish the energy of the intestine commotions of the triplets, and of the motions of the centres of inertia, both of the triplets and of the single atoms, reducing each triplet to very nearly the condition of equilibrium (§ 16 above) at the corners of an equilateral triangle of side ζ with a slow translatory motion through space.

§ 37. By similar dynamical considerations we see that the general tendency of collisions between doublets and triplets, or between triplets and triplets, must be to form quartets, quintets, and sextets of atoms; and that when such groups, carrying away large kinetic energies from the generative collisions, subsequently collide with slowly-moving single atoms, the general tendency must be to diminish their kinetic energies, and reduce them more and more nearly to groups in one or other configuration of equilibrium, with slow motion of their centres of inertia through space.

§ 38. But now consider a collision between a slowly-moving triplet or quartet or more-multiple group, and a slowly-moving single atom. Even with the triplet the case will not be rare in which the single atom will remain in combination, and the result yielded be a quartet having considerable intestine commotion, and moving slowly through space. In collisions between a quartet and a single atom, the case will be relatively less rare, and with a quintet and single atom, still less rare for a single atom to remain in combination, and form a quintet or a sextet.

§ 39. If groups of large numbers of atoms in equilibrium, or slowly vibrating, have been thus formed, or are given ready formed, with single atoms slowly moving in the space all around them, each single atom colliding with a group will very frequently remain in the group; and in virtue of the exhaustion of potential energy thus effected the vibrational energy of the group will be slightly augmented. But in not rare cases either the single atom which collided, or one of the atoms of the group in the neighbourhood of the collision, will be driven off, and generally with much greater velocity than the colliding atom had before the collision. Thus the average kinetic energy of vibration per atom of the group may be

kept constant, while the group is gaining by the accession to it of more and more single atoms from without. But the exhaustion of potential energy due to the greater number falling into, than being thrown out from, the group would cause an augmentation of kinetic energy in the surrounding atmosphere of free atoms. To obviate this, let the atmosphere around the group be contained in a finite closed vessel, which, when left to itself, repels each atom that comes near enough to it, and sends it back inwards with unchanged energy. Now let portions of this bounding surface be movable, and let them be so moved by proper external appliances, that work shall be done upon them by the impinging atoms to just such a degree as to keep the average kinetic energy of the free atoms constant. We have thus a Boscovichian realisation of a crystal of ice (hoar-frost) or other substance growing by condensation of a surrounding atmosphere of the same substance. The process in nature requires the abstraction of what is called the latent heat of the vapour to allow it to condense. This in our Boscovichian system is performed by the arrangement for letting work be done outwards by the moving parts of the boundary.

§ 40. Even if there were no surrounding atmosphere of single moving atoms, our group, unless quite free from intestine commotion, would occasionally throw off an atom in virtue of the chance concurrence of different sets of component vibrations at some of the outlying atoms. Now let there be just enough of atoms moving about in the space around the group to cause as many fallings-in as throwings-out of atoms, and with just enough of kinetic energy to neither gain nor lose energy in the surrounding atmosphere through these changes. This will also cause the average kinetic energy of the group to remain constant. Thus we have a crystal surrounded by an atmosphere of vapour at its own temperature, and at the proper temperature to cause neither condensation of the vapour nor evaporation of the solid.

§ 41. Now by somehow applying force to the atoms of the group increase their vibrational energy. We must, by introducing atoms from the boundary, increase the density of the atmosphere around it to cause as many atoms to enter the group as are thrown off from it. Continue this process until the inter-atomic oscillations in the group become so great that the atoms begin to pass from one con-

figuration of equilibrium to another, and back; as, for instance, the two configurations of § 46 (footnote) below. The group may still retain its form as a solid, and something of its rigidity as a solid.

§ 42. Now reverse the operations at the boundary so as to diminish the inter-atomic oscillatory energy of the group. The atoms *may* fall back into their previous positions of equilibrium. But they may *not;* and instead they may fall into another configuration more readily taken in a settlement from internal agitation than the previous configuration which was arrived at by growth from the boundary. This (with true molecules of matter instead of the ideal Boscovich atoms) seems to me, without doubt, the explanation of Madan's* beautiful discovery regarding chlorate of potash, and the change of crystalline structure, by which Lord Rayleigh† has shown that the optical phenomena presented in it are to be explained. Virtually the same view to explain other changes of crystalline structure by differences of temperature or applications of pressure seems to have been given by M. Mallard,‡ who is quoted by Madan in the article above referred to. In a future communication to the Royal Society I hope to include considerations regarding the effect of inter-atomic forces and motions in guiding to one or other of the two configurations described in § 54 and footnote on § 46 below.

§ 43. Once more communicate and continue communicating energy to the group by forces applied directly to its constituent atoms, and, at the same time, keep introducing fresh atoms from the outer boundary into the atmosphere surrounding the group to prevent the number of atoms in the group from diminishing. The intestine commotion will become so great that all configurations of equilibrium are utterly departed from, but still the atom is surrounded by neighbours well within the region of its attractive influence (a shell bounded by two concentric surfaces of radius I and ζ respectively) and constantly crossing and recrossing the spherical surface of radius ζ, or into and out of the sphere of repulsive force. If the region of attractive force be sufficiently thick, and the augmentation of the repulsive force from zero towards infinity be sufficiently rapid, it is

* "On the Effect of Heat in Changing the Structure of Crystals of Potassium Chlorate," *Nature*, May 20, 1886.

† *Philosophical Magazine*, 1888.

‡ *Bulletin de la Société Minéralogique*, 1882, and December 1885.

clear that our original group which was a crystal and is now fluid will remain more dense than the surrounding atmosphere of free atoms until we have imparted to the group far more of energy than was required to dislodge its constituent atoms from configurations of equilibrium. There then is a mass of liquid surrounded by an atmosphere of its vapour, and in thermal equilibrium with the vapour if we cease the action on its atoms by which we imparted energy to it. A little farther consideration would no doubt give us the virtual surface-tension of the liquid exactly according to Laplace's theory of capillary attraction; but we must not pause over this at present.

§ 44. Recommence applying forces to the atoms of the group, now liquid, and introducing fresh atoms into the surrounding atmosphere. The density of the atmosphere becomes greater, while that of the group becomes less. Go on till the two densities become equal: thus we reach the Caignard de la Tour and Andrews' critical point. If we continue now imparting energy to our original group, or to any of the atoms of the assemblage, we simply have a homogeneous assemblage in a state of homogeneous intestine commotion all through; the Boscovich realisation of a fluid raised higher and higher above its critical temperature.

On Molecular Tactics of Crystals and of the Artificial Twinning of Iceland Spar (§§ 45–60).

§ 45. (*a*) . . . (*j*). *Summary of Bravais' Doctrine of a Homogeneous Assemblage of Bodies.*

(*a*) The bodies must be equal, similar, and homochiral.*

(*b*) They must be all similarly oriented.

(*c*) They must be so distanced mutually that any point in one

* This will be more easily and not less thoroughly understood from illustrations than from a definition in general terms. Of an externally symmetrical man, the two hands are *allochirally* similar. Either is the *pervert* of the other; or they are *mutual perverts*. Two men of exactly equal and similar external figures would be allochirally similar if *one* holds out his right hand and *the other* his left; homochirally similar if each holds out his right hand or each his left. (We ignore at present the monochiral anti-symmetry of one heart on one side; of interior structure of intestinal canal *not* in the plane bisecting the exterior symmetric figure, &c., &c.). Looking to § (*i*) below, we

of them, and the corresponding points in all the others form a homogeneous assemblage of points. If this condition is fulfilled for any one chosen point of one body, (*a*) and (*b*) imply it for any other; and *vice versa* if this condition is fulfilled for three points of one body chosen arbitrarily but not in one line, (*b*) is a necessary consequence.

(*d*) A homogeneous assemblage of points means, and cannot mean other than, an assemblage which presents the same aspect and the same absolute orientation when viewed from different points of the assemblage. Some confusion of ideas has been introduced by leaving the generalised simplicity of Bravais, and considering an assemblage of double points, or triple points, or quadruple points, without noticing its being resolvable into two, or three, or four similar homogeneous assemblages of single points.

(*e*) *Rows of Points in a Homogeneous Assemblage.*—Through any two points of the assemblage draw a straight line, and produce it indefinitely in both directions. All points on this line at intervals successively equal to the distance between the two chosen points, are points of the assemblage. The interval between each point and the next to it on either side in the line is called by Bravais the *parameter* of the row.

(*f*) *Planes of Points* ("réseaux") *in a Homogeneous Assemblage.* —Take at random any three points of the group. The case of there being other points of the assemblage on the sides or within the area of the triangle of the chosen points may be excluded. Along the line of each side of the triangle produced in both directions, mark off in succession lengths equal to the side, and through each division draw parallels to the other two sides. The plane of the triangle extended indefinitely in all directions is thus divided into equal and homochirally similar triangles turned alternately in opposite directions. At every angle of

see two tetrahedrons, OPQR, OP′Q′R′, which are equal, and allochirally similar, being parallel perverts, either of the other, or parallel mutual perverts. From every point P of a body or group of points, draw a line through any one point O, and produce to P′, making OP′ = PO. The group of points (P′) is a parallel pervert of the group (P). The groups (P) and (P′) are parallel mutual perverts. Turn (P′) 180° round any line OK. In the position thus reached, it is the image of (P) in a plane mirror through O, perpendicular to OK. In their present positions they are mutual perverts inverted relatively to the line OK. Mutual perverts are allochirally similar.

each of these triangles a point of the assemblage is found. No point of the assemblage is to be found elsewhere in the same plane. Fig. 1 shows a homogeneous distribution of points in a plane. In the diagram they are joined by lines, determinately chosen according to § (*i*), so that all the angles of triangles formed by them are acute. Closely related to this triangular arrangement are three others. One of these is obtained by omitting PQ and its parallels

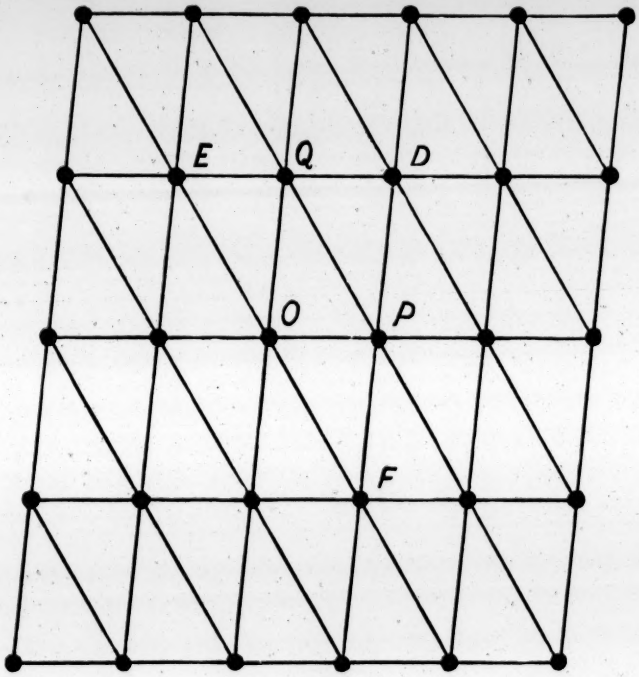


Fig. 1.

and taking instead the other diagonal OD of the parallelogram QOPD and drawing parallels to it through all the points. The two others are obtained similarly by omitting OQ and taking instead the other diagonal PE of the parallelogram QPOE; and by omitting OP and taking instead the other diagonal QF of the parallelogram PQOF.

(*g*) All the points of the assemblage lie in equidistant planes parallel to the plane of (*f*); similarly placed at the angles of triangles equal, similar, and similarly oriented to the triangles of (*f*). The distance between each of these planes, and the next plane to it, is easily proved to be equal to the reciprocal of the product of twice the area of the triangle into the number of points per unit volume.

In fig. 2 the points PQOP′Q′ and their congeners represent a homogeneous distribution in one plane. The orthogonal projection on this plane of the points in the two nearest parallel planes are represented respectively by R and its congeners, black dots (•), and by R′ and its congeners, white dots (∘). Thus explained, the diagram (fig. 2) is a complete specification of the whole homogeneous assemblage throughout space.

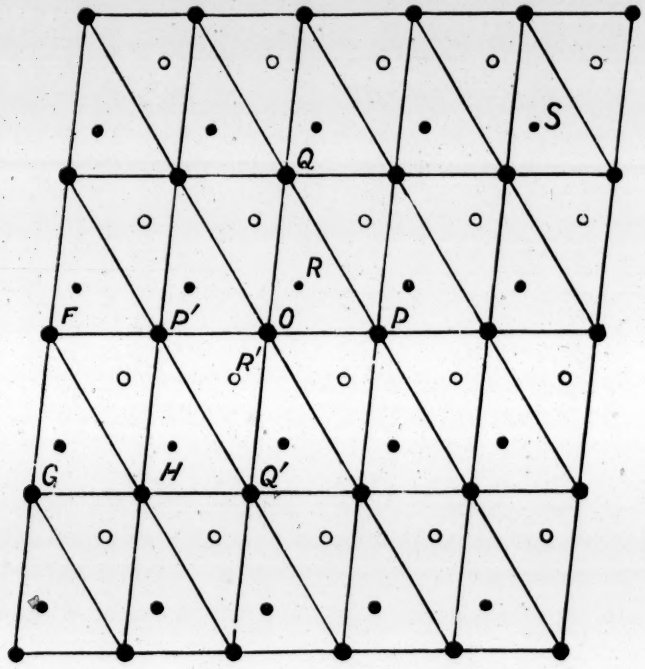


Fig. 2.

(*h*) *Tetrahedronal Grouping.*—Choose any one of the triangles (OPQ), and any point (S) in the nearest plane of points on either side of it; and imagine a tetrahedron of which these (OPQS) are the four corner points. By similarly dealing with all the triangles of all the planes, con-orientational with the first chosen triangle, and the points corresponding to the first chosen point in the neighbouring plane, we form a homogeneous assemblage of equal homochirally similar, samely oriented, tetrahedrons. Thus, for example, take the triangle FGH which is con-orientational with QOP. The tetrahedron on the base FGH corresponding to SQOP is RFGH.

Each point of the distribution is the common corner point of

eight of those tetrahedrons; of which the twelve edges meeting in it lie in the lines of six rows of points which intersect in that point.

(*i*) *Best conditioned Tetrahedronal Grouping. No Obtuse Angles.* —Instead of choosing our first two points and our first triangle at random, take any point O and its nearest neighbour on either side, P; and its next-nearest neighbour Q on the side making the angle QOP acute. The two other angles of this triangle are obviously, as Bravais remarks, acute. The only other way of thus finding *best conditioned* triangles is by taking O's other nearest neighbour, P′, and its other next-nearest, Q′. The triangles Q′OP′ and QOP are equal, homochirally similar, and oppositely oriented; and thus we find the only other possible best conditioned triangular grouping. Every other triangle of the points in the same plane, having none of the points within its area, has, as Bravais remarks, an obtuse angle. Consider now the nearest parallel plane of points on one side of the plane of QOP. Let R and its congeners (• black dots) be the orthogonal projections of its points on the plane of QOP. Let R′ and its congeners (◦ white dots) be the projections of the points of the nearest parallel plane on the other side of QOP. These projections will be situated relatively to the triangle P′OQ′ and its congeners as are the former projections (• black dots) relatively to the triangle QOP.

R being, of the projections on the plane of POQ of all the points of the two parallel planes, the one which lies within the area of the triangle QOP, we have in OPQR a best conditioned tetrahedronal grouping. OP′Q′R′ is another and the only other best conditioned tetrahedronal grouping. It is a parallel pervert of OPQR [*see* footnote on § 45*a* above]. Hence a homogeneous assemblage of single points is essentially free from monochiral anti-symmetry; or it is dichirally symmetrical.

(*j*) The tetrahedron found by taking, with O, P, Q, any other point than R in the plane through it parallel to QOP, has an obtuse angle along one, or obtuse angles along two, of its three edges, OP, PQ, QO: and so with O, P′, Q′, and any other point than R′ in the other parallel plane.

Closest Packing of one Homogeneous Assemblage of Equal and Similar Globes or Ellipsoids.*

§ 46. Take our tetrahedron OPQR, and by homogeneous distortional strain convert it into an equilateral tetrahedron ABCD, of equal volume. Take four globes, of diameters equal to the edges of this tetrahedron and place them with their centres at its corner points A, B, C, D. Alter this assemblage of globes by homogeneous strain till their centres, ABCD, become again the corner points of the original tetrahedron, OPQR. The globes have now become ellipsoids. Dealing thus with the whole original homogeneous assemblage of points, we find a closest packed homogeneous distribution of equal and similar ellipsoids through space.

§ 47. To find every possible closest packed homogeneous assemblage of given equal and similar ellipsoids, take a tetrahedron of four equal globes. Choose any three mutually perpendicular directions, and, by elongations and shrinkages of the group parallel to these directions, convert each globe into an ellipsoid equal and similar to

* There is another closest packing of globes or ellipsoids which has the same density as, and might without careful attention be mistaken for, the closest *homogeneous* packing. For simplicity think only of globes, and take a plane covered with globes touching one another in equilateral triangular order. Look at the accompanying diagram, fig. 6 of § (55) below, and see that there are two ways of placing a second layer on the first to continue the formation of an assemblage. The globes of the second layer may be placed, all of them over the black dots (•) or all of them over the white dots (∘). But having once chosen the position of the second layer there is no more freedom to choose in adding on layer after layer *if we are to make a single homogeneous assemblage.* Of the two positions which might be chosen for the third layer we must choose the one in which the globes are *not* over the globes of the first layer. The position of the fourth layer must be the one of which the globes are *not* over the globes of the second layer, but *are* over those of the first layer, and so on.

If on the contrary we place the globes of the third layer over the globes of the first, the globes of the fourth layer over those of the second, and so on, we have a peculiar and symmetrical grouping which was first, so far as I know, described by Mr William Barlow (*Nature*, December 20 and 27, 1883). This grouping is not one homogeneous assemblage. It consists of two homogeneous assemblages, one of them constituted by the first, third, fifth, seventh, &c., layers; the other the second, fourth, sixth, eighth, &c., layers. The consideration of this peculiar mode of grouping may be of great interest in the dynamical investigations to form the subject of my next communication to the R.S.E. (July 15), and, as Barlow has pointed out, may be of great importance in the theory of natural crystalline structure. I must, however, leave it for the present.

the given ellipsoid. Every possible configuration of closest homogeneous packing of the given ellipsoids is clearly to be thus found; and is specified in terms of three independent variables,—the three orientational coordinates, relative to the equilateral tetrahedron of the system of rectangular lines.

§ 48. In §§ 46, 47 we have a solution of the problem *given four points*, O, P, Q, R, *not in one plane, to place con-orientationally four equal and similar ellipsoids with their centres at the four points, and the surface of every one touching the surface of each of the three others.* From it we have the following perfectly simple construction for the answer. Bisect OP, OQ, OR, in F, G, H, and QR, RP, PQ, in F′, G′, H′, and join FF′, GG′, HH′. These three lines meet in one point S. The planes GSH, HSF, FSG are parallel to conjugate diametral planes of the required ellipsoids. These ellipsoids touch one another in the points F, F′, G, G′, H, H′. To construct them, first make four parallelepipeds, having a common corner at S, and their half-edges which meet in S, and their centres, as follows:—

Half-edges.	Centres.	Half-edges.	Centres.
SF, SG, SH	O	SH′, SF′, SG	Q
SG′, SH′, SF	P	SF′, SG′, SH	R

Inscribe within the twelve edges of each parallelepiped an ellipsoid, touching them at their middle points. This construction is interesting as showing, in the middle points of the twelve edges of the parallelepiped, the twelve points of contact of the ellipsoid with its twelve next neighbours.

The ellipsoid touching the twelve edges is, it need scarcely be remarked, similar to the inscribed ellipsoid touching the surfaces, but of $\sqrt{2}$ times the linear dimensions.

§ 49. To understand the configuration of a closely packed homogeneous assemblage of ellipsoids, it is convenient to consider the assemblage of globes to which it is reduced by strain (geometrical distortion), in § 46. The assemblage of ellipsoids has all characteristic features the same, except the inequalities of lines and angles involved in the distortional transition from one configuration to the other.

§ 50. In the close homogeneous assemblage of globes, we may first remark, that each globe is touched by its neighbours, at twelve points, being the points in which its surface is cut by diameters parallel to the six edges of the tetrahedron. If we place a number of small globes (boys' marbles, or billiard balls) on a table in close triangular order, and three as close as they can be together above them, we see nine of the twelve points of contact on the ball below the middle of the triangle of these three; six points on the circle in which it is cut by a horizontal plane through its centre, and three symmetrically

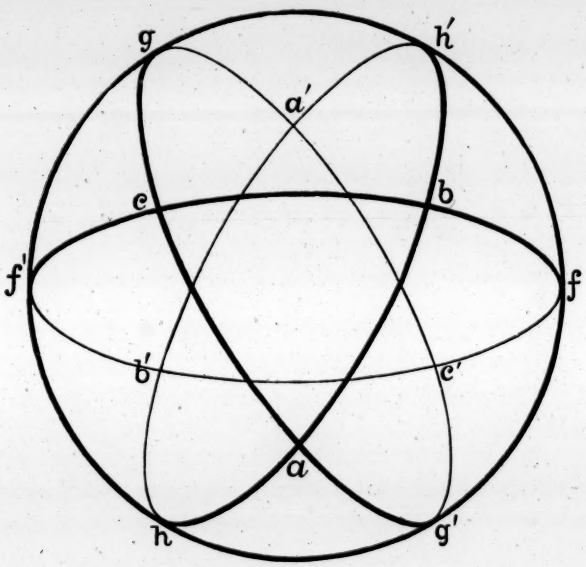


Fig. 3.

ranged on a small circle above it. The other ends of the diameters* through these three are the remaining three of the twelve. Or if we join the upper three by great circles, making a spherical triangle of 60° side, and complete these circles, they make another spherical triangle of 60° side, whose angular points are the lower three of the twelve contact points. The three great circles thus drawn cut the horizontal great circle in the first six points. Thus we see that the

* In the compound assemblage of two homogeneous assemblages described in the preceding footnote, there are twelve points of contact on each globe, of which nine are placed as those described in the text for the homogeneous single assemblage, and the remaining three are not "at the other ends of the diameters" as described in the text, but are at the opposite points of the small circle on which lie the ends of the diameters referred to.

twelve points are the intersections of four great circles, which divide the spherical surface into eight equilateral triangles, and six squares; all with arcs of 60° for boundaries. Fig. 3 shows an orthogonal projection of these circles on the plane of one of them; each an ellipse whose minor axis is $\frac{1}{3}$ of its major axis. The eight equilateral spherical triangles are *abc*, *ahg'*, *bfh'*, *cgf'*, *a'b'c'*, *a'h'g*, *b'f'h*, *c'g'f*. The six squares are *bcgh'*, *cahf'*, *abfg'*, *b'c'g'h*, *ca'h'f*, *a'b'f'g*.

§ 51. Draw planes through the centre of the sphere, parallel to the pairs of planes of the angular points of the eight spherical triangles; these are four planes, *the* four planes in which the assemblage is found in close triangular order. They are parallel to the sides of the tetrahedron ABCD.

§ 52. Draw planes through the centre of the sphere, parallel to the pairs of planes of the angular points of the six spherical squares; these are three planes, *the* three planes, in which the assemblage is found in square order. They are parallel to the pairs (AB,CD), (AC,BD), (AD,BC) of the edges of the tetrahedron; and are mutually orthogonal.

§ 53. Take a cube of the assemblage, having its sides parallel to the planes of § 51. It will present on every side, arrangement of the globes in square order, with *rows* along and parallel to the diagonals of the square sides of the cube. This I call the primitive cube of

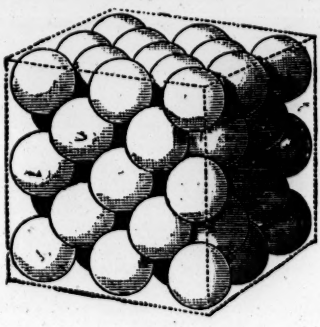


Fig. 4.

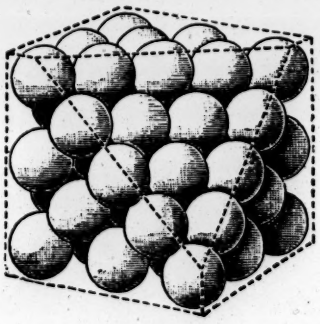


Fig. 5.

a homogeneous assemblage of closely packed globes. It is seen in fig. 4 taken from a paper published in *Nature* (Dec. 20, 1883), by Mr Barlow, who, so far as I know, was the first to show a cubic part of the close-packed homogeneous assemblage of equal globes.

§ 54. Bevel the corners of the primitive cube perpendicularly to its four line-diagonals as shown for one only of the corners bevelled in

fig. 5, which also is taken from Mr Barlow's paper. We thus get eight equilateral triangular facets, each showing close triangular grouping of the globes appearing in it. The four pairs of planes of these facets are, of course, parallel to the four faces of the tetrahedron, ABCD. If we make the bevelling of each corner deep enough, nothing is left of the cube but a regular octohedron, whose eight faces are parallel to the eight faces of the tetrahedron.

§ 55. If in building a triangular pyramid we commence with globes in close triangular order on a horizontal plane, and place the second layer above it over the white dots (∘) of the diagram (fig. 6),

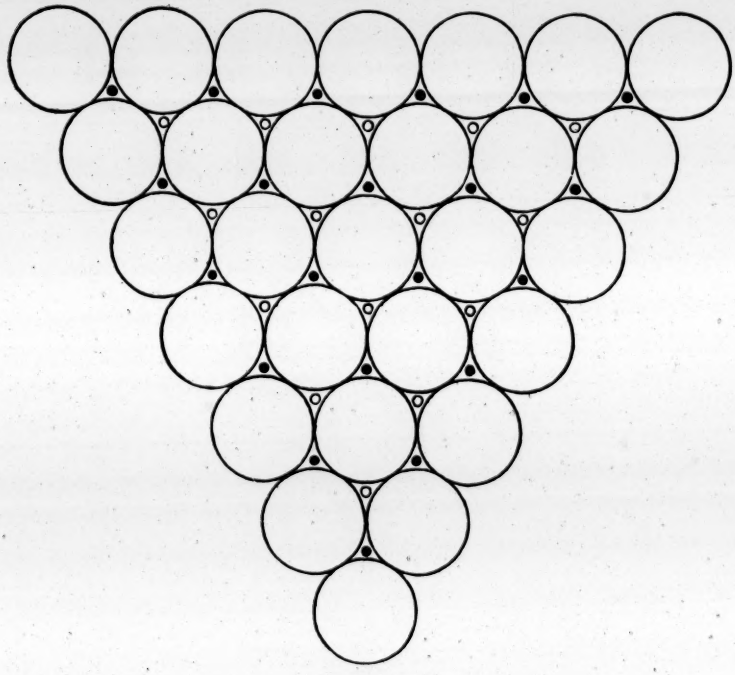


Fig. 6.

the third layer over the inner triangle of black dots (•), and the fourth a single globe over the centre of the diagram, we build up precisely the portion bevelled off the primitive cube in § 54. Thus we have a triangular pyramid whose three sides are isosceles right-angled triangles meeting at right angles along the three slant edges. The globes in these three faces are in square order. The lines of globes in contact in these faces are parallel and perpendicular to the bounding edges of the base. In the pyramid corresponding to the actual diagram, or any other with an odd number of globes in each

edge of the base, there are three lines of globes in contact along the lines bisecting the three vertical angles of the sides of the pyramid and ending in the single crowning globe.

§ 56. If instead of building the second layer as in § 55, we place a second layer over all the black dots (•), a third layer over all the white dots (◦), a fourth layer over centres of globes of the first layer, a fifth over black dots (•) again; a sixth over white dots (◦), and the last a single globe as in § 55, we make an ordinary triangular pyramid having three equilateral triangles for its slant sides and a fourth for base; and having the globes arranged in equilateral triangular order not only in the base as in § 55, but also in each of the three slant sides.

§ 57. The ordinary square pyramid of globes has for its base the same square order structure as the slant sides of the triangular pyramid of § 55, while its four slant sides have the same equilateral triangular structure as each of the three slant sides and the base of the pyramid of § 56. If we divide the ordinary square pyramid into four parts by two diagonal vertical planes through its centre, and turn one of these parts over till it rests on its triangular slant side, it becomes the triangular pyramid of § 55.

§ 58. In considering Baumhauer's splendid discovery of the artificial twinning of Iceland spar, by means of a knife, published about 22 years ago, soon after Reusch's fundamental discovery (1867) of the artificial twinning of Iceland spar by pressure, I endeavoured to picture to myself the molecular tactics called into play in the wonderful change of shape thus produced. It was necessary first to suppose known the molecular arrangement in the natural crystal. Two distinct hypotheses presented themselves, each perfectly definite; and it seems certain that the structure is one or other of these two.

Hypothesis (1).—Imagine an equilateral tetrahedron of a close packed homogeneous assemblage of globes. To avoid circumlocution let one of its faces rest on a horizontal plane. Let the whole system be shrunk homogeneously in lines perpendicular to this plane till the originally acute trihedral angle of the triangular pyramid of globes becomes the obtuse trihedral angle of the rhomb of Iceland spar. The shrinkage ratio required to do this would be exactly $\sqrt{8}$ to 1 if the inclination of each slant face to the base were exactly

45°* in the triangular pyramid obtained by truncating the obtuse trihedral angle of Iceland spar perpendicularly to the "axis" (or line equally inclined to the three edges meeting in the trihedral angle).

Hence if, instead of globes to begin with we take oblate ellipsoids of revolution, each having its equatorial diameter $\sqrt{8}$ (= 2·83) times its polar axis, and make a pyramid of them by laying a number of them flat on a horizontal plane and putting them together and building others up on them according to the rule of § 56, we have an obviously conceivable structure for Iceland spar. This is Hypothesis (1).

This Hypothesis I now find was given 200 years ago by Huyghens in his *Traité de la Lumière* (Leyden, 1690), and independently by Wollaston in the Bakerian Lecture for 1812, *Philosophical Transactions Royal Society* for the year 1813, Part 1, but with priority attributed to Huyghens. I had thought of it independently, but did not feel altogether satisfied with it, in the first place because of the great internal commotion which it would imply in the tactics of Baumhauer's twinning. Then it occurred to me to think of the subject thus. It seems as if the æolotropic quality of Iceland spar, according to which there are differences of quality for directional actions along and perpendicular to the shortest line-diagonal of the rhomb, may be naturally supposed to depend on the rhomb not being a cube; and that the change from a cube to the Iceland spar rhomb should be looked to as the cause of the æolotropy. If this is so we must begin with a cube which is isotropic in respect to its four line-diagonals. This is the case with the cube described in § 53, but it is not the case with the cube which we find if in the shrinkage† of Hypothesis (1) we pause at the stage in which the acute trihedral angle of the equilateral tetrahedron is rectangular on its way to becoming obtuse; on the contrary, in this configuration each globe is

* At ordinary temperatures the angle is 44° 36′·6 (Phillips, Brooke, and Miller's *Mineralogy*, § 407); and at temperature 300° it is almost exactly 45°. Huyghens must have taken it as exactly 45°, as he gave $\sqrt{8}$ for the ratio of the equatorial to the polar diameter in the statement of his hypothesis.

† The shrinkages to pass from the equilateral triangular pyramid to the pyramid with rectangular vertex and to the triangular pyramid for Iceland spar, will be understood in a moment by remarking that the tangents of inclinations of slant sides to base in the three cases are respectively $\sqrt{8}$, $\sqrt{2}$, and 1; and therefore the distances of vertex from base are as these numbers, the base being unchanged in the simple shrinkage specified in the text.

an oblate with equatorial diameter twice as long as the polar axis. Hence I have been led to think it probable that the molecular structure of Iceland spar is not that of Hypothesis (1), but is as follows.

§ 59. *Hypothesis* (2).—Take a primitive cube § 53 of the assemblage and distort it by shrinkage along any one of its four line-diagonals, with (for simplicity) no change of length in directions perpendicular to it. This reduces each globe to an oblate ellipsoid of revolution, and the cube to a rhomb, which is the rhomb of Iceland spar if the shrinkage-ratio is $\sqrt{2}$ to 1.

§ 60. Let FG (fig. 7) be one edge and G one of the two obtuse trihedral angles of a rhomb of Iceland spar; and F′, G′ the corners

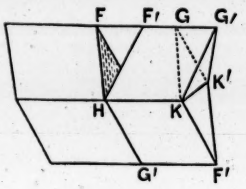


Fig. 7.

opposite to F, G (so that G′G is the optic axis). Let HKK′H′ be a diagonal plane parallel to FG and F′G′; HK, H′K′ being parallel to FG, F′G′. Now consider rows of oblates parallel to KK′ and HH′. The oblates are in contact at the ends of equatorial diameters in the lines of these rows. Turn the oblates of each row round the line of the row, in the half of the assemblage above HKK′H′ (supposing this plane horizontal and G to the right) all through equal angles against the motion of the hands of a watch till their equators become horizontal. The assemblage of centres shears to the right (with rotation *in the direction* of the hands of a watch), FG moving rightwards, till the angle between FG and the end face through F becomes a right angle. Simultaneously with this shearing motion there is a shrinkage of the assemblage in the direction perpendicular to the plane HKK′H′, entailed by the fact of the equatorial planes of the oblates turning from their primary inclined positions, with equatorial planes perpendicular to the optic axis, to their present horizontal positions. This is most readily seen by confining attention to the single row of oblates which initially had their centres and points of mutual contact in the short diagonal GF′ of the right-hand end face. The

shrinkage of the assemblage perpendicular to the plane HKK'H' implies elongation in lines parallel to FG, because the volume remains constant, and there is clearly neither elongation nor shrinkage perpendicular to the plane of the diagram. Now to fit the tactics of Baumhauer's twinning by the knife, we must have no change of dimensions of the assemblage in the plane HKK'H'. Hence while the turning and shearing motions described above are taking place, there must be a continual elongation of the substance of each oblate perpendicular to this plane, and shrinkage parallel to FG,* to just such an extent as to prevent the centre of each oblate from coming nearer to the plane HKK'H', but instead to cause all the centres to move in lines parallel to FG. The oblates are now no longer figures of revolution but are ellipsoids with three unequal axes: the shortest, vertical; the longest, perpendicular to the plane of the diagram; and the mean axis parallel to FG. To complete the process, proceed as follows:—

§ 61. Turn the oblates farther on in the same direction (opposite to the motion of the hands of a watch, as that in which they were turned in § 60), and through the same angle; and while, in consequence, the assemblage of centres shears to the right, give to the substance of each oblate a gradual shrinkage perpendicular to the plane HKK'H' and elongation parallel to the line FG, so as to cause the rightward shearing motion of the assemblage of centres to be still exactly parallel to the initial position of the line FG. The whole movement of which the first half has been described in § 60, and the second half in § 61, constitutes exactly what is done in Baumhauer's artificial twinning of an end portion of a prism of Iceland spar, by a knife applied at F, with its edge perpendicular to the plane of the diagram, and pressed against the edge FG of the obtuse angle between the two upper faces of the prism before and behind the plane of the diagram.

* Perhaps the simplest way of looking at the affair is found by considering that the elliptic section of each ellipsoid in the plane HKK'H' must remain constant; and so also must the horizontal and vertical axes of the elliptic section in the plane of the diagram. Hence, while the principal axes of the elliptic section turn in the manner described in §§ 60, 61, the ellipse itself must remain inscribed in a constant rectangle of vertical and horizontal sides in the plane of the diagram, while the third axis of the ellipsoid, which is perpendicular to the plane of the diagram, remains constant.

On the Equilibrium of a Homogeneous Assemblage of mutually Attracting Points (§§ 62–71).

§ 62. The chief object of this communication is to find the simplest possible way of realising, by means of an assemblage of points acting upon one another with forces in the lines joining them, and depending merely on the lengths of the joining lines, an elastic solid which shall not be subject to Poisson's restriction of the bulk-modulus to be exactly $\frac{4}{3}$ of the rigidity-modulus; but which may on the contrary have, with given rigidity, any magnitude of bulk-modulus through the whole range from $-\frac{4}{3}$ of the rigidity to $+\infty$, shown to be imaginable by Green. That the thing can be done I showed in my Baltimore Lectures (1884), and I gave an easily conceived although a somewhat complex way of doing it. I now find that the next-to-the-simplest-possible mode of arranging an assemblage of points to produce an elastic solid realises Green's ideal; while the very simplest possible is restricted by Poisson's limitation.

§ 63. The simplest possible arrangement of points to make a homogeneous elastic solid, is a single homogeneous assemblage as defined in § 45 *a–d* above. In the first place, for simplicity we shall suppose it to be elastically isotropic, or as nearly isotropic as we can make it.

§ 64. To make the solid as nearly as may be isotropic, the unstrained equilibrium distribution must be the equilateral homogeneous assemblage of § 21 above. Consider now a finite assemblage containing a very great number of points thus distributed. To take the very simplest possible case, let there be no force exerted between others than nearest neighbours. For the case of equilibrium, no force acts from without on any of the points, whether on the boundary or in the interior; and therefore clearly there is no mutual action between any of the points according to our present supposition of forces between nearest neighbours only. Suppose now the assemblage to be in equilibrium under the influence of forces acting on points in the boundary, giving rise to infinitesimal deviations from the equilateral homogeneous grouping. Instead of zero force in each shortest distance, there will now be a force which, for stability of equilibrium, must be pull or thrust, according as the distance is greater or less than that which we had in the zero-equilibrium.

Thus if, to help ideas, we look to a Boscovich curve, the distance between nearest neighbours for zero-equilibrium, which for brevity we shall call ζ, must be a point in which the curve cuts the line of abscissas with slope corresponding to repulsions for less distances and attractions for greater, and shows zero-force for all distances not less than $\zeta\sqrt{2}$.

§ 65. To investigate moduluses of elasticity, we must suppose the forces applied from without to the points on the boundary to be such as to produce homogeneous strain throughout the assemblage. The working out of this statical problem, to be given in a future communication, shows that the solid so constituted is not elastically isotropic; but that, on the contrary, it has essentially two different rigidities. It is in fact a cubical isotropic body with its two rigidities (article "Elasticity," *Encyclopædia Britannica*, ninth edition, or vol. iii. of my Collected Papers) not equal. An extension of the investigation to include the supposition of forces not only between nearest neighbours, but between nearest and next nearest neighbours and none farther, gives of course the two rigidities generally not equal; but it allows them to be equalised by a certain definite relation between forces and variations of forces at the two distances ζ and $\zeta\sqrt{2}$. Imposing this condition, we have elastic isotropy; and I find the compressibility to be essentially $\frac{5}{3}$ of the rigidity. The solid thus constituted is therefore subject to Poisson's restriction; and it will no doubt be found that this restriction is valid for any single equilibrated homogeneous distribution of points, with mutual forces according to Boscovich, and sphere of influence not limited to nearest and next-nearest neighbours, but extending to any large, not infinite, number of times the distance between nearest neighbours.

§ 66. Having thus failed to produce a solid free from Poisson's restriction, go back to the very simplest case, and try for another way of leaving its simplicity by which we may succeed. Try first to realise an incompressible elastic solid. When this is done we shall see, by an inevitably obvious modification, how to give any degree of compressibility we please without changing the rigidity, and so to realise an elastic solid with any given positive rigidity, and any given positive or negative bulk-modulus (stable without any surface constraint, only when the bulk-modulus is positive).

§ 67. To aid conception, make a tetrahedronal model of six equal straight rods, jointed at the angular points in which three meet, each having longitudinal elasticity with perfect anti-flexural rigidity. These constitute merely an ideal materialisation of the connection assumed in the Boscovich attractions and repulsions. A very telling *realisation* of the system thus imagined is made by taking six equal and similar bent bows and jointing their ends together by threes. The jointing might be done accurately by a ball and double socket mechanism of an obvious kind, but it would not be worth the doing. A rough arrangement of six bows of bent steel wire, merely linked together by hooking an end of one into rings on the ends of two others, may be made in a few minutes; and even its defects are not unhelpful towards a vivid understanding of our subject. We have now an element of elastic solid which clearly has an essentially definite ratio of compressibility to reciprocal of either of the rigidities (§ 27 above), each being inversely proportional to the stiffness of the bows. Now we can obviously make this solid incompressible if we take a boss jointed to four equal tie-struts, and joint their free ends to the four corners of the tetrahedron; and we do not alter either of the rigidities if the length of each tie-strut is equal to distance from centre to corners of the unstressed tetrahedron. If the tie-struts are shorter than this, their effect is clearly to augment the rigidities; if longer, to diminish the rigidities. The mathematical investigation proves that it diminishes the greater of the rigidities more than it diminishes the less, and that before it annuls the less it equalises the greater to it.

§ 68. If for the present we confine our attention to the case of the tie-struts longer than the un-strained distance from centre to corners, simple struts will serve; springs, such as bent bows, capable of giving thrust as well as pull along the sides of the tetrahedron, are not needed; mere india-rubber elastic filaments will serve instead, or ordinary spiral springs, and all the end-jointings become much simplified. A realised model accompanies this communication.

§ 69. The model being completed, we have two simple homogeneous Bravais assemblages of points; reds and blues, as we shall call them for brevity; so placed that each blue is in the centre of a tetrahedron of reds, and each red in the centre of a tetrahedron of blues. The other tetrahedronal groupings (Molecular Tactics,

§§ 45, 60) being considered each tetrahedron of reds is vacant of blue, and each tetrahedron of blues is vacant of reds.*

§ 70. Imagine the springs removed and the struts left; but now all properly jointed by fours of ends with perfect frictionless ball-and-socket triple-joints. We have a perfectly non-rigid three-dimensional skeleton frame-work, analogous to idealised plane netting consisting of stiff straight sides of hexagons perfectly jointed in threes of ends.

§ 71. Leaving mechanism now, return to the purely ideal mutually attracting points of Boscovich.

The group is placed at rest in simple equilateral homogeneous distribution:—shortest distance ζ. It will be in stable equilibrium, constituting a solid with the compressibility, and the two rigidities referred to in § 27 above. Condense it to a certain degree to be found by measurements made on the Boscovich curve, and it will become unstable. Let there be some means of consuming energy, or carrying away energy; and it will fall into a stable allotropic condition. The Boscovich curve may be such that this condition is the configuration of absolute minimum energy; and may be such that this configuration is the double homogeneous assemblage of reds and blues described above. Though marked red and blue, to avoid circumlocutions, these points are equal and similar in all qualities.

The mathematical investigation must be deferred for a future communication, when I hope to give it with some further developments.

* An interesting structure is suggested by adding another homogeneous assemblage, marked green; giving a green in the centre of each hitherto vacant tetrahedron of reds. It is the same assemblage of triplets as that described in § 24 above. It does not (as long as we have mere jointed struts of constant length between the greens and reds) modify our rigidity-modulus, nor otherwise help us at present, so, having inevitably noticed it, we leave it.

Continued Observations on the Progression and Rotation of Bivalve Molluscs and of detached Ciliated Portions of them.

By D. M'Alpine, Esq. *Communicated by* Dr Sims Woodhead. (With two Plates.)

Part II.—In Fresh-Water Mussel (*Unio*).

(*Abstract.*)

(Read May 6, 1889.)

In the fresh-water mussel the general results are much the same as in Mytilus. Movement of the animal is as a whole right-handed and slightly forward, though this is not invariably the case.

The movement appears to be brought about by the contraction and expansion of the foot, which is wedge-shaped, and larger in proportion than in the sea-mussel. When portions are detached, however, the same four parts, or pieces of them, exhibit decided movement as in the sea-mussel. The palps rotate, the gills and mantle-lobes also move, but remarkably slowly, while the ventral margin of the foot is pre-eminently active; it is *par excellence* the highly motile detached portion. Where not otherwise stated, the palps, gills, and mantle-lobes are always laid out with their inner surface uppermost, and for convenience of diagrammatic representation, as well as for clearness of explanation, the palps are always supposed to start from the half-round of the clock face. Thus right-handed rotation will have the successive positions of quarter to, hour, quarter past, ending with original position at half past.

I. The Labial Palps are somewhat transverse, triangular flaps on each side of the mouth, situated between the anterior adductor. Each pair of polyps, on either side of the mouth, is united along the line of attachment of the body, and this forms a groove between them leading to the mouth; they are fawn-coloured, and finely striated on their apposed faces.

1. *Right Labial Palps*, together rotate slowly to the left at different rates, from one round in 6 hours to one in 21 minutes; there is also slight forward movement.

2. *The Inner Palps* rotate in opposite directions, the right rotating left-handed, and the left right-handed; the left inner palp

being relatively the weaker,—the right moving at an average rate of about one round in about 14¼ minutes, and continuing for about 10 days, the left moving considerably more slowly, and continuing for over 7 days.

3. *Outer Palps.*—The right is very fitful in its movement, and varies considerably in different cases, both as regards rate of movement and direction of rotation; but the typical specimen rotates slowly left-handed, at an average rate of one round in 8¾ minutes; this movement going on for 52 hours, the rate of rotation diminishing gradually to one round in 3 hours and 10 minutes. The left invariably rotates right-handed, and the average rate was one round in 5 minutes.

4. *Labial Palps generally.*—The palps, normally, rotate inwards or towards their attached margin, and both on the same side rotate in the same direction. In some cases abnormal rotation occurs away from the attached margin. The rate varies from 5 minutes to 14½ minutes per round. The two outer palps seem to be possessed of nearly equal motive power, while of the two inner, the right is relatively twice as powerful as the left. The duration of movement is usually about a week, but the right inner continued for 10 days. It must be remembered, however, that when visible or even microscopic movement of the palp has ceased, the cilia are still in active motion.

The constancy of direction of the palps was well maintained. The right inner and left outer never varied from their normal. The right outer, in four recorded specimens, rotated in its normal left-handed direction, but one of them (the first too) began right-handed. The left inner also rotated normally right-handed in four specimens, but one of them on the fifth day was found moving in the reversed direction.

It would be interesting to determine how far the rotation of the palps of the Swan Anodon of Britain (*Anodonta cygnea*) agrees with the above.

II. GILLS.—The entire right outer gill moves extremely slowly in the direction of its cut surface (¼ inch in 5 days), and there is slight rotation of the anterior end. There is no movement of the left gill at all. A piece of the left inner gill in one case exhibited slight forward movement (1 inch in 24 hours), a second moved forwards

and rotated through a quarter of a circle on its posterior end in $2\frac{1}{3}$ hours. A piece of the left outer gill rotated round the posterior end, whilst a second portion travelled about $\frac{1}{4}$ inch in 3 hours.

The sluggish movements of the entire gill in Unio contrast strongly with those in Mytilus. It may be that more extended observation will reveal a greater capacity for movement, but the gill certainly does not possess that readiness of movement characteristic of the sea-mussel.

III. Mantle-Lobes.—Both right and left mantle-lobes were detached and laid down with their inner or ciliated surface uppermost, but no movement of any kind was detected. Then the right from another mussel was laid down with the outer or non-ciliated surface uppermost, and it moved away gently at once. It progressed in the direction of its cut surface, rotated around its posterior end (the opposite of the right outer gill), and travelled $\frac{1}{4}$ inch in 6 hours. Finely powdered charcoal placed upon the inner surface of the mantle was carried towards the free ventral margin and posteriorly. Two small pieces were taken from the body of the mantle, the one (*a*) with its inner surface uppermost, the other (*b*) with its outer surface uppermost. The latter was taken from a thin part of the mantle, and moved; while the former (*a*), which was thicker, did not move.

The movement (*b*) was exceedingly slow, and could only be registered at distant intervals. In $15\frac{1}{2}$ hours (at night) it had only covered $\frac{1}{16}$ of an inch, but in three hours during the day it travelled an equal distance. In 27 hours from the start it had progressed exactly half an inch altogether. After this it moved laterally as well as forward, so the exact distance traversed cannot be given.

A strip of the muscular free margin, about one inch in length, was next detached from the posterior end. The two ends immediately came together, and a coil was formed, but there was no further movement; while a similar piece from the anterior end remained perfectly still.

The muscular margin was then cut up into very small pieces, about $\frac{1}{8}$ of an inch in length. These fragments were observed for several days, and only exhibited a very slight change of position from day to day. There was an evident absence of that motive-

power which enabled similar pieces of the sea-mussel to roam about in all directions.

It will be observed that the mantle-lobe, in whole or in part, moved with its outer or non-ciliated surface uppermost.

IV. FOOT.—The foot is dark grey in colour and keel-shaped. It is capable of considerable expansion, and possesses great flexibility. Within the shell it is wedge-shaped, but when protruded it may enlarge to almost the size of the shell, $2\frac{1}{4}$ inches from a shell measuring $2\frac{7}{8}$ inches in length. Particles placed on the free ventral margin are carried inwards towards the body and then posteriorly. Although the entire detached foot does not move, the ventral margin is highly motile. A small strip from the anterior end moved forward $1\frac{7}{8}$ inch in 5 minutes. A piece detached from the central portion of the margin moved and wriggled about in various directions. Both ends were very sensitive, and in its contortions it travelled all over the plate.

A strip from the whole margin moved irregularly over the plate, covering $7\frac{1}{2}$ inches in $7\frac{1}{2}$ hours. Every part of the ventral margin—anterior, middle, and posterior—was seen under the microscope to be richly ciliated.

The anterior end of the foot moved in the direction of the free margin, rotating right-handed, $1\frac{1}{8}$ inch forward and $\frac{5}{8}$ inch to the left. It would follow from the ciliary current being inward and posterior on the foot, that it would, when detached, move in the opposite direction, or towards the free margin. It is probable that the direction, when necessary, can be reversed, as indicated in the forward and backward movement mentioned above. And this applies not only to the palps and foot, but possibly to the gills and mantle-lobes; for Tryon, in his *Structural and Systematic Conchology*, mentions a common case of the "inhalent" becoming an "exhalent" current:—"If an Anodonta be placed in a vessel of water into which some fine sand is introduced, the particles will be seen entering the incurrent siphon, and repelled from the orifice of the excurrent one; but after the animal has had enough of the unpalatable and irritating food, it will close its valves, forcing out the water, and with it the sand."

Part III.—In the Oyster.

Ostrea glomerata, Glas.; *O. edulis*, Linn., var. *purpurea*, Hanley.

In the oyster the left valve of the shell is thick and convex; it is on this side that the oyster usually rests. When unattached, however, there is some difference of opinion as to its position. In a recent number of *Nature* (12th April 1888) a summary of a *Report from the British Consul at Baltimore on the Oyster Fisheries of Maryland* is given, in which the oyster is said to feed twice a day, always at the still moment preceding the turn of the tide; and at no other time, except when feeding, or rather taking in food, does it open its shell. It feeds on the liquor in the shell, this habit necessitating the convex or left valve being lower, to retain the liquor in sufficient quantity.

Even when the shell is closed the liquid inside will be in circulation, owing to the action of the cilia, but with the important difference that the energy of the cilia, under these circumstances, is probably less, for, as will be fully shown in the sequel, the cilia-bearing part can increase or decrease the expenditure of energy. Professor Semper mentions (*Animal Life*, p. 147) having eaten oysters, which although with a salt flavour and bathed with brackish water at high tide, at ebb tide were surrounded with a rapid stream of drinkable fresh water, and opened their shells to it. He gives a drawing of this oyster living in spots where the water is quite fresh, and it is noteworthy that he should find that these oysters opened their valves at the turn of the tide. The most highly prized oysters in New South Wales are likewise those taken from beds where fresh and salt water mingles at certain seasons of the year. The movements of the parts of the oyster may help to explain some of its habits and to show the differences between the lower and upper side. In the following brief account of the three motile parts—palps, gills, and mantle-lobes—as well as of the entire shell-less animal, the rock oyster will always have precedence, since the results were decided and satisfactory on the whole; the movements of the mud oyster will be added as confirmatory. Any general description of parts is likewise given from the rock oyster.

The rock oyster of Sydney is *O. glomerata*, Gould. "The

rock oysters, although usually known under several different names, are now by most conchologists admitted to be only localised varieties of one and the same species, *Ostrea glomerata.*" The best oysters are obtained from the shallowest beds, and the very best are dry at low water. The mud oyster of New Zealand, as well as of Australia, is *O. edulis*, Linn., var. *purpurea*, Hanley; *O. chiloensis*, Sowerby, is identical with the small form known as the "native" in the London market. I am informed by Sir James Hector of New Zealand, that the common name of mud oyster given in Hutton's *Manual* is misleading, and ought rather to be the deep-water oyster of New Zealand, since it is found in from 3 to 12 fathoms of water, and usually on a shelly bottom.

Movements of the Oyster as a whole.—A specimen of the oyster laid out in pure salt water, with the gills fully expanded in front and posteriorly, was found to have moved to the left at the posterior end 1 inch in 8½ hours; while the hooded head, or anterior end, remained relatively fixed, as it is too shut in to allow the free play of the cilia. After a time the gills were drawn in, and the whole animal shrunk considerably.

Labial Palps.—The labial palps are a pair of roughly triangular or fan-shaped bodies of a pale flesh colour lying on each side of the mouth; the inner and outer on each side passing respectively into the lower and upper lips. When detached its shape is often lost completely. The line of attachment to the body is somewhat inclined to its long axis, and is just a continuation to the angle of the mouth of the line of attachment of the gills.

Of the palps together, the left was comparatively inactive, whilst the right exhibited considerable activity. The outer rotated right-handed, taking 42½ minutes for the first round, and an average of 23 minutes for the whole, and 3 minutes for the quickest round. The inner began to rotate right-handed very slowly. It then reversed its direction of rotation several times, always moving very slowly. The left inner palp rotated right-handed, and gradually left the outer palp, which was stationary. The first round took 36 minutes, after which the motion was very slow indeed, and was not recorded. The two inner palps together with left uppermost gave evidence of very slight left-handed rotation. Of the right palps, the inner rotates left-handed pretty

constantly, but the rate varied considerably in the different specimens. One specimen with the outer surface uppermost began to move bodily to the right and to rotate right-handed; the first round was as usual the slowest, taking 19½ minutes, the 30th taking only 3 minutes. Next morning it took 16 minutes. On the 3rd day, 38 minutes; on the 4th, 10½ and 13½ minutes; on the 6th day three rounds took 5, 9½, and 10½ minutes respectively; on the 8th day lateral movement was observed, but no definite rotation and no further movement was noted. Another specimen, with the inner surface uppermost, on the 2nd day was rotating left-handed at a rate of a round in 2½ minutes; on the 6th day in 1¾ to 2½ minutes, now right-handed; on the 7th day the movement became much slower, and was again left-handed and more irregular, and it now began to move in a straight line. The movements ended on the 9th day. Reckoning from the last of the continuous rotation to the last of the recorded rounds, after which there was a little left-handed rotation, the time was exactly 1 day 6 hours 52 minutes, so that the palp had been rotating more or less continuously for nearly 8 days of 24 hours each.

On the 10th day, examining under the microscope, the cilia were seemingly as active as ever, but there was no movement of the palp; on the 11th day, however, left-handed rotation was again resumed after renewal of the sea water, and half a round was completed, and the palp had moved ⅜ of an inch higher up. On the 12th day another round was completed, one quarter during the night, the other during the day, that of the night taking about 10 hours, and that of the day about 9 hours. During the night there was little change of position while rotating, but during the day, or last quarter round, there was a deal of progressive movement combined with the rotation. In less than 2 hours the half of the last quarter round was about completed when the palp began to move in a curve in the direction of the tip, with very little if any rotation for some time. The distance thus traversed varied for different periods of the course. The entire curve measured about 2½ inches, while for a given period of 1¼ hour the distance was $\frac{1}{16}$ of an inch, and for the last 2 hours 22 minutes it was ⅝ of an inch. At the close of the round the palp was almost exactly 1½ inch to the left of its first position, and the same lower down.

On the 13th day the curve of progression was not passed through, and movement ceased altogether on the 14th day.

The right inner palp, with a renewal of water once, was actually engaged in rotation for the space of 11 days, and, if the rotating detached palps of oyster and sea-mussel were kept in constantly moving sea water, it is hard to say how long they might not continue their movements.

The day after rotation ceased the palp was examined under the microscope, and the cilia were still in active motion except at the tip end, where the long cilia moved but sluggishly, and were almost still. Now that rotation had ended the next point to determine was, how long the "ciliary motion" might last after such a display of energy. On the 15th day, as just mentioned, the cilia were in motion all round, but most actively in the curved part. Here material was being passed along in the direction of the arrow and thrown off at the anterior end. Later on, this end was found to move a little backward, while the tip end remained fixed.

On the morning of the 16th day the anterior end was found to have moved back nearly $\frac{1}{8}$ inch, while the tip end was stationary. Microscopic appearances explained this, for ciliary motion was now seen to be confined to the curve *a*, where the cilia were still working actively. On the evening of the same day no further movement had taken place, only a slight movement at the tip end. Under the microscope, the tip was seen to have tucked itself in a little (hence the motion), and the cilia were still active in the curve, more particularly towards the very anterior end. On the morning of the 17th day, there was no change of position, and under the microscope no ciliary motion was observed. The cilia stood out like a fringe in the curve, perfectly still; the cilia had thus ceased to move not all round at once, but in patches, as it were, and as far as observed in the specimen in the following order:—At the tip end first, round the outer margin next, and on the attached margin nearest to the mouth last of all, where the cilia had a powerful appearance.

The ciliary motion lasted at least up to the evening of the 16th day, while the rotatory motion ended on the 14th day. And taking the last observation for the ciliary motion, it exceeded the other by 2 days 9 hours.

The palp, as it lies in the clear sea water, still retains its colour,

a pale brown, and there are no external indications of disruption or decay.

Three specimens were taken from the mud oyster, but the movement was quite insignificant, though it was always left-handed, commencing slowly, and gradually attaining the speed of one round in $2\frac{3}{4}$ minutes, after which the movement was gradually lost, the same irregularities as in the left inner palp, though less marked, being still observed. A specimen from a mud oyster exhibited no rotatory motion, but it progressed about $\frac{1}{2}$ an inch, and altered its shape most materially. A left inner palp rotated right-handed at once, and after moving upwards for a little, the tip became stationary, and it rotated regularly, the tip being elevated and bent back upon the body of the palp, pointing in an opposite direction to that of the movement. The slowest of fifty rounds was done in $4\frac{1}{2}$ minutes, and the quickest in $2\frac{1}{2}$ minutes, the general average giving one round in $3\frac{1}{4}$ minutes. At the end of fifty rounds the palp was only $\frac{3}{8}$ inch lower down, and $\frac{1}{4}$ inch to the left of the original position. This rotation continued for 24 hours. In the mud oyster the movement was very indefinite.

In the left outer path the rotation was right-handed and very irregular, but it continued for over 53 hours. Another specimen, tried with the outer surface uppermost, at first moved laterally to the right and with slight rotation to the left; the speed steadily increasing, the slowest round taking 62 minutes, the last and quickest (when movement was accidentally stopped) $7\frac{1}{4}$ minutes. Being again started it began to move bodily to the right, and after a little irregular movement this was continued, attaining a speed of one round in $3\frac{3}{4}$ minutes, and an average of one round in $6\frac{1}{5}$ minutes. This continued, gradually diminishing in speed, until the fifth day, when two complete rounds were followed, taking $14\frac{3}{4}$ and $23\frac{3}{4}$ minutes respectively. Shortly after, the palp reached the margin of the water, where it remained.

A specimen from a mud oyster moved upwards, and rotated right-handed, but very slightly. During the second and third days it moved very little to the right, without any turning, and then ceased.

Palps generally.—When the palps of either side were laid down together they rotated in the direction of their cut margins, and

when the palps were taken separately they still followed the same direction, with the exception of the right outer palp. But it will be remembered that it also rotated left-handed as well as right-handed, with its inner surface uppermost, so that, generally speaking, the palps may be said to rotate in the direction of their cut margins. And since the palps rotate generally in this direction, it might be assumed that the cilia work in the opposite way, and this is found to be the case. Thus it is no uncommon sight to see matter sweeping across the palp, whilst it is rotating from the attached to the free margin, there to be got rid of, and this shows unmistakably in what direction the cilia act, at least on that surface. Having settled these two general facts, that the palps rotate usually in the direction of their cut margins, and that the cilia work in the opposite direction, we may now enter a little into detail.

Of the two right palps the inner is particularly active and persistent, going at the average rate of $2\frac{1}{2}$ minutes per round, and continuing its movements for 13 days, while the outer has likewise a good record of 3 minutes per round. The direction of rotation in both is slightly variable.

Of the two left palps, the inner is more active, going at the average rate of 3 minutes per round; while the outer is not only slower, but the slowest of all the palps. There was here also variation in direction of rotation, and it may be assumed that all the palps are capable of it.

The palps on the two sides of the body are thus pretty evenly balanced, as far as movement is concerned, but the advantage on the whole evidently lies with the right side.

Function.—In detached palps it is very easy to observe the direction in which matters are carried by them. In the course of rotation it was observed, more particularly in the left inner and left outer palp, both lying as they naturally do with their inner surface uppermost. In the left outer the dirty matter was sent spinning across from the inner to the outer margin, where it formed into slimy threads along the outer edge. There it gradually became parted off from the body of the palp, separating from the tip end first, and latterly it formed a streamer, carried round by the revolving palp. Ultimately the matter was got rid of entirely, and

the same process was repeated as occasion arose. In the left inner, matter was also seen passing rapidly across from the inner to the outer margin, gradually getting loosened from the body of the palp, and ultimately being thrown off from the tip backward. There was none of that literal shaking off of rubbish met with in the palp of Mytilus, but it separated almost insensibly. In the face of the above facts, it does sound strange to read in a lecture delivered by Professor Huxley at the Royal Institution, on "Oysters and the Oyster Question" (*English Illustrated Magazine*, vol. i. p. 52)—"The anterior ends of each pair of hemi-branchiæ are attached between the two palps of the side to which they belong. The applied surfaces of the palps, between which lies the commencement of the mouth-cleft, are ridged and richly ciliated, so that anything brought by the ciliary current of the gills is led directly into the oral cavity." Instead of that, it is generally led directly away from it, and we have already seen that in the sea-mussel it is the same when attached, so that the onus of proof must rest with those who make such a statement as the above, in future.

The larval oyster is unprovided with palps, and so it would seem to have its mouth unprotected at an age when such protection was most needed. But there is an apparent substitute for them on the relatively large oval ciliated disc or velum, which overlies the larval mouth. Whatever development may say as to the future of the velum, it probably functions partly in the larva, as do the palps in the adult, *i.e.*, in addition to the locomotive, and possibly respiratory, function, it has the function of guarding the mouth against unsuitable food as it swims about with its velum in front. And thus the suggestion of Loven, that the velum becomes the palps, may at least have a functional basis.

It may be suggested that the palps act both as guards and guides to the mouth, seeing that they can vary their direction of rotation, and consequently the direction of the ciliary current, the latter when feeding, and the former at other times. Still the unguarded statement must not henceforth be made, that the cilia of the palps act constantly and mechanically in the direction of the mouth.

Since the palps are all capable of reversing the direction of their rotation, and since a palp (the right inner) has been actually

observed to send materials towards the mouth end when detached, it is rendered extremely probable that the palps in the oyster exercise the double function of guarding the mouth, and of guiding food materials towards it, at the proper time.

Right Inner Gill-Plate.—In the rock oyster the entire gill exhibited only faint indication of movement in the direction of its cut surface. Pieces of the gill-plate of the mud oyster laid on the inner surface moved forward and to the left, then one began to rotate on its anterior end, then the posterior end became the pivot, and in 57 minutes it had returned to its original transverse position, but to the left and about one inch higher up. Placed in its original position again, the movements were exactly reversed. It retained very slight power of movement for five days.

Right Outer Gill-Plate—(*a*) Entire rock oyster.—The movements were very irregular and slight, the only definite result being that the gill-plate moved as a whole $\frac{1}{32}$ inch in 6 hours. (*b*) Pieces of the corresponding gill-plate of the mud oyster exhibited no movement of any kind.

Left Inner Gill-Plate—(*a*) Entire rock oyster.—One specimen moved in the direction of the cut surface, moving more at the ends than at the centre. It thus moved forward in this irregular manner $1\frac{11}{16}$ of an inch in 4 hours 50 minutes. The free edge travelled $1\frac{7}{16}$ inch, whilst the cut edge had worked through only $1\frac{5}{6}$ inch during the same period. (*b*) Pieces (of mud oyster) gave absolutely negative results.

Left Outer Gill-Plate—(*a*) Entire rock oyster.—Beyond slight movement in the direction of the cut surface, there was no change in position. (*b*) Pieces (mud oyster). A small piece half an inch in length, completed a revolution in 3 hours 12 minutes, and moved slightly forward as a whole; the third and fourth quarter revolutions were performed in 30 and 32 minutes respectively.

Mantle-Lobes.—Each mantle-lobe of the oyster contracts considerably on being separated from the shell, hence on the opened side it is greatly shrunken, but it may be removed from the remaining valve in a fair condition for laying out. Thus the mantle-lobe when detached has not the compact form of that of Mytilus, but is elongated, curved, more or less puckered, and the portion extending from the mouth end, and sweeping round the adductor

muscle, is much broader than the remainder. The one end will be called the anterior and the other the posterior, for purpose of description. The mantle-lobe is attached during life to the inside of the valve. In the right mantle-lobe of the rock oyster there was not the slightest movement, whilst there was only slight movement of the two ends of the left when its lobe was laid down with its cut surface uppermost. It appeared as though the contraction of the centre brought about curling inwards of the two ends.

Pieces of the mascular margin about a quarter of an inch in length can turn on themselves and change their position to a very slight extent.

If the movements of the three forms, Mytilus, Unio, and Ostrea, be compared in their natural condition, Unio possesses the greatest activity, and Ostrea, as far as known, the least; but if the progressive and rotatory movements due to cilia are in question, then Mytilus undoubtedly takes the lead. Unio, with its relatively large keel-shaped foot, can move along, either upright or on its side, in a way that neither Mytilus, with its byssus-secreting foot, nor Ostrea, without a foot at all, can approach; but when the animal, divested of its shell, is placed in its native element, there is, perhaps, contrary to expectation, a power of progression and rotation in Mytilus only slightly shared by Unio, and almost absent from Ostrea.

The general table will show at a glance the contrast between the nature, direction, and rate of movement in each of the various parts; and it will likewise be noticed that each of the three forms has a distinct and specially active part, suggestive of underlying differences. There is the gill in Mytilus, the ventral margin of the foot in Unio, and the labial palp in Ostrea. In Mytilus, no doubt, the palps are specially active, but they are eclipsed in rapidity and readiness of movement by the gills, so that the latter form the more prominent motile parts.

Palps.—The palps have all a combined rotatory and progressive motion, normally, in the direction of the cut margin. As between Mytilus and Unio, the main difference lies in the two fellow palps on either side of the former, rotating in *opposite* directions, and in the latter in the *same* direction. Ostrea may be said to agree with Mytilus, only the left outer or lowermost palp rotates

like its fellow, or it may be said to agree with Unio, in the palps of both sides rotating alike, only the right outer or uppermost can turn either way, by preference, the opposite to that of Unio. The rate on the whole is in favour of the oyster, the general average being 3¼ minutes per round; next comes Mytilus, with a general average of 4; and lastly Unio, with a general average of 8¼. It will be remarked that the average of the corresponding palps agree pretty closely in Ostrea and Mytilus, and that it is the left outer which is the slowest in both.

Gills.—As regards the movement of the gills in their entirety, there is hardly any comparison between those of Mytilus and the other two. All agree indeed in moving, and in moving in the direction of the cut margin, but beyond that they have little in common. The noticeable feature in the gill of Mytilus is its readiness, when detached and laid out, to move away; and so energetic is it that even the anatomists have not failed to notice the swimming movement of parts of it, and as far as known to me, this is the only mollusc in which such has been noticed. It is the only part of any mollusc examined so far which can be absolutely relied upon to move steadily and immediately when detached, hence its great suitability for investigations concerning cilia and their movements.

The palps as a rule could be trusted to rotate after a longer or shorter resting period; but it was noticeable in the oyster, that of the two fellow palps detached from either side, only one of them usually rotated, so much so, that after spending much time at first in detaching single specimens, I got into the habit latterly of detaching and laying out two together, with the sure and certain hope that one of them, at least, would not disappoint me.

The average rate of movement for the gills of the sea-mussel is 2 minutes per inch for those of the fresh-water mussel; there is no general average, but ⅛ inch done in 3 hours; and for those of the oyster the average of about 13 minutes per ⅛ of an inch, or 85 minutes for one measured inch; or, to express it in a comparative way, 1 inch is traversed by the gill of Mytilis, Ostrea, and Unio in 2 minutes, 1½ hours, and 24 hours respectively.

As there was no definite progressive movement of the entire lobe observed in the oyster, the comparison lies between the sea and fresh-

water mussel. There was rotation as well as progression in both, but if we compare the latter, then the mantle-lobe of Mytilus has the decided advantage. That of Mytilus went at the rate of 1 inch in 50 minutes, and that of Unio an inch in 24 hours. The foot of Mytilus is so entirely different from that of Unio that comparison will not be attempted. Suffice it to say, that the slow-moving, definite-shaped, steadily directed foot of Mytilus, when attached, is a perfect contrast to the same when detached, while the free margin of the foot of Unio appears to possess all the activity of the keel-shaped mass.

The cilia of the sea-mussel are almost instantly arrested in their movements by fresh water, just as those of the fresh-water mussel are by salt water, but it is interesting to note that each may be gradually accustomed to the changed conditions. Professor Semper, in his *Animal Life*, mentions the case of a Unio living within reach of the flood tide, of the sea-mussel in perfectly fresh water, and of the edible oyster in brackish water. This may help to explain how the cilia originally became gradually adapted to salt water in the one mussel, and fresh water in the other.

There are still two important considerations suggested by the preceding observations—the resting of the cilia and the relative and absolute rate of movement of detached parts.

It is pointed out that the cilia are supposed to continue their work without any rest, and it may be conceded that microscopic examination goes to prove this, but from the observations made on the variation in direction of the movements of pieces of gills, &c., within very short periods, such can scarcely be the case; and it must be argued that different series of cilia are brought into play at different times. It may be imagined, in a structure like the gill, with its innumerable cilia, that they rest in relays without interfering much, if at all, with the general effect; and I have often had to point out, as in the rotating palps of the oyster, that in the course of long spells of rotation they "rested half a minute" or so. To the observer this looks exactly like "stopping to take breath," or the wearied rower laying down his oar for a minute.

In the course of these investigations an important distinction was noticed between the action of the cilia and the movement of the cilia-bearing mass. The movement of the mass might cease, and

yet the cilia themselves, when examined under the microscope, would be in active motion. The cilia in themselves are thus not the cause of movement; there has to be co-operation or co-ordination of some sort before the ciliary motion can give rise to movement of the part bearing the cilia. There are, therefore, two motions connected with cilia to be distinguished—one, the ordinary so-called "ciliary motion," which creates currents in the liquid, and keeps up a constant stream; another, which may be called ciliary motive-power, which is sufficiently powerful to move the cilia-bearing mass. The practical bearing of the distinction is evident in the investigation of the action of drugs, &c., upon ciliary movement. It will be necessary in future, not only to determine what arrests ciliary motion, but also what affects their motive power, apart from their own proper movement.

The absolute and relative rate of movement of the detached parts is a subject replete with interest, and the movements of the gill of the sea-mussel, slow as they may seem when compared with the dashing Infusorian, can actually hold its own; but, as pointed out by Nageli, quoted by Sachs in his Lectures on the Physical Plants—'Whether the movements of a body appears to us rapid or slow, however, depends also on the relation between its size and the space passed over in a definite time. If an elephant and a mouse travel an equal distance in the same time, we call the first slow, the second quick. A man in walking passes over somewhat more than half his length in one second. The most rapid swarm-cell travels, in the same time, a distance which is $2\frac{1}{2}$ times as great as its diameter. Judged by this standard, the gill of Mytilus only traverses its own height in about a minute, and so far is relatively slow.

In both the Odontophora and the Lamellibranchiata the cilia on the velum cause a rotation of the embryo within the egg-capsule, but it is a curious fact that the cilia do not always act. Thus on the development of Sepia, it is noted that "the whole embryo now becomes ciliated, though the ciliation does not cause the usual rotation; while, in such a closely allied form as Loligo, it does occur. Loligo differs mainly from Sepia in the early enclosure of the yolk by the blastoderm and in the embryo exhibiting the phenomena of rotation within the egg-capsule so characteristic of other Mollusca.* The

* Balfour's *Embryology*, p. 247.

ciliated non-rotating embryo of Sepia recalls the cases of the non-rotating ciliated detached parts, even although the cilia themselves are in active motion.

After these general references, the rotation of the embryos of Mytilus, Unio, and Ostrea will now be considered. The development of *Mytilus edulis*, L., has been recently and specially studied by John Wilson, Demonstrator of Zoology, University of St Andrews, and he has found that cilia cover the greater part of the surface of the embryo, causing it to rotate actively, but no idea is given of rotation.*

In Anodonta and Unio there is likewise rotation of the embryo.† In *Unio litoralis*, when the rotation is most active, 7 or 8 revolutions are said to be observed per minute,‡ but this is presumably, as seen under the microscope, and therefore of no value without the magnification. In *Anodonta intermedia* the rotation is at the rate of from 4 to 1 rounds per minute (15 to 79 seconds),§ but the same remark probably applies.

Although the development of the oyster has been recently and carefully studied, there is no mention of rotation of the embryo within the egg, although there is a double oval ring of cilia. Probably it does occur, as in the larva of Cardium, which it otherwise closely resembles.

The rotation of the embryo in the ovum of the frog is described as being from right to left, at a rate of from 5 to 12 minutes per round (Pfluger's *Archiv*, 1870, Heft 2 and 3). Even at the highest rate, it is slow, as compared with some of the detached parts already considered, and with the embryo of Unio, for instance. For proper comparison, however, the relative sizes would require to be taken into account. The rate of rotation, so seldom given, is worthy of attention, particularly in a case like Mytilus, where the adult still retains a power only possessed by the embryos of higher forms.

The movements of the detached parts of the three chosen forms have thus yielded valuable results, and to a certain extent these forms are representative for our present purpose. There is the free

* *First Annual Report of Scottish Fisheries Board.*

† Balfour's *Comparative Embryology*, vol. i. p. 266.

‡ Owen, *Lectures on Invertebrate Animals*, p. 526.

§ Bronn's *Thier-reich.*

General Table for Comparison of Results as to Direction and Rate of Movement.

		Direction.			Rate.		
		Mytilus.	Unio.	Ostrea.	Mytilus.	Unio.	Ostrea.
LABIAL PALPS—							
Right—	Inner	Left-handed	L. H.	L. H.	2½ mins. per round.	6½ mins. per round.	2½ mins. per round.
	Outer	Right-handed	L. H.	R. H.	3 ,,	7 ,,	3 ,,
Left—	Inner	Right-handed	R. H.	R. H.	3⅛ ,,	14⅜ ,,	3 ,,
	Outer	Left-handed	R. H.	R. H.	7½ ,,	5 ,,	4⅝ ,,
				Average Rate,	4 ,,	8¼ ,,	3¼ ,,
GILLS—							
Right—	Inner	...	...	...	2 mins. per inch.	...	...
	Outer	...	...	...	1½ ,,	⅓ inch traversed in 3 hours.	...
Left—	Inner	...	...	...	2 ,,	...	1 inch in 1½ hours.
	Outer	...	...	...	2½ ,,	...	...
				Average Rate,	2 ,,	...	...
MANTLE-LOBES—							
Right—		...	...	...	...	24 hours per inch.	...
Left—		...	...	...	50 mins. per inch.	...	...
FOOT—		...	...	...	1 hour per inch.	5 mins. per inch (ventral margin).	...

UNIO.

Fig. 1.
Labial Palps detached.
Showing successive variations in shape.
a. Right Outer
b. Left Outer
c. Right Outer (another specimen)

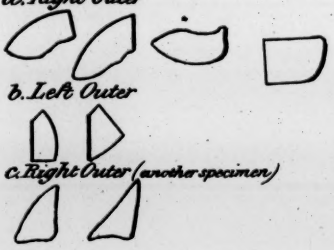


Fig. 2.
Left Labial Palps.
Showing opposed faces.
a One Specimen detached
Anterior end
Outer Palp
Inner Palp
Line of attachment
Posterior end
b Another Specimen attached
Anterior
Outer
Fine Striae
Inner
Posterior

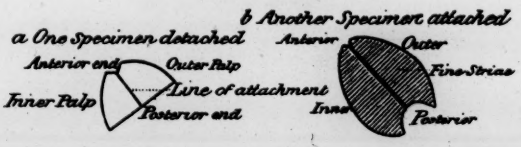


Fig. 3.
Right Outer Gill,
Rotating on Anterior end
Inner Surface uppermost.
Anterior
Attached margin
Posterior

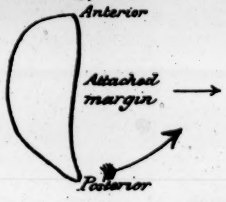
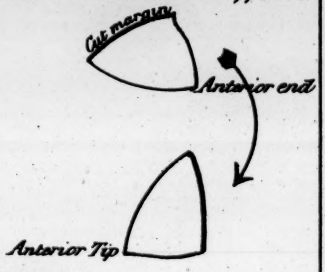


Fig. 4.
Rotating Piece of Left Inner Gill
Inner Surface uppermost.
Posterior
Anterior
Free margin

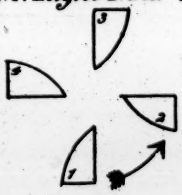


Fig. 5.
Anterior Portion of Foot snipped off
Cut margin
Anterior end
Anterior Tip

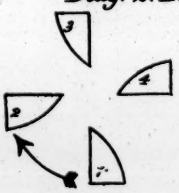


DIAGRAMS AND SPECIMENS OF ROTATING PALPS.

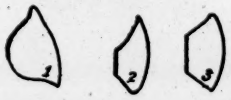
Diag. 1. Right Inner - Left-handed.



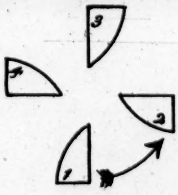
Diag. 2. Left Inner - (a) Right-handed



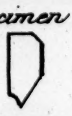
Specimen (d) at three different time



Diag. 3. Right Outer - (a) Left-handed.



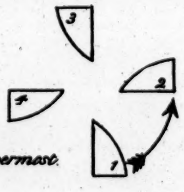
Specimen (e)



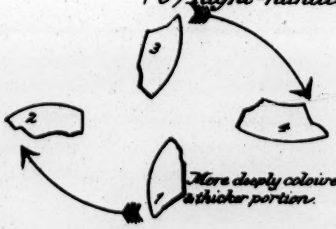
Specimen (d) Outer Surface uppermost.



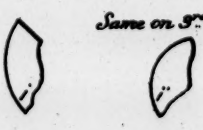
(b). Left handed.



(b) Right-handed



Same on 3rd day.



Diag. 4. Left Outer - Right-handed.



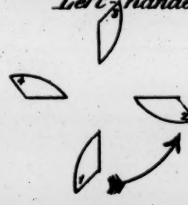
Specimen (d)



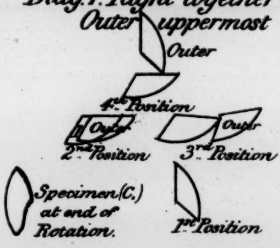
ROTATION & SPECIMENS OF PALPS OF ROCK OYSTERS.

RIGHT

Diag. 3. Right Inner - Left-handed.

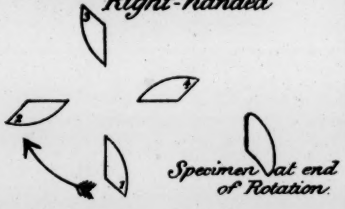


Diag. 1. Right together - Outer uppermost

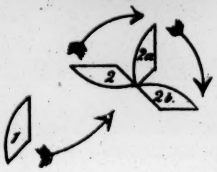


LEFT.

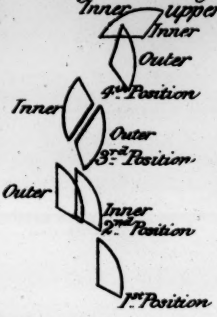
Diag. 6. Left Inner - Right-handed



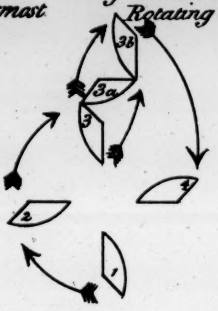
Diag. 4. Right Inner Rotating on Tip.



Diag. 2. Left together Inner uppermost



Diag. 7. Left Inner - Rotating on Tip.



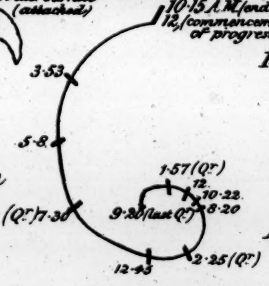
Diag. 8. Left Outer - Right-handed



Diag. 5. Right Outer - (a) Right-handed.

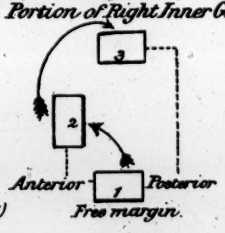


Diag. 9. Curve formed by tip of Palp.

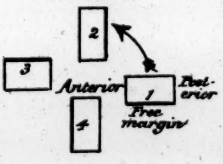


ROTATION OF PORTIONS OF GILL.

Diag. 10. Portion of Right Inner Gill.



Diag. 11. Portion of Left Outer Gill.



OYSTER. (OSTREA.)

Fig. 2. Labial Palps detached. (Mud Oyster) Showing successive variations in shape.

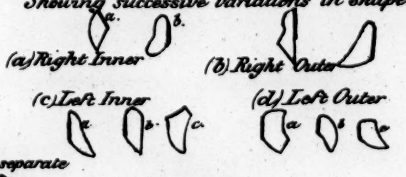


Fig. 1. Right Labial Palps. (a) attached & showing apposed faces.



(b) detached & separate



Fig. 4. Right Inner Palp.

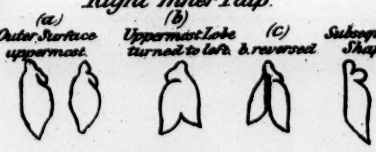
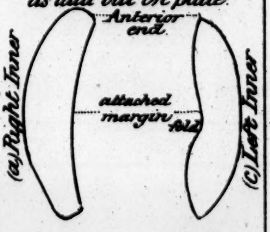


Fig. 5. Right Gills. - Outer View.



Fig. 6. Gills, - Inner Surface uppermost, as laid out on plate.



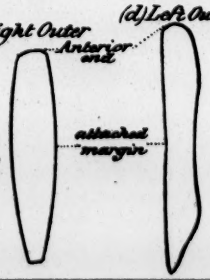
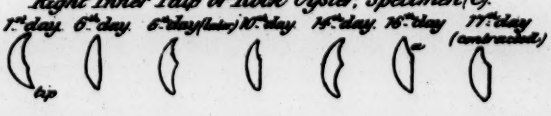


Fig. 3. Right Inner Palp of Rock Oyster; Specimen (c).



form, such as the Unio, wallowing in the soft mud, or roaming freely about in running or stagnant fresh water; the anchored form, such as the Mytilus, riding securely at anchor on the shore-line by means of its byssus; and the attached form, such as the Ostrea, normally fixed by one valve, and living in sea water, often several fathoms deep. The free form, the anchored form, and the attached form represent different stages of freedom of movement in the adult condition, and when observations are made on a sufficient number of species, we may expect to have fresh light thrown upon what is often referred to as the mysterious movements of the cilia.

Further, while there are many points of interest suggested by this investigation, the following may perhaps claim special attention: —The rapidity and readiness of movement by the gill of Mytilus—rapid as the free-swimming Infusorian, and ready at the moment of detachment besides to move, not only horizontally, but vertically and inverted; the duration of movement of the palp of Ostrea; and the distinction between ciliary movement and ciliary motive-power.

These facts are all capable of being turned to useful account apart from the immediate organisms supplying and elucidating them.

Strophanthus hispidus—*continued:* Pharmacological Action. By Dr Thomas R. Fraser.

(*Abstract.*)

(Read June 3, 1889.)

Strophanthus extract and Strophanthin are substances of great pharmacological activity, as, by subcutaneous administration, Strophanthin produces death in average-sized frogs with a dose of $\frac{1}{1000}$th of a grain, and in rabbits weighing about 3 lbs. with a dose of $\frac{1}{100}$th of a grain. The kind of action is the same with both substances, and therefore Strophanthin may properly be regarded as the active principle of *Strophanthus hispidus.* In my preliminary communications I have already described the more important characteristics of the action, and, therefore, to-night I shall content myself with drawing attention to a few only of these characteristics.

I have not obtained evidence of any primary action on the brain, medulla oblongata, spinal cord, nor motor nerves. Sensibility, however, is slightly affected; one of the most conspicuous evidences of which is the insensibility of the cornea following the application of dilute solutions to the eye-ball.

In the preliminary communications I have stated that the chief action is that exerted upon the heart, produced when large toxic doses are given by a powerful action upon the heart-muscle.

This action upon the heart-muscle is one which Strophanthus exerts upon all the other striped muscular fibres of the body. These muscles are affected by twitching movements, their tonicity is increased, and, finally, their contractility is destroyed. They are not then, however, flaccid, soft, and alkaline, but hard and acid in reaction. In fact, the condition of the muscles is one of *rigor mortis*, in which the alkaline reaction of the living muscle has given place to the acid reaction of rigor; and, just as the acid reaction of ordinary *rigor mortis* continues until the muscle becomes flaccid in the process of putrefaction, so the acid and hard condition produced by the pharmacological action of Strophanthus remains until putridity causes the muscle again to become alkaline, soft, and flaccid. This prevention of the initial muscular flaccidity of death and precipitation, as it were, of *rigor mortis*, to which I drew attention so long ago as 1870,* has since been recognised by other observers as events in the pharmacological action of digitalis, and of several members of the digitalis group.

The twitches produced in the muscles of the body have been further examined in detached muscles, immersed in normal saline solution to which Strophanthus had been added, and connected with a lever recording the movements on a revolving cylinder. The movements in the detached muscle are thus seen to be very remarkable. At first a faint twitch occurs in an individual muscular fibre, then simultaneous and independent contractions of different fibres rapidly succeed each other, until by and by a perfect tumult of contractions occurs in rapid succession in different muscular fibres, and the lever attached to the muscle is kept in almost constant motion, its excursions being altogether irregular both in time and in extent. [Tracings were shown.] While the muscle is so affected,

* *Proc. Roy. Soc. Edin.*, vol. vii., 1869–70, p. 102.

electric stimulation of its motor nerve fails to cause any effect. After the twitching contractions have ceased, the motor nerve regains its influence; and the contractions of the entire muscle, which are now produced by stimulating the nerve, illustrate the condition of increased tonicity produced by Strophanthus. The muscle curve is altogether different from the normal curve. The muscle contracts actively, but, after having contracted, it relaxes with great tardiness; and it is only after several revolutions of a rapidly revolving cylinder that the curve gradually falls to the abscissa. [Curves were shown.]

When Strophanthus is administered by subcutaneous injection to a frog, one of whose muscles, without otherwise deranging its normal relations, is attached to a lever writing on a revolving cylinder, similar changes in contractility are observed, but they are less in degree. [Curves were shown.]

Owing partly to the circumstance that a larger quantity of any substance introduced into the blood is in any given time conveyed to the heart than to any other individual organ or structure of the body, the action of Strophanthus is exerted with the greatest energy and activity upon the heart. With very minute doses, its contractions are rendered slower and more perfect and complete; and with larger doses, the diastolic dilatation of its chambers is reduced until dilatation disappears altogether, and the heart ceases to beat because its muscle can no longer relax. The condition of the muscle becomes the same as that of the other striped muscles under the influence of large doses. It is hard, non-contractile under stimulation, and acid in reaction,—the condition of true *rigor mortis* having been produced as the ultimate stage in the sequence of events in the pharmacological action of Strophanthus. These changes occur even although all the nerve connections of the heart are severed, or the vagus nerve is paralysed by the previous administration of atropine.

The power of an extremely minute quantity of Strophanthus to produce these effects on the heart was illustrated in a series of experiments, in which an attempt was made to determine the minimum quantity required to paralyse the frog's heart. The heart was attached to an apparatus allowing it to pump from a reservoir a fluid which sustained its nutrition for many hours,

and to this fluid a certain dose of Strophanthus was added. The apparatus further recorded the individual contractions of the heart, so as not only to show the rate of contraction, but also the amplitude of each contraction. With very minute doses, and in the early stages of the action of larger doses, the contractions became slower, and an increased volume of fluid was projected from the heart at each contraction. It was found that when the circulating fluid contained Strophanthin in the proportion of one part in ten millions, the characteristic changes were produced. Even the almost inconceivably minute dose which was brought in contact with the heart when a solution of *one in six millions* was used, produced complete stoppage of the heart, in extreme systolic contraction, in less than half an hour. *One part in fifteen millions, one part in eighteen millions*, and even *one part in twenty millions* also produced well-marked effects; but these extreme dilutions did not always arrest the heart's contractions, and, as contrasted with the changes produced by less dilute solutions, the slowing of the heart was due to delay during its diastole more than during its systole.

When the exposed heart is observed *in situ* after the administration of Strophanthin, even when the dose is only a minimum lethal one, the changes that are seen are usually those indicating a great increase in the strength and in the duration of systolic contraction, and the ultimate standstill of the heart is, as before described, brought about by the systolic contraction becoming persistent and passing immediately into *rigor mortis*. This increased duration of contraction, with consequent lessening of the dilatation of the heart and of the capacity of its chambers, is not, however, the action which is likely to be serviceable in weak conditions of the organ or in the existence of disabling lesions.

I accordingly made some experiments in which Strophantin was given in rather less than minimum lethal doses—in some experiments by subcutaneous administration, and in others by direct application of a solution to the heart's surface. When care was taken to prevent any irritant, even the air in motion, from reaching the heart, a minute dose produced slowing and a great increase in the amplitude of dilatation, with strong systolic contractions. In some experiments the diastolic pause was so greatly lengthened that the heart remained motionless for two or three minutes, with its

ventricle very large and full of blood; but yet the interrupting systolic contractions were strong, and they completely and deliberately emptied the ventricle of its large accumulation of blood. It was found, however, to be extremely difficult, by any adjustment of the dose, to produce a standstill of the heart in diastole. The heart either recovered altogether, or, in the course of time, the diastolic pauses became briefer, and the systole predominated until the heart ceased to beat in systolic rigidity.

The experiments demonstrated that with doses below the minimum lethal—that is to say, with such doses as would be employed in the therapeutic administration of Strophanthus—both the diastole and the systole of the heart were rendered more perfect, and that the action, therefore, was greatly to increase the working capacity of the heart.

An endeavour was next made to determine upon what structures Strophanthus acts in order to produce the changes that follow the administration of doses less than lethal. The above experiments were repeated after all the nerve connections of the heart with the central nervous system had been divided, and also after the vagus inhibitory apparatus had been paralysed by atropine; but the action that has been described was produced equally well after these modifications had been made. It could not, therefore, be explained by any action on the central nervous system, nor on the cardio-inhibitory influence of the vagus either within or outside the heart.

As there are several points in the physiology of the heart that yet remain unsolved, it would be hazardous to adopt any theoretical explanation, without considerable reserve. It, however, appears probable that at least two structures are involved in the action of Strophanthus on the heart, namely, the muscular fibre itself, and a portion of the intra-cardiac nerve apparatus. The action of Strophanthus upon the muscle of the heart explains the prolonged and strengthened systolic contraction, and the ultimate standstill in extreme systole, following the administration of lethal and of toxic doses. The action upon a part of the intra-cardiac nerve apparatus explains the increased amplitude of dilatation and the prolonged diastole, which, under nice adjustments of small doses, may become permanent, and actually cause a standstill of the heart in extreme diastole. The two actions are, in a sense, antagonistic:

and when minute or therapeutic doses are given, the muscle action does not assert itself so conspicuously as the nerve action; but when the doses are large, the action on the muscle of the heart over-rides the less powerful action upon the intra-cardiac nervous structure.

Whether this latter structure can further be defined by experiment or not, the most important consideration is that the action upon it contributes greatly to enhance the value, for therapeutic purposes, of the action of Strophanthus upon the muscle of the heart; as the two actions in combination render the contractions of the heart stronger and more ample than they could be rendered by either action alone.

In thus presenting to the Society a synopsis or sketch of the observations made, during a series of years, on the Natural History, Chemistry, and Pharmacology of Strophanthus, I have endeavoured to select what appear to be the more important of the results that have been arrived at. In the pharmacological part of the observations these are probably the results that relate to the action upon the circulation. The nature of this action, determined *by pharmacological experiments*, rendered it obvious that Strophanthus would produce beneficial effects in many forms of disease of the heart. It was, therefore, employed for that purpose, and the most sanguine anticipations of its value have now been amply confirmed.

The Theory of Determinants in the Historical Order of its Development. By Thomas Muir, M.A., LL.D.

PART I. *Determinants in General* (1841–44).

(Continued from p. 448 of Vol. XVI.)

It is next pointed out that the transformation of the primitive permutation into any other may be accomplished by interchanges only, because by this means any given letter may be made to occupy the first place, then any other given letter to occupy the second place, and so on. From this also it follows that any system of circular substitutions may be replaced by a system of interchanges. Should the transformation of one permutation into another be effected by interchanges, the number of these will be

even or odd according as the two permutations belong to the same or different classes; for, by the above theorem, every interchange makes only one group more or one group less, and consequently the total number of interchanges, and the net increase or diminution of the number of groups, must be both even or both odd. The counting of *interchanges* may thus be substituted for the counting of cycles.

Finally, Cramer's rule is introduced, in which, as we know, it is neither cycles nor interchanges that are counted, but *inverted pairs*, or, as Cauchy, like Gergonne, calls them, *inversions*. To establish the rule, it is clear that two courses were open, viz., to connect inversions directly with cycles or to connect them with interchanges. The latter course is taken, the requisite connecting theorem being that *the interchange of two elements of a permutation increases or diminishes the number of inversions by an odd number*, an odd number of interchanges thus corresponding to an odd number of inversions, and an even to an even. The proof is not direct, like Rothe's, being effected with the help of a fourth related entity, the difference-product. The order of thought in it is as follows:—If we define the difference-product of the primitive permutation $a, b, c, d, \ldots$ to be

$$(a-b)(a-c)\ldots.(b-c)\ldots.,$$

then it is clear that in the difference-product of any derived permutation there will be found exactly as many factors with changed sign as there are inversions of order in the permutation. A change of sign in the difference-product thus becomes a test for the existence of an odd number of inversions, and consequently, instead of the theorem just enunciated, it will suffice to show that *the interchange of two elements of a permutation alters the sign of the difference-product.* This Cauchy says must be true, for, the elements being h and k, it is manifest that the factor which involves them both,

$$h-k \quad \text{or} \quad k-h,$$

must change sign, but that the factors which involve them and any third element s constitute a partial product

$$(h-s)(k-s) \quad \text{or} \quad (h-s)(s-k),$$

the sign of which cannot change. (III. 37)

Of the three memoirs, the first and third, like Jacobi's third and second, do not at present require attention. A slight reference to the first—on alternating functions—is, however, necessary, because Cauchy, unlike Jacobi, makes determinants a special class of alternating functions, and it is therefore of importance to see the exact position he assigns to them. It will be remembered that in 1812 he partitioned symmetric functions into permanent and alternating, and made determinants a class of the latter; that is to say, his scheme of logical relationship was

$$\text{Functions}\begin{cases}\text{(A) Symmetric}\\ \text{(B)}\end{cases}\begin{cases}(a)\ \text{Alternating}\begin{cases}(a)\ \text{Determinants.}\\ \end{cases}\\ (b)\ \text{Permanent}\end{cases}$$

The memoirs we have now come to indicate a departure from this, both verbal and substantial. The change is made too without any reason being assigned; indeed, there is not even a word to imply that any change had taken place. Alternating functions are, as in his *Cours d'analyse*, put on the same level as symmetric functions; the term *permanent* is dispensed with; a new entity, *alternating aggregates*, is introduced; what were formerly called determinants are made a class of these alternating aggregates; and for the name determinant *resultant* is substituted. The scheme of relationship is thus transformed into

$$\text{Functions}\begin{cases}\text{(A) Alternating}\begin{cases}(a)\ \text{Alternating Aggregates}\begin{cases}(a)\ \text{Resultants.}\\ (\beta)\end{cases}\\ (b)\end{cases}\\ \text{(B) Symmetric}\\ \text{(C)}\end{cases}$$

Neither scheme, we must at the same time remember, is really as simple as here indicated, being complicated by the fact that a function may be alternating in more than one way. This is brought out much more explicitly and clearly in the present memoirs than in that of 1812, as the following quotations will show. We have first of all (p. 151), an *alternating function of several variables.*

> "Une fonction alternée de plusieurs variables x, y, z, . . . , est celle qui change de signe, en conservant, au signe près, la même valeur, lorsqu'on échange deux de ces variables entre elles."

Next we have an *alternating function with respect to several indices* (p. 155):—

"Quelquefois on représente ces mêmes variables par une seule lettre affectée de divers indices

$$0,\ 1,\ 2,\ 3,\ \ldots\ ,\ n,$$

et l'on peut dire alors que la fonction ou la somme dont il s'agit est *alternée par rapport à ces indices.* Ainsi, par exemple, le produit

$$(x_0 - x_1)(x_0 - x_2)(x_1 - x_2)$$

est une fonction alternée par rapport aux variables

$$x_0,\ x_1,\ x_2,$$

ou, ce qui revient au même, par rapport aux indices

$$0,\ 1,\ 2."$$

This example being an alternating function according to the first definition, it would seem that here we have a mere abbreviation or variation of language. There are, however, it must be borne in mind, functions which are alternating with respect to indices, and are not alternating according to the first definition. For example, any determinant, like

$$a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2,$$

is alternating with respect to all the indices involved, but is not alternating with respect to all or any other number of the variables $a_1,a_2,a_3,\ b_1,b_2,b_3,\ c_1,c_2,c_3$. Strange to say, Cauchy makes no mention of this, but goes on to a third definition, by means of which alternating functions are made in another way to include determinants. He says (p. 156):—

"On pourrait obtenir aussi des fonctions qui seraient *alternées par rapport à diverses suites*, c'est à dire, des fonctions qui auraient la propriété de changer de signe, en conservant, au signe près, la même valeur quand on échangerait entre eux les termes correspondants de ces mêmes suites. Considérons, par exemple, m suites différentes composées chacune de n termes qui se trouvent représentés, pour la première suite, par

$$x_0,\ x_1,\ \ldots\ ,\ x_{n-1},$$

pour la seconde suite, par

$$y_0,\ y_1, \ldots\ldots, y_{n-1},$$

pour la troisième suite, par

$$z_0,\ z_1, \ldots\ldots, z_{n-1},$$

etc., ... ; et soit

$$f(x_0, x_1, \ldots\ldots, x_{n-1};\ y_0, y_1, \ldots\ldots, y_{n-1};\ z_0, z_1, \ldots\ldots, z_{n-1};\ \ldots\ldots)$$

une fonction donnée de ces divers termes. Si à cette fonction l'on ajoute toutes celles que l'on peut en déduire, à l'aide d'un ou de plusieurs échanges opérés entre les lettres

$$x, y, z, \ldots\ldots$$

prises deux à deux, chacune des nouvelles fonctions étant prise avec le signe + ou avec le signe −, suivant qu'elle se déduit de la première par un nombre pair, ou par un nombre impair d'échanges; le résultat de cette addition sera une somme alternée par rapport aux suites dont il s'agit."

It is a little unfortunate that this definition proceeds on different lines from the others, being rather indeed a *rule for the formation* of an alternating function with respect to several sets of variables than a definition of such a function. It would have been much more appropriate and instructive to have said that a function was called *alternating with respect to two or more sets of the same number of variables* when the interchange of each member of a set with the corresponding member of another set altered the function in sign merely. Examples like the following could then have been given to make the two usages of the term perfectly clear, and to show the exact relation between them. To illustrate the first usage, the expressions

$$ac - bc,$$

$$(a-b)(c-d),$$

$$(a-b)(a-c)(b-c),$$

might be taken, where $ac - bc$ is an alternating function with respect to the variables a, b; $(a-b)(c-d)$ an alternating function with respect to a, b, and also with respect to c, d; and $(a-b)(a-c)(b-c)$ an alternating function with respect to a, b, with respect to a, c, and

with respect to b, c, or, shortly, an alternating function of all its variables. On the other hand, the expressions

$$a^2b - c^2d,$$
$$a\,b - c\,d,$$

would illustrate the second usage; $a^2b - c^2d$ being an alternating function with respect to the sets of variables ab, cd; and $ab - cd$ an alternating function with respect to the sets ab, cd, and also with respect to the sets ac, bd. In a word, the alteration which produces change of sign is, in the case of the first usage, interchange of two individual elements; in the case of the second usage it is interchange of two ranks or sets of elements.

The entity to which the new name *somme alternée* is given is explained as follows (p. 160):—

"Soit

$$\mathrm{f}(x, y, z, \ldots)$$

une fonction quelconque de n variables

$$x, y, z, \ldots$$

et ajoutons à cette fonction toutes celles qu'on peut en déduire par la transposition des variables, ou, ce qui revient au même, par un ou plusieurs échanges opérés chacun entre deux variables seulement, chaque nouvelle fonction étant prise avec le signe + ou le signe −, suivant qu'elle se déduit de la première à l'aide d'un nombre pair ou impair de semblables échanges. La somme s ainsi obtenue sera la *somme alternée* que nous représentons par la notation

$$\mathrm{S}\,[\pm \mathrm{f}(x, y, z, \ldots)].$$

Ou trouvera, par exemple, en supposant $n = 2$,

$$s = \mathrm{f}(x, y) - \mathrm{f}(y, x);$$

en supposant $n = 3$,

$$s = \mathrm{f}(x,y,z) - \mathrm{f}(x,z,y) + \mathrm{f}(y,z,x) - \mathrm{f}(y,x,z) + \mathrm{f}(z,x,y) - \mathrm{f}(z,y,x),$$

etc."

The only matter now remaining for explanation is the mode of transition from *sommes alternées* to *résultantes*, the difficult point

being, as in the memoir of 1812, to include all kinds of the latter as special cases of the former. The two pages which Cauchy devotes to the subject are curious to read, and deserve a little attention. He says (p. 161):—

"Concevons maintenant que la fonction

$$f(x,y,z,\ldots)$$

se reduise au produit de divers facteurs dont chacun renferme une suite des variables

$$x,\ y,\ z,\ldots.$$

en sorte que l'on ait, par exemple,

$$f(x,y,z,\ldots)=\phi(x)\chi(y)\psi(z)\ldots.$$

alors, pour obtenir la somme alternée

$$s=S[\pm\phi(x)\chi(y)\psi(z)\ldots]$$

il suffira . . ."

and having shown the mode of formation, and given the examples

$$s=\phi(x)\chi(y)-\phi(y)\chi(x),$$

$$s=\phi(x)\chi(y)\psi(z)-\phi(x)\chi(z)\psi(y)+\ldots$$

he adds

"Les sommes de cette espèce sont celles que M. Laplace a désignées sous le nom de *résultantes*."

In regard to this the first comment clearly must be that it is not a little misleading. The sums referred to are only a very special class of those functions which Laplace called resultants; they belong, in fact, to that peculiar type for which in later times the name *alternant* was coined. In the second place, Cauchy's virtual renunciation of his own word "determinant" must be noted,—a renunciation all the more curious when we consider that the word had now been adopted by Jacobi, and had thereby become the recognised term in Germany. It may be that Laplace's word "resultant" had proved more acceptable in France, and that Cauchy merely bowed to the fact; but there is little or no evidence to support this.*

* Liouville, in a paper published in the same year as Cauchy's memoirs, uses *resultant*, but adds in a footnote, "Au lieu du mot *résultante*, les géomètres emploient souvent le mot *déterminant*" (*Liouville's Journ.*, vi. p. 348).

In the paragraph following the above Cauchy proceeds, as it were, to rectify matters. He says (p. 162):—

"Les formes des fonctions désignées par

$$\phi(x),\ \chi(x),\ \psi(x),\ \text{etc.}$$

étant arbitraires, aussi bien que les variables

$$x,\ y,\ z,\ \ldots,$$

permettent aux divers termes qui composent le tableau (2) d'acquérir des valeurs quelconques, et représentons ces variables à l'aide de lettres diverses

$$x,\ y,\ z,\ \ldots\ldots,\ t$$

affectés d'indices différents

$$0,\ 1,\ 2,\ \ldots,\ n-1,$$

dans les diverses lignes verticales. Alors, au lieu du tableau (2), on obtiendra le suivant

$$(5)\qquad \begin{cases} x_0,\ x_1,\ x_2,\ \ldots\ldots,\ x_{n-1} \\ y_0,\ y_1,\ y_2,\ \ldots\ldots,\ y_{n-1} \\ z_0,\ z_1,\ z_2,\ \ldots\ldots,\ z_{n-1} \\ \quad\ldots\ldots\ldots \\ t_0,\ t_1,\ t_2,\ \ldots\ldots,\ t_{n-1} \end{cases}$$

et la résultante s des termes dans ce dernier tableau sera

$$s = \mathrm{S}[\pm x_0 y_1 z_2 \ldots t_{n-1}]."$$

The general determinant is doubtless here reached, but the transition requisite for the attainment of it, viz., from $\phi(x)$, $\chi(x)$, $\psi(x)$, ... to the perfectly independent x_0, x_1, x_2, ... is not made without considerable strain. This is all the more surprising, too, when we consider, that a much less troublesome and less objectionable mode of bringing determinants under alternating aggregates lay ready to Cauchy's hand. Bearing in mind the definition given above, of *fonctions alternées par rapport à diverses suites*, we see that a determinant of the nth order could have been made to appear as an alternating function with respect to n ranks of n variables each. For example, the determinant

$$a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2,$$

could have been introduced as a function alternating with respect to any two of the three ranks,

$$a_1 \quad a_2 \quad a_3,$$
$$b_1 \quad b_2 \quad b_3,$$
$$c_1 \quad c_2 \quad c_3;$$

and indeed, as we know, it is alternating also with respect to any two of the ranks

$$a_1 \quad b_1 \quad c_1,$$
$$a_2 \quad b_2 \quad c_2,$$
$$a_3 \quad b_3 \quad c_3,$$

that is to say, according to another phrase of Cauchy's, used above, it is alternating with respect to the indices 1, 2, 3.

The fourteen pages (pp. 163–176) which follow, are taken up with the properties of determinants as thus defined and with the application of them to the solution of simultaneous linear equations. Most of the matter is already familiar to us, and may be altogether passed over. One of the theorems it is necessary to give verbatim, not because of its importance, but because it serves to make evident the untenable position Cauchy had taken up in so peculiarly bringing determinants under the head of alternating aggregates. The theorem is (p. 164):—

> "Si, avec les variables comprises dans le tableau (5), on forme une fonction entière, du degré n, qui offre, dans chaque terme, n facteurs dont un seul appartienne à chacune des suites horizontales de ce tableau, et qui soit alternée par rapport à ces mêmes suites, la fonction entière dont il s'agit devra se réduire, au signe près, à la résultante s."

This not only justifies the definition proposed above to be substituted for Cauchy's, but it also entitles us to say that Cauchy having started by including determinants among alternating functions of one kind, viz., functions alternating with respect to every pair of n variables, soon succeeds in showing that they are alternating functions of an entirely different kind, viz., functions alternating with respect to every pair of *n ranks* of variables.

The only other noteworthy matter is a theorem in regard to

the solution of a set of simultaneous equations. Viewing the equations

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= \xi \\ a_2x + b_2y + c_2z &= \eta \\ a_3x + b_3y + c_3z &= \zeta \end{aligned}\right\}$$

as giving each of the three variables ξ, η, ζ, in terms of the other three x, y, z, we see that on solving for x, y, z, we obtain a converse system, that is to say, a system giving each of the three x, y, z, in terms of ξ, η, ζ. The latter system is, as we know,

$$\left.\begin{aligned} x &= \frac{A_1}{\Delta}\xi + \frac{A_2}{\Delta}\eta + \frac{A_3}{\Delta}\zeta, \\ y &= \frac{B_1}{\Delta}\xi + \frac{B_2}{\Delta}\eta + \frac{B_3}{\Delta}\zeta, \\ z &= \frac{C_1}{\Delta}\xi + \frac{C_2}{\Delta}\eta + \frac{C_3}{\Delta}\zeta, \end{aligned}\right\}.$$

where Δ is the determinant of the original system, and

$$A_1, B_1, C_1, A_2, \ldots,$$

are the cofactors in Δ of $a_1, b_1, c_1, a_2, \ldots$, respectively. Multiplying the determinants of the two systems, we obtain the determinant of the quantities

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}.$$

Hence (p. 176):—

"Si, n variables

$$x, y, z, \ldots, t,$$

étant liées à n autres variables

$$\mathrm{x}, \mathrm{y}, \mathrm{z}, \ldots, \mathrm{t},$$

par n équations linéaires, on suppose les unes exprimées en fonctions linéaires des autres, et réciproquement; les deux résultantes formées avec les coefficients que renfermeront ces fonctions linéaires dans les deux hypothèses, offriront un produit équivalent à l'unité." (xxi. 6)

RETROSPECT OF THE PERIOD, 1813–1841.

The characteristics of this period are best brought out by comparison with those of the preceding period, it being carefully borne in mind, in making the comparison, that the two are markedly unequal in length, the period of pioneering, as we may term it, extending to 120 years, and the next to only about 30.

In the first place, then, the evidence shows that as time went on there was considerable increase of interest in the subject, and a more widely spread knowledge of it; for, whereas to the longer period there belong 20 papers by 13 writers, for the shorter period the corresponding numbers are 35 and 18. Among the 18 writers, too, are represented nationalities which had previously not put in an appearance, viz., English, Italian, and Polish.

In the second place, we have proof that the early period was by far the more fruitful in original results. The pioneers had mapped out most of the prominent features of the new country; their successors had consequently to concern themselves in a considerable degree with filling in the details. During the second period one finds the fundamental propositions of the first period reproduced in new varieties of form; also, there are not awanting new proofs, extensions, and specialisations of old theorems; but of absolutely fresh departures there are comparatively few. An examination of the results numbered XLV.–LVIII. will show the character of these departures. It will be seen that they are due to Desnanot, Scherk, Schweins, Jacobi, Sylvester, and Cauchy. The most notable name of the period is Jacobi's, and next to it that of Schweins. There is no one name, however, which stands out in this period so conspicuously as Cauchy's does in the first period. Sylvester, unlike the others, it must be remembered, was only beginning his career, and we have yet to see him in the fulness of his power.

In the next place, the second period contrasts with the first in that during it important work was done on the subject of *special forms* of determinants. Here, again, the noteworthy names are those of Jacobi and Schweins.

Lastly, it having been noted in the retrospect of the first period that the subject of determinants was almost entirely a creation of the French intellect, we must not fail to take cognisance now of the

fact that in the second period the pre-eminence belongs to Germany, France, however, taking still a fairly good second place.

CAYLEY (1841).

[On a theorem in the geometry of position. *Cambridge Math. Journ.*, ii. pp. 267–271; or *Collected Math. Papers*, i. pp. 1–4.]

Of the two English mathematicians whose names are inseparably associated with the development of what has been called *Modern Higher Algebra*, Sylvester, as we have seen, was the first to direct public attention to the functions then partially known as determinants, but called by him in the heat of supposed discovery "zetaic products of differences." Cayley it was, however, who gave the great impetus to the study of them—an impetus due to two different causes, the choice of an exceedingly apt notation and the masterly manner in which he put the functions to use. How he obtained his knowledge we know not. It may be that Sylvester's two early papers had directed his attention to the matter, and that he had then read some of the authors who preceded Cauchy; but, whether this be true or not, it is certain that by his own independent research he had attained in 1841 a powerful and comprehensive grasp of the subject. The little paper to which we have now come is ample evidence of this. A peculiar interest attaches to it also, as being the first fruits of Cayley's genius, the earliest of that long and varied series of papers which has done so much to extend the bounds of pure mathematics.*

With characteristic directness and concision he opens as follows:—

> "We propose to apply the following (new?) theorem to the solution of two problems in Analytical Geometry.

* In a strictly chronological arrangement Cayley's paper would not follow, but precede the papers of Craufurd, Cauchy, and Jacobi of the same year. It was published in February: Cauchy's note was presented to the Academy on 8th March, and Jacobi's memoir bears the date 17th March, though not published for more than two months afterwards. As Cayley's first appearance, however, marks the beginning of a new epoch, and as the other papers referred to belong by their character to the preceding epoch, a slight deviation from the chronological order seems warranted.

"Let the symbols

$$|\alpha|\ ,\quad \begin{vmatrix}\alpha, & \beta\\ \alpha', & \beta'\end{vmatrix}\ ,\quad \begin{vmatrix}\alpha, & \beta, & \gamma\\ \alpha', & \beta', & \gamma'\\ \alpha'', & \beta'', & \gamma''\end{vmatrix},\ \&c. \qquad \text{(VII. 10)}$$

denote the quantities

$$\alpha,\quad \alpha\beta'-\alpha'\beta,\quad \alpha\beta'\gamma''-\alpha\beta''\gamma'+\alpha'\beta''\gamma-\alpha'\beta\gamma''+\alpha''\beta\gamma'-\alpha''\beta'\gamma,\quad \&c.$$

the law of whose formation is tolerably well known, but may be thus expressed,

$$|\alpha|=\alpha,\quad \begin{vmatrix}\alpha, & \beta\\ \alpha', & \beta'\end{vmatrix}=\alpha|\beta'|-\alpha'|\beta|,$$

$$\begin{vmatrix}\alpha, & \beta, & \gamma\\ \alpha', & \beta', & \gamma'\\ \alpha'', & \beta'', & \gamma''\end{vmatrix}=\alpha\begin{vmatrix}\beta', & \gamma'\\ \beta'', & \gamma''\end{vmatrix}+\alpha'\overline{\begin{vmatrix}\beta'', & \gamma''\\ \beta, & \gamma\end{vmatrix}}+\alpha''\begin{vmatrix}\beta, & \gamma\\ \beta', & \gamma'\end{vmatrix},\quad \&c.$$

the signs + being used when the number of terms in the side of the square is odd, and + and − alternately when it is even. Then the theorem in question is

$$\begin{vmatrix}\rho\,\alpha+\sigma\,\beta+\tau\,\gamma\ldots, & \rho\,\alpha'+\sigma\,\beta'+\tau\,\gamma'\ldots, & \rho\,\alpha''+\sigma\,\beta''+\tau\,\gamma''\ldots\\ \rho'\alpha+\sigma'\beta+\tau'\gamma\ldots, & \rho'\alpha'+\sigma'\beta'+\tau'\gamma'\ldots, & \rho'\alpha''+\sigma'\beta''+\tau'\gamma''\ldots\\ \rho''\alpha+\sigma''\beta+\tau''\gamma\ldots, & \rho''\alpha'+\sigma''\beta'+\tau''\gamma'\ldots, & \rho''\alpha''+\sigma''\beta''+\tau''\gamma''\ldots\\ \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots\end{vmatrix}$$

$$=\begin{vmatrix}\rho, & \sigma, & \tau\ldots\\ \rho', & \sigma', & \tau'\ldots\\ \rho'', & \sigma'', & \tau''\ldots\\ \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots\end{vmatrix}\begin{vmatrix}\alpha, & \beta, & \gamma\ldots\\ \alpha', & \beta', & \gamma'\ldots\\ \alpha'', & \beta'', & \gamma''\ldots\\ \ldots & \ldots & \ldots\\ \ldots & \ldots & \ldots\end{vmatrix}$$

"This theorem admits of a generalisation which we shall not have occasion to make use of, and which therefore we may notice at another opportunity."

Here then we have for the first time in the notation of determinants the pair of upright lines so familiar in all the later work. The introduction of them marks an epoch in the history, so important to the mathematician is this apparently trivial matter of notation. By means of them every determinant became representable, no matter how heterogeneous or complicated its elements might be; and the most disguised member of the family could be exhibited in its true lineaments. While the common characteristic of previous notations is their ability to represent the determinant of such a system as

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{vmatrix}$$

and failure to represent in the case of systems like

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}, \quad \begin{vmatrix} a & b & c \\ 1 & a & b \\ 0 & 1 & a \end{vmatrix}, \quad \begin{vmatrix} 4 & 5 & 6 \\ 3 & 2 & 7 \\ 8 & 1 & 0 \end{vmatrix};$$

Cayley's notation is equally suitable for all. To illustrate by analogy,—the infinitesimal calculus supplied with Lagrange's notation for the differential coefficient of $\phi(x)$, but unable to symbolise the differential coefficients of such a special function as $ax^3 + bx^2$, or $\log(1-x)/(1+x)$ would be in the exact predicament of the theory of deteminants prior to Cayley.

Of less importance is the fact, which the quotation indicates, that Cayley had discovered for himself the multiplication-theorem, but characteristically hesitated to proclaim it *new*: also, that, probably following Vandermonde, he took the recurrent law of formation for his definition, making the signs all + in one case and + and − alternately in the next, exactly as Vandermonde did.

He then proceeds to the seemingly geometrical problem:—

> "To find the relation that exists between the distances of five points in space.
>
> "We have, in general, whatever x_1, y_1, z_1, w_1, &c., denote,

$$\begin{vmatrix} x_1^2+y_1^2+z_1^2+w_1^2, & -2x_1, & -2y_1, & -2z_1, & -2w_1, & 1 \\ x_2^2+y_2^2+z_2^2+w_2^2, & -2x_2, & -2y_2, & -2z_2, & -2w_2, & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_5^2+y_5^2+z_5^2+w_5^2, & -2x_5, & -2y_5, & -2z_5, & -2w_5, & 1 \\ 1, & 0, & 0, & 0, & 0, & 0 \end{vmatrix}$$

multiplied into

$$\begin{vmatrix} 1, & x_1, & y_1, & z_1, & w_1, & x_1^2+y_1^2+z_1^2+w_1^2 \\ 1, & x_2, & y_2, & z_2, & w_2, & x_2^2+y_2^2+z_2^2+w_2^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1, & x_5, & y_5, & z_5, & w_5, & x_5^2+y_5^2+z_5^2+w_5^2 \\ 0, & 0, & 0, & 0, & 0, & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \overline{x_1-x_1}^2+\overline{y_1-y_1}^2+\overline{z_1-z_1}^2+\overline{w_1-w_1}^2, & \overline{x_1-x_2}^2+\ldots, & \overline{x_1-x_3}^2+\ldots, & \overline{x_1-x_4}^2+\ldots, & \overline{x_1-x_5}^2+\ldots, & 1 \\ \overline{x_2-x_1}^2+\ldots, & \overline{x_2-x_2}^2+\ldots, & \overline{x_2-x_3}^2+\ldots, & \overline{x_2-x_4}^2+\ldots, & \overline{x_2-x_5}^2+\ldots, & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \overline{x_5-x_1}^2+\ldots, & \overline{x_5-x_2}^2+\ldots, & \overline{x_5-x_3}^2+\ldots, & \overline{x_5-x_4}^2+\ldots, & \overline{x_5-x_5}^2+\ldots, & 1 \\ 1, & 1, & 1, & 1, & 1, & 0 \end{vmatrix}.$$

Putting the w's equal to 0, each factor of the first side of the equation vanishes, and therefore in this case the second side of the equation becomes equal to zero. Hence x_1,y_1,z_1, x_2,y_2,z_2, &c., being the coordinates of the points 1, 2, &c., situated arbitrarily in space, and $\overline{12}^2$, $\overline{13}^2$, &c., denoting the squares of the distances between these points, we have immediately the required relation

$$\begin{vmatrix} 0, & \overline{12}^2, & \overline{13}^2, & \overline{14}^2, & \overline{15}^2, & 1 \\ \overline{21}^2, & 0, & \overline{23}^2, & \overline{24}^2, & \overline{25}^2, & 1 \\ \overline{31}^2, & \overline{32}^2, & 0, & \overline{34}^2, & \overline{35}^2, & 1 \\ \overline{41}^2, & \overline{42}^2, & \overline{43}^2, & 0, & \overline{45}^2, & 1 \\ \overline{51}^2, & \overline{52}^2, & \overline{53}^2, & \overline{54}^2, & 0, & 1 \\ 1, & 1, & 1, & 1, & 1, & 0 \end{vmatrix} = 0,$$

which is easily expanded, though from the mere number of terms the process is somewhat long."

Than this no better example could have been chosen to illustrate what has just been said above regarding the great advantages of Cayley's notation. As is well known, the result arrived at had been given in forms, lengthy and forbidding, many years before by Lagrange and Carnot. What Cayley did was to rob it of all disguise, by expressing it as the vanishing of an elegantly formed determinant; and secondly, to show that the said determinant vanished because it was eight times the square* of another determinant whose zero character could not be overlooked. As has been implied, the result is purely algebraical, its geometrical character only appearing when x, y, z are taken to denote the coordinates of a point.

The corresponding identities for the cases of four points in a plane and three points in a straight line are given; and the latter of the two is most interestingly shown to be deducible also from the general theory of elimination. This is done as follows:—

"Let

$$x_{,,} - x_{,,,} = \alpha, \quad x_{,,,} - x_{,} = \beta, \quad x_{,} - x_{,,} = \gamma;$$

then

$$\overline{12}^2 = \gamma^2, \quad \overline{23}^2 = \alpha^2, \quad \overline{31}^2 = \beta^2, \quad \text{and } \alpha + \beta + \gamma = 0;$$

from which α, β, γ are to be eliminated. Multiplying the last equation by $\beta\gamma$, $\gamma\alpha$, $\alpha\beta$, and reducing by the three first,

$$\begin{aligned}
0 .\alpha &+ \overline{12}^2.\beta &+ \overline{31}^2.\gamma &+ \alpha\beta\gamma &= 0,\\
\overline{12}^2.\alpha &+ 0 .\beta &+ \overline{23}^2.\gamma &+ \alpha\beta\gamma &= 0,\\
\overline{31}^2.\alpha &+ \overline{23}^2.\beta &+ 0 .\gamma &+ \alpha\beta\gamma &= 0,\\
\alpha &+ \beta &+ \gamma &+ 0.\alpha\beta\gamma &= 0;
\end{aligned}$$

from which, eliminating $\alpha, \beta, \gamma, \alpha\beta\gamma$ by the general theory of simple equations

$$\begin{vmatrix}
0, & \overline{12}^2, & \overline{13}^2, & 1\\
\overline{21}^2, & 0, & \overline{23}^2, & 1\\
\overline{31}^2, & \overline{32}^2, & 0, & 1\\
1, & 1, & 1. & 0
\end{vmatrix} = 0."$$

* The first factor being 16 times the second, and the w's unnecessary.

The conviction that the identity ought to come out as a result of elimination, and the ingenious fulfilment of it by using the identity $\alpha+\beta+\gamma=0$ after the manner of Sylvester's paper of 1840 are very noteworthy.

It is finally noticed that "the additional equation that exists between the distances of five points on a sphere" can be similarly obtained, and the process is given.

GRUNERT (1842).

[Ueber die Theorie der Elimination. *Archiv der Math. u. Phys.*, ii. pp. 76–105, 345–377.]

This paper, extending to more than sixty pages, is little else than an amplified reproduction of work by Cauchy. Nine pages at the beginning concern simultaneous linear equations; the rest is entirely taken up with the various modes of eliminating x between two algebraical equations, $\phi(x)=0$, $\psi(x)=0$.

In the former part, which seems based on the third chapter of the *Cours d'Analyse*, the only fresh matter is a lengthy proof of the proposition that *the difference-product of any number of quantities changes sign when two of the quantities are transposed.* It will suffice to note in regard to it that the so-called inductive method is followed, and that two cases have to be considered, viz. (1) when the new quantity is not one of the two which are interchanged, (2) when it is. (III. 38)

The second part follows closely Cauchy's memoir of 1840.

TERQUEM (1842).

[Notice sur l'élimination. Formules de Cramer. *Nouv. Annales de Math.*, i. pp. 125–131.*]

This is merely a simply written exposition of Cramer's rule, and of Bezout's rule of 1779, and contains nothing noteworthy. It is curious, however, to observe the reason given for directing attention to Cramer's rule,—"Comme ce procédé ne se trouve

* The continuation intimated at the close (p. 131) was never made.

décrit, que je sache, que dans un seul ouvrage élémentaire français, peu répandu (*Manuel d'Algèbre*, p. 80, 2e édition, 1836)." This indicates a sad contrast to the state of matters attested to by Gergonne,* showing that there is a fashion which changeth even in things mathematical. The new favourite, it also appears, was Bezout's rule of 1764; for in passing this over, in order to give an account of the same author's rule of later date, Terquem says in regard to it, " Comme ce procédé est décrit dans tous les ouvrages à l'usage des classes, nous ne nous y arrêterons pas."

CAYLEY (1843).

[Demonstration of Pascal's Theorem. *Cambridge Math. Journ.*, iv. pp. 18–20; or *Collected Math. Papers*, i. pp. 43–45.]

At the outset of this paper two lemmas are given, the second of which stands as follows:—

" Lemma 2. Representing the determinants

$$\begin{vmatrix} x_1, & y_1, & z_1 \\ x_2, & y_2, & z_2 \\ x_3, & y_3, & z_3 \end{vmatrix}, \text{ \&c.}$$

by the abbreviated notation $\overline{123}$, &c.; the following equation is identically true:

$$\overline{345}\,.\,\overline{126} - \overline{346}\,.\,\overline{125} + \overline{356}\,.\,\overline{124} - \overline{456}\,.\,\overline{123} = 0.$$

This is an immediate consequence of the equations

$$\begin{vmatrix} . & . & x_3, & x_4, & x_5, & x_6 \\ . & . & y_3, & y_4, & y_5, & y_6 \\ . & . & z_3, & z_4, & z_5, & z_6 \\ x_1, & x_2, & x_3, & x_4, & x_5, & x_6 \\ y_1, & y_2, & y_3, & y_4, & y_5, & y_6 \\ z_1, & z_2, & z_3, & z_4, & z_5, & z_6 \end{vmatrix} = \begin{vmatrix} . & . & x_3, & x_4, & x_5, & x_6 \\ . & . & y_3, & y_4, & y_5, & y_6 \\ . & . & z_3, & z_4, & z_5, & z_6 \\ x_1, & x_2, & . & . & . & . \\ y_1, & y_2, & . & . & . & . \\ z_1, & z_2, & . & . & . & . \end{vmatrix} = 0."$$

(XXIII. 13)

* The passage in question, which we quoted under Cramer, is to be found in the *Annales de Math.*, xx. p. 45.

The identity is readily recognisable as Bezout's (1779). The mode of arriving at it, however, is fresh, and worthy of every attention. The determinant of the sixth order on the left is shown to be equal to zero; and it is implied that the identity is got by transforming the said vanishing determinant into an aggregate of products of pairs of determinants by means of Laplace's expansion-theorem. The method is far-reaching in its application, and manifestly Cayley could have used it to produce a host of identities of similar kind.

The equatement of the two determinants of the sixth order deserves also to be noted, and may be taken as evidence that Cayley was familiar with the theorem that a determinant is not altered if each element of one row be diminished by the corresponding element of another row. No such theorem had been formulated or used before his time. (LIX.)

Lastly, it may be pointed out that we have here the first instance of a practice which afterwards became very general, viz., putting a dot instead of a zero element when writing a determinant.

The other lemma and the main body of the paper are geometrical; but as an important determinant identity is implicitly established in the course of the investigation, and as it is of the greatest historical importance to make evident the wonderful command which Cayley with his new notation had suddenly obtained over determinants, we shall give the full text of these portions also, at least up to a certain point.

> "Lemma 1. Let $U = Ax + By + Cz = 0$ be the equation of a plane passing through a given point taken for the origin, and consider the planes
>
> $$U_1 = 0, \quad U_2 = 0, \quad U_3 = 0, \quad U_4 = 0, \quad U_5 = 0, \quad U_6 = 0;$$
>
> the condition which expresses that the intersections of the planes (1) and (2), (3) and (4), (5) and (6), lie in the same plane, may be written down under the form *

* The commas which Cayley prints after the elements in a determinant we omit here and henceforth.

$$\begin{vmatrix} A_1 & A_2 & A_3 & A_4 & . & . \\ B_1 & B_2 & B_3 & B_4 & . & . \\ C_1 & C_2 & C_3 & C_4 & . & . \\ . & . & A_3 & A_4 & A_5 & A_6 \\ . & . & B_3 & B_4 & B_5 & B_6 \\ . & . & C_3 & C_4 & C_5 & C_6 \end{vmatrix} = 0.$$

"Consider now the points 1, 2, 3, 4, 5, 6, the coordinates of these being respectively $x_1, y_1, z_1, \ldots\ldots, x_6, y_6, z_6$. I represent, for shortness, the equation to the plane passing through the origin, and the points 1, 2, which may be called the plane $\overline{12}$, in the form

$$x\,\overline{12}_x + y\,\overline{12}_y + z\,\overline{12}_z = 0;$$

consequently the symbols $\overline{12}_x$, $\overline{12}_y$, $\overline{12}_z$ denote respectively $y_1z_2 - y_2z_1$, $z_1x_2 - z_2x_1$, $x_1y_2 - x_2y_1$, and similarly for the planes $\overline{13}$, &c. If now the intersections of $\overline{12}$ and $\overline{45}$, $\overline{23}$ and $\overline{56}$, $\overline{34}$ and $\overline{61}$ lie in the same plane, we must have by lemma (1) the equation

$$\begin{vmatrix} 12_x & 45_x & 23_x & 56_x & . & . \\ 12_y & 45_y & 23_y & 56_y & . & . \\ 12_z & 45_z & 23_z & 56_z & . & . \\ . & . & 23_x & 56_x & 34_x & 61_x \\ . & . & 23_y & 56_y & 34_y & 61_y \\ . & . & 23_z & 56_z & 34_z & 61_z \end{vmatrix} = 0.$$

Multiplying the two sides of this equation by the two sides respectively of the equation

$$\begin{vmatrix} x_6 & x_1 & x_2 & . & . & . \\ y_6 & y_1 & y_2 & . & . & . \\ z_6 & z_1 & z_2 & . & . & . \\ . & . & . & x_3 & x_4 & x_5 \\ . & . & . & y_3 & y_4 & y_5 \\ . & . & . & z_3 & z_4 & z_5 \end{vmatrix} = \overline{612} \,.\, \overline{345},$$

and observing the equations

$$x_6\overline{12}_x + y_6\overline{12}_y + z_6\overline{12}_z = \overline{612}\,, \quad \overline{112} = 0\,, \quad \&c.$$

this becomes

$$\begin{vmatrix} 612 & . & . & . & . & . \\ 645 & 145 & 245 & . & . & . \\ 623 & 123 & . & . & 423 & 523 \\ . & 156 & 256 & 356 & 456 & . \\ . & . & . & . & . & 534 \\ . & . & . & 361 & 461 & 561 \end{vmatrix} = 0\,,$$

reducible to

$$\overline{612}\,.\,\overline{534} \begin{vmatrix} \overline{145} & \overline{245} & . & . \\ 123 & . & . & 423 \\ 156 & 256 & 356 & 456 \\ . & . & 361 & 461 \end{vmatrix} = 0\,;$$

or, omitting the factor $\overline{612}\,.\,\overline{534}$, and expanding

$$\overline{145}\,.\,\overline{256}\,.\,\overline{423}\,.\,\overline{361} + \overline{245}\,.\,\overline{123}\,.\,\overline{456}\,.\,\overline{361}$$

$$- \overline{245}\,.\,\overline{123}\,.\,\overline{356}\,.\,\overline{461} - \overline{245}\,.\,\overline{156}\,.\,\overline{423}\,.\,\overline{361} = 0.\text{"}$$

The purely algebraical identity involved in this is in later notation

$$\begin{vmatrix} |y_1z_2| & |y_4z_5| & |y_2z_3| & |y_5z_6| & . & . \\ |z_1x_2| & |z_4x_5| & |z_2x_3| & |z_5x_6| & . & . \\ |x_1y_2| & |x_4y_5| & |x_2y_3| & |x_5y_6| & . & . \\ . & . & |y_2z_3| & |y_5z_6| & |y_3z_4| & |y_6z_1| \\ . & . & |z_2x_3| & |z_5x_6| & |z_3x_4| & |z_6x_1| \\ . & . & |x_2y_3| & |x_5y_6| & |x_3y_4| & |x_6y_1| \end{vmatrix} = \begin{vmatrix} |x_1y_2z_3| & . & . & |x_4y_2z_3| \\ |x_1y_4z_5| & |x_2y_4z_5| & . & . \\ |x_1y_5z_6| & |x_2y_5z_6| & |x_3y_5z_6| & |x_4y_5z_6| \\ . & . & |x_3y_6z_1| & |x_4y_6z_1| \end{vmatrix}.$$

HESSE (1843).

[Ueber die Bildung der Endgleichung, welche durch Elimination einer Variabeln aus zwei algebraischen Gleichungen hervorgeht, und die Bestimmung ihres Grades. *Crelle's Journal*, xxvii. pp. 1–5.]

Hesse, at this time, must have been unaware of Richelot's paper (dated from the same University), and Grunert's paper, not to speak of writings published outside Germany, for the method which he gives of finding the final equation is nothing more nor less than Sylvester's dialytic method. His exposition, to say the least, is not preferable to Grunert's, and the determinant of the $(m+n)^{\text{th}}$ order which he prints is misleading in points of detail.

GRASSMANN (June 1844).

[Die Wissenschaft der extensiven Grösse, oder die Ausdehnungslehre, eine neue mathematische Disciplin dargestellt und durch Anwendungen erläutert. Erster Theil, die lineale Ausdehnungslehre enthaltend. xxxii + 279 pp. Leipzig, 1844.]

A quite peculiar form of the law of formation of a determinant had its origin with Grassmann. Grassmann, it will be remembered, was one of the most distinguished of the mathematicians who occupied themselves with the search for an *Algebra of directed quantities*, or with the allied problem of the geometrical interpretation of the so-called imaginary expressions of ordinary algebra. By the beginning of the third decade of the century, the way had been gradually, though intermittingly, prepared for important discoveries on the subject by the writings of Wallis (1685), Buée (1805), Argand (1806), Servois (1813), Mourey (1828), Warren (1828), and Gauss (1831).* With Hamilton and Grassmann important discoveries came. Hamilton, whose writings of 1833 and 1835 show that even then he had meditated to some purpose on the matter, announced in 1843 his great invention of Quaternions. In 1844 Grassmann followed with the first part of the Ausdehnungslehre.

* See art. "Quaternions," by Professor Tait, in *Encyclopædia Britannica*; or Hamilton's *Lectures on Quaternions.*

In his preface Grassmann explains the steps by which he had been led to his theory. First, there was the question of the addition of directed straight lines (Strecken), or *vectors*, to use Hamilton's widely accepted term. This it was unnecessary to linger over, as his predecessors had already dealt satisfactorily with it. Then came the question of multiplication of vectors. Seeing that when a and b represent two lines in magnitude only, in other words, are scalars and not vectors, the product ab represents the rectangle of which a and b are adjacent sides, Grassmann ventured to denote by the product ab, when a and b are vectors, a parallelogram having the vectors for adjacent sides. This definition of multiplication manifestly entailed the result

$$a^2 = 0\,;$$

and along with the definition of addition required further that

$$a(b+c) = ab+ac\,.$$

These two again involved a third, viz.,

$$ab = -ba$$

for from the two we have

$$\begin{aligned} 0 &= (a+b)^2, \\ &= (a+b)a+(a+b)b, \\ &= a^2+ba+ab+b^2, \\ &= ba+ab\,. \end{aligned}$$

The remaining steps of the building up of the theory need not be told, as these laws of *outer* multiplication ("*äussere Multiplication*") suffice for the purpose we have in view.

The exposition of the theory itself is broken up into an introduction and nine chapters, all of them marked by ability and much originality. It is the second chapter which deals specially with outer multiplication, and at the end of it (pp. 70–73) occurs the application which concerns determinants. The matter is introduced by a sentence or two pointing out that it is scarcely to be expected that outer multiplication can be so directly applied to ordinary algebra as to geometry and dynamics, because in ordinary algebra the quantities are essentially alike (*gleichartige*, in the sense of the Ausdehnungslehre), and outer multiplication presupposes the idea

of unlikeness. In certain circumstances, however, we are told that we may impose distinctions upon the quantities, and then outer multiplication may be applied with notable results.

"Um hiervon eine Idee zu geben, will ich n Gleichungen ersten Grades mit n Unbekannten setzen, von der Form

$$\begin{aligned} a_1x_1 + a_2x_2 + \ldots + a_nx_n &= a_0, \\ b_1x_1 + b_2x_2 + \ldots + b_nx_n &= b_0, \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ s_1x_1 + s_2x_2 + \ldots + s_nx_n &= s_0, \end{aligned}$$

wo $x_1, \ldots, x_n$ die Unbekannten seien. Hier können wir die Zahlencoefficienten, welche verschiedenen Gleichungen angehören, sofern wir diese Verschiedenheit an ihrem Begriff noch festhalten, als verschiedenartig ansehen, und zwar alle als an sich verschiedenartig, d. h. als unabhängig in dem Sinne unserer Wissenschaft, die einer und derselben Gleichung als unter sich in derselben Beziehung gleichartig. Addiren wir nun in diesem Sinne alle n Gleichungen und bezeichnen die Summe des Verschiedenartigen in dem Sinne unserer Wissenschaft mit dem Verknüpfungszeichen $\dot{+}$, indem die gleichen Stellen in den so gebildeten Summenausdrücken immer dem Gleichartigen zukommen sollen, so erhalten wir

$$\begin{aligned} (a_1 \dot{+} b_1 \dot{+} \ldots \dot{+} s_1)x_1 \dot{+} (a_2 \dot{+} b_2 \dot{+} \ldots \dot{+} s_2)x_2 \\ \ldots \dot{+} (a_n \dot{+} b_n \dot{+} \ldots \dot{+} s_n)x_2 = a_0 \dot{+} b_0 \dot{+} \ldots \dot{+} s_0, \end{aligned}$$

oder bezeichnen wir $(a_1 \dot{+} b_1 \dot{+} \ldots \dot{+} s_1)$ mit p_1, und entsprechend die übrigen Summen, so haben wir

$$p_1x_1 \dot{+} p_2x_2 \dot{+} \ldots \dot{+} p_nx_n = p_0.$$

Aus dieser Gleichung, welche die Stelle jener n Gleichungen vertritt, lässt sich nun auf der Stelle jede der Unbekannten, z. B. x_1 finden, wenn wir die beiden Seiten mit dem äusseren Produkte aus den Coefficienten der übrigen Unbekannten äusserlich multipliciren, also hier mit $p_2p_3 \ldots p_n$. Da nämlich, wenn man die Glieder der linken Seite einzeln multiplicirt, nach dem Begriff des äusseren Produktes, alle

Produkte wegfallen, welche zwei gleiche Factoren enthalten, so erhält man

$$p_1 p_2 p_3 \cdot \cdot \cdot p_n x_1 = p_0 p_2 p_3 \cdot \cdot \cdot p_n .$$

Also da beide Produkte, als demselben System n-ter Stufe angehörig einander gleichartig sind, so hat man

$$x_1 = \frac{p_0 p_2 p_3 \cdot \cdot \cdot p_n}{p_1 p_2 p_3 \cdot \cdot \cdot p_n} ."$$

The method is thus seen to consist in the deduction of a new equation by addition, and in the elimination of all the unknowns, except one, from the equation, by multiplying both sides by the product of the coefficients of the other unknowns,—the multiplication in question being "outer," and for the purposes of the multiplication, any two coefficients of one and the same equation being considered as "like," and any two belonging to different equations as "unlike." For example, in the case of $n = 3$ we have

$$x_1 = \frac{(a_0 + b_0 + c_0) \,.\, (a_2 + b_2 + c_2) \,.\, (a_3 + b_3 + c_3)}{(a_1 + b_1 + c_1) \,.\, (a_2 + b_2 + c_2) \,.\, (a_3 + b_3 + c_3)},$$

$$= \frac{(a_0 a_2 + a_0 b_2 + a_0 c_2 + b_0 a_2 + b_0 b_2 + \ldots) \,.\, (a_3 + b_3 + c_3)}{(a_1 a_2 + a_1 b_2 + a_1 c_2 + b_1 a_2 + b_1 b_2 + \ldots) \,.\, (a_3 + b_3 + c_3)},$$

$$= \frac{(a_0 b_2 + a_0 c_2 + b_0 a_2 + b_0 c_2 + c_0 a_2 + c_0 b_2) \,.\, (a_3 + b_3 + c_3)}{(a_1 b_2 + a_1 c_2 + b_1 a_2 + b_1 c_2 + c_1 a_2 + c_1 b_2) \,.\, (a_3 + b_3 + c_3)},$$

since $a_0 a_2 = b_0 b_2 = \ldots = c_1 c_2 = 0$; and finally

$$x_1 = \frac{a_0 b_2 c_3 - a_0 b_3 c_2 + a_2 b_3 c_0 - a_2 b_0 c_3 + a_3 b_0 c_2 - a_3 b_2 c_0}{a_1 b_2 c_3 - a_1 b_3 c_2 + a_2 b_3 c_1 - a_2 b_1 c_3 + a_3 b_1 c_2 - a_3 b_2 c_1},$$

"worin wir, da alles entsprechend geordnet ist, wieder die gewöhnliche Multiplicationsbezeichnung einführen konnten." (III. 39)

All this semblance of demonstration is of little moment compared with the fact sought to be demonstrated, viz., that a determinant is expressible as the outer product of the sums of the elements of its columns. Grassmann, however, makes no reference to determinants.

In a paragraph of a subsequent chapter (p. 129), he takes up the problem of elimination between two equations of the mth and n^{th} degrees. What it contains is a reproduction of Sylvester's dialytic method, without any reference to the author of the method.

On the Scalar Relations connecting Six Vectors. By the **Rev. M. M. U. Wilkinson**, Reepham Vicarage, near Norwich. *Communicated by* Professor TAIT.

(Read July 15, 1889.)

A. *Introduction.*

1. In the case of two Vectors, α, β, the Scalars $\alpha^2, \beta^2, S\alpha\beta$ are connected by no relation. In other words, the Tensors of two straight lines, and the angle between them, are three independent quantities. In this case every other Scalar Function of α and β can be expressed in terms of $\alpha^2, \beta^2, S\alpha\beta$. Thus,

$$S\alpha\beta\alpha\beta = 2S^2\alpha\beta - \alpha^2\beta^2.$$

For convenience we shall call Scalars of the form $-\alpha^2$, Tensor Scalars, and Scalars of the form $S\alpha\beta$, Primary Scalars.

2. The introduction of a third Vector, γ, introduces three fresh Scalars, as the Tensor of γ, and its two inclinations to the Vectors α, β. In this case we have six independent Scalars, in terms of which every other Scalar involving only the three Vectors can be expressed. Thus, $\alpha^2, \beta^2, \gamma^2, S\alpha\beta, S\beta\gamma, S\gamma\alpha$, are independent Scalars. All other Scalar functions of α, β, γ, connect themselves with these by equations. Thus,

$$S^2\alpha\beta\gamma = \begin{vmatrix} S\alpha\gamma, & S\alpha\beta, & \alpha^2 \\ S\beta\gamma, & \beta^2, & S\alpha\beta \\ \gamma^2, & S\beta\gamma, & S\alpha\gamma \end{vmatrix} \quad \ldots\ldots (1)$$

3. In general, if we have n Vectors, we have $3(n-1)$ independent Scalars, as each fresh Vector introduces three fresh Scalars, namely, its Tensor and its inclination to any two of the other Vectors. All other Scalars involving the n Vectors can be expressed in terms of the $3(n-1)$ independent Scalars. Now, with n Vectors we have n Tensors, and $\frac{n(n-1)}{1.2}$ Primary Scalars, making $\frac{n(n+1)}{1.2}$ Scalars in all. As of these $3(n-1)$ are independent, it follows that $\frac{(n-2)(n-3)}{1.2}$ independent equations connect the Tensors and Primary Scalars.

All Scalars, it is clear, express in terms of Tensors and Primary Scalars.

Thus, when $n=4$, we have one such independent equation, as

$$\begin{vmatrix} a^2 , & S\alpha\beta , & S\alpha\gamma , & S\alpha\delta \\ S\alpha\beta , & \beta^2 , & S\beta\gamma , & S\beta\delta \\ S\alpha\gamma , & S\beta\gamma , & \gamma^2 , & S\gamma\delta \\ S\alpha\delta , & S\beta\delta , & S\gamma\delta , & \delta^2 \end{vmatrix} = 0 \quad . \quad . \quad . \quad (2)$$

Equations such as

$$S\alpha\delta S\alpha\beta\gamma = a^2 S\beta\gamma\delta + S\alpha\beta S\gamma\alpha\delta + S\alpha\gamma S\alpha\beta\delta \quad . \quad . \quad . \quad (3)$$

are not independent equations, as they can be obtained from (2) by means of equations of the form (1).

4. In the case of $n=5$ there are three relations connecting the five Tensors and ten Primary Scalars. Here various problems present themselves, of this character; having given twelve of these Scalars, to find equations connecting them with the other three. Of course the twelve must be so selected as not to contain ten which are functions of only four vectors, and which would, therefore, be connected by an equation (2).

In the case of $n=6$, we have six Tensors and fifteen Primary Scalars, connected by six independent equations. So if fifteen of these, so selected as to be a set of fifteen independent Scalars, are given, six equations sufficient to determine the remaining six, can be found.

The problem we aim at discussing is, in the case of six Vectors, having given the fifteen Primary Scalars to express in terms of them the Tensors, and other Scalars.

B. *Principal Formulæ.*

5. Our Vectors are α, β, γ, δ, ϵ, ζ.

We have at once,

$$S\alpha\beta\gamma\delta = S\alpha\beta S\gamma\delta - S\alpha\gamma S\beta\delta + S\alpha\delta S\beta\gamma ; \quad . \quad . \quad . \quad (4)$$

and, since,

$$S\alpha\beta\gamma S\delta\epsilon\zeta = S\alpha\beta(V\delta\epsilon S\gamma\zeta + V\epsilon\zeta S\delta\gamma + V\zeta\delta S\gamma\epsilon) ;$$

we have the important formula,

$$S\alpha\beta\gamma S\delta\epsilon\zeta = \begin{vmatrix} S\alpha\zeta, & S\alpha\epsilon, & S\alpha\delta \\ S\beta\zeta, & S\beta\epsilon, & S\beta\delta \\ S\gamma\zeta, & S\gamma\epsilon, & S\gamma\delta \end{vmatrix}; \quad . \quad . \quad . \quad (5)$$

of which (1) is a particular case.
Also, since,

$$S\alpha\beta\gamma\delta\epsilon\zeta = S\alpha\beta\gamma S\delta\epsilon\zeta + S\alpha\beta\gamma(\delta S\epsilon\zeta - \epsilon S\delta\zeta + \zeta S\delta\epsilon); \quad . \quad (6)$$

we see that the expression for $S\alpha\beta\gamma\delta\epsilon\zeta$ in terms of Primary Scalars contains $6 + 3 \times 3 = 15$ terms in all.

6. Representing the 10 determinants (of which $S\alpha\beta\gamma S\delta\epsilon\zeta$ is one) as follows:—

$$\left.\begin{array}{ccc} & S\alpha\beta\gamma S\delta\epsilon\zeta = V; & \\ S\alpha\beta\zeta S\gamma\delta\epsilon = \lambda_1^2 V; & S\alpha\beta\epsilon S\gamma\zeta\delta = \lambda_2^2 V; & S\alpha\beta\delta S\gamma\epsilon\zeta = \lambda_3^2 V; \\ S\gamma\alpha\zeta S\beta\delta\epsilon = \mu_1^2 V; & S\gamma\alpha\epsilon S\beta\zeta\delta = \mu_2^2 V; & S\gamma\alpha\delta S\beta\epsilon\zeta = \mu_3^2 V; \\ S\beta\gamma\zeta S\alpha\delta\epsilon = \nu_1^2 V; & S\beta\gamma\epsilon S\alpha\zeta\delta = \nu_2^2 V; & S\beta\gamma\delta S\alpha\epsilon\zeta = \nu_3^2 V; \end{array}\right\} \quad (7)$$

we have, at once,

$$\left.\begin{array}{l} 1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \mu_1^2 + \mu_2^2 + \mu_3^2 = \nu_1^2 + \nu_2^2 + \nu_3^2 = \\ \quad = \lambda_1^2 + \mu_1^2 + \nu_1^2 = \lambda_2^2 + \mu_2^2 + \nu_2^2 = \lambda_3^2 + \mu_3^2 + \nu_3^2; \end{array}\right\} \quad . \quad . \quad (8)$$

Now define as follows,

$$W = S\alpha\gamma\epsilon S\alpha\delta\zeta S\beta\epsilon\zeta S\beta\gamma\delta - S\alpha\gamma\delta S\alpha\epsilon\zeta S\beta\delta\zeta S\beta\gamma\epsilon; \quad . \quad . \quad (9)$$

a little consideration will show that, by permuting the Vectors, there are only two expressions of the form W, and that W^2 is a symmetrical function of the Vectors. Thus, since,

$$S\beta\gamma\delta S\beta\epsilon\zeta + S\beta\gamma\epsilon S\beta\zeta\delta + S\beta\gamma\zeta S\beta\delta\epsilon = 0;$$
$$S\alpha\gamma\delta S\alpha\epsilon\zeta + S\alpha\gamma\epsilon S\alpha\zeta\delta + S\alpha\gamma\zeta S\alpha\delta\epsilon = 0;$$

we have,

$$S\beta\gamma\delta S\beta\epsilon\zeta S\alpha\gamma\epsilon S\alpha\zeta\delta - S\alpha\gamma\delta S\alpha\epsilon\zeta S\beta\gamma\epsilon S\beta\zeta\delta$$
$$= S\alpha\gamma\zeta S\alpha\delta\epsilon S\beta\gamma\epsilon S\beta\zeta\delta - S\beta\gamma\zeta S\beta\delta\epsilon S\alpha\gamma\epsilon S\alpha\zeta\delta.$$

Calling the permutation of any two Vectors one permutation, an odd number of permutations changes the sign of W merely. The formulæ (7), (8), readily show that W^2 is symmetrical.

7. We have,

$$W^2 + 4\mu_2^2\mu_3^2\nu_2^2\nu_3^2V^4 = V^4(\mu_1^2\nu_1^2 - \mu_2^2\nu_2^2 - \mu_3^2\nu_3^2)^2 \text{ ; or,}$$

$$W^2 = V^4(\mu_1^4\nu_1^4 + \mu_2^4\nu_2^4 + \mu_3^4\nu_3^4 - 2\mu_2^2\mu_3^2\nu_2^2\nu_3^2 - 2\mu_3^2\mu_1^2\nu_3^2\nu_1^2 - 2\mu_1^2\mu_2^2\nu_1^2\nu_2^2) \quad . \quad (10)$$

The symmetry of this is obvious, for

$$\{\mu_3^2\nu_3^2 - (\mu_1\nu_1 + \mu_2\nu_2)^2\}\{\mu_3^2\nu_3^2 - (\mu_1\nu_1 - \mu_2\nu_2)^2\} =$$

$$= \{1 - \mu_1^2 - \mu_2^2 - \nu_1^2 - \nu_2^2 + (\mu_1\nu_2 - \mu_2\nu_1)^2\}\{1 - \mu_1^2 - \mu_2^2 - \nu_1^2 - \nu_2^2 + (\mu_1\nu_2 + \mu_2\nu_1)^2\} \text{ ;}$$

$$\{\lambda_3^2 - (\nu_1\mu_2 + \nu_2\mu_1)^2\}\{\lambda_3^2 - (\nu_1\mu_2 - \nu_2\mu_1)^2\} =$$

$$= \{-1 + \mu_1^2 + \mu_2^2 + \nu_1^2 + \nu_2^2 - (\mu_1\nu_2 + \mu_2\nu_1)^2\}\{-1 + \mu_1^2 + \mu_2^2 + \nu_1^2 + \nu_2^2 - (\mu_1\nu_2 - \mu_2\nu_1)^2\}$$

When W vanishes, be it observed, equations (8) and (10) give us the well-known rectangular system.

8. It will assist brevity in the calculations to define as follows :—

$$\left.\begin{array}{lll} b_1 = S\alpha\zeta S\beta\epsilon S\gamma\delta \text{ ;} & c_1 = S\alpha\zeta S\beta\delta S\gamma\epsilon \text{ ;} & e_1 = S\alpha\zeta S\beta\gamma S\delta\epsilon \text{ ;} \\ b_2 = S\alpha\epsilon S\beta\zeta S\gamma\delta \text{ ;} & c_2 = S\alpha\epsilon S\beta\gamma S\delta\zeta \text{ ;} & e_2 = S\alpha\epsilon S\beta\delta S\gamma\zeta \text{ ;} \\ b_3 = S\alpha\delta S\beta\gamma S\epsilon\zeta \text{ ;} & c_3 = S\alpha\delta S\beta\zeta S\gamma\epsilon \text{ ;} & e_3 = S\alpha\delta S\beta\epsilon S\gamma\zeta \text{ ;} \\ b_4 = S\alpha\gamma S\beta\delta S\epsilon\zeta \text{ ;} & c_4 = S\alpha\gamma S\beta\epsilon S\delta\zeta \text{ ;} & e_4 = S\alpha\gamma S\beta\zeta S\delta\epsilon \text{ ;} \\ b_5 = S\alpha\beta S\gamma\delta S\epsilon\zeta \text{ ;} & c_5 = S\alpha\beta S\gamma\epsilon S\delta\zeta \text{ ;} & e_5 = S\alpha\beta S\gamma\zeta S\delta\epsilon \end{array}\right\} . \quad (11)$$

$$\left.\begin{array}{lll} B_1 = c_1 - e_1 \text{ ;} & C_1 = e_1 - b_1 \text{ ;} & E_1 = b_1 - c_1 \text{ ;} \\ B_2 = c_2 - e_2 \text{ ;} & C_2 = e_2 - b_2 \text{ ;} & E_2 = b_2 - c_2 \text{ ;} \\ B_3 = c_3 - e_3 \text{ ;} & C_3 = e_3 - b_3 \text{ ;} & E_3 b_3 - c_3 \text{ ;} \\ B_4 = c_4 - e_4 \text{ ;} & C_4 = e_4 - b_4 \text{ ;} & E_4 = b_4 - c_4 \text{ ;} \\ B_5 = e_5 - c_5 \text{ ;} & C_5 = b_5 - e_5 \text{ ;} & E_5 = c_5 - b_5 \text{ ;} \end{array}\right\} \quad . \quad . \quad (12)$$

so that

$$0 = B_1 + C_1 + E_1 = B_2 + C_2 + E_2 = \ldots = B_5 + C_5 + E_5 \quad . \quad (13)$$

9. Then we have,

$$\left.\begin{array}{lll} & V = E_1 + C_2 + B_3 \text{ ;} & \\ \lambda_1^2V = -B_2 - C_3 - E_4 \text{ ;} & \lambda_2^2V = -E_3 - B_1 - C_4 \text{ ;} & \lambda_3^2V = -C_1 - E_2 - B_4 \text{ ;} \\ \mu_1^2V = -E_5 - E_2 - E_3 \text{ ;} & \mu_2^2V = -C_1 - C_5 - C_3 \text{ ;} & \mu_3^2V = -B_1 - B_2 - B_5 \text{ ;} \\ \nu_1^2V = E_1 + E_4 + E_5 \text{ ;} & \nu_2^2V = C_5 + C_2 + C_4 \text{ ;} & \nu_3^2V = B_4 + B_5 + B_3 \end{array}\right\} \quad (14)$$

Whence, among many other expressions for W^2, we have,

$$\begin{aligned} W^2 = \quad & (B_5+B_1+B_2)^2(B_5+B_3+B_4)^2 \\ & +(C_5+C_1+C_3)^2(C_5+C_2+C_4)^2 \\ & +(E_5+E_1+E_4)^2(E_5+E_2+E_3)^2 \end{aligned}$$

$$\begin{aligned} & -2(C_5+C_1+C_3)(C_5+C_2+C_4)(E_5+E_1+E_4)(E_5+E_2+E_3) \\ & -2(E_5+E_1+E_4)(E_5+E_2+E_3)(B_5+B_1+B_2)(B_5+B_3+B_3) \\ & -2(B_5+B_1+B_2)(B_5+B_3+B_4)(C_5+C_1+C_3)(C_5+C_2+C_4)\,; \quad (15) \end{aligned}$$

we postpone for the present the consideration of the expansion of this, which will, of course, be symmetrical in form as well as in reality.

10. From the known formula for 8 vectors,

$$\begin{vmatrix} S\alpha\alpha_1, & S\beta\alpha_1, & S\gamma\alpha_1, & S\delta\alpha_1 \\ S\alpha\beta_1, & S\beta\beta_1, & S\gamma\beta_1, & S\delta\beta_1 \\ S\alpha\gamma_1, & S\beta\gamma_1, & S\gamma\gamma_1, & S\delta\gamma_1 \\ S\alpha\delta_1, & S\beta\delta_1, & S\gamma\delta_1, & S\delta\delta_1 \end{vmatrix} = 0\,; \quad . \; . \quad (16)$$

putting $\alpha_1=\alpha$, $\beta_1=\beta$, $\gamma_1=\epsilon$, $\delta_1=\zeta$, we find

$$D_1\alpha^2\beta^2 + D_2\alpha^2 + D_3\beta^2 + D_4 = 0\,; \quad . \; . \; . \quad (17)$$

where

$$D_1 = \begin{vmatrix} S\gamma\epsilon, & S\delta\epsilon \\ S\gamma\zeta, & S\delta\zeta \end{vmatrix} = S\gamma\delta\nabla\zeta\epsilon\,; \quad . \; . \; . \; . \; . \quad (18)$$

$$D_2 = \begin{vmatrix} 0 \quad , & S\beta\gamma, & S\beta\delta \\ S\beta\epsilon, & S\gamma\epsilon, & S\delta\epsilon \\ S\beta\zeta, & S\gamma\zeta, & S\delta\zeta \end{vmatrix};$$

Now,

$$S\beta\zeta S\beta\delta\gamma\epsilon - S\beta\epsilon S\beta\delta\gamma\zeta =$$

$$S\beta\zeta S\beta\delta S\gamma\epsilon - S\beta\zeta S\beta\gamma S\delta\epsilon - S\beta\epsilon S\beta\delta S\gamma\zeta + S\beta\epsilon S\beta\gamma S\delta\zeta\,;$$

So that

$$D_2 = -(S\beta\zeta S\beta\delta\gamma\epsilon - S\beta\epsilon S\beta\delta\gamma\zeta)\,; \; . \; . \; . \; . \quad (19)$$

So too,

$$D_3 = -(S\alpha\zeta S\alpha\delta\gamma\epsilon - S\alpha\epsilon S\alpha\delta\gamma\zeta)\,; \; . \; . \; . \; . \quad (20)$$

Again,

$$S\alpha\gamma\beta\zeta S\alpha\delta\beta\epsilon - S\alpha\gamma\beta\epsilon S\alpha\delta\beta\zeta =$$

$$(S\alpha\gamma S\beta\zeta + S\alpha\zeta S\beta\gamma - S\alpha\beta S\gamma\zeta)(S\alpha\delta S\beta\epsilon + S\alpha\epsilon S\beta\delta - S\alpha\beta S\delta\epsilon)$$

$$- (S\alpha\gamma S\beta\epsilon + S\alpha\epsilon S\beta\gamma - S\alpha\beta S\gamma\epsilon)(S\alpha\delta S\beta\zeta + S\alpha\zeta S\beta\delta - S\alpha\beta S\delta\zeta) =$$

$$- S^2\alpha\beta \begin{vmatrix} S\gamma\epsilon\,, & S\delta\epsilon \\ S\gamma\zeta\,, & S\delta\zeta \end{vmatrix} - S\alpha\beta S\alpha\zeta \begin{vmatrix} S\beta\gamma\,, & S\beta\delta \\ S\gamma\epsilon\,, & S\delta\epsilon \end{vmatrix} - S\alpha\beta S\alpha\epsilon \begin{vmatrix} S\gamma\zeta\,, & S\delta\zeta \\ S\beta\gamma\,, & S\beta\delta \end{vmatrix}$$

$$+ S\alpha\beta S\alpha\gamma \begin{vmatrix} S\beta\epsilon\,, & S\delta\epsilon \\ S\beta\zeta\,, & S\delta\zeta \end{vmatrix} + S\alpha\beta S\alpha\delta \begin{vmatrix} S\beta\zeta\,, & S\gamma\zeta \\ S\beta\epsilon\,, & S\gamma\epsilon \end{vmatrix} + \begin{vmatrix} S\alpha\gamma\,, & S\alpha\delta \\ S\beta\gamma\,, & S\beta\delta \end{vmatrix} \begin{vmatrix} S\alpha\epsilon\,, & S\alpha\zeta \\ S\beta\epsilon\,, & S\beta\zeta \end{vmatrix} =$$

$$\begin{vmatrix} 0\,, & S\alpha\beta\,, & S\alpha\gamma\,, & S\alpha\delta \\ S\alpha\beta\,, & 0\,, & S\beta\gamma\,, & S\beta\delta \\ S\alpha\epsilon\,, & S\beta\epsilon\,, & S\gamma\epsilon\,, & S\delta\epsilon \\ S\alpha\zeta\,, & S\beta\zeta\,, & S\gamma\zeta\,, & S\delta\zeta \end{vmatrix};$$

Hence,

$$D_4 = S\alpha\gamma\beta\zeta S\alpha\delta\beta\epsilon - S\alpha\gamma\beta\epsilon S\alpha\delta\beta\zeta\,; \quad . \quad . \quad . \quad . \quad (21)$$

Equations (17) to (21), and considerations of symmetry, show that we have these three equations,

$$\left.\begin{aligned} &\alpha^2\beta^2 S\gamma\delta V\zeta\epsilon - \alpha^2(S\beta\zeta S\beta\delta\gamma\epsilon - S\beta\epsilon S\beta\delta\gamma\zeta) - \beta^2(S\alpha\zeta S\alpha\delta\gamma\epsilon - S\alpha\epsilon S\alpha\delta\gamma\zeta) \\ &\qquad + S\alpha\gamma\beta\zeta S\alpha\delta\beta\epsilon - S\alpha\gamma\beta\epsilon S\alpha\delta\beta\zeta = 0\,; \\ &\alpha^2\beta^2 S\gamma\epsilon V\delta\zeta - \alpha^2(S\beta\delta S\beta\epsilon\gamma\zeta - S\beta\zeta S\beta\epsilon\gamma\delta) - \beta^2(S\alpha\delta S\alpha\epsilon\gamma\zeta - S\alpha\zeta S\alpha\epsilon\gamma\delta) \\ &\qquad + S\alpha\gamma\beta\delta S\alpha\epsilon\beta\zeta - S\alpha\gamma\beta\zeta S\alpha\epsilon\beta\delta = 0\,; \\ &\alpha^2\beta^2 S\gamma\zeta V\epsilon\delta - \alpha^2(S\beta\epsilon S\beta\zeta\gamma\delta - S\beta\delta S\beta\zeta\gamma\epsilon) - \beta^2(S\alpha\epsilon S\alpha\zeta\gamma\delta - S\alpha\delta S\alpha\zeta\gamma\epsilon) \\ &\qquad + S\alpha\gamma\beta\epsilon S\alpha\zeta\beta\delta - S\alpha\gamma\beta\delta S\alpha\zeta\beta\epsilon = 0\,; \end{aligned}\right\} \quad (22)$$

These three equations, if added together, give $0 = 0$, as is obvious. Eliminating $\alpha^2\beta^2$, by virtue of the identity,

$$S\gamma\delta S\epsilon\zeta S\gamma\delta V\zeta\epsilon + S\gamma\epsilon S\zeta\delta S\gamma\epsilon V\delta\zeta + S\gamma\zeta S\delta\epsilon S\gamma\zeta V\epsilon\delta = 0\,;$$

we have an equation of the form,

$$F_1\alpha^2 + F_2\beta^2 + F_3 = 0\,; \quad . \quad . \quad . \quad . \quad . \quad (23)$$

11. Equations (22) give two independent equations to find α^2, β^2, leading to a quadratic equation. The solution of this quadratic

equation will involve W. For, as appears from (9), we can form an equation containing α^2, β^2, and W as follows. We have,

$$W = \begin{vmatrix} S\alpha\zeta, & S\alpha\delta, & \alpha^2 \\ S\gamma\zeta, & S\gamma\delta, & S\alpha\gamma \\ S\epsilon\zeta, & S\delta\epsilon, & S\alpha\epsilon \end{vmatrix} \begin{vmatrix} S\beta\delta, & S\beta\gamma, & \beta^2 \\ S\delta\epsilon, & S\gamma\epsilon, & S\beta\epsilon \\ S\delta\zeta, & S\gamma\zeta, & S\beta\zeta \end{vmatrix}$$

$$- \begin{vmatrix} S\alpha\zeta, & S\alpha\epsilon, & \alpha^2 \\ S\gamma\zeta, & S\gamma\epsilon, & S\alpha\gamma \\ S\delta\zeta, & S\delta\epsilon, & S\alpha\delta \end{vmatrix} \begin{vmatrix} S\beta\epsilon, & S\beta\gamma, & \beta^2 \\ S\delta\epsilon, & S\gamma\delta, & S\beta\delta \\ S\epsilon\zeta, & S\gamma\zeta, & S\beta\zeta \end{vmatrix};$$

On expanding this, the coefficient of $\alpha^2\beta^2$ vanishes, and we have an equation which we may write,

$$G_1\alpha^2 + G_2\beta^2 + G_3 = W; \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Equations (23) and (24) are sufficient to determine α^2 and β^2 in terms of the Primary Scalars.

C. *Expressions for* F_1, F_2, F_3, G_1, G_2, G_3.

12. We have,

$$\begin{aligned} F_1 = & - S\gamma\delta S\epsilon\zeta(S\beta\zeta S\beta\delta S\gamma\epsilon - S\beta\epsilon S\beta\delta\gamma\zeta) \\ & - S\gamma\epsilon S\delta\zeta(S\beta\delta S\beta\epsilon\gamma\zeta - S\beta\zeta S\beta\epsilon\gamma\delta) \\ & - S\gamma\zeta S\delta\epsilon(S\beta\epsilon S\beta\zeta\gamma\delta - S\beta\delta S\beta\zeta\gamma\epsilon) \\ = & S\beta\zeta S\beta\delta\gamma\epsilon S.\gamma\zeta V\delta\epsilon \\ & + S\beta\delta S\beta\epsilon\gamma\zeta S.\gamma\delta V\epsilon\zeta \\ & + S\beta\epsilon S\beta\zeta\gamma\delta S.\gamma\epsilon V\zeta\delta; \quad . \quad . \quad . \quad (25) \end{aligned}$$

$$\begin{aligned} F_2 = & S\alpha\zeta S\alpha\delta\gamma\epsilon S.\gamma\zeta V\delta\epsilon \\ & + S\alpha\delta S\alpha\epsilon\gamma\zeta S.\gamma\delta V\epsilon\zeta \\ & + S\alpha\epsilon S\alpha\zeta\gamma\delta S.\gamma\epsilon V\zeta\delta; \quad . \quad . \quad . \quad . \quad (26) \end{aligned}$$

$$\begin{aligned} F_3 = & S\alpha\gamma\beta\zeta S\alpha\delta\beta\epsilon S.\gamma\zeta V\epsilon\delta \\ & + S\alpha\gamma\beta\delta S\alpha\epsilon\beta\zeta S.\gamma\delta V\zeta\epsilon \\ & + S\alpha\gamma\beta\epsilon S\alpha\zeta\beta\delta S.\gamma\epsilon V\delta\zeta; \quad . \quad . \quad . \quad (27) \end{aligned}$$

$$\begin{aligned}
G_1 &= S.\gamma\epsilon V\delta\zeta.\{S\beta\delta S.\gamma\beta V\zeta\epsilon + S\beta\gamma S.\delta\beta V\epsilon\zeta\} \\
&\quad - S.\gamma\delta V\epsilon\zeta.\{S\beta\epsilon S.\gamma\beta V\zeta\delta + S\beta\gamma S.\epsilon\beta V\delta\zeta\} \\
&= S\gamma\delta S\epsilon\zeta(S\beta\delta S\beta\epsilon S\gamma\zeta + S\beta\gamma S\beta\zeta S\delta\epsilon - S\beta\delta S\beta\zeta S\gamma\epsilon - S\beta\gamma S\beta\epsilon S\delta\zeta) \\
&\quad + S\gamma\epsilon S\delta\zeta(S\beta\epsilon S\beta\zeta S\gamma\delta + S\beta\gamma S\beta\delta S\epsilon\zeta - S\beta\delta S\beta\epsilon S\gamma\zeta - S\beta\gamma S\beta\zeta S\delta\epsilon) \\
&\quad + S\gamma\zeta S\delta\epsilon(S\beta\delta S\beta\zeta S\gamma\epsilon + S\beta\gamma S\beta\epsilon S\delta\zeta - S\beta\epsilon S\beta\zeta S\gamma\delta - S\beta\gamma S\beta\delta S\epsilon\zeta) \\
&= S\gamma\delta S\epsilon\zeta(S\beta\epsilon S\beta\delta\gamma\zeta - S\beta\zeta S\beta\delta\gamma\epsilon) \\
&\quad + S\gamma\epsilon S\delta\zeta(S\beta\zeta S\beta\epsilon\gamma\delta - S\beta\delta S\beta\epsilon\gamma\zeta) \\
&\quad + S\gamma\zeta S\delta\epsilon(S\beta\delta S\beta\zeta\gamma\epsilon - S\beta\epsilon S\beta\zeta\gamma\delta);
\end{aligned}$$

or

$$F_1 = G_1; \quad \ldots\ldots\ldots \quad (28)$$

and, in precisely the same way, we have,

$$F_2 = -G_2; \quad \ldots\ldots\ldots \quad (29)$$

Again, in the same way as we obtained Equations (19) and (20), we have,

$$\begin{vmatrix} 0 & S\alpha\zeta & S\alpha\delta \\ S\alpha\gamma & S\gamma\zeta & S\gamma\delta \\ S\alpha\epsilon & S\epsilon\zeta & S\delta\epsilon \end{vmatrix} = S\alpha\gamma S\alpha\delta\zeta\epsilon - S\alpha\epsilon S\alpha\delta\zeta\gamma;$$

$$= S\alpha\delta\zeta V.\,\alpha V\gamma\epsilon;$$

$$\begin{vmatrix} 0 & S\beta\delta & S\beta\gamma \\ S\beta\epsilon & S\delta\epsilon & S\gamma\epsilon \\ S\beta\zeta & S\delta\zeta & S\gamma\zeta \end{vmatrix} = S\beta\epsilon S\beta\gamma\delta\zeta - S\beta\zeta S\beta\gamma\delta\epsilon;$$

$$= S\beta\gamma\delta V.\,\beta V\epsilon\zeta$$

&c. &c.

So that we have

$$G_3 = S\alpha\delta\zeta V.\,\alpha V\gamma\epsilon.\,S\beta\gamma\delta V.\,\beta V\epsilon\zeta - S\alpha\epsilon\zeta V.\,\alpha V\gamma\delta.\,S\beta\gamma\epsilon V.\,\beta V\delta\zeta; \quad (30)$$

All our results show the power which Quaternion Expressions have of representing in a simple manner results which cannot be otherwise than complicated. We give such formulæ for F_1, F_2, F_3, G_3, as naturally present themselves, without entering upon the question whether simpler can be found.

D. *Development of* W.

13. Consideration of symmetry show that, in the expansion (15) of W^2, terms such as B_5^4, $2B_1B_5^3$ will disappear. But it will not be

amiss to show how this at once appears from the algebraic expression (15) itself. For, if we assume

$$\left.\begin{aligned} p_1+q_1&=B_5+B_1+B_2\,; & p_1-q_1&=B_5+B_3+B_4\,;\\ p_2+q_2&=C_5+C_1+C_3\,; & p_2-q_2&=C_5+C_2+C_4\,;\\ p_3+q_3&=E_5+E_1+E_4\,; & p_3-q_3&=E_5+E_2+E_3\,; \end{aligned}\right\} \quad . \quad (31)$$

we have, by (13),

$$p_1+p_2+p_3=0\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

so that

$$p_1^4+p_2^4+p_3^4-2p_2^2p_3^2-2p_3^2p_1^2-2p_1^2p_2^2=0\,,$$

and,

$$\begin{aligned} W^2=q_1^4+q_2^4+q_3^4&-2q_2^2q_3^2-2q_3^2q_1^2-2q_1^2q_2^2\\ &-4p_2p_3q_1^2-4p_3p_1q_2^2-4p_1p_2q_3^2\,; \quad . \quad . \quad . \quad . \quad (33), \end{aligned}$$

from which terms of the form B_5^4, $2B_1B_5^3$, &c., have disappeared.

14. It is now evident that we may write,

$$W^2=\Sigma\omega_{r,s}+\Sigma\omega_t+\Sigma\omega_{r,s,t}\,; \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where

$$\begin{aligned} \Sigma\omega_{r,s}&=\omega_{5,1}+\omega_{5,2}+\omega_{5,3}+\omega_{5,4}+\omega_{4,1}\\ &\quad+\omega_{4,2}+\omega_{4,3}+\omega_{3,1}+\omega_{3,2}+\omega_{2,1}\,; \quad . \quad . \quad . \quad (35)\\ \Sigma\omega_t&=\omega_1+\omega_2+\omega_3+\omega_4+\omega_5\,; \quad . \quad . \quad . \quad . \quad (36)\\ \Sigma\omega_{r,s,t}&=\omega_{5,1,2}+\omega_{5,1,3}+\omega_{5,1,4}+\omega_{5,3,4}+\omega_{5,2,4}+\omega_{5,2,3}\\ &\quad+\omega_{4,1,2}+\omega_{4,1,3}+\omega_{4,1,5}+\omega_{4,3,5}+\omega_{4,2,5}+\omega_{4,2,3}\\ &\quad+\ldots\\ &\quad+\omega_{1,2,3}+\omega_{1,2,4}+\omega_{1,2,5}+\omega_{1,4,5}+\omega_{1,3,5}+\omega_{1,3,4}\,; \quad . \quad (37) \end{aligned}$$

Any one of the terms in each of these equations being found, all the rest may be found by simple permutation in various ways. But, as in such process mistakes are likely to be made, a table of the values $\omega_{5,1}$, &c.; ω_1, &c.; and $\omega_{5,1,2}$, &c., will be given.

15. The table may be constructed as follows, or in many other ways:—

$$\begin{aligned} \omega_{5,1}&=B_1^2B_5^2+C_1^2C_5^2+E_1^2E_5^2-2C_1E_1C_5E_5-2E_1B_1E_5B_5-2B_1C_1B_5C_5\\ &=(B_1B_5-C_1C_5)^2+(B_1+C_1)(B_5+C_5)[(B_1+C_1)(B_5+C_5)-2C_1C_5-2B_1B_5]\\ &=(B_1B_5-C_1C_5)^2-(B_1^2-C_1^2)(B_5^2-C_5^2)=(B_1C_5-B_5C_1)^2\,;\\ \omega_5&=4B_1B_2B_3B_4+4C_1C_2C_3C_4+4E_1E_2E_3E_4-2C_1C_2E_3E_4-2E_1E_2C_3C_4\\ &\quad-2E_1B_2E_3B_4-2B_1E_2B_3E_4-2B_1C_2C_3B_4-2C_1B_2B_3C_4\\ &=2B_1B_2B_3B_4+2C_1C_2C_3C_4+2E_1E_2E_3E_4\,: \end{aligned}$$

For

$$2B_1B_2B_3B_4 - 2B_1B_4C_3C_2 - 2B_1B_3E_2E_4 = -2B_1B_3B_4E_2 + 2B_1B_4E_3C_2 - 2B_1B_3E_2E_4$$
$$= 2B_1(B_3E_2C_4 + B_4E_3C_2);$$

$$2C_1C_2C_3C_4 - 2C_1C_2E_3E_4 - 2B_2B_3C_1C_4 = -2C_1C_2C_4B_3 + 2C_1C_2E_3B_4 - 2B_2B_3C_1C_4$$
$$= 2C_1(B_3C_4E_2 + C_2E_3B_4);$$

$$2E_1E_2E_3E_4 - 2C_3C_4E_1E_2 - 2E_1E_3B_2B_4 = -2E_1E_2E_3C_4 + 2E_1E_3B_4C_2 - 2C_3C_4E_1E_2$$
$$= 2E_1(B_3C_4E_2 + E_3B_4C_2);$$

and

$$\omega_{4,1,2} = 2B_1B_2B_4^2 - 2C_1E_2C_4E_4 - 2B_1E_2E_4B_4 - 2B_2C_1B_4C_4$$
$$= -2B_1B_2B_4C_4 - 2B_1B_2E_4B_4 - 2C_1E_2C_4E_4 - 2B_1E_2E_4B_4 - 2B_2C_1B_4C_4$$
$$= 2E_1B_2B_4C_4 - 2C_1E_2C_4E_4 + 2B_1C_2E_4B_4.$$

16. So we have,

$$\left.\begin{aligned}
&\omega_{5,1} = (B_1C_5 - B_5C_1)^2 = (C_1E_5 - C_5E_1)^2 = (E_1B_5 - E_5B_1)^2 \\
&\quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
&\omega_{4,1} = (B_1B_4 - C_1C_4)^2 = (E_1C_4 - B_1E_4)^2 = (C_1E_4 - E_1B_4)^2 \\
&\omega_{4,2} = (B_2B_4 - E_2E_4)^2 \\
&\omega_{4,3} = (C_3C_4 - E_3E_4)^2 \\
&\omega_{3,1} = (B_1B_3 - E_1E_3)^2 \\
&\omega_{3,2} = (B_2B_3 - C_2C_3)^2 \\
&\omega_{2,1} = (C_1C_2 - E_1E_2)^2
\end{aligned}\right\} \quad . \quad (38)$$

$$\left.\begin{aligned}
&\omega_5 = 2(B_1B_2B_3B_4 + C_1C_2C_3C_4 + E_1E_2E_3E_4); \\
&\omega_4 = 2(B_1C_2E_3C_5 + C_1E_2B_3B_5 + E_1C_3B_2E_5); \\
&\omega_3 = 2(C_1B_2E_4C_5 + E_1C_2B_4B_5 + B_1E_2C_4E_5); \\
&\omega_2 = 2(E_1B_3C_4C_5 + C_1E_3B_4E_5 + B_1C_3E_4B_5); \\
&\omega_1 = 2(E_2C_3B_4C_5 + B_2E_3C_4B_5 + C_2B_3E_4E_5);
\end{aligned}\right\} \quad . \quad . \quad . \quad (39)$$

Where we observe the identity,

$$\omega_5 = 2(C_1C_2 - E_1E_2)(C_3C_4 - E_3E_4) + 2(E_1E_3 - B_1B_3)(E_2E_4 - B_2B_4)$$
$$+ 2(B_1B_4 - C_1C_4)(B_2B_3 - C_2C_3); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

&c.

17. The other values are,

$$\omega_{5,1,2} = 2(E_1E_2B_5C_5 - B_1B_2C_5E_5 + C_1C_2E_5B_5);$$
$$\omega_{5,1,3} = 2(B_1B_3C_5E_5 - C_1C_3E_5B_5 + E_1E_3B_5C_5);$$
$$\omega_{5,1,4} = 2(C_1C_4E_5B_5 - E_1E_4B_5C_5 + B_1B_4C_5E_5);$$
$$\omega_{5,3,4} = 2(E_3E_4B_5C_5 - B_3B_4C_5E_5 + C_3C_4E_5B_5);$$
$$\omega_{5,2,4} = 2(B_2B_4C_5E_5 - C_2C_4E_5B_5 + E_2E_4B_5C_5);$$
$$\omega_{5,2,3} = 2(C_2C_3E_5B_5 - E_2E_3B_5C_5 + B_2B_3C_5E_5);$$
$$\omega_{4,1,2} = 2(E_1B_2B_4C_4 - C_1E_2C_4E_4 + B_1C_2E_4B_4);$$
$$\omega_{4,1,3} = 2(E_1C_3B_4C_4 - B_1E_3E_4B_4 + C_1B_3C_4E_4);$$
$$\omega_{4,1,5} = 2(C_1B_5C_4E_4 - E_1E_5B_4C_4 + B_1C_5E_4B_4);$$
$$\omega_{4,3,5} = 2(C_3E_5B_4C_4 - B_3B_5C_4E_4 + E_3C_5E_4B_4);$$
$$\omega_{4,2,5} = 2(B_2E_5B_4C_4 - C_2C_5E_4B_4 + E_2B_5C_4E_4);$$
$$\omega_{4,2,3} = 2(E_2B_3C_4E_4 - B_2C_3B_4C_4 + C_2E_3E_4B_4);$$
$$\omega_{3,1,2} = 2(B_1E_2B_3C_3 - E_1C_2C_3E_3 + C_1B_2E_3B_3);$$
$$\omega_{3,1,4} = 2(C_1E_4E_3B_3 - B_1C_4B_3C_3 + E_1B_4C_3E_3);$$
$$\omega_{3,1,5} = 2(B_1E_5B_3C_3 - C_1C_5E_3B_3 + E_1B_5C_3E_3);$$
$$\omega_{3,4,5} = 2(C_4E_5B_3C_3 - B_4B_5C_3E_3 + E_4C_5E_3B_3);$$
$$\omega_{3,2,5} = 2(B_2C_5E_3B_3 - E_2E_5B_3C_3 + C_2B_5C_3E_3);$$
$$\omega_{3,2,4} = 2(C_2B_4C_3E_3 - B_2E_4E_3B_3 + E_2C_4B_3C_3);$$
$$\omega_{2,1,3} = 2(B_1C_3C_2E_2 - E_1B_3E_2B_2 + C_1E_3B_2C_2);$$
$$\omega_{2,1,4} = 2(B_1E_4C_2E_2 - C_1B_4B_2C_2 + E_1C_4E_2B_2);$$
$$\omega_{2,1,5} = 2(C_1E_5B_2C_2 - B_1B_5C_2E_2 + E_1C_5E_2B_2);$$
$$\omega_{2,4,5} = 2(B_4E_5B_2C_2 - C_4C_5E_2B_2 + E_4B_5C_2E_2);$$
$$\omega_{2,3,5} = 2(B_3C_5E_2B_2 - E_3E_5B_2C_2 + C_3B_5C_2E_2);$$
$$\omega_{2,3,4} = 2(E_3B_4B_2C_2 - C_3E_4C_2E_2 + B_3C_4E_2B_2);$$
$$\omega_{1,2,3} = 2(B_2E_3C_1E_1 - C_2B_3B_1C_1 + E_2C_3E_1B_1);$$
$$\omega_{1,2,4} = 2(C_2E_4B_1C_1 - E_2B_4E_1B_1 + B_2C_4C_1E_1);$$
$$\omega_{1,2,5} = 2(C_2E_5B_1C_1 - B_2B_5C_1E_1 + E_2C_5E_1B_1);$$
$$\omega_{1,4,5} = 2(B_4C_5E_1B_1 - E_4E_5B_1C_1 + C_4B_5C_1E_1);$$
$$\omega_{1,3,5} = 2(B_3E_5B_1C_1 - C_3C_5E_1B_1 + E_3B_5C_1E_1);$$
$$\omega_{1,3,4} = 2(B_3E_4B_1C_1 - E_3C_4C_1E_1 + C_3B_4E_1B_1).$$

In connection with which equations we observe the identities,

$$2(B_1C_5 - C_1B_5)(E_1E_2 - C_1C_2) = 2(E_2C_5E_1B_1 + C_2E_5B_1C_1 + B_2B_5C_1E_1);$$

&c.

In writing down the complete out-spread of W^2, it will not be necessary to avail ourselves of more than a few of these results.

18. From what proceeds we have

$$W^2 = U_0 + U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7 + U_8 + U_9;$$

where

$$U_0 = \Sigma S^2 a\beta S^2 \beta\gamma S^2 \gamma\delta S^2 \delta\epsilon S^2 \epsilon\zeta S^2 \zeta a;$$

$$U_1 = -2\Sigma S^2 a\beta S^2 a\zeta S^2 \beta\gamma S^2 \delta\epsilon S\gamma\delta S\epsilon\zeta S\delta\zeta S\gamma\epsilon;$$

$$U_2 = -2\Sigma S^2 a\beta S^2 a\zeta S^2 \delta\epsilon S^2 \gamma\delta S\epsilon\zeta S\beta\epsilon S\gamma\zeta S\beta\gamma;$$

$$U_3 = 2\Sigma S^2 a\beta S^2 a\zeta S^2 \delta\epsilon S\beta\gamma S\beta\delta S\gamma\epsilon S\gamma\zeta S\gamma\delta S\epsilon\zeta;$$

$$U_4 = 2\Sigma S^2 a\beta S^2 \gamma\delta S^2 \epsilon\zeta Sa\epsilon S\delta\epsilon S\beta\delta S\beta\gamma S\gamma\zeta Sa\zeta;$$

$$U_5 = -4\Sigma S^2 a\beta S^2 \gamma\delta S^2 \epsilon\zeta Sa\zeta S\delta\zeta Sa\delta S\beta\gamma S\gamma\epsilon S\beta\epsilon;$$

$$U_6 = 2\Sigma S^2 a\beta S^2 \gamma\epsilon S^2 \delta\zeta Sa\epsilon S\delta\epsilon S\beta\delta S\beta\gamma S\gamma\zeta Sa\zeta;$$

$$U_7 = 4\Sigma S^2 a\beta S^2 \gamma\delta Sa\epsilon Sa\zeta S\beta\epsilon S\beta\zeta S\gamma\epsilon S\gamma\zeta S\delta\epsilon S\delta\zeta;$$

$$U_8 = -2\Sigma S^2 a\beta S^2 \gamma\delta Sa\epsilon Sa\zeta S\gamma\epsilon S\gamma\zeta S\beta\epsilon S\delta\zeta S\beta\delta S\epsilon\zeta;$$

$$U_9 = 4\Sigma Sa\zeta Sa\delta Sa\gamma Sa\beta S\beta\gamma S\epsilon\zeta S\gamma\epsilon S\delta\zeta S\beta\zeta S\beta\epsilon S\gamma\delta S\delta\epsilon.$$

In this expansion for W^2 it is to be observed that

U_9 contains 15 terms, U_0, U_5, U_6, each contain 60 terms,

U_7 contains 45 terms, U_1, U_4 each contain 180 terms,

U_2 contains 90 terms, and U_3, U_8 360 terms each.

Counting the weight of each term as 1, 2, or 4, according to its coefficient, we have,

$$\text{weight of + terms} = 60 + 2\times 360 + 2\times 180 + 2\times 60 + 4\times 45 + 4\times 15$$
$$= 60(1 + 12 + 6 + 2 + 3 + 1) = 1500;$$

$$\text{weight of - terms} = 2\times 180 + 2\times 90 + 4\times 60 + 2\times 360$$
$$= 60(6 + 3 + 4 + 12) = 1500.$$

The same number, as should manifestly be the case.

19. If we had expressed W^2 in terms of b_1, c_1, e_1, &c., it should have been borne in mind that these Scalars are connected by five

relations, contained in the following 10 equations, five of which are easily obtained from the other five.

$$e_1e_3b_5 = b_1b_3e_5\,; \qquad e_1e_2c_5 = c_1c_2e_5\,;$$
$$e_2e_4b_5 = b_2b_4e_5\,; \qquad c_2e_3b_4 = e_2b_3c_4\,;$$
$$c_2c_3b_5 = b_2b_3c_5\,; \qquad c_1b_3e_4 = e_1c_3b_4\,;$$
$$e_3e_4c_5 = c_3c_4e_5\,; \qquad b_1c_2e_4 = e_1b_2c_4\,;$$
$$b_1b_4c_5 = c_1c_4b_5\,; \qquad b_1e_2c_3 = c_1b_2e_3\,.$$

20. It can be readily shown that the expression for W^2 cannot be square rooted so as to express W in the form

$$\Sigma(-1)^r S\alpha\beta S\beta\gamma S\gamma\delta S\delta\epsilon S\epsilon\zeta S\zeta\alpha.$$

For a single permutation of any two letters changes the sign of W, while the successive permutations of α, β; α, γ; α, δ; α, ϵ; α, ζ, being 5 in number, do *not* change the sign of

$$S\alpha\beta S\beta\gamma S\gamma\delta S\delta\epsilon S\epsilon\zeta S\zeta\alpha.$$

So the expression for W would not change its sign for a permutation which would change the sign of W.

E. *Formulæ for* $S\alpha\beta\gamma$, &c.

These may be obtained in various ways. Thus we have

$$S\alpha\gamma\delta S\alpha\epsilon\zeta = a^2 S\gamma\delta V\epsilon\zeta + S\alpha\gamma S\alpha\delta V\zeta\epsilon + S\alpha\delta S\alpha\gamma V\epsilon\zeta\,;$$
$$S\alpha\gamma\epsilon S\alpha\zeta\delta = a^2 S\gamma\epsilon V\zeta\delta + S\alpha\gamma S\alpha\epsilon V\delta\zeta + S\alpha\epsilon S\alpha\gamma V\zeta\delta\,;$$
$$S\alpha\gamma\zeta S\alpha\delta\epsilon = a^2 S\gamma\zeta V\delta\epsilon + S\alpha\gamma S\alpha\zeta V\epsilon\delta + S\alpha\zeta S\alpha\gamma V\delta\epsilon\,;$$

whence

$$S\gamma\delta S\epsilon\zeta S\alpha\gamma\delta S\alpha\epsilon\zeta + S\gamma\epsilon S\zeta\delta S\alpha\gamma\epsilon S\alpha\zeta\delta + S\gamma\zeta S\delta\epsilon S\alpha\gamma\zeta S\alpha\delta\epsilon$$
$$= S\gamma\delta S\epsilon\zeta(S\alpha\zeta S\alpha\delta\gamma\epsilon - S\alpha\epsilon S\alpha\delta\gamma\zeta)$$
$$+ S\gamma\epsilon S\delta\zeta(S\alpha\delta S\alpha\epsilon\gamma\zeta - S\alpha\zeta S\alpha\epsilon\gamma\delta)$$
$$+ S\gamma\zeta S\delta\epsilon(S\alpha\epsilon S\alpha\zeta\gamma\delta - S\alpha\delta S\alpha\zeta\gamma\epsilon) = -F_2 \quad . \quad . \quad . \quad (41)$$

as appears from (26).

We have also

$$S\alpha\gamma\delta S\alpha\epsilon\zeta + S\alpha\gamma\epsilon S\alpha\zeta\delta + S\alpha\gamma\zeta S\alpha\delta\epsilon = 0\,,$$

and it easily results from (10) that we obtain a set of thirty formulæ, from which selecting,

$$2S\alpha\gamma\zeta S\alpha\epsilon\delta S\beta\gamma\epsilon S\beta\zeta\delta = V^2(\mu_3^2\nu_3^2 - \mu_1^2\nu_1^2 - \mu_2^2\nu_2^2) + W\ ;$$
$$2S\alpha\gamma\delta S\alpha\epsilon\zeta S\beta\delta\zeta S\beta\gamma\epsilon = V^2(\mu_1^2\nu_1^2 - \mu_2^2\nu_2^2 - \mu_3^2\nu_3^2) - W\ ;$$

we have, see (7)

$$\frac{S\alpha\gamma\zeta S\alpha\delta\epsilon}{S\alpha\gamma\epsilon S\alpha\zeta\delta} = \frac{V^2(\mu_3^2\nu_3^2 - \mu_1^2\nu_1^2 - \mu_2^2\nu_2^2) + W}{2\mu_2^2\nu_2^2V^2}\ ;$$
$$\frac{S\alpha\gamma\delta S\alpha\epsilon\zeta}{S\alpha\gamma\epsilon S\alpha\zeta\delta} = \frac{V^2(\mu_1^2\nu_1^2 - \mu_2^2\nu_2^2 - \mu_3^2\nu_3^2) - W}{2\mu_2^2\nu_2^2V^2}\ ;$$

we have thus an expression for $S\alpha\gamma\epsilon S\alpha\zeta\delta$, and by permuting we can form, in like manner, expressions for

$$S\alpha\gamma\epsilon S\beta\zeta\gamma \quad \text{and} \quad S\alpha\zeta\delta S\beta\zeta\gamma\,,$$

and so obtain $S^2\alpha\gamma\epsilon$. In fact we have,

$$[\{V^2(\mu_3^2\nu_3^2 - \mu_1^2\nu_1^2) + W\}S\gamma\epsilon V\delta\zeta + \mu_2^2\nu_2^2V^2(S\gamma\zeta V\delta\epsilon + S\gamma\delta V\zeta\epsilon)]S\alpha\gamma\epsilon S\alpha\zeta\delta$$
$$= -2\mu_3^2\nu_2^2V^2F_2\,.$$

It will be observed how Quaternion methods enable us to express simply forms which, without this powerful analysis, would present formidable complexities.

Report on Atmospheric Circulation, based on the Observations made on Board H.M.S. "Challenger" 1873–76.

By **Alexander Buchan, LL.D.**

(Part I. read April 16, 1888; Part II. read May 6, 1889.)*

(*Abstract.*)

In these papers the meteorological observations taken during the voyage of the "Challenger" are discussed; and, from data collected from all parts of the world, fifty-two maps have been prepared, showing for each month of the year the distribution of temperature and pressure over the globe and the prevailing winds. Part I.

* These papers were submitted by permission of the Lords Commissioners of Her Majesty's Treasury. For the Report itself see "Report of the Scientific Results of the Voyage of H.M.S. 'Challenger,'" *Physics and Chemistry*, vol. ii. part vi.

deals with the diurnal, and Part II. with the seasonal phenomena of meteorology.

Diurnal Phenomena.—An examination of the temperatures observed by the "Challenger" proves that nowhere over the ocean does the mean daily fluctuation of the temperature of the surface amount to a degree Fahrenheit, the extremes being from about 0°·3 in high latitudes to 0°·9 in the tropics. Thus the atmosphere over the ocean rests on, or blows over, a surface the temperature of which is practically uniform at all hours of the day. This small variation is a prime factor in meteorology, particularly in those discussions which relate to the diurnal phenomena of atmospheric pressure and winds.

The temperature of the air over the open sea shows a daily variation of 3°·2, being four times greater than that of the sea over which it lies; but when the "Challenger" was near land, the variation rose still further to 4°·4. This larger variation in the daily temperature of the air, as compared with that of the sea, is a point of much significance in atmospheric physics, from the light it casts on the relations of the atmosphere and its aqueous vapour to solar and terrestrial radiation.

The phases of the elastic force of vapour over the open sea occur at the hours of the maximum and minimum temperatures of the sea and the air. On nearing land, however, this no longer holds good; but owing to the influence of the land breeze, the time of minimum humidity is delayed from 4 to 6 A.M., and owing to the sea breeze and its effects, the amount of the aqueous vapour falls to a secondary minimum from noon to 2 P.M. As regards the relative humidity, the maximum occurs from midnight to 4 A.M., and the minimum about 2 P.M., this curve being thus inverse to that of the temperature; and it may be added, that this is substantially the curve of the relative humidity for all climates and seasons.

The phenomena of the double diurnal barometric tide appear in their simplest form in the centre of the Pacific, or in the midst of the largest water surface of the globe. The following are the variations of pressure from September 1 to 12, 1875, in mean lat. 1° 8′ S. and long. 150° 40′ W., the mean pressure for the time being 29,928 inches:—

	Inch.		Inch.
2 A.M.	−0·012	2 P.M.	−0·043
4 „	−0·022	4 „	−0·055
6 „	0·003	6 „	−0·028
8 „	0·028	8 „	0·004
10 „	0·032	10 „	0·013
Noon,	0·006	Mid.	0·012

from which it is observed that the amplitude of the range from the morning maximum to the afternoon minimum amounts to 0°·087 inch.

Latitude for latitude, the smallest variations over the open sea occur in the anticyclonic regions of the different oceans. Thus about lat. 36°, and the time of the year when the sun is highest in the heavens, the amounts are—for the South Pacific, 0·036 inch; North Pacific, 0·025 inch; South Atlantic, 0·024 inch ; and North Atlantic, 0·014. It thus appears that these amplitudes diminish as the ocean is more land-locked with continents.

In the open ocean the morning minimum of pressure is largest in equatorial regions, and it diminishes with latitude; but the rate of diminution with latitude, through anticyclonic and other regions, is generally less, and is more uniform than is the case with the afternoon minimum. Further, in high latitudes over the open sea, the diurnal barometric tide shows only one maximum and one minimum; and also in continental situations in high latitudes there occurs in summer only one maximum and one minimum, but the phases of their occurrence are the reverse of each other.

In middle and higher latitudes in summer, proximity to the sea, conspicuously so when the places are situated on the west coasts of continents and islands, delays the time of occurrence of the morning maximum and the afternoon minimum; whilst in continental situations the morning maximum occurs much earlier than in lower latitudes, and the afternoon minimum nearly as late as at places near the sea. But, as seen from the "Challenger" observations, these peculiarities of the curves do not occur over the open sea in the higher latitudes. The retardation of the time of occurrence of the morning maximum is greatest in situations which, while strongly insular in character, are at the same time on, or not far from, an

extensive tract of land to eastward or south-eastward. A table was given showing, for fourteen stations, a gradual retardation of this phase of the diurnal pressure in June, from 7 A.M. at Culloden, to 11 A.M. at St Petersberg, and finally to 3 P.M. at Sitka.

As regards the land surfaces of the globe, the great range hitherto observed between the morning maximum and the afternoon minimum is nearly two-tenths of an inch in the arid climate of Jacobabad. At Aden, where the climate at all seasons is dry, it is 0·084 inch in January, whereas in August it amounts to 0·163 inch, or nearly double that of January, when the sun occupies a much lower place in the sky. On the other hand, at Bombay, during the dry season in January, the range is 0·119 inch, but during the wet season in July, though the sun's position is then nearly vertical, the range is only 0·067 inch.

The "Challenger" observations show that the atmosphere over the open sea rests on a floor or surface, subject to a diurnal range of temperature so small as to render the temperature practically constant both day and night, and also that the diurnal oscillations of the barometer occur over the open sea equally as over the land surfaces of the globe. This consideration leads to the all-important conclusion that the diurnal oscillations of the barometer are not caused by the heating and cooling of the earth's surface by solar and terrestrial radiation, and by the effects which follow these diurnal changes in the temperature of the surface, but are primarily caused by the direct heating by solar radiation, and cooling by nocturnal radiation of the molecules of the air and its aqueous vapour, and the dust particles suspended in it, these changes of temperature being instantaneously communicated through the whole mass of the atmosphere, from its lowermost stratum resting on the surface to the extreme limit of the atmosphere. The all-important bearing of these considerations of the theory of the diurnal oscillations of the barometer was explained at length.

The peculiarities of the diurnal barometric tides in deep valleys, and those at high-level observatories, such as Obirgipfel and Ben Nevis, were described and discussed.

During the cruise, observations of the force of the wind were made on 1202 days, at least twelve times each day, 650 of the days being on the open seas and 552 near land. As regards the open sea,

the diurnal variation of the force of the wind is exceedingly small, the difference between the hour of least and greatest velocity being less than a mile per hour; and as the hours of occurrence of these very small maxima and minima vary with the different oceans, they cannot be regarded as true maxima and minima.

Quite different is it with the winds observed by the "Challenger" near land, the force of the wind there giving a curve as pronouncedly marked as the diurnal curves of temperature, pressure, or humidity. The minimum occurs from 2 to 4 A.M. and the maximum from noon to 4 P.M., the highest velocity being at 2 P.M. The curves from each of the five great oceans give one and the same result, viz., a curve closely congruent with that of the diurnal temperature. The differences between the hour of least and greatest velocity are these:—Southern Ocean, 6½ miles; South Pacific, 4½ miles; South Atlantic, 3½ miles; and North Atlantic and North Pacific, 3 miles per hour. Another point of considerable importance is that in no case does the maximum velocity, attained near land about or shortly after noon, reach the velocity of the wind on the open sea.

The diurnal variation in the amount of cloud is very small. There are, however, indicated two maxima, one about sunrise and the other early in the afternoon; and two minima, one at noon and the other from sunset to midnight, the differences not exceeding 6 per cent. of the whole sky. The observation of the diurnal occurrence of rain on the open sea is inversely as the temperature, 684 days' observations giving 96 cases in the seven hours, from 9 A.M. to 4 P.M., but 135 in the two hours from midnight to 2 A.M., these being respectively the times of minimum and maximum occurrence.

Of the forty-five thunderstorms recorded, twenty-six occurred over the open sea and nineteen near land. Of those over the open sea twenty-two occurred during the ten hours from 10 P.M. to 8 A.M., but during the remaining fourteen hours of the day only four were recorded. Hence, the important conclusion is arrived at, that over the open sea thunderstorms are essentially phenomena of the night, and occur mostly during the morning minimum of temperature and pressure, squalls reaching the daily maximum at the same time; the phases of the curve of distribution during the twenty-four hours being thus the reverse of what obtains over the land surfaces of the globe. On the other hand, the maximum in the

diurnal curve of lightning over the open sea is closely coincident with the evening maximum of pressure. The phases of the diurnal curves of the electric phenomena are these:—Thunderstorms over land, 2 to 6 P.M.; lightning over land, 8 P.M. to midnight; lightning over the open sea, 8 P.M. to 4 A.M.; and thunderstorms over the open sea, 10 P.M. to 8 A.M.

Monthly, Annual, and Recurring Phenomena.—The following among other tables have been published with the Report:—Table IV., showing the Mean Diurnal Variation of Atmospheric Pressure at 147 Stations; Table VI., the Mean Monthly Height of the Barometer at 1365 Stations; Tables VII. and VIII., the Mean Monthly Direction of the Prevailing Winds at 746 Stations; and Table IX., the Mean Monthly Temperature at 1620 Stations. The results of these data are represented on fifty-two large maps, giving for the months and the year the distribution over the globe of the temperature and pressure of the atmosphere, and the prevailing winds. These results were stated in some detail, from which the following broad conclusions were drawn:—This investigation shows in the clearest and most conclusive manner, that the distribution of the pressure of the earth's atmosphere is determined by the geographical distribution of land and water in their relations to the varying heat received from the sun through the months of the year; and since the relative pressure determines the direction and force of the prevailing winds, and these in their turn the temperature, moisture, rainfall, and in a very great degree the surface currents of the ocean, it is plain there is here a principle applicable not only to the present state of the earth, but also to different distributions of land and water in past times. In truth, it is only by the aid of this principle that any rational attempt, based on causes having a purely terrestrial origin, can be made in explanation of those glacial and warm geological epochs through which the climates of Great Britain and other countries have passed. Hence the geologist must familiarise himself with the nature of those climatic changes, which necessarily result from different distributions of land and water, especially those changes which influence most powerfully the life of the globe.

On the Stomach of the Narwhal (*Monodon monoceros*).

By **G. Sims Woodhead**, M.D., and **Robert W. Gray**, *Student of Anatomy, the University, Edinburgh.* (With Four Plates.)

(Read March 18, 1889.)

Although numerous most admirable descriptions of the stomach of various species of Delphinidæ (the family of toothed Whales to which the Narwhal belongs) have from time to time appeared from the pens of most able observers (of whom a list will be found in the references appended), we have found it impossible to find anything more than a mere indication of the histological structure of the various portions of the walls of the peculiar digestive apparatus of these animals.* As one of us had an opportunity of obtaining material in a comparatively fresh condition, we decided to make arrangements for preserving it properly, so that it might be subjected to microscopic examination on being brought to this country.† With all the care that was taken some portions of the mucous membrane have suffered slightly, but in all cases the changes are so slight that we are enabled to speak positively on the points to which reference is made.

In the Narwhal, as in other cetaceans, the stomach is of a complex nature, and has been compared by some observers to that of the ruminants. In the present state of our knowledge, however, it is difficult to give any definite opinion on the morphological affinities and relations of the viscus, though it certainly approaches the carnivorous type much more nearly than that of the herbivorous ruminants. The subject is one in which great difficulties are involved, and although we think that, eventually, the histological structure which we now give may throw some light on the matter, we are not prepared to say in what group of animals the nearest morphological structure is found. We now propose to describe in

* Since this was read, we have seen Professor Max Weber's admirable paper, in which a description of the histological structure of the stomach is given, *Morph. Jahr.*, 1887–1888, p. 637 *et seq.;* and Sir Wm. Turner's paper on "Additional Observations on the Stomach in the Ziphioid and Delphinoid Whales," *Jour. Anat. and Phys.*, vol. xxiii. p. 466 *et seq.*

† The stomach obtained was that of an adult female, 14 feet in length, killed in the Greenland Sea during the summer of 1888.

detail the several cavities or compartments into which the stomach is subdivided, giving in each instance both the naked-eye appearance, and, as far as possible, the microscopic structure, and, in some cases, offering a few observations on the more interesting features.

The Œsophagus.

The œsophagus terminates by expanding into a wide dilatation constituting a cavity of considerable size, which may be called "the œsophageal paunch." The orifice of communication is wide and patent, having a diameter of 3 inches. The mucous membrane presents an opaque, whitish appearance, and is thrown into well-marked longitudinal folds, besides being slightly corrugated transversely. Numerous minute follicular-looking depressions are evident, scattered over the surface, which, however, must not be looked upon as the openings of glands.

Structure of Coats.—1. The outer or fibrous coat is a layer of tough areolar tissue, containing a few bundles of yellow elastic fibres.

2. The muscular coat consists of an outer thin layer of non-striped muscular fibres running longitudinally, and a thick, well-developed layer fully $\frac{1}{8}$th inch in thickness, in which the fibres are disposed circularly.

3. The submucous or areolar coat is of very considerable thickness, and is formed of somewhat dense connective tissue, in which numerous well-formed vessels are seen. Non-striped muscle fibres are present under the mucous membrane, running for the most part longitudinally, forming a layer of considerable thickness. More externally the connective tissue is looser; here and there are a few bundles of muscle fibres running circularly, also large blood-vessels, both arteries and veins, the venous channels being especially large and numerous.

4. The mucous membrane is remarkable for its thickness, and for the dense and horny nature of the epithelium. Superficially the cells are arranged in regular lamellæ, closely resembling the cells of the stratum corneum of the human skin, their nuclei, however, never actually disappear as in the stratum lucidum, and there is no space round those nuclei near the free surface. In some cases

small granules of pigment may be seen around the nuclei. More deeply a regular rete Malpighii obtains, the cells of which are polyhedral in shape, and have short "prickles" at the margin. The deepest layer of the epithelium consists of cubical or slightly columnar cells (Plate II. fig. 1), the nuclei of which stain deeply; some appear to have a single nucleolus, surrounded by a clear space, others again a delicate intra-nuclear plexus. The numerous papillæ which extend the surface for the production of epithelial cells, appear to project upwards as delicate filiform processes, ramifying somewhat irregularly, as is evidenced by the great subdivision of the mass of epithelium in the deeper parts. This subdivision is frequently so marked that masses of epithelium seem to be cut off from the rest, presenting the appearance of gland acini, for which, in allied species, they have by some observers been mistaken.

The Œsophageal Paunch.

This, the dilated termination of the œsophagus, consists of a single compartment, constricted in such a manner that two cavities are formed, viz., a main portion and an outgrowth or diverticulum ending blindly. Both these lie in the direct line of the œsophagus, and the diverticulum or appendage may therefore be regarded as its cæcal termination. The main cavity, which is somewhat rounded in form, measures internally 9 inches in its longest diameter by $5\frac{1}{2}$ inches in its shortest, whilst the appendage or diverticulum, cylindrical throughout, is 11 inches by 4 inches. Within the main cavity, some 3 inches from its œsophageal end and opening at right angles to the line of the œsophagus, is found the orifice leading into the first true digestive cavity of the stomach. The diameter of this opening is $2\frac{1}{2}$ inches, notwithstanding the elevation of the mucous membrane round its margin to be afterwards mentioned. Throughout the main cavity, the mucous membrane, as in the œsophagus, is white and opaque in appearance, but, at the opening leading into the second compartment, its character changes abruptly, and forms a ring-like band leading to the summit of an elevated fold of mucous membrane, fully 1 inch in height, which may perform, to some extent, the function of a valve. The longitudinal folds of mucous membrane already mentioned as occurring in the œsophagus

may be traced downwards into the cavity of the first compartment, where, however, their character becomes more complex, and they present a very remarkable appearance. At first they are elevated into crests, afterwards they are broken up into rows of nipple-like processes (fig. 1), which gradually get lower and converging towards the apical end of the appendage, finally disappear. They



Fig. 1.

are exceedingly numerous round the orifice leading to the first true digestive cavity, which they may assist in occluding, or at least they may act in preventing the passage of large morsels of food through it.

Throughout the first compartment the mucous membrane presents a somewhat honeycombed appearance, due to the presence of small blind depressions which are most numerous in the nipple-like processes mentioned.

Structure of Coats.—1. The outer coat, continuous with the peritoneum, consists of dense areolar tissue with numerous blood-vessels and some yellow elastic fibres. The structure of this coat is similar to that in the walls of the other compartments.

2. The muscular coat consists of two layers arranged as in the

wall of the œsophagus, viz., a thin outer longitudinal and a thick inner circular layer, separated by loose areolar tissue, containing blood-vessels and a small amount of yellow elastic tissue; this is an arrangement which obtains throughout the entire stomach. In this compartment the longitudinal fibres converge towards the apical end of the appendage.*

3. The submucous coat is in the form of a layer of loose areolar tissue of considerable thickness, in the outer part of which small masses of fat are found, and throughout which a few yellow elastic fibres are dispersed. Numerous blood-vessels are present, and immediately under the mucous membrane the areolar tissue becomes denser, and contains numerous muscle fibres running, for the most part, longitudinally. The elements of this coat are similarly arranged in the walls of all the compartments.

4. The mucous membrane is similar in structure to that of the œsophagus, and consists of a dense layer of laminated horny epithelium.

A section through one of the nipple-like processes of this compartment shows a core of connective tissue surrounded by epithelium (Plate I. fig. 1, and Plate II. fig. 1). In the centre of the core the connective tissue is loose and open, but nearer the epithelium it becomes much more dense. In the core numerous bundles of muscular tissue are seen, arranged circularly and longitudinally in the peripheral portion, circularly nearer the centre. The blood-vessels are extremely well developed, the larger trunks running in the centre, the smaller at the periphery. The arteries, which are numerous, have the ordinary structure, with this exception, that their walls are very stout, especially the tunica adventitia; the veins have in like manner a well-developed adventitious coat. A complex system of lymphatics is present, especially under the epithelium, where they assume the form and arrangement of large sinuses.

From the observations we have made, we feel convinced that the so-called first compartment is in no way to be looked upon as part of the stomach proper. It is rather a somewhat globular dilatation

* It is just possible that this may have to be looked upon as the direct continuation of the œsophagus, as we find that the longitudinal muscles converge in a most peculiar manner at the apex where the wall is exceedingly thin, and there is formed a small stellate mass of fibrous tissue at this point, into which the particular fibres are inserted as it were.



FIG. 1.—Transverse section of one of the nipple-like processes from the wall of the œsophageal compartment. Small papilliform and filiform processes covered with squamous epithelial cells. × 15.

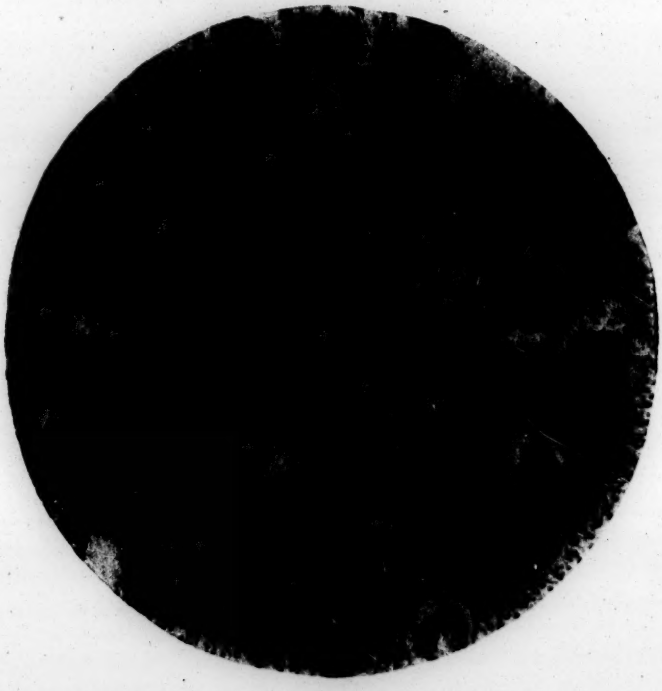


FIG. 2.—Transverse section of ducts and glands from the "pyloric" end of the "intermediate" division, showing cellular connective tissue and the single layer of columnar or axial epithelial cells. × 180.



FIG. 2.—Vertical section of mucous membrane of the first true digestive (cardiac) cavity. In the deeper part the glandular tubes have both principal or axial and peripheral cells. Nearer the surface they have only a single layer of columnar epithelium. × 15.

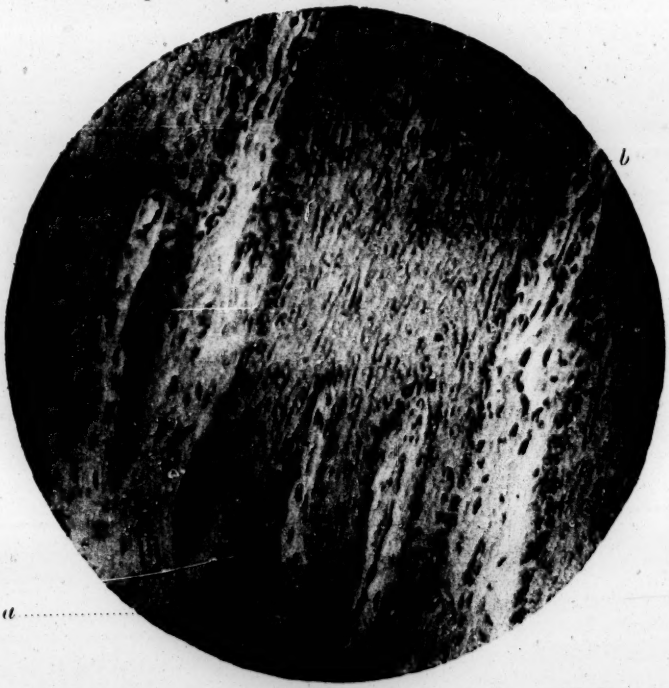


FIG. 1.—Vertical section through the deep layer of the epithelium on the surface of one of the finger-like processes from the œsophageal chamber. Cubical (*a*) and polyhedral (*b*) cells well seen. × 200.

of the lower end of the œsophagus with a lateral diverticulum. In this connection we may quote Home's description of the stomach of *Delphinus delphis*, where he says—"The first stomach lies in the direction of the œsophagus, which is continued into it. . . . . There is a canal, between the first and second cavities, 3 inches long, which opens into the second by a projecting orifice, and the cuticular covering of the first stomach terminates immediately beyond this orifice, which is $2\frac{1}{2}$ inches in diameter." This certainly bears out the theory that the so-called first cavity, with its lateral diverticulum, is merely a modification of the œsophageal tube; in the Narwhal and in most others of the Delphinidæ, at and near the extremity of the tube, but in Home's porpoise at some little distance from the extremity.



Fig. 2.

We may here again mention the distinct line of demarcation between the squamous epithelium of the œsophageal cavity and the glandular secreting epithelium of the first true digestive cavity (fig. 2).

Those observers who have looked for the salivary gland in the whale have failed to find anything more than a mere rudiment, though Morrison Watson and Young say that in the Beluga which they examined, they detected an apparently glandular body which occupied the usual position of the submaxillary gland. They could, however, find no duct, and the parts were not well enough preserved to allow of any very accurate observations being made. Macalister* could find no trace of a salivary gland in *Globiocephalus*

* *Proc. Zool. Soc.*, 1867, p. 480.

svineval, so that it may be assumed that the glands are at any rate not constant.* This, taken along with the width of the œsophagus, the peculiar expansions at its termination, and the absence of any but the most elementary masticatory apparatus, points to two facts —(*a*) that the food is swallowed immediately, and it is not in any way mixed with a salivary secretion; (*b*) as there are no secreting glands in the œsophageal sacs, any digestive fluid that is mixed with the food in all probability regurgitates from the first true gastric compartment. In such case we should have to look upon the œsophageal sac as a "mixer" at all events, whatever may be its functions as a grinder; and it might further be suggested that hard parts of crustaceans, and other indigestible matter, may be strained out in the diverticulum, and that they may then be ejected through the wide opening of the œsophagus.

A somewhat similar process goes on in some of the carnivorous birds, *e.g.*, the owl, in which, however, the grinding and mixing apparatus is below or beyond the secreting portion of the gastric apparatus. We know that in birds, in which the gizzard with its strong muscular wall and horny epithelium is below the secreting area, the food is both ground and mixed with gastric juice, and it seems to be at least possible that the same process may go on in the dilated and strengthened portion of the œsophagus of the Narwhal, the gastric juice being from time to time driven back from the first true stomach.

In connection with this question, we may refer to Turner's paper, in which he speaks of Tyson's observations on the contents of the stomach (œsophageal pouch?) and œsophagus of the porpoise, as bones and other partially digested substances. Cleland also mentions the presence of perfectly clean bones in the stomach of the white-beaked dolphin. One of us has on several occasions observed partially digested cuttle-fish in the mouth cavity of the recently killed Narwhal, so that the Narwhal has at least the power of vomiting its food under certain conditions, and it is also probable that indigestible material may be similarly ejected.

* In the Greenland whale, *Belœna mysticetus*, near the extremity of the upper jaw, a blindly-ending depression is found on each side of the middle line within the cavity of the mouth, which, as Eschricht and Reinhardt suggest ("Recent Memoirs on the Cetacea," *Memoirs of the Ray Society*), probably represent the rudiments of Stenson's duct.

In most of the stomachs examined, the first compartment contained the less digestible portions of the cuttle-fish (*Gonatus fabricii*), the horny mandibles and the crystalline lenses, the latter sometimes several hundreds in number. There were also found the dermal skeletons of crustaceans, usually *Pasiphaë tarda* (Kroyer) and *Hymourdora glacialis* (Buch.).

Scoresby mentions the remains of fishes as found in the œsophageal receptacle.

The fact that we have these considerable accumulations of indigestible matter in the œsophageal cavity points in the direction we have indicated.

Remembering the absence of any œsophageal dilatation in the Ziphioid whales, *e.g.*, Hyperoodon, and the fact, as observed by one of us, that Hyperoodon, like the Narwhal, feeds mainly on the cuttle-fish, *Gonatus fabricii*, it becomes evident that this difference in structure cannot be accounted for by any difference in the nature of the food, as Turner suggests. We should suggest rather, that the œsophageal paunch and the wide gullet are probably associated with a habit of bolting and storing up the food, digestion being allowed to go on at leisure.

The Cardiac Cavity.

The Second Compartment, but the *First True Digestive Cavity* of the stomach, is cylindrical in form, measuring internally 9 inches by $3\frac{1}{2}$ inches. At the œsophageal end is situated the opening already referred to, by which this cavity communicates with the previous compartment; at the distal or pyloric end is the opening leading into the third compartment, a constricted orifice which scarcely admits the forefinger. The mucous membrane of this compartment has a florid and vascular appearance, is soft to the touch, and is thrown into irregularly convoluted folds, which have a slight tendency to converge towards either end of the compartment. Numerous minute follicular depressions are evident, especially between the ridges of mucous membrane.

Structure of the Coats.—The fibrous, muscular, and submucous coats are as in the preceding compartment. The mucous membrane is exceedingly soft, and nearly as thick as the whole of the remaining portion of the wall of the compartment (Plate II. fig. 2). It

contains elongated tubular glands embedded in a small amount of connective tissue, in which is a system of delicate blood-vessels and lymphatics. Between the tubular glands in the deeper layer the amount of connective tissue is very slight indeed, but near the surface epithelium, where the gland ducts proper are situated, it becomes more abundant. The glands are simple unbranched tubes arising each from a duct of its own, and running almost straight from the epithelial surface to the submucosa, where it ends in a short hooked extremity. The secreting portion of the walls of these glands is formed of a double layer of cells (Plate II. fig. 2, and Plate III. figs. 1 and 2). Internally, a layer of cells (viz., the central cells of the gland) are for some distance down columnar in form, but they become smaller in size and distinctly cubical as we come to the deeper part of the gland, the amount of protoplasm in each cell gradually becoming smaller. Outside this is a second layer of cells (the parietal cells of the gland), completely investing the tubule, and not occurring merely at intervals as in the cardiac glands of the stomach of the dog and human subject. The cells are large nucleated protoplasts, irregular in shape, slightly flattened, pyramidal, &c., each lodged in a distinct cavity formed by a framework of delicate connective tissue on which small flattened nuclei are seen (Plate IV. fig. 1). These cells are largest in the deepest part of the mucosa; near the free surface they become smaller in size, more flattened, and eventually they disappear.

This second compartment corresponds with the first compartment of the Ziphioid whales as described by Turner* in *Mesoplodon bidens* and in *Hyperoodon rostratus*, and is in the Narwhal somewhat larger than the first compartment without the diverticulum, though scarcely so much as one would gather from Meckel's description. Murie,† on the other hand, says that in Globiocephalus the first gastric cavity is by far the largest.

With it commences the true secreting portion of the stomach, and it may be compared to the cardiac end of the dog's stomach. The point of special interest is that the peripheral cells are specially numerous, forming a layer completely investing the central or axial columnar epithelial cells. The reticulum, in which the large parietal cells are embedded, corresponds apparently

* (5) Page 247. † (6) Page 247.

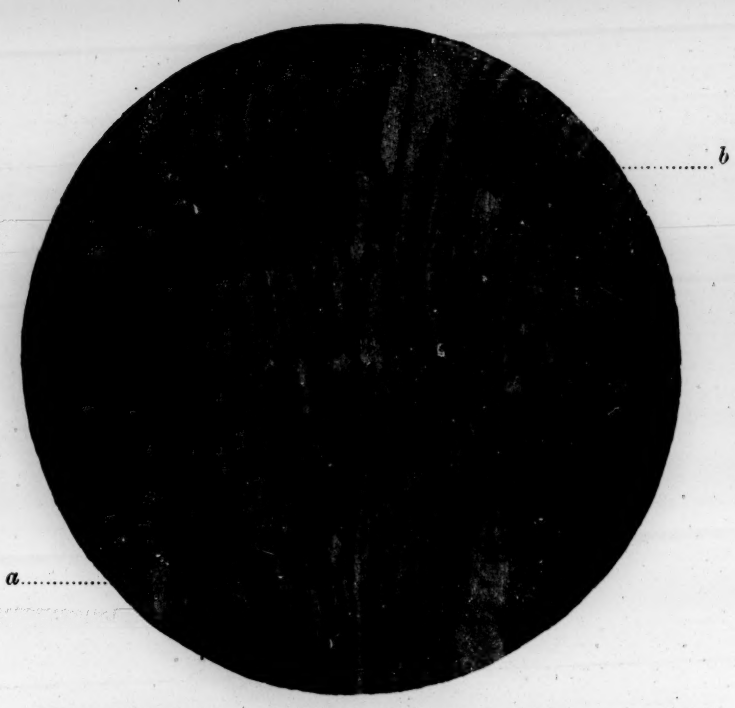


FIG. 1.—Point of junction between gland tube, with double layer of epithelium (*a*) and duct with single layer (*b*). Plate II. fig. 2, more highly magnified. × 60

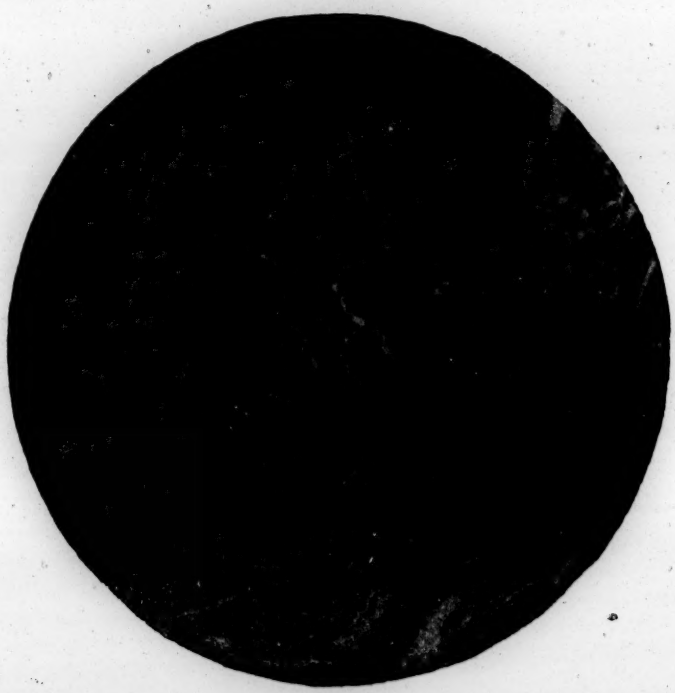


FIG. 2.—Vertical section through the mucous membrane of the "cardiac" division. The two kinds of gland cells are both present. × 60.

to that in the same position in the pig and in the porpoise. It appeared in many cases as though the large parietal cells were entirely surrounded by the delicate strands, but it is probable that there is the same arrangement here that there is in the animals mentioned, and that there is a small orifice in the inner wall of the space, bringing the two sets of cells into direct communication. Here and there flattened nuclei may be seen lying on the strands of this reticulum.

The Second True Digestive Cavity.

This cavity, externally concealed, almost, between the second and fourth compartments, is cylindrical in form, is curved abruptly upon itself, through a right angle, and measures 3 inches in length by 1 inch in diameter. The orifice at its proximal end has already been referred to in connection with the last compartment; the orifice at the distal end, leading into the next compartment, is also small in size, being less than 1 inch in diameter. At neither end is there a valve-like sphincter. The mucous membrane, florid and vascular in appearance, is smooth and almost devoid of rugæ. Although small in size, this cavity is not a mere canal forming the means of communication between adjoining cavities, but must be regarded as a distinct compartment, since it possesses both a proximal and a distal orifice and walls proper to itself, with slight structural peculiarities.

Structure of the Walls.—The fibrous, muscular, and submucous coats are arranged as in previous compartments, but in the areolar tissue of the fibrous coat, which in this case is not continuous with the peritoneum (this compartment lying concealed between the adjoining portions of the stomach, and not being invested by a serous coat), there are numerous large ganglion nerve-cells, and the circular fibres of the muscular coat are here especially well developed.

Mucous Membrane.—As in the previous compartment, the glands are mostly simple elongated tubules, arranged vertically to the free surface, though in some cases they divide. They do not, however, hook round at their extremities, and from the presence in most cases of only a single set of cells, there is a distinct difference in this structure from those already described. The glands of the second com-

partment were lined by a double set of cells—the "central" and "parietal." Of these in this cavity, in most cases ordinary columnar epithelial secreting cells, resting on a basement membrane of flattened nucleated cells, and corresponding to the "central" cells, alone are present (Plate I. fig. 2); but in a few glands, at what may be called the cardiac extremity, a few of the large parietal cells are found in the deeper part of the tube (Plate IV. fig. 2). The framework of connective tissue supporting the glands is much more abundant and stronger than in the previous compartment.

It is generally agreed that in Delphinidæ this portion of the stomach is much narrower and more intestine-like than any of the other compartments. Meckel, speaking of this portion of the stomach of the Narwhal, describes it as much narrower and longer, commencing with a small cæcum, which lies backwards and to the left, first taking a turn forwards, then quickly backward, then again forward and somewhat to the right, in order to join by a very narrow sphincter orifice the duodenum. (This can scarcely correspond to the cavity above described in anything but shape.) In the case of Globiocephalus, Murie * describes it as a communicating passage, which "leaves the second stomach on its right inferior wall, an inch below the wide aperture which connects the first and second, and it enters the third stomach above and behind. There is no true sphincter at either end."

Turner,† referring to Murie's description, says that "in *Globiocephalus melas*, both Dr Jackson and I have described five compartments, though Dr Murie regards the compartment which I have numbered three not as a true digestive sac, but only as a communicating canal. With this interpretation, however, I am unable to agree, and still adhere to my opinion that it is a true, though small, gastric compartment." Morrison Watson and Young give their reasons for dissenting from Murie's view briefly, as follows:—

They point out that in many animals, *e.g.*, birds, certain limited areas of the stomach are set apart for special glandular secretion.

* *Trans. Zool. Soc. Lond.*, vol. viii. part 4, p. 257.

† "Anatomy of Sowerby's Whale," *Jour. Anat. and Phys.*, October 1885, p. 154.



FIG. 2.—Transverse section through the deep portion of the mucous membrane of the "intermediate" compartment, from near the cardiac cavity. The cardiac type of gland is well seen. Peripheral cells (*a*); axial cells (*b*); delicate intertubular connective tissue (*c*). × 200.

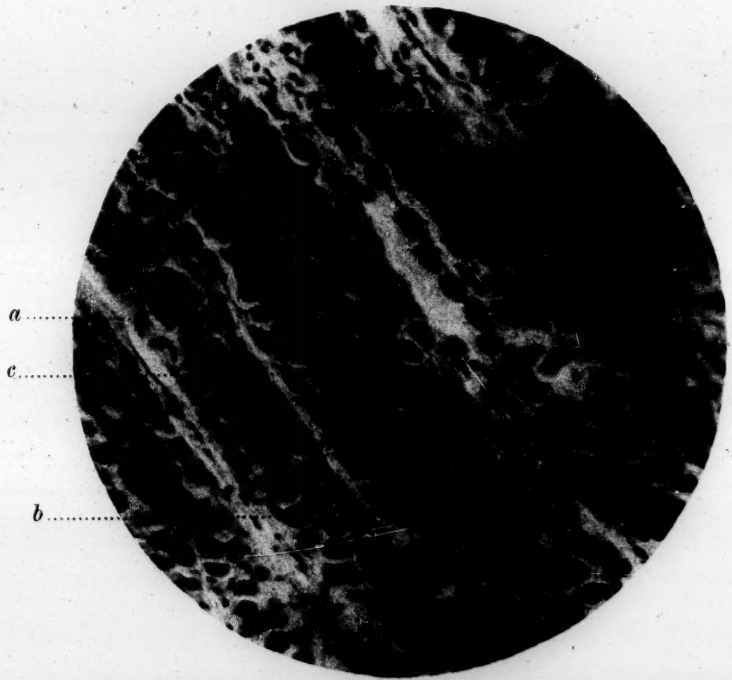


FIG. 1.—Fig. 2 of Plate III. × 200. Delicate intertubular connective tissue (*a*). Peripheral cells contained in meshes of fine stroma. Axial (*b*) and peripheral cells (*c*) both present. Between the rows of peripheral cells lymph and blood channels are seen. × 200.

The interpolation of a single simple communicating tube, devoid of gastric function, would be an anomaly; and they adduce further argument, that for the purpose of communication a mere orifice, similar to those between the other compartments, would serve equally well.

On these grounds, we are inclined to agree with those who consider this to be a distinct gastric compartment, in addition to which we might adduce the following :—

The mucous membrane consists of a series of glandular tubes lined with distinct secreting epithelium, as already described. Although there is no regular valvular fold of mucous membrane at either orifice, these orifices will neither of them admit of the passage of more than a single finger, and the circular muscular fibres around the constricted portions will, in these positions, have greater contracting power than around the more dilated portions, and they must exert a sphincter action rather than assist in producing peristaltic movement, more especially as they lie at each end of the dilated portions of the tract.

The Third True Digestive Cavity.

This cavity, in certain respects, resembles the human stomach, being somewhat cylindrical and crescentic in form, and slightly curved upon itself lengthwise, with a well-marked fundus at the proximal end. Internally it measures, while moderately distended, 7 inches in its longest diameter by 3 inches in its shortest. Both orifices lie in its lesser curvature, the distance between them being 3 inches. Of these the first has already been noticed; while the second, leading into the fourth true gastric compartment, is slightly larger, being fully 1 inch in diameter. The mucous membrane is raised into imperfectly marked folds throughout the interior of the cavity; it also forms a slightly elevated ridge at the distal orifice, but there is no regular valve present.

Structure of the Walls.—The fibrous, muscular, and submucous coats are arranged as in the previous compartments, and the ganglion cells, although less numerous, are also present in the areolar tissue of the submucous and peritoneal coats. In these situations there are also small collections of lymph follicular tissue.

Mucous Membrane.—In this we find only one kind of glands

(Plate I. fig. 2). These are lined by cubical epithelial cells, and the secreting tubules branch and run irregularly in the deeper part of the membrane. The connective tissue, which is somewhat altered near the surface, probably by digestion, is dense, and in an intermediate zone consists very largely of cellular connective or lymphoid tissue. In the deeper parts the lymphoid character is not so well marked, although nuclei are still numerous. The terminations of the glands rest on a connective tissue mucosa, in which numerous blood-vessels are found.

The Fourth True Digestive Cavity.

This, the last of the cavities into which the stomach is subdivided, is cylindrical in form, and measures $9\frac{1}{2}$ inches in length by $2\frac{3}{4}$ inches in diameter. The orifice by which it communicates with the intestine, viz., the pyloric, is capable of being distended to a diameter of three-fourths of an inch, and is situated about an inch from the distal end of the cavity, a short cæcal pouch being thus formed. The mucous membrane is similar in appearance to that of the previous compartments, but is smooth, and almost devoid of rugæ. The pylorus is represented by a well-marked elevation of the mucous membrane, which, however, is blunt, and rounded at its free edge.

Structure of the Walls.—The outer coats are much the same as in the previous compartments, but the longitudinal layer of fibres of the muscular coat is very poorly developed, and, with the exception of a few fibres, is, in most parts, absent. Nerve ganglion cells are present as in the wall of the two previous compartments. The arteries in the submucous coat are numerous, and are remarkable for the thickness of their muscular, and the denseness of their adventitious, coats.

Mucous Membrane.—The glands are very similar in their structure to those of the previous compartment (Plate I. fig. 2); the lower parts of the glands frequently branch, and form transverse or acute bendings. Small patches of lymphoid tissue are present, and there are delicate bundles of muscular fibre lying parallel to the axes of the glands.

The third and fourth true digestive cavities, though separated by a somewhat narrow opening, are very similar to one another in all

respects. The glandular epithelium is, in both, entirely of the principal or pyloric variety, and the gland tubes are alike in other respects. The only valve-like structure with a sphincter muscle is met with at the distal end of the fourth compartment, and from this we conclude that at this point we have the termination of the true gastric function. From this description it is evident that, as Tyson and others assert, the first true digestive cavity corresponds in every way to the cardiac end of the stomach of the carnivora and man; whilst the following portions, with the exception, perhaps, of what we have called the econd true digestive cavity, must be looked upon as essentially pyloric in structure, and probably also in function.*

The Duodenum.

The arrangement of the glandular tissue in the duodenum does not resemble any form we have seen described, except that met with in the intermediate glands between the true pyloric glands and Brunner's glands in other animals. They form part of the mucosa proper, but between them are seen well-developed bands of muscular and lymphoid tissue apparently prolonged upwards, so that they come to be embedded in a kind of submucous tissue. The ducts are comparatively short, and are surrounded in their whole length by bands of non-striped muscular fibre. In addition to this set of bands of muscle, a well-developed layer of muscular fibre was observed parallel to the surface, and at the level of and surrounding the orifices of the ducts of these glands. It will at once suggest itself that the fibres running parallel to the gland tubes, during contraction, would drive out the secretion, but that those surrounding the mouths would constrict them regularly or intermittently, and so lead to a temporary storing up of the duodenal secretion. Such a suggestion is entirely hypothetical, except in so far as it is borne out by the arrangement of the bundles of non-striped muscular fibre, and by what we know of the process of digestion in other animals.

It was unfortunate that the duodenum was cut off so short. We

* Turner, *loc. cit.*, would speak of the cardiac cavity as that immediately following the œsophageal paunch; then there would be two *intermediate* cavities; and lastly, a simple pyloric cavity.

made a most careful search for the bile duct in both the wall of the last gastric cavity and the duodenum, but could find no trace of it; we therefore came to the conclusion that it must open lower down in the duodenum.* Of this part of the alimentary canal some 6 inches attached to the stomach has been examined. Immediately after the constriction at the pylorus, the intestine expands to a diameter of $1\frac{1}{4}$ inches; the mucous membrane is at first thrown into longitudinal ridges, which become irregularly convoluted at a distance of 5 inches from the pylorus.

The intestine measures from pylorus to anus 97 feet in an adult animal, so that the length of the intestine is to that of the body as 7 is to 1—about the usual proportion in Delphinidæ. There is no separation into large and small intestine.

The structure of the wall of the duodenal portion has been already mentioned. In the intestine the muscular layers attain considerable thickness, both the outer longitudinal and the inner circular layers of the muscular coat being well formed, whilst the muscularis mucosæ is exceedingly thick, and is thrown into folds from which bundles of muscular tissue extend between the gland follicles.

The glands, as we have seen, are in the form of bundles of follicles, separated by well-marked septa of fibrous tissue. The secreting tubes appear to have a somewhat simple acinal arrangement, and present the appearance of modified Brunner's glands:—several tubules opening into a single duct, which in turn opens on the surface. The secreting cells lining the tubules have the ordinary appearance of columnar epithelium. The supporting framework of connective tissue is highly cellular, and here and there small masses of lymph follicular tissue are seen.

* Since the above was read, Professor Sir Wm. Turner has published an account of the stomach of a full-time fœtal Narwhal (*Jour. of Anat. and Phys.*, vol. xxiii. p. 486, "On the Stomach of Ziphioid and Delphinoid Whales"), in which he states that "the duodenum arose from the right and posterior aspects of the fifth compartment. It was somewhat dilated at its commencement, but soon became a cylindriform tube. . . . . The mucous lining of the dilated portion was smooth, that of the cylindriform tube was elevated into valvulæ conniventes. Close to where the cylindriform part of the duodenum began was a semilunar fold of mucous membrane, which bounded the orifice of the hepatico-pancreatic duct."

Literature on the Stomach of Delphinidæ.

1. Home, Structure of the Different Cavities of the Stomach of the Whale, *Phil. Trans.*, 1807.
2. Meckel, *System der vergleichenden Anatomie*, vol. iv. p. 527, 1829.
3. Owen, *Anatomy of Vertebrates*, vol. iii. p. 45.
4. Jackson, *Boston Journal of Natural History*, vol. iv. p. 167, 1845.
5. Turner, (*a*) A Contribution to the Anatomy of the Pilot Whale, *Jour. of Anat. and Phys.*, vol. ii. p. 70, 1867; (*b*) Further Observations on the Stomach of Cetacea, *ibid.*, vol. iii., 1868; (*c*) Anatomy of Sowerby's Whale (*Mesoplodon bidens*), *ibid.*, 1885; (*d*) On the Stomach in Ziphioid and Delphinoid Whales, *ibid.*, vol. xxiii. p. 466.
6. Murie, On the Organisation of the Caaing Whale (*Globiocephalus melas*), *Trans. Zool. Soc. Lond.*, vol. viii., 1867.
7. Clark, *Delphinus albirostris*, *Proc. Zool. Soc. Lond.*, 1876.
8. Watson and Young, Anatomy of the Northern Beluga (*Beluga catodon*), *Trans. Roy. Soc. Edin.*, vol. xxix., 1879.
9. Cleland, On the Viscera of the Porpoise and White-headed Dolphin, *Jour. of Anat. and Phys.*, vol. xviii., 1884.

For a more complete bibliography, see Watson and Young's paper on *Beluga catodon*, above referred to.

Meetings of the Royal Society—Session 1888–89.

Monday, 26th November 1888.

General Statutory Meeting. Election of Office-Bearers. *P.* xvi. 1.

Monday, 3rd December 1888.

Sir Douglas Maclagan, M.D., Vice-President, in the Chair.

1. The Chairman gave an Opening Address. *P.* xvi. 2.

The following Communications were read:—

2. On the Ostracoda collected by H. B. Brady, Esq., F.R.S., in the South Sea Islands. By Geo. S. Brady, Esq., F.R.S. Communicated by Dr John Murray. *T.* xxxv. 489.

3. On *Pseudalius alatus*, Leuck., collected by Mr Robert Gray in the Arctic Seas, and other Species of the Genus. By Dr O. v. Linstow. Communicated by Dr John Murray. *P.* xvi. 15.

4. Restatement of the Theory of Organic Evolution. By Professor Patrick Geddes. Part I.—Botanical and Zoological.

The following Candidates for Fellowship were balloted for, and declared duly elected Fellows of the Society:—

WILLIAM SOMERVILLE, Esq., B.Sc., of Comiston.
DAVID PATRICK, Esq., M.A.
ALEXANDER JAMES, M.D.
RALPH STOCKMAN, M.D.
A. H. F. BARBOUR, M.D.
A. CRICHTON MITCHELL, B.Sc.

Monday, 17th December 1888.

Sir William Thomson, President, in the Chair.

The following Communications were read:—

1. On some Fossil Plants from Teilia Quarry, Gwaenysgor, near Prestatyn, Flintshire. By R. KIDSTON, Esq., F.G.S. *T.* xxxv. 419.

2. An Abstract of the Results of an Inquiry into the Causation of Asiatic Cholera. (*a*) General. By NEIL M'LEOD, Esq., M.D. (Edin.), and WALTER J. MILLES, Esq., F.R.C.S. Eng. (*b*) With special regard to the Reproduction of the Disease. By NEIL M'LEOD, Esq., M.D. Communicated by Dr G. SIMS WOODHEAD. *P.* xvi. 18.

3. Preliminary Observations with a Large Rotatory-Polarization Spectroscope. By Professor TAIT.

4. Further Remarks on the Curve

$$y = \varepsilon^{-x^2} \int_0^x \varepsilon^{x^2} dx .$$

By Professor TAIT.

5. Preliminary Remarks on the Homologies of the Mesenteries in Antipatharia and other Anthozoa. By GEORGE BROOK, Esq., F.L.S. *P.* xvi. 35.

Monday, 7th January 1889.

Professor Chrystal, Vice-President, in the Chair.

The Chairman, in terms of the Laws, communicated to the Society the Names proposed by the Council to complete the lists of British and Foreign Honorary Fellows.

At the request of the Council, Professor TAIT gave an Address On the Compressibility of Glass, Mercury, Water, and Solutions of Common Salt. Illustrative Experiments were made. (*Challenger Report;* Physics and Chemistry, vol. ii. part iv.)

JAMES DALRYMPLE DUNCAN, Esq., F.S.A. Scot., was balloted for, and declared duly elected a Fellow of the Society.

Monday, 14th January 1889.

Sir William Thomson, President, in the Chair.

The following Communications were read:—

1. On certain Bodies, apparently of Organic Origin, from a Quartzite Bed near Inveraray. By his Grace The Duke of Argyll, K.G., K.T. Specimens were exhibited. *P.* xvi. 39.
2. On the Deformation of Rocks by Mechanical Movements. By Archibald Geikie, Esq., LL.D., Director-General of the Geological Survey.

Monday, 21st January 1889.

Professor Sir Douglas Maclagan, M.D., Vice-President, in the Chair.

The following Communications were read:—

1. The History and Theory of Heredity. By J. Arthur Thomson, Esq., M.A. *P.* xvi. 91.
2. Note on the Transformation of Ciliated into Stratified Squamous Epithelium, as a result of the application of Friction. By John Berry Haycraft, Esq., M.D., D.Sc., and E. W. Carlier, Esq., M.B., B.Sc. Accompanied by a Demonstration. *P.* xvi. 119 (*Abstract*).
3. On the Virial Equation. By Professor Tait. *P.* xvi. 65 (*Abstract*).
4. A remarkable Fog-Bow seen from Ben Nevis on 4th December 1888. By R. T. Omond, Esq.
5. A new Type of Dimorphism found in certain Antipatharia. By George Brook, Esq., F.L.S. *P.* xvi. 78.
6. A Method of Demonstrating the presence of Uric Acid in the Contractile Vacuoles of some Lower Organisms. By Dr A. B. Griffiths, F.C.S. *P.* xvi. 131.

Monday, 4th February 1889.

The Rev. Professor Flint, D.D., Vice-President, in the Chair.

The following Communications were read:—

1. On *Strophanthus hispidus:* its Natural History, Chemistry, and Pharmacology. Part I. By Professor Thomas R. Fraser, M.D., F.R.S., *P.* xvi. 73 (*Abstract*); *T.* xxxv.
2. On Improvements in the Apparatus for Counting the Dust Particles in the Atmosphere. By John Aitken, Esq., F.R.S. *P.* xvi. 135.
3. The Electrotonic Variation with strong Polarizing Currents. By

G. N. STEWART, Esq. Communicated by Professor RUTHERFORD, F.R.S. *P.* xvi. 234.

The Council recommended, as Honorary Fellows, the following:—

Foreign Honorary Fellows.

MARCELLIN PIERRE EUGÈNE BERTHELOT, Member of the Institute of France.

Dr ERNST CURTIUS, Professor of Philosophy in the University of Berlin.

His Excellency JAMES RUSSELL LOWELL, Esq., Hon. D.C.L. Oxford, and LL.D. Cambridge and Edinburgh.

Dr GEORG HERMANN QUINCKE, Professor of Physics in the University of Heidelberg.

British Honorary Fellow.

Sir ROBERT STAWELL BALL, LL.D., Royal Astronomer of Ireland.

A Ballot to take place on 18th February.

Monday, 18th February 1889.

Dr Thomas Muir, Vice-President, in the Chair.

The following Communications were read:—

1. On the Prolonged Action of Sea Water on Pure Natural Magnesium Silicates. By ALEXANDER JOHNSTONE, Esq., F.G.S. *P.* xvi. 172.

2. On the so-called "Liver" of *Carcinus moenas.* By Dr A. B. GRIFFITHS, F.C.S. *P.* xvi. 178.

3. Differentiation of any (Scalar) Power of a Quaternion. By ALEXANDER M'AULAY, Esq., Ormonde College, Melbourne. Communicated by Professor TAIT. *P.* xvi. 201.

4. Note on the preceding Paper. By Professor TAIT. *P.* xvi. 205.

5. The Change in the Thermo-electric Properties of Wood's Fusible Metal at its Melting Point. By ALBERT CAMPBELL, Esq., B.A. Communicated by Professor TAIT. *P.* xvi. 83.

6. On the Anatomy and Physiology of Phreoryctes. By FRANK E. BEDDARD, Esq., M.A. *P.* xvi. 117 (*Abstract*); *T.* xxxv. 629.

A Photograph of an Apparatus, by Professor R. J. ANDERSON of Galway, to illustrate the Five Systems of Crystals, and also a Model of the Hexagonal System, was laid on the Table.

The following Gentlemen were balloted for, and declared duly elected Foreign Fellows of the Society:—

MARCELLIN PIERRE EUGÈNE BERTHELOT, Member of the Institute of France.

Dr ERNST CURTIUS, Professor of Philosophy in the University of Berlin.

His Excellency JAMES RUSSELL LOWELL, Esq., Hon. D.C.L. Oxford, and LL.D. Cambridge and Edinburgh.

Dr GEORG HERMANN QUINCKE, Professor of Physics in the University of Heidelberg.

The following Gentleman was balloted for, and declared duly elected as a British Honorary Fellow of the Society:—

Sir ROBERT STAWELL BALL, LL.D., Royal Astronomer of Ireland.

Monday, 4th March 1889.

Sir William Thomson, F.R.S., President, in the Chair.

The President announced that the Council had awarded

The Keith Prize, for the Period 1885–87, to Mr J. Y. BUCHANAN.

The Makdougall-Brisbane Prize, for the Period 1884–86, to Dr JOHN MURRAY.

The Makdougall-Brisbane Prize, for the Period 1886–88, to Dr ARCHIBALD GEIKIE.

The following Communications were read:—

1. Deductive Evidence of a Uterine Nerve Centre, and of the Location of such in the Medulla Oblongata. By Dr J. OLIVER. Communicated by Dr G. SIMS WOODHEAD. *P.* xvi. 175.
2. Diagram illustrating the History of Determinants. By Dr MUIR.
3. Note on the Relation between the Mutual Distances of Five Points in Space. By THOMAS MUIR, LL.D. *P.* xvi. 86.
4. The Relation among Four Vectors. By Professor TAIT. *P.* xvi. 88.
5. On a Gyrostatic Model of a medium capable of transmitting Waves of Transverse Vibration. By the PRESIDENT.
6. Observations on the Metabolism of Man during Starvation. By D. NOEL PATON, M.D., and RALPH STOCKMAN, M.D. *P.* xvi. 121.

The following Candidates were balloted for, and declared duly elected Fellows of the Society:—

BOVERTON REDWOOD, Esq.
Rev. JAMES LINDSAY, M.A., B.D., B.Sc., F.G.S.

Monday, 18th March 1889.

Dr John Murray, Vice-President, in the Chair.

The following Communications were read:—

1. A Contribution to the Chromatology of the Bile. By JOHN B. HAYCRAFT, M.D., D.Sc., and Dr HAROLD SCOFIELD. *P.* xvi. 188.

2. On a Relation between Two Groups of Four Vectors. By Professor TAIT. *P.* xvi. 89.

3. Description of a Portable Apparatus for Counting the Dust Particles in the Atmosphere. By JOHN AITKEN, Esq., F.R.S. *P.* xvi. 169.

4. On the Stomach of the Narwhal (*Monodon monoceros*). By G. SIMS WOODHEAD, M.D., and R. W. GRAY, Esq. *P.* xvi. 792.

5. Researches on Micro-Organisms, &c. Part. III. By Dr A. B. GRIFFITHS, F.C.S.

6. By permission of the Meeting, there were laid on the table Additional Remarks on the Virial of Molecular Force, by Professor TAIT.

Monday, 1*st April* 1889.

Sir William Thomson, President, in the Chair.

The following Communications were read:—

1. On the Duration of Impact. By Professor TAIT.

2. On New Forms of Magneto-Static Current- and Volt-meters, and on an Electro-Static Voltmeter, with Multiple Voltaic Pile facilitating Graduation. By Sir W. THOMSON. (*The Instruments were exhibited.*)

3. On the Properties of Manganese Steel. By A. CRICHTON MITCHELL, Esq. *T.* xxxv.

4. On an Improved Method of Measuring small Rotations of the Plane of Polarization by ordinary Apparatus. By W. PEDDIE, D.Sc.

5. On the Relations among the Line-, Surface-, and Volume-Integrals. By Professor TAIT. *P.* xvi. 257.

The following Candidates for Fellowship were balloted for, and declared duly elected Fellows of the Society:—

JOHN ALISON, Esq., M.A.
R. W. PHILIP, M.D., M.A., F.R.C.P.E.
T. EDGAR UNDERHILL, M.D., F.R.C.S.E.

Monday, 15*th April* 1889.

Sir Douglas Maclagan, M.D., Vice-President, in the Chair.

PRIZES.

The Chairman presented the Keith Prize, for 1885–87, to Mr J. Y. Buchanan, for a valuable Series of Communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, &c., two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic

Ocean," were communicated to the Society, and printed in its *Proceedings* during the period 1885–87.

Dr Buchan, on presenting the prize, made the following statement:—

In our *Transactions* for 1880 there is a noteworthy paper by Mr Buchanan on the "Compressibility of Glass."

The methods hitherto employed for measurement of the compressibility of solids have usually been *indirect*, depending for the most part on the determination of Young's modulus by traction, flexure, or acoustical processes. This modulus is a function of the rigidity and the compressibility jointly, so that the rigidity had to be determined independently, say, by torsional methods. Thus the values of compressibility hitherto obtained have been more or less uncertain.

Buchanan's process is a simple and *direct* one. He measures the linear compression of a rod under hydrostatic pressure, and (assuming the material to be isotropic) obtains thus one-third of the compressibility.

His apparatus, especially the device by which the ends of the rod can be microscopically examined while it is exposed to very high pressures, is an ingenious one, and it is easy to work with; so that it is to be hoped that he will soon extend his results to a large number of the more common solids, for our information on this subject is still extremely meagre and unsatisfactory.

From the results of Mr Buchanan's experiments on "Ice and Brines," it is shown that the first ice formed on the sea in Arctic and Antarctic regions consists of pure ice, retaining, however, a large quantity of the residual sea-water in its interstices; and during the winter this enclosed liquor solidifies in the interstices of the crystals to ice and cryohydrates, in so far as the temperature and the nature of the salts in solution permit.

Some years previously, Dr Otto Pettersson incidentally remarked, in his work on the *Properties of Water and Ice*, that a thermometer immersed in a mixture of snow and sea-water, which is constantly stirred, indicates 28°·8, and therefore we may regard this as the upper limit of the freezing and the nether limit of the melting temperatures of sea-water. Mr Buchanan's experiments confirm this statement, and he has the merit of applying the fact that ice melts in ordinary sea-water at a temperature of from 29°·1 to 28°·8, according to its saltness, to explain successfully the anomalous distribution of temperature in Antarctic waters. The warmer and denser water of lower latitudes bathes the under surfaces of the

icebergs, which in many cases are at a depth of 1500 to 1800 feet. The temperature of the mixture at the surface of contact falls; the heat abstracted from the sea-water melts a portion of the iceberg, and a saline solution is produced, less salt, and therefore lighter than the water not in contact with the iceberg. This solution, being lighter, necessarily flows up along the sides of the iceberg to the surface, and its place is taken by new undiluted sea-water, which in its turn is cooled, diluted, and transferred to the surface, thus giving a practically constant temperature of 29°·0 in the neighbourhood of the icebergs. The result is the production of an energetic engine of circulation and means of cooling and equalising the temperature of the water within the reach of icebergs.

Another valuable research of Mr Buchanan's deals with "Similarities in the Physical Geography of the Great Oceans,"* in which he shows that the distribution of temperature from the surface of the ocean downwards follows the difference of density due to evaporation and dilution of the surface water. Wherever the density of the surface water of the sea is high, there the deeper are high temperatures found. On the other hand, where the density on the surface is low, as happens, for example, near rivers, there the high temperature of the surface penetrates downward but a little way. Hence the important bearing of the surface waters of dry and rainy regions of the sea respectively on oceanic circulation.

In the same paper are marked off, with a precision not previously attained, the low inshore temperature of the sea on the west of Africa from the Straits of Gibraltar to Cape Verde, and again from about Loando to the mouth of the Orange River; on the west of North America, from Vancouver Island to Cape St Lucas; and of South America, from the Gulf of Guayaquil to Chiloè. From Humboldt downwards these stretches of cold water have been generally attributed to the flow equatorwards of cold oceanic currents from Antarctic and Arctic regions. Mr Buchanan shows, from the observations of navigators, the deep olive-green and often almost black appearance of the sea, and the abnormally low and practically uniformly distributed temperature of the sea through many degrees of latitude, that this cannot be the true explanation. He then points out that over all these extensive sea-boards the winds blow from the land westwards over the ocean, with the result that the cold water of greater depths is raised to the surface to take the place of the warmer waters of the surface, which are driven far to

* *Proc. Roy. Soc.*, vol. xxiii. p. 123, 1874, and *Proc. Roy. Geog. Soc.*, Dec. 1886.

westward by the prevailing winds in a manner analogous to what Dr Murray, in a recent communication, showed to prevail in the Scottish lochs as the effect of winds.

In the wide field of oceanic research, Mr Buchanan has further contributed materially to the improvement of instruments and methods of observation; has made original observations on currents, temperatures, and salinities; commenced the work of contouring the bed of the sea off the coast of Africa; drawn attention to the remarkable manner in which deposits are being laid down from the mouth of the Congo for 600 miles westwards into the Atlantic; and made valuable suggestions as to the effect of the great Atlantic waves which accompany the storms of north-western Europe on the land and bottom of the sea immediately adjoining.

In reference to fresh water lakes, he has shown, from observations made in Loch Lomond and Linlithgow Loch in 1878–79, that the opinion generally held up to that time, that the bulk of the fresh water of a frozen lake presented a temperature of 39°2, was erroneous.

The Chairman presented the Makdougall-Brisbane Prize, for the period 1884–86, to Dr John Murray, for his Papers on the Drainage Areas of Continents and Ocean Deposits, the Rainfall of the Globe and Discharge of Rivers, the Height of the Land and Depth of the Ocean, and the Distribution of Temperature in the Scottish Lochs as affected by the Wind.

Professor Crum Brown made the following statement:—

These papers are of great geographical, meteorological, and zoological value. By careful selection of the most trustworthy data, well-devised and laborious measurements, and judicious estimations where certainty was unattainable, Dr Murray has arrived at closely approximate results for the correct mean heights of the land of each of the great divisions of the earth, and of the mean depths of the sea in each ocean. From these he has calculated the total volume of solid land above, and the total volume of water below sea-level.

He has made careful measurements of the drainage areas of the world, and shown that a very large proportion of the solid débris washed down by rivers is deposited in shallow land-locked seas, and that comparatively little, if any, finds its way to the great depths of the ocean which are distant from the land. The deposits in the depths of the ocean proceed at an excessively slow rate, and, if we except the earbones of whales and sharks' teeth, which withstand

the powerfully decomposing forces at these depths, they may be restricted to the materials contributed by the ash of meteors and the scoriæ of volcanoes.

The paper on the influence of wind on the salt and fresh-water lochs of Scotland is highly important from a zoological and economical, as well as from a meteorological, point of view. In it Dr Murray shows how the wind, by producing first horizontal and then vertical currents, influences the distribution of temperature through the whole depths of the enclosed lochs, whether fresh or salt. He has further described crustacea dredged up from the depths of our salt-water lochs which have a peculiar novel interest both on account of their remarkable luminous organs, and, as he has been the first to show, on account of their being the chief food of the herring. We now know what becomes of the herring when they are no longer to be seen or caught near the surface,—they are feeding in the depths of the lochs or sea adjoining, and in all probability the extent of their migrations is from the depths of the sea shorewards during the spawning season.

These papers form the justification of the Council's award, and they represent but a small part of Dr Murray's scientific work. The records of the "Challenger" Expedition, now so nearly finished, will always remain a proof that he is a skilful organiser, as well as an able, ingenious, and successful investigator.

The Chairman then presented the Makdougall-Brisbane Prize, for the period 1886–88, to Dr Archibald Geikie, for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," printed in the Society's *Transactions*.

Mr B. N. Peach, in explaining the grounds of the award, said:—

Mr Archibald Geikie, from a lifetime devoted to active study in the field, from the laborious manner in which he has collated almost everything that has been written upon the subject, and from his charming style as a writer, has done more than any other living person to spread a knowledge of geology in this country. Besides his excellent official work in connection with the geological survey of these islands, of which he is now the Director-General, he has written many separate works, and has contributed numerous weighty original papers to various scientific societies and journals. Among these contributions, some of the more important have appeared in the *Transactions* of our Society, and for one of them—his monograph on the "Old Red Sandstone of Western Europe"—he

has already received the award of the Makdougall-Brisbane medal.

Though their literary grace will doubtless make many of his writings classics in the literature of the science, it is perhaps on account of his original work in the study and classification of the igneous rocks of Scotland that he will most deserve the recognition and gratitude of geologists. With the exception of Maclaren's excellent detailed work among the rocks of Arthur's Seat and the Pentland Hills, all previous writers had looked upon the volcanic rocks of Scotland as mere eruptive mineral masses. Mr Geikie was the first to treat them, as a whole, from a chronological point of view as the records of a long succession of volcanic eruptions, the relative geological dates of which could be certainly fixed. In 1861 he contributed to this Society a paper giving a first sketch of a classification of the volcanic phenomena of Scotland, which may be said to have struck the keynote of all his subsequent researches in this subject. The more detailed studies of later years have indeed enabled him to modify some of the views there enunciated, but the main broad outlines remain as they were originally traced. Year by year he has filled in these outlines from a detailed study in the field. He was thus enabled to add a new chapter to the volcanic history of his native country by the discovery of the remains of volcanoes of Permian age in the counties of Ayr and Dumfries, of the existence of which he was ignorant in 1861.

After many years of investigation he gave to this Society his paper upon the "Carboniferous Volcanic Rocks of the Basin of the Forth," which was published in our *Transactions* in 1879. It was doubtless from the exhaustive manner in which the microscopic structures of these rocks was there worked out that the eyes of Continental geologists were opened to the fact, long insisted upon by him, that there is really no essential difference between the products of Palæozoic volcanoes and those of the present day.

Some of Mr Geikie's earliest labours among the volcanic rocks were devoted to the great basaltic tracts of the Inner Hebrides. They seem to have had a peculiar fascination for him, for we find him returning to them again and again during more than thirty years. In the midst of other researches, he has never lost sight of the claims of these Western Islands. To gain further insight into their geological history he has made excursions to Auvergne, the Eifel country, Vesuvius, the Lipari Islands, and to Western America, and now at length, gathering up the results of this long-continued research, he has presented them to the Royal Society of Edinburgh as the memoir on the "History of Volcanic Activity during the Tertiary

Period in Britain," which was published last year in the *Transactions*. The most striking features of this memoir are the new views it presents of the grand and varied sequence of events in the protracted volcanic activity of Tertiary time, and more especially the recognition of the plateau type of volcanic eruption in this country, and the connection of the successive sheets of basalt that form the terraced hills of our Western Islands with the almost innumerable dykes which cross the country as far as Yorkshire. For this patient and successful research, culminating in an essay, which is probably the most memorable which the author has yet written, the Council has again awarded him the Makdougall-Brisbane medal.

The following Communications were read:—

1. Obituary Notice of R. M. Smith. By Professor SWAN. *P.*
2. On the Air's Resistance to an Oscillating Body (its Influence on Timekeepers). By E. SANG, LL.D. *P.* xvi. 181.
3. A New and Easy Method for the Rapid and Sure Detection of Mercury. By A. JOHNSTONE, Esq., F.G.S. Communicated by Professor CRUM BROWN.

Monday, 6th May 1889.

Mr A. Beatson Bell in the Chair.

The following Communications were read:—

1. Obituary Notices:—
   (*a*) Of Dr Charles Edward Wilson. By WILLIAM JOLLY, Esq., F.G.S., H.M. Inspector of Schools. *P.*
   (*b*) Of William Dickson, Esq. By JOSIAH LIVINGSTON, Esq. *P.*
2. Analysis of the "Challenger" Meteorological Observations. Second Paper. By Dr BUCHAN, Secretary to the Scottish Meteorological Society. *P.* xvi. 786.
3. On the Secretion of Carbonate of Lime by Animals. Part II. By ROBERT IRVINE, F.C.S., and Dr G. SIMS WOODHEAD, F.R.C.P.Ed. *P.* xvi. 324.
4. Continued Observations on the Progression and Rotation of Bivalve Molluscs and of detached Ciliated Portions of them. Part II. In Fresh Water Mussel (*Unio*). Part III. In the Osyter (*Ostrea*). By D. M'ALPINE, Esq. Communicated by Dr G. SIMS WOODHEAD. *P.* xvi. 725.
5. Preliminary Note on the Composition of Sea Water. By Dr JOHN GIBSON.

Dr WILLIAM MORSE GRAILY HEWITT, F.R.C.P., was balloted for, and declared duly elected a Fellow of the Society.

Monday, 20th May 1889.

Sheriff Forbes Irvine in the Chair.

The following Communications were read:—

1. Obituary Notice: Professor Madvig. By Professor SELLAR. *P.*

2. On the Identity of Hofmann's "Dibenzyl Phosphine" with Oxide of Tri-benzyl-Phosphine, and on some other Points connected with the Phosphorized Derivatives of Benzyl. By Professor LETTS and R. F. BLAKE, Esq., Queen's College, Belfast. *P.* xvi. 193.

3. The Development of Diarthrodial Joints in Birds and Mammals. By D. HEPBURN, Esq., M.B. Communicated by Sir WILLIAM TURNER. *P.* xvi. 258.

4. Observations on the Progressive Movement of Detached Ciliated Portions of Frogs and Tortoises. By D. M'ALPINE, Esq. Communicated by Dr G. SIMS WOODHEAD.

5. Observations on the Progression, Pulsation, and Quivering of Excised Hearts of Fish, Frogs, Reptiles, Birds, and Mammals. By D. M'ALPINE, Esq. Communicated by Dr G. SIMS WOODHEAD.

6. On the Solubility of Carbonate of Lime in Fresh and Sea Water. By W. S. ANDERSON, Esq. Communicated by Dr JOHN MURRAY. *P.* xvi. 319.

Monday, 3rd June 1889.

Sheriff Forbes Irvine in the Chair.

1. Photographs of Mirage were exhibited by Lord MACLAREN.

The following Communications were read:—

2. On *Strophanthus hispidus:* its Natural History, Chemistry, and Pharmacology. Part II. By Professor T. R. FRASER, M.D. *P.* xvi. 743 (*Abstract*); *T.* xxxv.

3. On the Compressibility of Mercury. By Professor TAIT.

4. Notice of Fundamental Tables in Trigonometry and Astronomy, arranged according to the Decimal Division of the Quadrant. By E. SANG, LL.D. *P.* xvi. 249.

5. Quaternion Note on a Geometrical Problem. By Professor TAIT. *P.* xvi. 315.

6. On the Non-oscillating Pendulum. By Professor TAIT.

7. On Bravais' Uniform Distribution of Points. By Sir WILLIAM THOMSON.

The following Candidates for Fellowship were balloted for, and declared duly elected Fellows of the Society:—

GEORGE H. GEDDES, Esq., C.E.
WILLIAM PECK, Esq., F.R.A.S.
ROBERT WILSON, Esq., M.Inst.C.E.
The Rev. ROBERT MUNRO, M.A., B.D., F.S.A. Scot.

Monday, 17*th June* 1889.

The Hon. Lord M'Laren, Vice-President, in the Chair.

The following Communications were read:—

1. Obituary Notice of the late Dr Samuel Drew. By Dr ARTHUR JACKSON, Sheffield. Communicated by T. ANDREWS, Esq., F.R.S. *P.*

2. On the Behaviour of the Hydrates and Carbonates of the Alkali Metals, and of Barium, at High Temperatures, on the Properties of Lithia, and the Atomic Weight of Lithium. By Professor DITTMAR. *T.* XXXV. 429.

3. On the Hydrokinetic Equations. By Professor TAIT.

4. On an interesting Family of Glissettes. By Professor TAIT.

5. A Revision of the Genus *Coscinodiscus* and some allied Genera. By JOHN RATTRAY, Esq., M.A., B.Sc., F.R.S.E. *P.* xvi. 449.

Monday, 1*st July* 1889.

John Murray, LL.D., Vice-President, in the Chair.

The following Communications were read:—

1. On the Determination of the Curve, on one of the Cöordinate Planes, which forms the Outer Limit of the Positions of the Point of Contact of an Ellipsoid of Revolution which always touches the Three Planes of Reference. By Dr G. PLARR. Communicated by Professor TAIT. *T.* XXXV. 471.

2. On the Thermal Conductivity and the Specific Heat of Manganese Steel. By A. CRICHTON MITCHELL, B.Sc. *T.* XXXV.

3. On the Placentation of the Halicore Dugong. By Sir WILLIAM TURNER, F.R.S. *P.* xvi. 264 (*Abstract*); *T.* XXXV. 641.

4. Does the Coefficient of Absorption depend upon the Intensity of Light? By W. PEDDIE, D.Sc.

5. On the Renal Organs of the Nematoidea. By Dr A. B. GRIFFITHS, F.C.S.

The following Candidates were balloted for, and declared duly elected Fellows of the Society:—

Professor T. D. COLLIS BARRY, M.R.C.S.E., F.Z.S.
ANDREW E. SCOUGAL, Esq., M.A.

Monday, 15*th July* 1889.

Professor Chrystal, Vice-President, in the Chair.

The following Communications were read:—

1. Theoretical Description of a New "Azimuth Diagram." By Captain P. WEIR. Communicated by the PRESIDENT. *P.* xvi. 354. With a Note by Professor TAIT. *P.* xvi. 359.

2. On Molecular Arrangement. By the PRESIDENT. *P.* xvi.

3. Electrification of Air by Flame. By the Same. *P.* xvi. 262.

4. The Geographical Distribution of some Tropical Diseases. By R. W. FELKIN, M.D., F.R.G.S. *P.* xvi. 266.

5. On the Compressibility of Solutions of Sugar. By Professor TAIT.

6. On the Coagulation of Egg and Serum Albumen, Vitellin and Serum Globulin. By JOHN BERRY HAYCRAFT, M.D., D.Sc., and Mr C. W. DUGGAN, M.B. *P.* xvi. 361.

7. Some New Points in Connection with the Latent Period of Muscle Contraction. By ALEXANDER JAMES, M.D. *P.* xvi. 385.

8. On the Time of Impact as depending on the Masses of the Impinging Bodies. By Professor TAIT.

9. On the Segmentation of the Nucleus of the Oculo-motor Nerve. By Dr ALEXANDER BRUCE.

10. On the Upward Continuation of the Spinal Cord. By the Same.

11. On the Skull and Visceral Arches of *Laemargus microcephalus*. By P. J. WHITE, M.B. Communicated by Professor EWART.

12. On the Scalar Equations which represent the Relations connecting n Points. By the Rev. M. M. U. WILKINSON. Communicated by Professor TAIT. *P.* xvi. 773.

13. On some Novel Quaternion Formulæ. By Professor TAIT. *T.* xxxv. 527.

14. On Benzyl Phosphines and their Derivatives. By Professor LETTS and Mr R. F. BLAKE. Part I. Benzyl Phosphines. Part II. Action of Alcohols upon a Mixture of Phosphorus and Iodide of Phosphorus. Part III. The Products of the Oxidation of Benzyl Phosphines.

15. Non-Alternate ± Knots, of Orders Eight and Nine. By Professor C. N. LITTLE. Communicated by Professor TAIT. *T.* xxxv. 663.

16. Review of the Session. By the CHAIRMAN.

Donations to the Library of the Royal Society from 1887 to 1889.

I. Transactions and Proceedings of Learned Societies, Academies, &c.

Adelaide.—Transactions and Proceedings of the Adelaide Philosophical Society. Vols. X., XI. 1886–89. 8vo.

University Calendar for 1888.

American Association for the Advancement of Science.—34th, 35th, 36th, and 37th Meetings (Ann Arbor, 1885; Buffalo, 1886; New York, 1887; Cleveland, 1888).

Amsterdam.—Verhandelingen der Kon. Akademie van Wetenschappen. Afd. Natuurkunde, Dl. XXVI., 1888. Afd. Letterkunde Dl., XVII. 1888. XVIII. 1889. Verslagen en Mededeelingen, Natuurkunde, 2e Rks., Dl. XIV. 1888; 3e Rks., Dl. III., IV., V., 1887–89. Letterkunde, 3e Rks., Dl. III., IV., V., 1887–88; Jaarboek, 1886–87–88. Poemata Latina. Catalogus der Boekerij der Akademie.

Kon. Zoologisch Genootschap "Natura Artis Magistra." Bijdragen, 14e, 15e, 16e Aflev., 1887–88.

Wiskundig Genootschap. Nieuw Archief voor Wiskunde XIV., 1887; XV. 1, 1888. Opgaven III. 4, 5, 1887; IV. 1.

Flora Batava: Afbeelding en Beschrijving van Nederlandsche Gewassen. Voortgezet door F. W. Van Eedden. 281–286 Afleveringen, 1887–89.—*From the Dutch Government.*

Baltimore.—Johns Hopkins University.—The American Journal of Mathematics. Professor Newcomb (Editor-in-chief). Vols. X., XI. 1887–89.

The American Chemical Journal. Edited by Professor Remsen. Vols. IX., X., XI. 1888–89.

The American Journal of Philology. Edited by Professor Gildersleeve. Vol. IX. 1888–89. 8vo.

Studies from the Biological Laboratory of the Johns Hopkins University. Vol. IV. 3, 4. 1888. 8vo.

University Studies in Historical and Political Science. 1887–89.

University Circulars. 1887–89.

Peabody Institute.—Catalogue of the Library of the Peabody Institute of the City of Baltimore. Part IV. 1889.

Basel.—Verhandlungen der Naturforschenden Gesellschaft. Bd. VIII. 2 1887. 8vo.

Batavia.—Observations made at the Magnetical and Meteorological Observatory. Vol. VI. (Supplement), VII., VIII., IX., X., XI. 1886–89. fol.

Regenwaarnemingen in Nederlandsch Indie, 9e Jaarg, 1887; 10e Jaarg, 1888. 8vo.

Batavia.—*The Batavian Society of Arts and Sciences.*—Verhandelingen van het Bataviaasch Genootschap van Kunsten en Wetenschappen. Deel XLVII. 1888; XLVIII. 1889. 8vo.

Tijdschrift voor Indische Taal-Land- en Volkenkunde. Deel XXXII. Nos. 1–6. 1887–88. 8vo.

Natuurkundig Tijdschrift voor Nederlandsch Indie, uitgeven door de Kon. Natuurkundig Vereeniging. 1887–89. 8vo.

Notulen, Deel XXV.–XXVII. 1887–89.

Belfast.—Proceedings of the Natural History and Philosophical Society for 1887–89.

Bergen.—Museum's Aarsberetning for 1887–88. 8vo.

Berlin.—Abhandlungen der Königlichen Akademie der Wissenschaften, 1887–88.—Sitzungsberichte, 1887–89. 8vo.

Fortschritte der Physik im Jahre 1882. Dargestellt von der physikalischen Gesellschaft zu Berlin. 1te Abtheil.—Allgemeine Physik, Akustik. 2te Abtheil.—Optik, Wärmelehre, Elektricitätslehre. 3e Abtheil.—Physik der Erde. Berlin, 8vo.

Verhandlungen der Physikalischen Gesellschaft im Jahre 1887–88.

Zeitschrift der Deutschen Meteorologischen Gesellschaft. 1887–89. 8vo.

Ergebnisse der Meteorologischen Beobachtungen im Jahre 1887, herausgegeben von dem Preussischen Meteorologischen Institut. 4to.

Königliche Technische Hochschule. 1887–89. 4to.

Bern.—Beiträge zur geologischen Karte der Schweiz, XXe–XXIIe Lief. Description Géologique des Préalpes du Canton de Vaud et du Chablais. Texte et Atlas, 1887; XXIVe Lief., Das Aarmassiv, nebst Abschnitt des Gatharmassivs. 4to.—*From the Commission Fédérale Géologique.*

Mittheilungen der Naturforschenden Gesellschaft in Bern. Nos. 1169–1214. 1887–88. 8vo.

Berwickshire.—Berwickshire Naturalists' Club, Proceedings of. Vol. XII. 1. 1887. 8vo.

Birmingham.—Proceedings of the Birmingham Philosophical Society. Vol. VI. 1, 2. 1888. 8vo.

Bologna.—Accademia d. Scienze dell' Istituto di Bologna.

Memorie, Ser. VI. Tom. VI., VII., VIII., IX. 1884–88.

Calendrier Universel et Méridien Universel: Rapport 1889.

Bombay.—Journal of the Bombay Branch of the Royal Asiatic Society. Vol. XVII.

Magnetical and Meteorological Observations made at the Government Observatory for 1886–87. Bombay. 4to.

Journal of the Natural History Society. Vol. IV. 1, 2.

Bonn.—Verhandlungen des Naturhistorischen Vereines der Preussischen Rheinlande und Westfalens. 1888–89. 8vo.

Bordeaux.—Mémoires de la Société des Sciences Physiques et Naturelles. 3e Sér. Tom. III. (2) 1886.

Bordeaux.—Observations Pluviométriques et Thermométriques dans la Gironde. 1886–87.

Bulletins de la Société de Géographie Commerciale. 1888–89. 8vo.

Boston.—Memoirs of the Boston Society of Natural History. Vol. IV. 1888. 4to.

Proceedings of the Boston Society of Natural History. Vol. XXIII. 1887–88. 8vo.

Proceedings of the American Academy of Arts and Sciences Vol. XXII. 1887.

Brera.—See *Milan.*

British Association for the Advancement of Science.—Report of the Meeting at Bath. 1888.

Brunswick.—Jahresbericht des Vereins für Naturwissenschaft. 1886–87.

Brussels.—Bulletin de l'Académie Royale des Sciences, des Lettres, et des Beaux-Arts de Belgique. Tomes LVII., LVIII. 1888–89. 8vo.

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PRINTED BY NEILL AND COMPANY, EDINBURGH.

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